Universidade Federal do Rio de Janeiro

Lista IV - Sistemas Lineares I

Alunos	Igor Abreu da Silva
DRE	112053874
Curso	Engenharia Eletrônica
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Professor	Natanael Nunes de Moura Junior

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1 Diagrama de Pólos e Zeros

1.1 Questão 1

1.1.1 Item a

$$\frac{1}{s+1} + \frac{1}{s+3} = \frac{(s+3)+(s+1)}{(s+1)(s+3)} = \frac{2s+4}{s^2+4s+4}$$

$$Zeros : 2s + 4 = 0 \rightarrow s = -2$$
$$Polos : s^2 + 4s + 4 = 0 \rightarrow s = -2$$

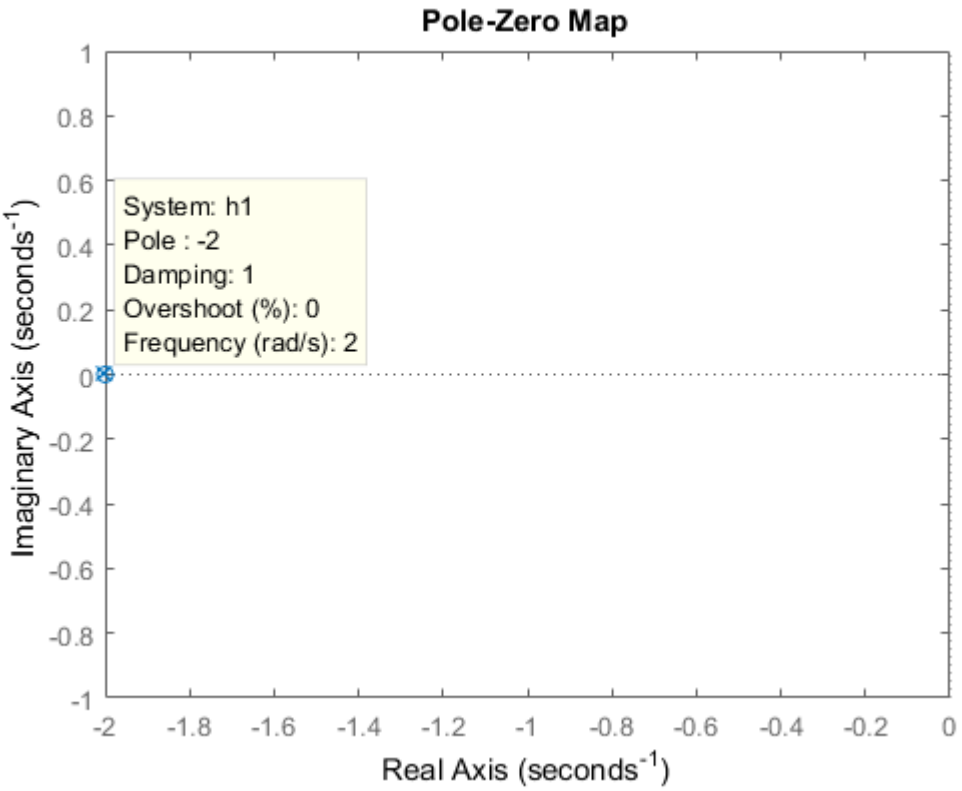

Figura 1: Pólos e Zeros - Item a

1.1.2 Item b

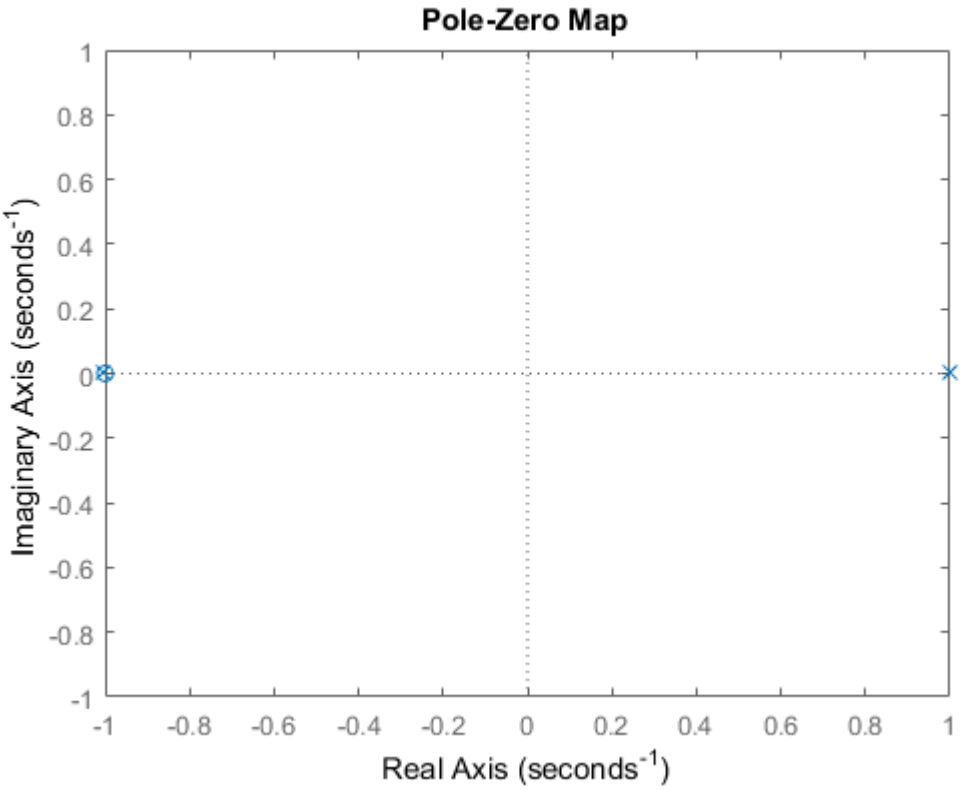

Figura 2: Pólos e Zeros - Item b

$$Zeros : s + 1 = 0 \rightarrow s = -1$$
$$Polos : s^2 + 1 = 0 \rightarrow s = -1; +1$$

1.1.3 Item c

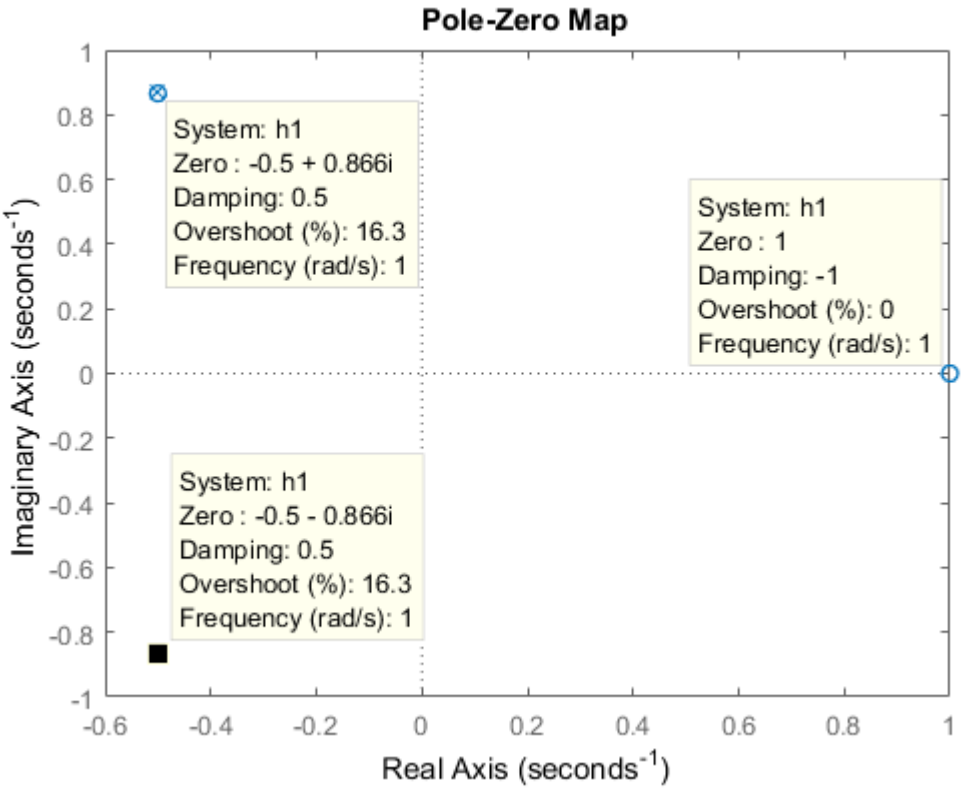

Figura 3: Pólos e Zeros - Item c

$$Zeros : s^3 - 1 = 0 \rightarrow s = \frac{-1+\sqrt{3}}{2}; \frac{-1-\sqrt{3}}{2}; 1$$

$$Polos : s^2 + s + 1 = 0 \rightarrow s = \frac{-1+\sqrt{3}}{2}; \frac{-1-\sqrt{3}}{2}$$

2 Propriedade da Transformada de Laplace

2.1 Questão 2

2.1.1 Item a

$$\int_{-\infty}^{+\infty} x(t-1)e^{-st}u(t)dt \rightarrow \int_{0}^{+\infty} x(\tau)e^{-s(\tau+1)}d\tau \rightarrow e^{-s}\int_{0}^{+\infty} x(\tau)e^{-s\tau}d\tau \Rightarrow e^{-s}X(s)$$

2.1.2 Item b

Pela propriedade da derivação de Laplace, temos:

$$s^3Xs - s^2x(0^-) + sx^{'}(0^-) - x^{''}(0^-)$$

2.1.3 Item c

Pela propriedade da integração de Laplace, temos:

$$\frac{X(s)}{s}$$

2.2 Questão 3

$$\int_{-\infty}^{+\infty} cos(\omega_0 t)u(t)e^{-st}dt \rightarrow = \frac{1}{2}\int_{0}^{\infty} e^{-t(s-\omega_0)} + e^{-t(s+\omega_0)}dt \rightarrow$$

$$\frac{1}{2(s-w_0)} + \frac{1}{2(s+w_0)} \rightarrow \frac{s}{s^2+\omega_0^2}$$

3 Resposta em Frequência

3.1 Questão 4

$$H(j\omega) = \frac{j\omega + 2}{-\omega^2 + 5j\omega + 4}$$

$$|H(j\omega)| = \frac{\sqrt{2^2+\omega^2}}{\sqrt{(4-\omega^2)^2 + (5\omega)^2}}$$

$$\angle H(j\omega) = arctan\left(\frac{w}{2}\right) - arctan\left(\frac{5w}{4-\omega^2}\right)$$

$$y(t) = |H(j\omega)|cos(\omega t + \phi + \angle H(j\omega))$$

3.1.1 Item a

$$y(t) = 5|H(j\omega)|cos(2t + 30^o + \angle H(j\omega))$$

$$|H(j\omega)| = \frac{\sqrt{4+4}}{\sqrt{(4-4)^2 + (10)^2}} \Rightarrow \frac{\sqrt{2}}{5}$$

$$\angle H(j\omega) = arctan\left(\frac{2}{2}\right) - arctan\left(\frac{10}{4-4}\right) \Rightarrow = -45^o$$

$$y(t) = \sqrt{2}cos(2t - 15^o)$$

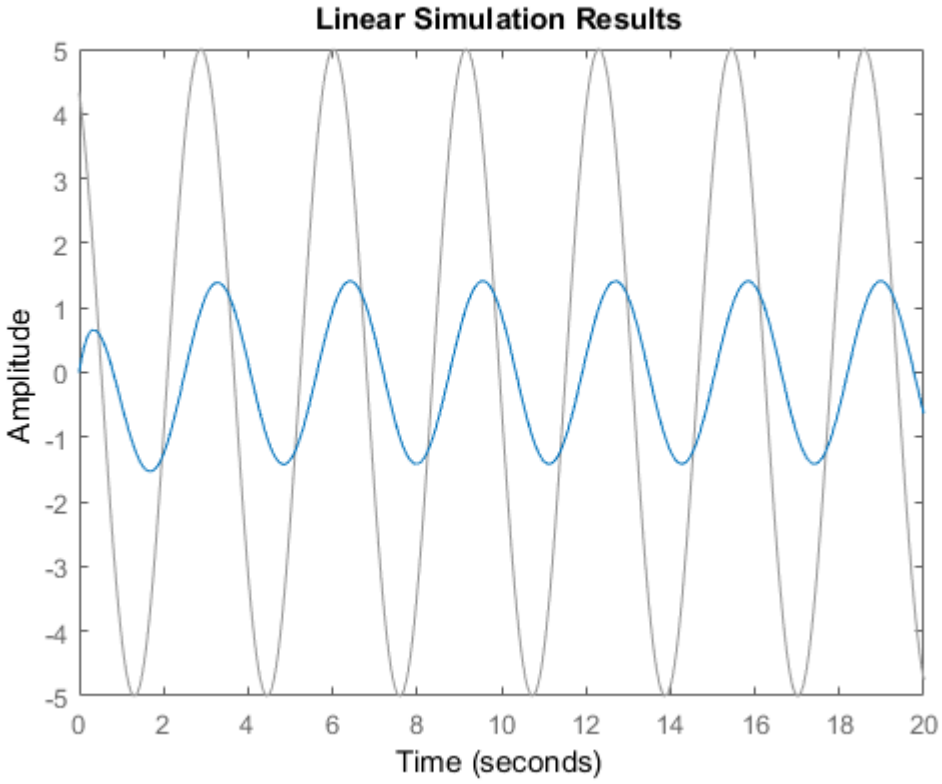

Figura 4: $y(t) = \sqrt{2}cos(2t - 15^o)$

3.1.2 Item b

$$y(t) = 10|H(j\omega)|sen(2t + 45^o + \angle H(j\omega))$$

$$|H(j\omega)| = \frac{\sqrt{4+4}}{\sqrt{(4-4)^2 + (10)^2}} \Rightarrow \frac{\sqrt{2}}{5}$$

$$\angle H(j\omega) = arctan\left(\frac{2}{2}\right) - arctan\left(\frac{10}{4-4}\right) \Rightarrow = -45^o$$

$$y(t) = 2\sqrt{2}sen(2t)$$

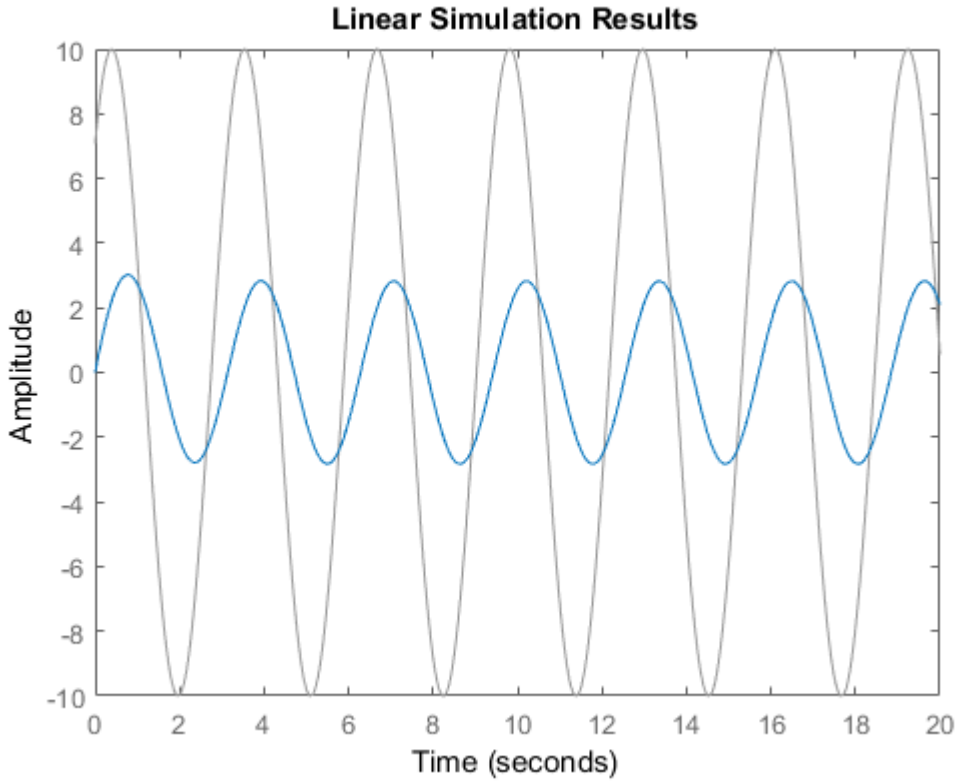

Figura 5: $y(t) = 2\sqrt{2}sen(2t)$

3.1.3 Item c

$$y(t) = 10|H(j\omega)|cos(4t + 40^o + \angle H(j\omega))$$

$$|H(j\omega)| = \frac{\sqrt{4+16}}{\sqrt{(4-16)^2 + (20)^2}} \Rightarrow 0.2$$

$$\angle H(j\omega) = arctan\left(\frac{4}{2}\right) - arctan\left(\frac{20}{4-16}\right) \Rightarrow = -63.4^o - 59^o = -122.4^o$$

$$y(t) = 2cos(4t - 82.4^o)$$

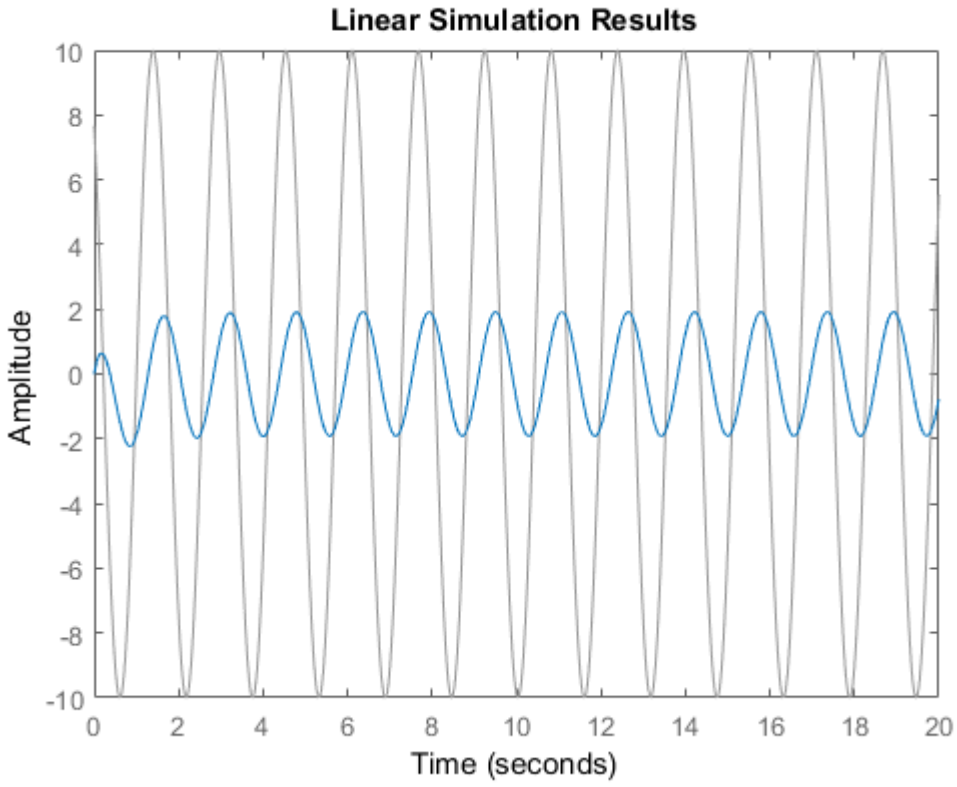

Figura 6: $y(t) = 2cos(4t - 82.4^o)$

3.2 Questão 5

$$H(j\omega) = \frac{10 - j\omega}{j\omega + 10}$$

$$|H(j\omega)| = \frac{\sqrt{10^2 + (-\omega)^2}}{\sqrt{(\omega)^2 + (10)^2}} \Rightarrow 1$$

$$\angle H(j\omega) = arctan\left(\frac{-w}{10}\right) - arctan\left(\frac{w}{10}\right)$$

$$y(t) = |H(j\omega)|cos(\omega t + \phi + \angle H(j\omega))$$

3.2.1 Item a

$$y(t) = cos(wt + \theta + \angle H(j\omega))$$

$$\angle H(j\omega) = arctan\left(\frac{-w}{10}\right) - arctan\left(\frac{w}{10}\right)$$

$$y(t) = cos(\omega t + \theta + \angle H(j\omega))$$

3.2.2 Item b

$$y(t) = cos(2 + \theta + \angle H(j\omega))$$

$$\angle H(j\omega) = arctan\left(\frac{-1}{10}\right) - arctan\left(\frac{1}{10}\right) \Rightarrow -11.4^o$$

$$y(t) = cos(t - 11.4^o)$$

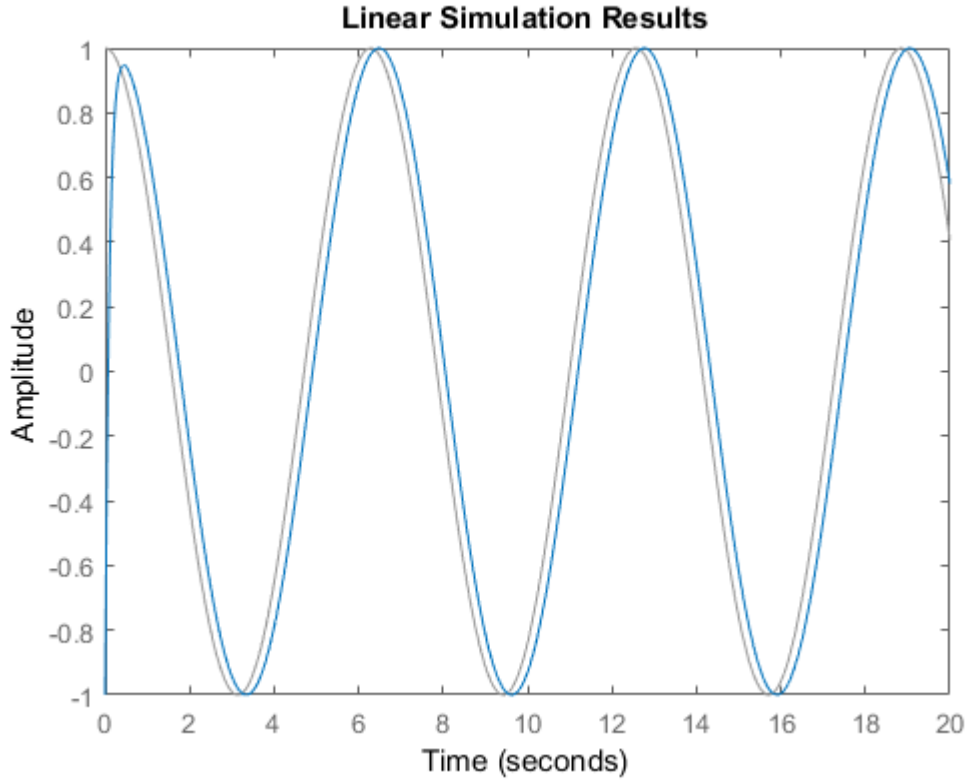

Figura 7: $y(t) = cos(t - 11.4^o)$

3.2.3 Item c

$$y(t) = cos(2t + \theta + \angle H(j\omega))$$

$$\angle H(j\omega) = arctan\left(\frac{-2}{10}\right) - arctan\left(\frac{2}{10}\right) \Rightarrow -22.6^o$$

$$y(t) = cos(2t - 22.6^o)$$

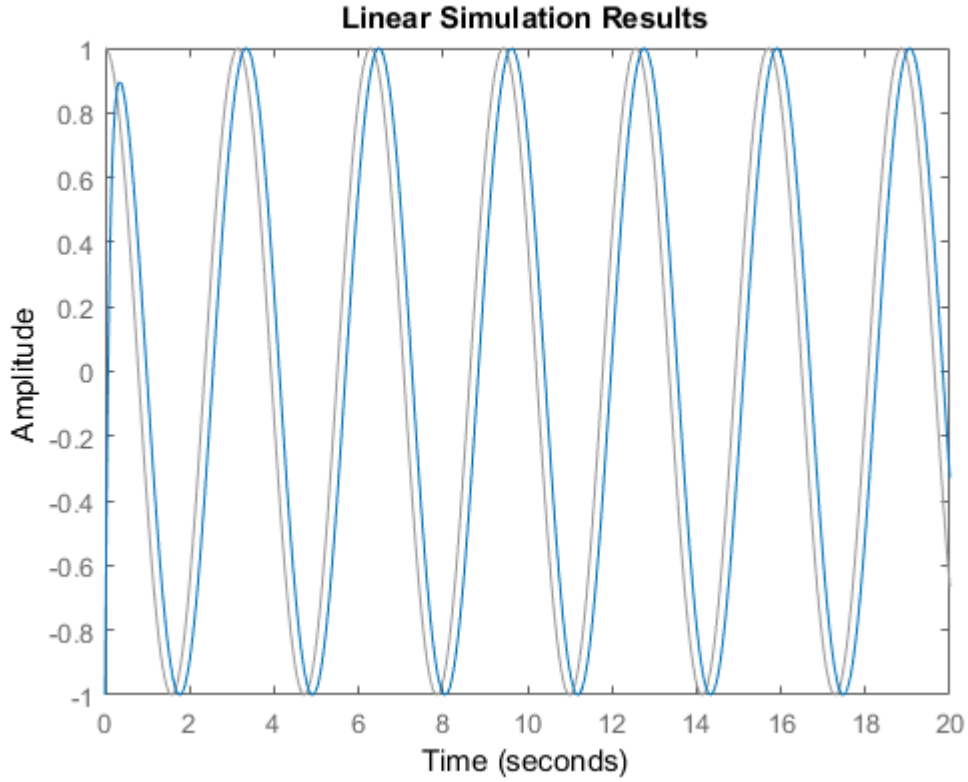

Figura 8: $y(t) = cos(2t - 22.6^o)$

3.2.4 Item d

$$y(t) = cos(10t + \theta + \angle H(j\omega))$$

$$\angle H(j\omega) = arctan\left(\frac{-10}{10}\right) - arctan\left(\frac{10}{10}\right) \Rightarrow -22.6^o$$

$$y(t) = cos(10t - 90^o)$$

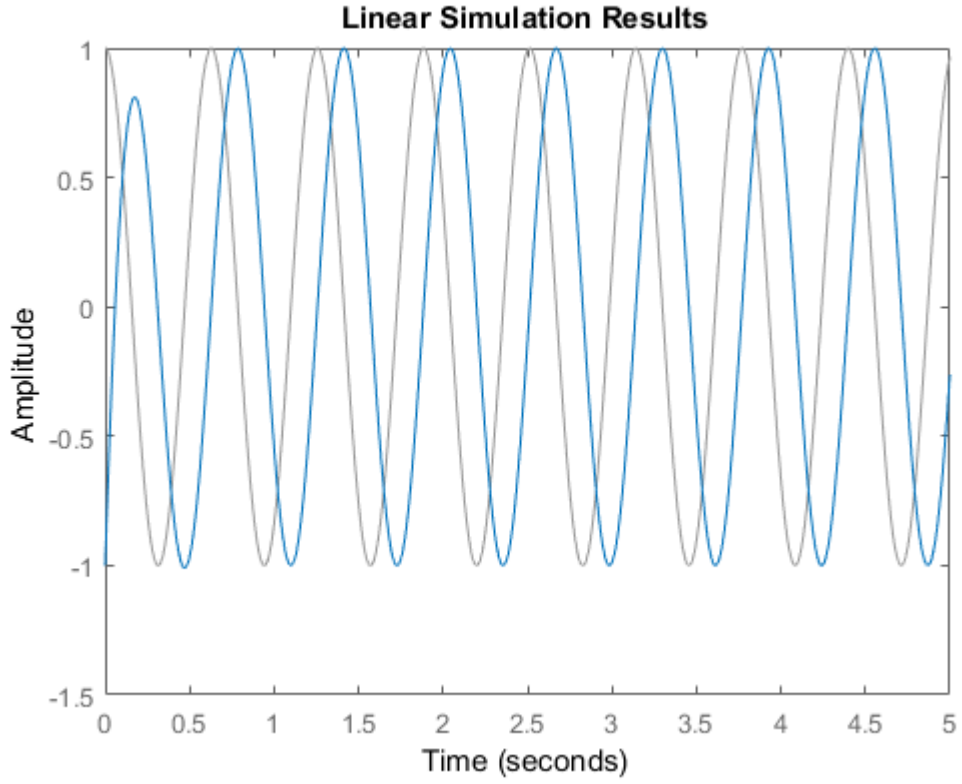

Figura 9: $y(t) = cos(10t - 90^o)$

3.2.5 Item e

$$y(t) = cos(100t + \theta + \angle H(j\omega))$$
$$\angle H(j\omega) = arctan\left(\frac{-100}{10}\right) - arctan\left(\frac{100}{10}\right) \Rightarrow -22.6^o$$
$$y(t) = cos(100t - 168.5^o)$$

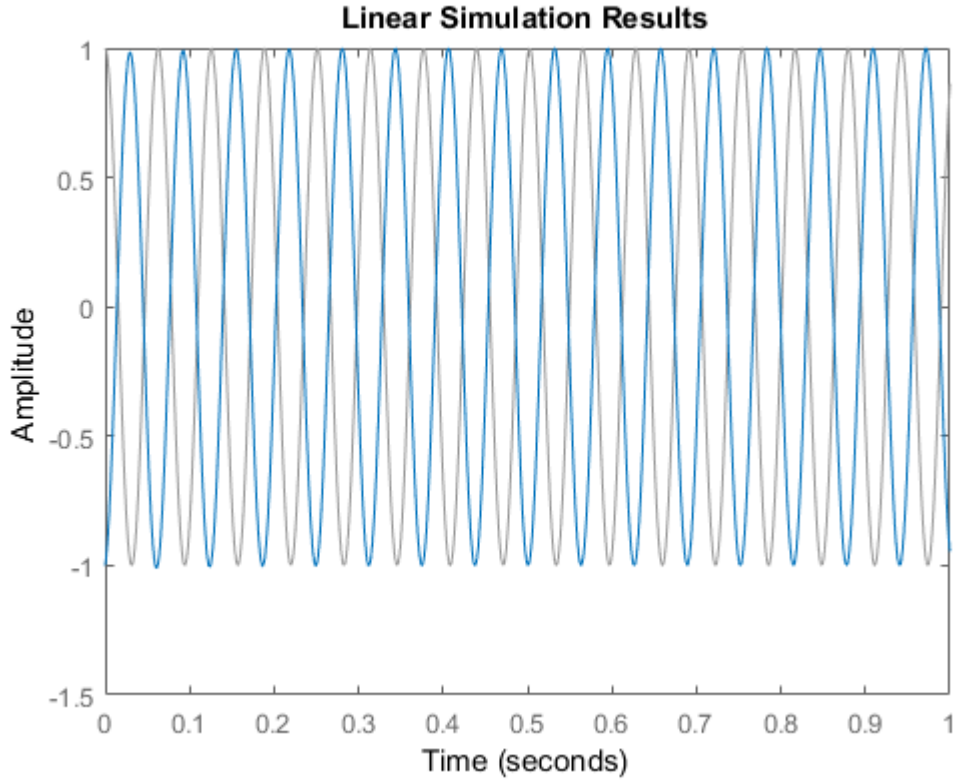

Figura 10: $y(t) = cos(100t - 168.5^o)$

3.3 Questão 6

3.3.1 Item a

É possível, uma vez que a resposta e um deslocamento de fase.

3.3.2 Item b

Não é possível, uma vez que ocorre uma alteração na frequência.

3.3.3 Item c

É possível, o sinal de saída e o mesmo que a entrada.

4 Diagrama de Bode

4.1 Questão 7

4.1.1 Item a

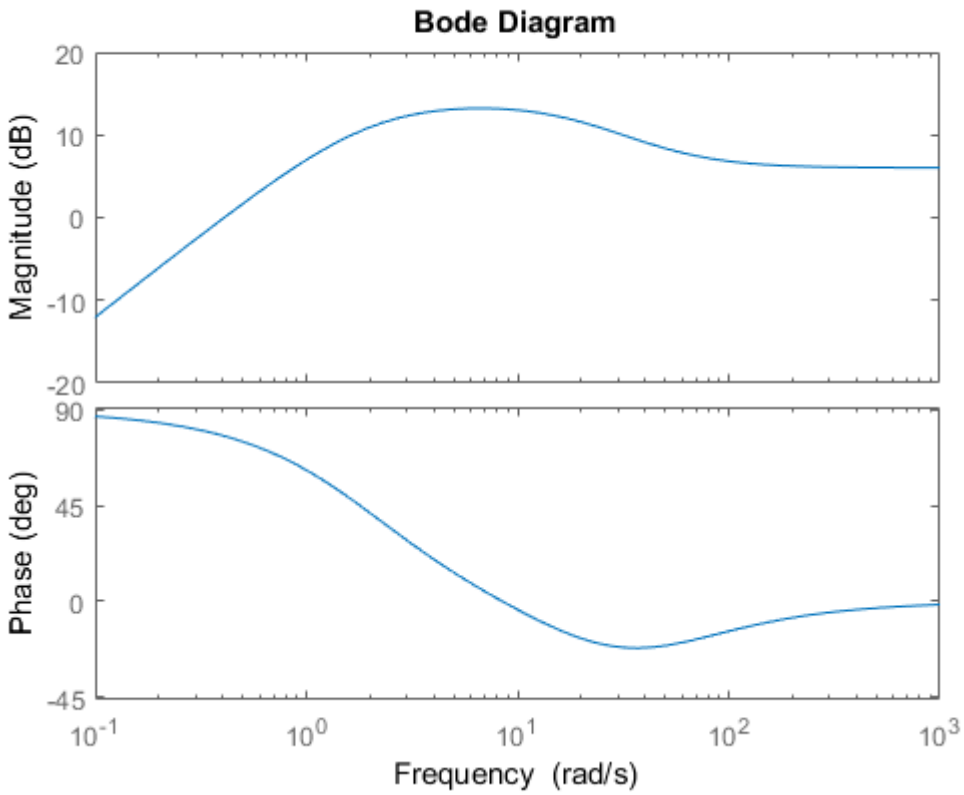

Figura 11: Diagrama de Bode - Item a

4.1.2 Item b

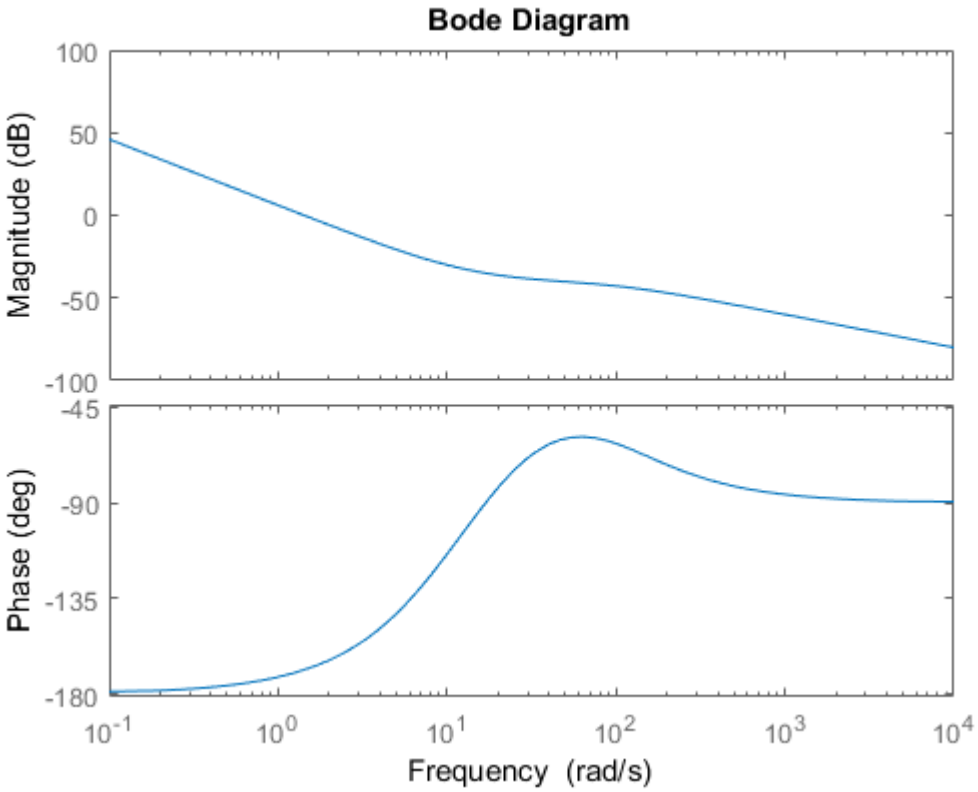

Figura 12: Diagrama de Bode - Item b

4.1.3 Item c

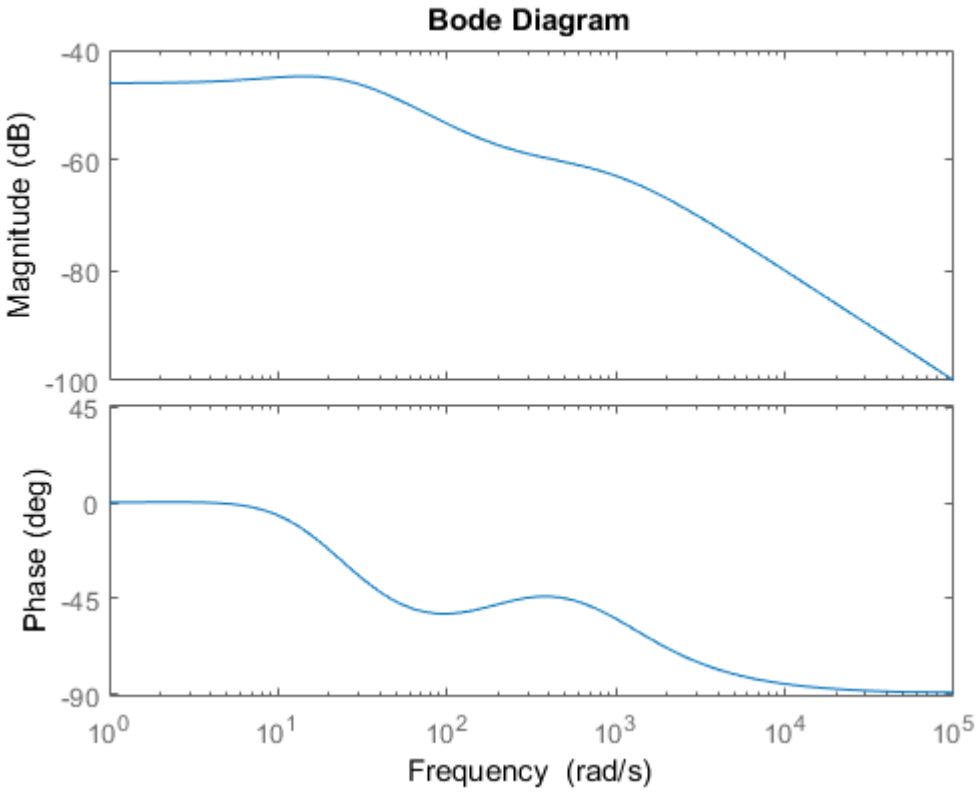

Figura 13: Diagrama de Bode - Item c

4.1.4 Item d

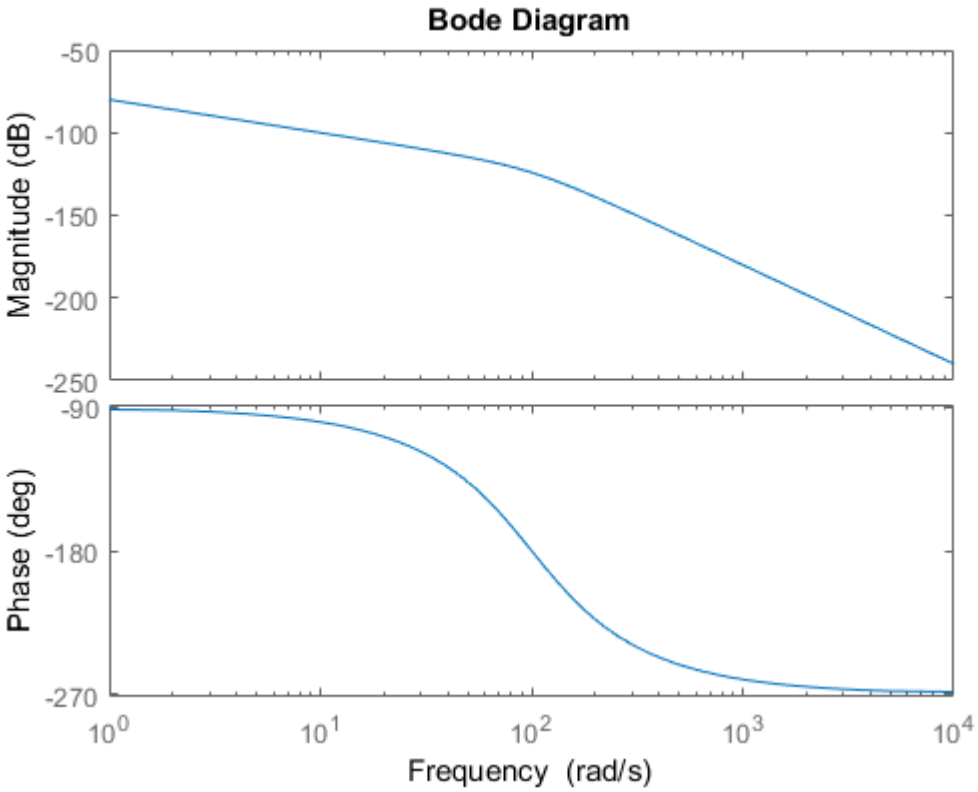

Figura 14: Diagrama de Bode - Item d

4.2 Questão 8

4.2.1 Item a

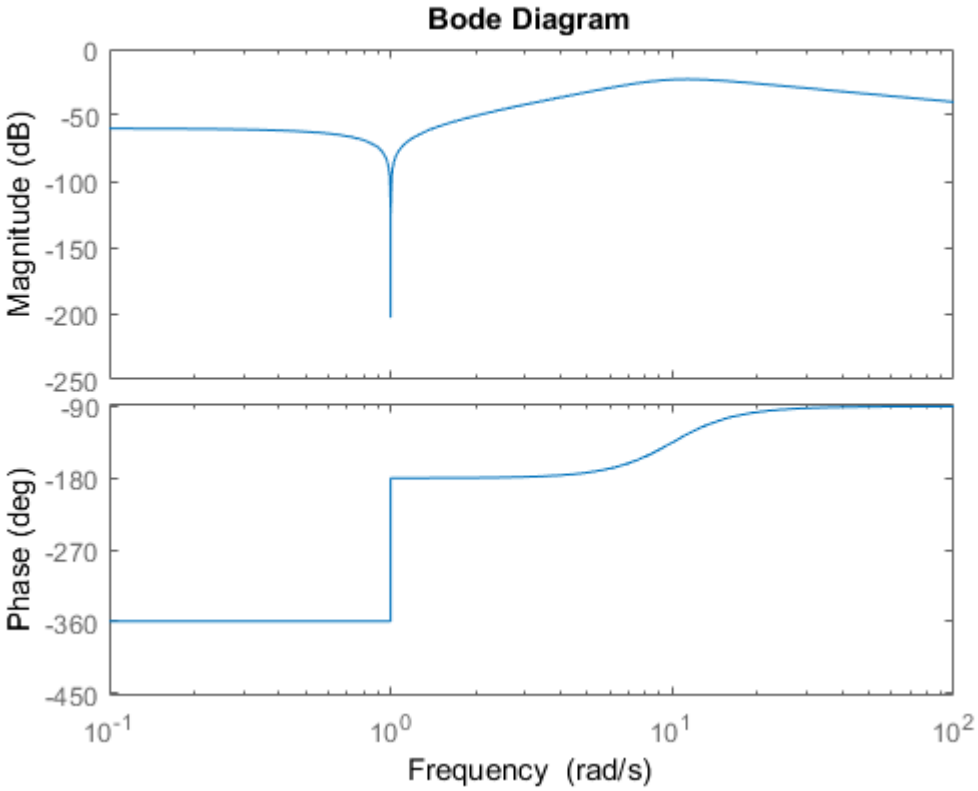

Figura 15: Diagrama de Bode - Item a

Nenhuma das figuras disponibilizadas.

4.2.2 Item b

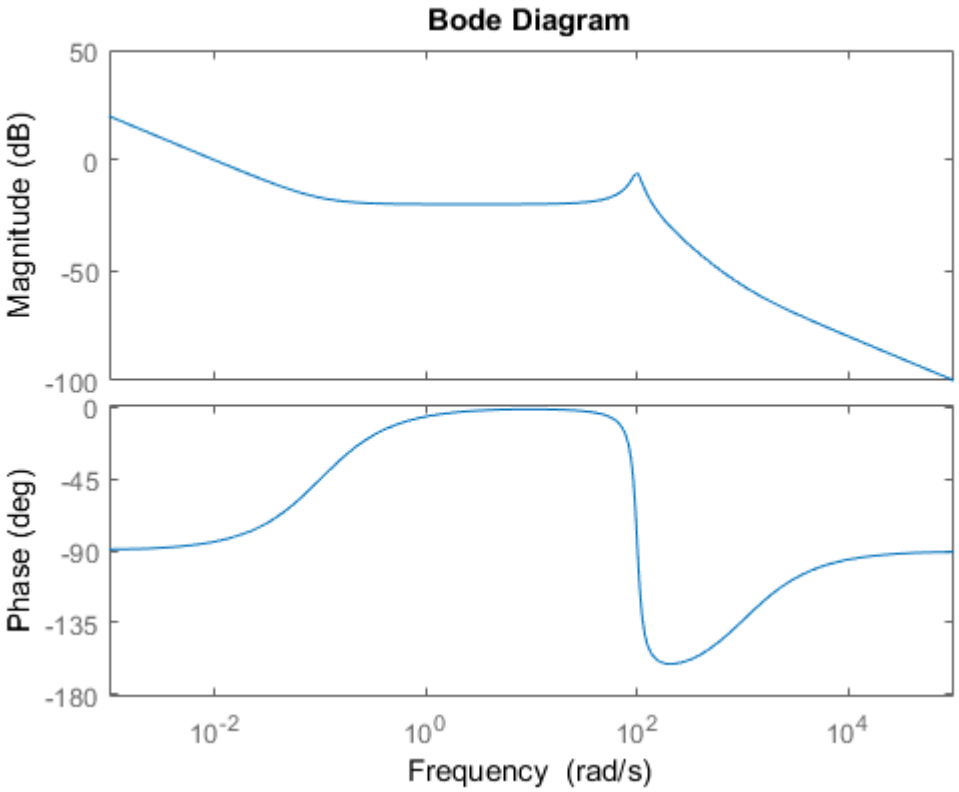

Figura 16: Diagrama de Bode - Item b

Resposta: **Figura 2**

4.2.3 Item c

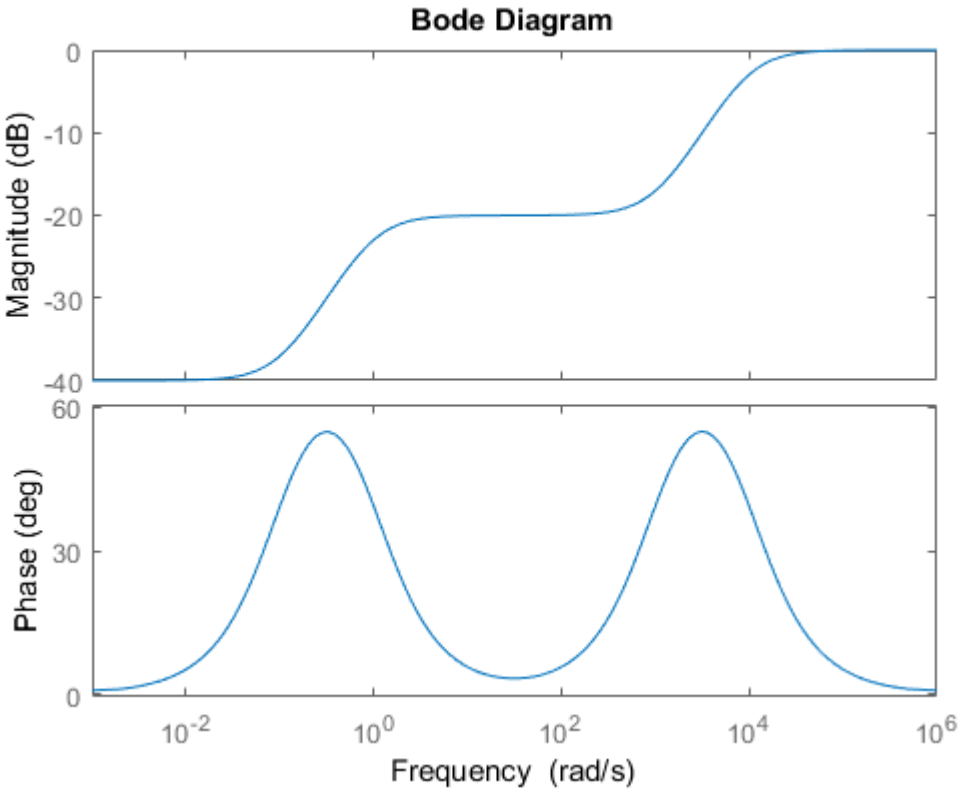

Figura 17: Diagrama de Bode - Item c

Resposta: **Figura 1**

4.2.4 Item d

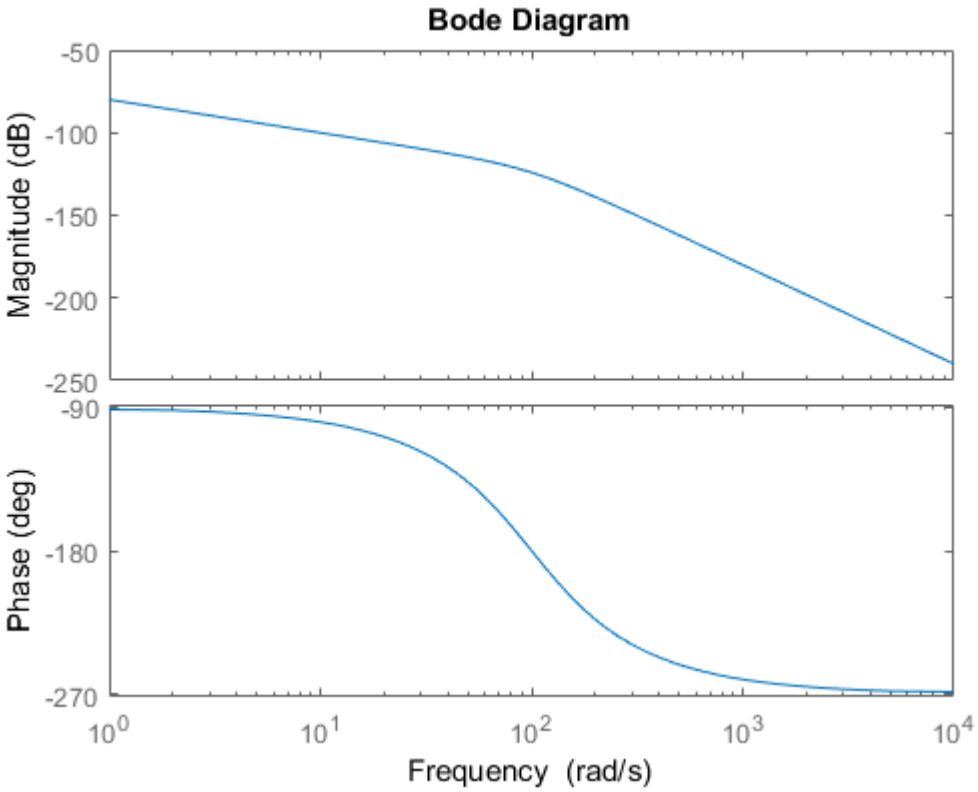

Figura 18: Diagrama de Bode - Item d

Resposta: **Figura 3**