Portfolio Construction for Long-Term Goals:

Integrating Markowitz Optimization, Monte Carlo Simulation, and Capital Accumulation Plans

Iacopo Idrissi Azzouzi

Bachelor Student in Mathematics for Finance and Insurance
University of Turin

Turin, 2025

Abstract

This paper explores an integrated framework for long-term portfolio construction, combining Modern Portfolio Theory, systematic capital accumulation plans, and Monte Carlo simulations. The proposed approach accounts for real-world constraints such as taxation, transaction costs, and annual rebalancing, assessing the probability of achieving predefined capital goals over a 20-year horizon. Results provide practical insights for long-term investors and financial planners, offering a probabilistic evaluation of success rates under different market scenarios.

Contents

1 Introduction

Constructing investment portfolios for long-term objectives represents one of the most critical challenges in contemporary financial planning. Private investors, pension funds, and wealth managers share the need to adopt strategies that enable them to achieve specific capital goals, such as funding education, accumulating retirement wealth, or preserving family assets. In this context, the traditional approach based on Modern Portfolio Theory (MPT), introduced by Markowitz in the 1950s, has long served as the primary framework for optimal diversification, bringing forward the concept of the efficient frontier and formalizing the trade-off between risk and expected return.

Over time, researchers such as Meucci and Fabozzi have pointed out the limitations of these traditional models when applied to realistic settings. They typically assume an investor with a static horizon and an immediately investable lump sum, overlooking practical constraints such as the need for periodic contributions—characteristic of systematic capital accumulation plans—as well as the impact of transaction costs and taxation. In other words, the classical model disregards the market frictions that shape real-world investment decisions, potentially leading to misleading estimates of the probability of achieving a predefined financial target.

To address these shortcomings, this work proposes an integrated approach that combines Markowitz portfolio optimization with a systematic capital accumulation plan, incorporating realistic elements such as transaction costs and taxation. To probabilistically assess the outcomes of this strategy, the analysis relies on Monte Carlo simulation techniques, which enable the modeling of the stochastic evolution of accumulated capital over time while accounting for return uncertainty and market volatility.

This study aims to answer a central question:

What is the probability of achieving a predefined capital goal using a Markowitz-optimized portfolio combined with a systematic capital accumulation plan, under realistic market constraints?

The contribution of this research lies in the integration of three tools—Markowitz optimization, capital accumulation plans, and Monte Carlo simulation—within a framework that closely reflects operational realities, offering valuable analytical insights for both academic research and long-term financial planning.

2 Literature Review

The starting point for modern portfolio construction theory lies in the seminal work of Harry Markowitz, who introduced a systematic approach to investment diversification based on the joint analysis of expected return and risk, measured through the variance of returns. By defining the efficient frontier, Markowitz formalized the set of portfolios that maximize expected return for each level of risk. His contribution shifted the focus from the selection of individual securities to the holistic management of portfolios, marking a radical turning point in modern finance. This approach, further consolidated over the following decades, remains the theoretical foundation for most strategic investment allocation decisions.

In the years that followed, several authors proposed extensions to address the intrinsic limitations of the Markowitz framework. The Black-Litterman model combined market equilibrium structures with investors' subjective views, providing a flexible framework for generating coherent return inputs. Bayesian techniques introduced probabilistic approaches to parameter estimation, addressing the uncertainty surrounding expected returns and covariances. In parallel, the literature on robust optimization, developed by scholars such as Meucci and Fabozzi, aimed at building portfolios less sensitive to estimation errors and more stable over time. These extensions expanded the scope of Modern Portfolio Theory, seeking to bridge the gap between theoretical modeling and the practical needs of institutional and private investors.

Another relevant strand of research concerns systematic capital accumulation plans, conceptually related to the dollar-cost averaging (DCA) strategy. This method consists of investing fixed amounts at regular intervals regardless of market conditions, with the aim of reducing market-timing risk and benefiting from cost averaging over the long term. Empirical studies have shown that this strategy can mitigate perceived volatility and support investors in maintaining investment discipline. However, the literature also highlights some limitations of DCA: while reducing the risk of entering at unfavorable times, it tends to deliver lower returns compared to lump-sum investing in long-term upward-trending markets. Moreover, few studies have explicitly integrated DCA into Markowitz-based optimization frameworks, leaving a gap that this study aims to partially fill.

To assess the probability of success for accumulation strategies, Monte Carlo simulations have become increasingly widespread. Glasserman provides a comprehensive treatment of these methods, describing their applications in risk assessment and financial planning. Monte Carlo simulations make it possible to model the stochastic evolution of returns and accumulated wealth, generating outcome distributions instead of point estimates. This probabilistic approach has proved particularly useful in retirement planning and wealth management, as it incorporates market uncertainty and allows for an assessment of the likelihood of achieving predefined financial goals.

Despite these theoretical advances, significant methodological challenges remain. Markowitz models and most of their extensions assume normally distributed returns and static portfolio weights, overlooking the dynamic nature of markets and the presence of fat tails in return distributions. Similarly, standard Monte Carlo simulations rely on parametric inputs that may underestimate the likelihood of extreme events, limiting their ability to capture adverse market scenarios. Finally, few studies account for real-world frictions such as transaction costs, taxation, and investment constraints, which significantly affect net long-term performance.

In light of these considerations, this study is positioned at the intersection of three research streams: classical portfolio optimization theory, systematic capital accumulation strategies, and probabilistic modeling through Monte Carlo simulation. Its objective is to propose an integrated framework that bridges part of the gap between theoretical models and practical financial planning, incorporating constraints and costs that better reflect the operational conditions faced by real-world investors.

3 Theoretical Framework

3.1 Expected Return, Variance, and Covariance

The construction of an optimal portfolio relies on a rigorous mathematical formalization of the relationships between expected return, risk, and the interdependence among assets. For a single financial instrument i, the random return is denoted by R_i, and its expected value is:

$$\mu_i = \mathbb{E}[R_i].$$

The risk associated with this asset is measured by its variance:

$$\sigma_i^2 = \mathrm{Var}(R_i) = \mathbb{E}\left[(R_i - \mu_i)^2\right],$$

which quantifies the dispersion of returns around their mean.

In portfolio construction, the focus shifts from the risk of a single asset to how assets interact with each other. The covariance between two assets i and j is defined as:

$$\mathrm{Cov}(R_i, R_j) = \mathbb{E}\left[(R_i - \mu_i)(R_j - \mu_j)\right] = \rho_{ij}\sigma_i\sigma_j,$$

where ρ_{ij} represents the correlation coefficient. Correlations lower than one are the foundation of diversification, enabling a reduction in overall portfolio risk without a proportional sacrifice in expected return.

For a portfolio of n assets with weights $w = (w_1, w_2, \ldots, w_n)^\top$, such that:

$$\sum_{i=1}^{n} w_i = 1 \quad \text{and} \quad w_i \geq 0 \ \forall i,$$

the expected portfolio return is:

$$\mu_p = w^\top \mu,$$

where $\mu = (\mu_1, \mu_2, \ldots, \mu_n)^\top$. The portfolio variance is:

$$\sigma_p^2 = w^\top \Sigma w,$$

where Σ denotes the $n \times n$ covariance matrix. This matrix formalism compactly captures all interactions between assets and constitutes the core of the Markowitz approach.

3.2 Markowitz Portfolio Optimization

The classical problem formulated by Markowitz consists in minimizing portfolio variance for a given target return. Formally:

$$\min_{w} \; w^\top \Sigma w$$

subject to:

$$w^\top \mu = \mu_p, \quad w^\top \mathbf{1} = 1, \quad w_i \geq 0 \; \forall i.$$

Neglecting the non-negativity constraints for analytical tractability, the solution can be derived using the method of Lagrange multipliers. The Lagrangian is:

$$\mathcal{L}(w, \lambda, \gamma) = w^\top \Sigma w - \lambda(w^\top \mu - \mu_p) - \gamma(w^\top \mathbf{1} - 1).$$

Differentiating with respect to w and setting the derivative to zero gives:

$$\frac{\partial \mathcal{L}}{\partial w} = 2\Sigma w - \lambda\mu - \gamma\mathbf{1} = 0,$$

which yields:

$$w = \frac{1}{2}\Sigma^{-1}(\lambda\mu + \gamma\mathbf{1}).$$

By imposing the budget constraint $w^\top \mathbf{1} = 1$ and the target return constraint $w^\top \mu = \mu_p$, the Lagrange multipliers λ and γ can be determined, leading to the optimal weights. Defining:

$$A = \mathbf{1}^\top \Sigma^{-1} \mathbf{1}, \quad B = \mathbf{1}^\top \Sigma^{-1} \mu, \quad C = \mu^\top \Sigma^{-1} \mu, \quad \Delta = AC - B^2,$$

the closed-form solution for the optimal weights is:

$$w^* = \Sigma^{-1} \left[\frac{C - B\mu_p}{\Delta} \mathbf{1} + \frac{A\mu_p - B}{\Delta} \mu \right].$$

Varying μ_p traces out the set of portfolios on the efficient frontier, representing the best achievable trade-off between risk and return.

3.3 Efficient Frontier and Special Portfolios

Two portfolios are particularly important on the efficient frontier: the global minimum-variance (GMV) portfolio and the tangency portfolio. The GMV portfolio is obtained by setting μ_p at the value that minimizes variance, yielding weights:

$$w_{GMV} = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^\top \Sigma^{-1} \mathbf{1}}.$$

The tangency portfolio, by contrast, arises in the presence of a risk-free asset with return r_f and maximizes the Sharpe ratio:

$$\max_{w} \frac{w^\top \mu - r_f}{\sqrt{w^\top \Sigma w}},$$

with the solution:

$$w_T = \frac{\Sigma^{-1}(\mu - r_f \mathbf{1})}{\mathbf{1}^\top \Sigma^{-1}(\mu - r_f \mathbf{1})}.$$

Geometrically, the efficient frontier in the risk-return plane is a convex curve; the inclusion of a risk-free asset produces the Capital Market Line, a straight line that dominates any combination of risky assets alone. This construction not only identifies the efficient combinations of risk and return but also provides an operational guide for selecting portfolios based on investor preferences and market constraints.

4 Data and Portfolio Construction

The definition of the dataset and the selection of financial instruments represent the preliminary and fundamental step of any portfolio optimization process. This section describes the characteristics of the selected assets, the sources of historical data, the procedures applied for cleaning and transforming the time series, and the estimation of the parameters used for the subsequent construction of the optimized portfolio.

4.1 Asset Selection

The portfolio is constructed using four global exchange-traded funds (ETFs), selected to ensure a high degree of diversification across asset classes and geographical areas. The selection was guided by three main criteria:

1. Broad global market coverage, including developed and emerging markets;
2. Diversification across asset classes, incorporating equities, bonds, real estate, and commodities;
3. Tax-efficient domiciliation, with funds primarily domiciled in Ireland or the United States.

The chosen instruments are:

- **Vanguard FTSE All-World UCITS ETF (VWRA)**: a global equity fund providing extensive geographical and sectoral diversification across developed and emerging markets;
- **iShares Core Global Aggregate Bond ETF (AGGG)**: a fixed-income fund offering exposure to global government and corporate bonds, aimed at providing stability and reducing overall portfolio volatility;
- **SPDR Gold Shares (GLD)**: a commodity ETF tracking the price of physical gold, serving as a hedge against inflation and a safe-haven asset during market downturns;
- **iShares Global REIT ETF (REET)**: an international real estate fund granting access to listed real estate investment trusts (REITs) in both developed and emerging markets.

4.2 Data Sources and Transformation

Historical data were retrieved from Yahoo Finance and Bloomberg (for VWRA and AGGG) and from State Street (for GLD). Adjusted closing prices, which account for reinvested dividends and stock splits, were used to ensure consistency in total return calculations.

Daily prices were aggregated to a monthly frequency, consistent with the contribution schedule of the PAC. Monthly logarithmic returns were computed as:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right),$$

where P_t represents the adjusted closing price at the end of month t.

The time series underwent a preliminary cleaning process, which included the removal of outliers and the imputation of missing observations through linear interpolation. The sample period covers the last 10 years (2014–2024), providing a sufficient number of observations to obtain reliable parameter estimates.

4.3 Parameter Estimation and Operational Constraints

Based on the monthly returns, the following parameters were estimated:

Annualized mean returns:

$$\mu_i^{\text{ann}} = 12 \cdot \bar{r}_i,$$

where $\bar{r}_i$ denotes the average monthly return of asset i.

Annualized covariance matrix:

$$\Sigma^{\text{ann}} = 12 \cdot \text{Cov}(r),$$

where $\text{Cov}(r)$ is the sample covariance matrix of monthly returns.

The risk-free rate was set at 1% per year, consistent with the average yields of short-term Swiss government securities over the sample period.

Finally, weight constraints were introduced to reflect practical considerations of diversification and risk control:

$$0 \leq w_i \leq 0.30 \quad \forall i.$$

5 Portfolio Optimization

Portfolio optimization lies at the core of Modern Portfolio Theory and serves as the foundation for analyzing the risk-return trade-off across different asset combinations. The objective of this section is to construct optimal portfolios based on the previously defined data and constraints, identifying two reference configurations: the **global minimum-variance (GMV) portfolio** and the **maximum-Sharpe portfolio**. Furthermore, the efficient frontier obtained through numerical simulation is presented to visualize the set of efficient combinations and to compare constrained solutions with their unconstrained theoretical counterparts.

5.1 The Global Minimum-Variance (GMV) Portfolio

The global minimum-variance portfolio is the combination of asset weights that minimizes overall volatility without imposing any constraint on expected return. Formally:

$$\min_{w} \; w^{\top} \Sigma w$$

subject to:

$$w^{\top} \mathbf{1} = 1 \quad \text{and} \quad 0 \leq w_i \leq 0.3 \; \forall i.$$

The optimization, solved numerically using the Sequential Least Squares Programming (SLSQP) method, returns a portfolio evenly distributed across the four ETFs, with weights of 0.25 each. The expected annualized return of the GMV portfolio is approximately 7.6%, with a volatility of about 5.3%.

5.2 The Maximum-Sharpe Portfolio

In the presence of a risk-free asset with return r_f, the portfolio that maximizes the Sharpe ratio—defined as the excess return over the risk-free rate per unit of risk—can be identified:

$$\max_{w} \; \frac{w^{\top} \mu - r_f}{\sqrt{w^{\top} \Sigma w}}$$

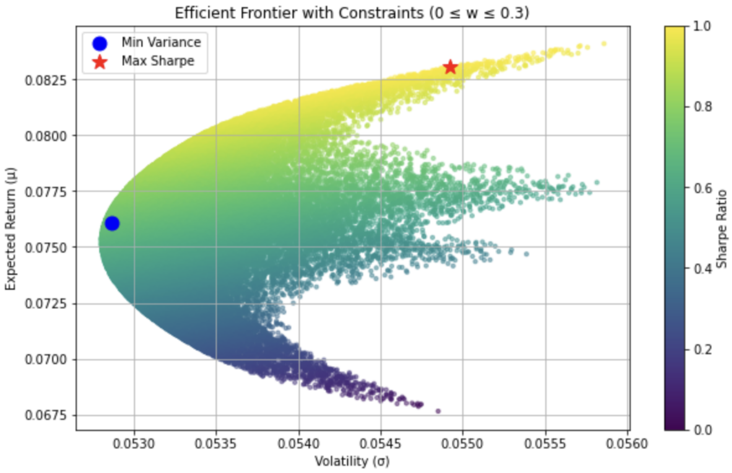
subject to:

$$w^{\top} \mathbf{1} = 1 \quad \text{and} \quad 0 \leq w_i \leq 0.3 \; \forall i.$$

The optimal solution exhibits higher allocations to high-return assets, particularly gold (30%) and real estate (29.3%), with lower allocations to global equities (13.5%) and bonds (27.1%). The maximum-Sharpe portfolio delivers an expected annualized return of approximately 8.3%, with a slightly higher volatility of about 5.5%.

5.3 Efficient Frontier and Interpretation

To illustrate the set of efficient combinations, a constrained efficient frontier was generated via Monte Carlo simulation of 50,000 random portfolios, respecting the weight limits. Figure 1 displays the risk-return plane, coloring each portfolio according to its Sharpe ratio. The two highlighted points correspond to the GMV portfolio (blue circle) and the maximum-Sharpe portfolio (red star).




```
Min Variance Portfolio Weights:
 VWRA    0.25
AGGG    0.25
GLD     0.25
REET    0.25
dtype: float64

Max Sharpe Portfolio Weights:
 VWRA    0.135111
AGGG    0.271724
GLD     0.300000
REET    0.293165
dtype: float64
```

Figure 1: Constrained efficient frontier ($0 \leq w \leq 0.3$) with GMV (blue circle) and maximum-Sharpe (red star) portfolios.

The analysis reveals that the constrained frontier is more compressed than the unconstrained theoretical frontier, reflecting the reduction in efficiency introduced by weight limits. The GMV portfolio is located at the lower-left end of the frontier, characterized by the minimum achievable volatility, while the maximum-Sharpe portfolio lies higher and slightly to the right, offering significantly greater return with only a marginal increase in risk.

6 Capital Accumulation Plan (PAC) Modeling

Integrating a Capital Accumulation Plan (PAC) into the portfolio optimization framework is a crucial step toward aligning the analysis with the operational reality of private investors. Unlike classical Markowitz models, which assume a lump-sum investment at time zero, a PAC involves periodic contributions that are progressively added to the invested capital. This mechanism reduces market-timing risk and facilitates systematic wealth accumulation over the long term. This section formalizes the mathematical representation of periodic contributions, the portfolio rebalancing rules, and the modeling of transaction costs and implicit tax burdens, thereby providing a framework that closely mirrors real-world market conditions.

6.1 Increasing Monthly Contributions

The PAC is modeled as a sequence of monthly contributions that may grow over time to account for an investor's increasing saving capacity. The contribution at time t (in months) is defined as:

$$C_t = C_0(1+\delta)^{\lfloor t/12 \rfloor},$$

where C_0 represents the initial monthly contribution and δ the annual growth rate of contributions. This mechanism introduces a more realistic dynamic in capital accumulation, simulating the progressive increase in disposable savings. Contributions are invested monthly into the selected ETFs according to the proportions determined by the optimized portfolio (GMV or maximum-Sharpe).

6.2 Portfolio Rebalancing

To maintain the target portfolio composition in the presence of periodic contributions and differential asset returns, an **annual rebalancing** policy is implemented. At the end of each year, the portfolio weights are restored to their optimized levels, mitigating the risk of drift toward unintended configurations. Although more frequent rebalancing could further reduce deviations from the target weights, it would increase transaction costs, undermining net performance. The choice of an annual frequency thus represents a reasonable compromise between strategic consistency and cost containment. Rebalancing serves two main purposes: preserving the chosen risk-return profile and supporting the "buy low, sell high" mechanism by reallocating capital toward underperforming assets while trimming positions in those that have outperformed.

6.3 Transaction Costs and Taxation

To account for market frictions, a **transaction cost of 0.1%** of the traded value is applied to contributions and rebalancing operations. This conservative assumption, consistent with average costs for ETF transactions through low-cost brokers, slightly reduces the effectively invested capital and, when compounded over time, may significantly affect final outcomes.

Regarding taxation, two distinct elements are considered:

1. **Withholding taxes on ETFs**: As the selected ETFs are primarily domiciled in Ireland and the United States, dividends are subject to withholding taxes (typically around 15%), already embedded in historical price data and therefore not modeled separately.

2. **Swiss withholding tax (Verrechnungssteuer, 35%)**: This tax applies to dividends distributed to Swiss-resident investors but is fully reclaimable through the tax return process. Consequently, it does not represent a permanent drag on returns and is not further deducted in the simulation.

Finally, it is important to note that **ETF management fees (TER)**, while not explicitly modeled, are already reflected in historical price data and thus implicitly incorporated in the returns used for simulation.

7 Monte Carlo Simulation

To evaluate the robustness of the constructed portfolio and the probability of achieving the predefined objectives, a Monte Carlo simulation was conducted, fully integrating the accumulation plan structure with the stochastic behavior of financial markets. The model generated **10,000 independent scenarios** over a 20-year horizon, assuming that asset returns follow a **multivariate normal distribution** parameterized by the historical mean returns and covariance matrix of the selected ETFs.

The portfolio dynamics for each simulated path follow the recursive relationship:

$$V_t = (V_{t-1} + C_t) \cdot (1 + r_p(t)) - \text{Costs}_t,$$

where:

- C_t denotes the monthly contribution of the accumulation plan (initially CHF1,000, with a 2% annual growth rate),
- $r_p(t)$ represents the simulated monthly portfolio return,
- Costs_t accounts for the annual rebalancing cost (0.1% of traded value).

7.1 Simulation Results

Over the 20-year horizon, for a cumulative invested capital of approximately **CHF292,054**, the final portfolio value exhibits:

- a **mean** of **CHF617,455** and a **median** of **CHF609,575**,
- a **5th–95th percentile range** between **CHF470,583** and **CHF788,808**.

These figures indicate that the portfolio more than doubles the invested capital even under conservative assumptions. Notably, the probability of ending the period with a portfolio value above the total invested capital reached **100%**, underscoring the robustness of the strategy.

7.2 Interpretation

The growing gap between the invested capital and the average simulated portfolio value becomes particularly pronounced in the second half of the investment horizon, reflecting the compounding effect of returns. While the dispersion of trajectories illustrates the inherent variability of market outcomes, the fact that nearly all simulations remain above the invested capital line highlights the resilience of the adopted strategy. The distribution of final portfolio values (Figure 3) is near-symmetric with a slight positive skewness, reflecting the potential for favorable outcomes. These findings reinforce the conclusion that

```
Mean: 617,455 CHF | Median: 609,575 CHF
5th–95th percentile: 470,583 – 788,808 CHF
Total contributed capital: 292,054 CHF
Success rate: 100.00%
```

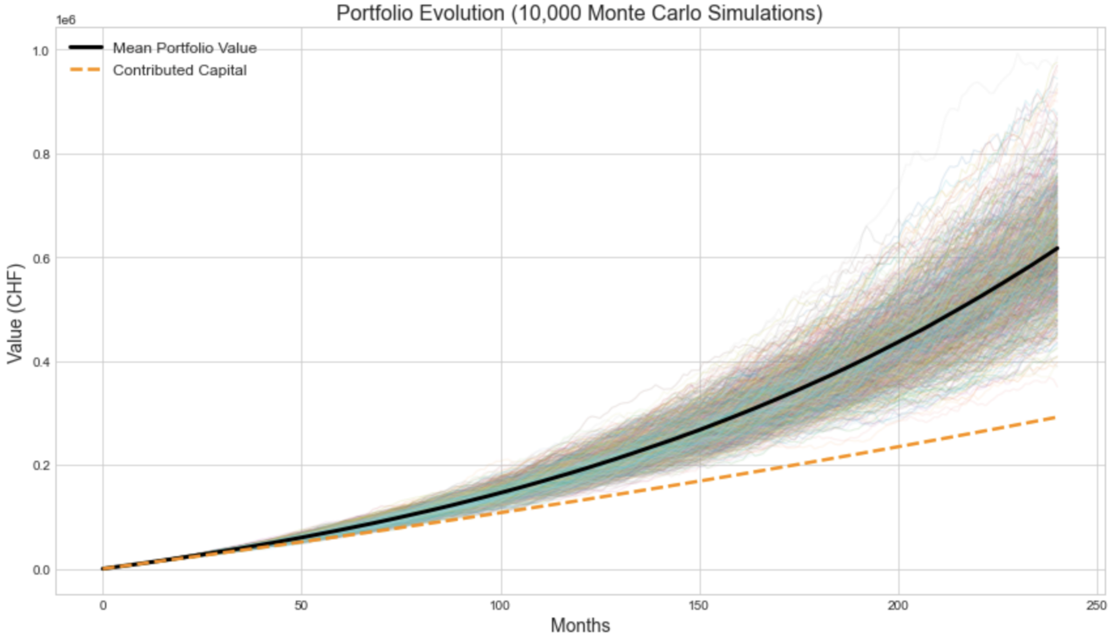


Figure 2: Simulated portfolio evolution across 10,000 scenarios. The black line represents the mean trajectory, while the dashed orange line indicates the cumulative invested capital.

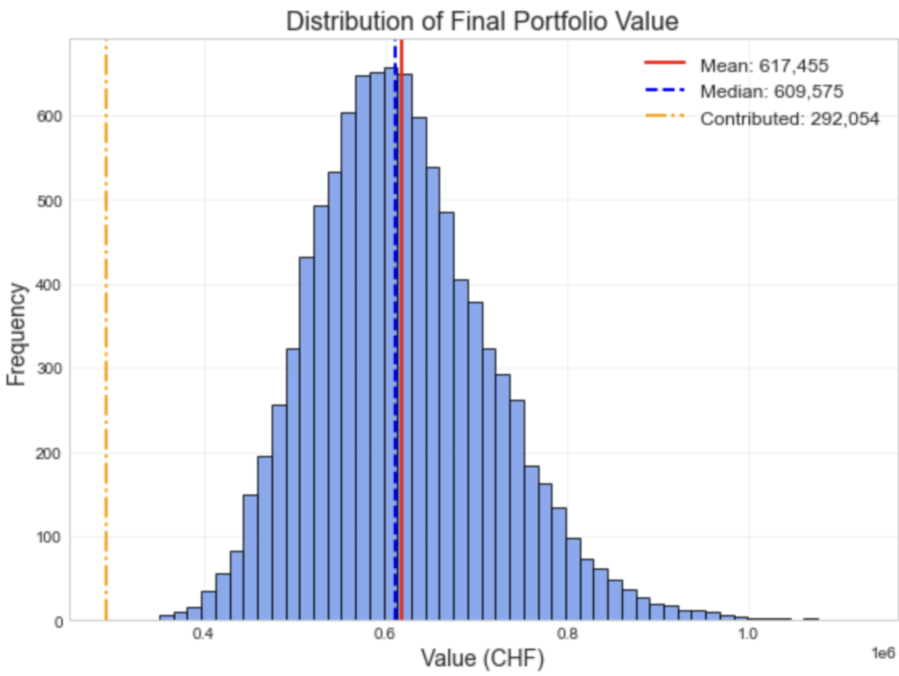


Figure 3: Distribution of final portfolio values after 20 years. Vertical lines indicate the mean (red), the median (blue dashed), and the cumulative invested capital (orange).

the combination of periodic contributions, annual rebalancing, and optimized allocation constitutes a solid strategy for achieving long-term financial goals.

8 Sensitivity Analysis

To assess the robustness of the model and its ability to maintain an acceptable risk-return profile under adverse conditions, a sensitivity analysis was conducted across three alternative scenarios: increased volatility, reduced contribution growth, and worsened cross-asset correlations. The goal is to measure the impact of each factor on the distribution of final portfolio outcomes, comparing them to the baseline scenario.

In the first stress test, portfolio volatility was increased by 25% relative to historical levels. As expected, the distribution widened, with a deterioration of the 5th percentile (from approximately CHF497,403 to CHF482,740) and an increase of the 95th percentile (from CHF823,171 to CHF848,686), illustrating how greater uncertainty expands the range of potential outcomes. However, the median declined only marginally (from CHF639,502 to CHF635,810), indicating the overall resilience of the strategy.

In the second scenario, the contribution growth rate of the accumulation plan was reduced from 2% to 1%, simulating an investor with lower savings capacity over time. This stress had a more pronounced effect on the median, which fell to approximately CHF591,330, and also reduced the 95th percentile to CHF766,413. Nevertheless, the portfolio still maintained final values well above the total capital contributed (CHF292,054), demonstrating that even with smaller contributions, the model preserves a high probability of success.

Finally, in the third scenario, cross-asset correlations were increased by 10%, reducing the effectiveness of diversification. This resulted in a slight deterioration of the lower tail (5th percentile at CHF481,609) and the median (CHF633,853), with limited impact on mean and upper-tail outcomes. This suggests that, while diversification remains a key element of the model, the portfolio retains a strong capacity for growth even in more correlated, less differentiated market environments.

The results are summarized in Table 1, which compares the main summary indicators for each scenario.

Table 1: Monte Carlo simulation results for the baseline scenario and stress tests.

Scenario	**Mean**	**Median**	**5th Percentile**	**95th Percentile**
Baseline	646,789	639,502	497,403	823,171
Volatility +25%	646,688	635,810	482,740	848,686
Contributions +1%	599,181	591,330	459,581	766,413
Correlations +10%	645,368	633,853	481,609	846,530

Overall, the sensitivity analysis confirms that the portfolio maintains considerable robustness even under adverse conditions, with means and medians remaining well above the total invested capital and a distribution that, while widening in the higher-volatility scenario, preserves a high likelihood of achieving positive outcomes. The integration of a capital accumulation plan with annual rebalancing and optimized allocation thus proves to be a resilient strategy, capable of absorbing market shocks and operational parameter changes without compromising long-term objectives.

9 Discussion and Conclusion

The results obtained confirm the validity of an approach that integrates Modern Portfolio Theory (MPT) with a Capital Accumulation Plan (PAC), enhanced by periodic rebalancing and realistic operational costs. The Monte Carlo simulation demonstrates that, over a 20-year horizon, the optimized portfolio not only preserves the invested capital but also generates substantial growth, with a high probability of significantly exceeding the total amount contributed. This represents a clear advantage over static allocation strategies, which, lacking rebalancing and variance-covariance optimization, tend to experience progressive imbalances and reduced long-term efficiency.

Conceptually, the model highlights how dynamic diversification — achieved through Markowitz-optimized portfolio construction and preserved via annual rebalancing — provides effective protection against adverse market scenarios. The inclusion of imperfectly correlated assets, such as bonds, gold, and real estate, reduces overall volatility, improving the risk-return profile even under the most extreme sensitivity tests. Furthermore, the accumulation plan offers both a behavioral and technical advantage: investing periodically in a diversified portfolio mitigates market-timing risk and leverages the cost-averaging effect over time.

Nevertheless, the model presents some inherent limitations. First, the assumption of normally distributed returns oversimplifies real market dynamics, which often exhibit fat tails and asymmetries not captured by a Gaussian framework. Second, the use of static weights between rebalancing periods does not account for potential dynamic management strategies, which could further improve portfolio efficiency. Finally, the cost and taxation assumptions, deliberately simplified to ensure interpretability, do not fully reflect the complexity of real-world conditions, where variable commissions, taxes, and other fees could materially affect net outcomes for investors.

These limitations open avenues for future research. Integrating GARCH models would allow for the inclusion of conditional volatility, improving risk representation in turbulent markets. Bootstrap simulations could generate scenarios based on historical resampling, capturing empirical return dynamics more effectively than parametric distributions. Finally, the adoption of dynamic asset-allocation strategies — adjusting weights in response to market signals or macroeconomic forecasts — could further enhance performance, albeit at the expense of greater operational complexity.

Conclusion

This study investigated the probability of achieving a long-term capital accumulation target through a Markowitz-optimized portfolio integrated with a systematic accumulation plan, assessing its performance via 10,000 Monte Carlo simulations over a 20-year horizon. The results show that this combination not only preserves invested capital but also delivers substantial growth, maintaining a high probability of success even in adverse market conditions.

Answering the initial research question, the probability of reaching the predefined capital target is high when adopting a strategy based on a Markowitz-optimized portfolio and regular capital contributions, even after incorporating transaction costs and rebalancing. This approach thus appears particularly well-suited for long-term investors — such as those focused on retirement planning, wealth accumulation, or funding future projects — who aim to balance growth and risk management.

From an operational perspective, the model offers valuable insights for wealth management: the importance of diversified allocation, the critical role of periodic rebalancing, and the need for a disciplined investment approach emerge as key elements for strategic success. Although further improvements are possible — including dynamic volatility modeling and more advanced simulation methodologies — the work presented here provides a practical and implementable framework for long-term financial planning, with relevant applications in both academic and professional contexts.

Bibliography

References

[1] Markowitz, H. M. (1952). *Portfolio Selection*. The Journal of Finance, 7(1), 77–91.

[2] Markowitz, H. M. (1959). *Portfolio Selection: Efficient Diversification of Investments*. Wiley, New York.

[3] Glasserman, P. (2003). *Monte Carlo Methods in Financial Engineering*. Springer, New York.

[4] Meucci, A. (2009). *Risk and Asset Allocation*. Springer, New York.

[5] Fabozzi, F. J. (2010). *Robust Portfolio Optimization and Management*. Wiley, New Jersey.

[6] Black, F., & Litterman, R. (1992). *Global Portfolio Optimization*. Financial Analysts Journal, 48(5), 28–43.