# ECE/CS 559 Neural Networks, Fall 2017 - Homework #7
# Due: 11/10/2017, the end of class.

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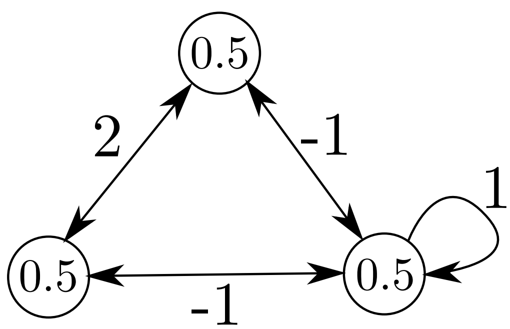
All the notes in the beginning of Homework #1 apply. As usual, please include the computer codes in your report.

1. **(40pts)** Let $\phi(x) = 1$ if $x \geq 0$, and $\phi(x) = -1$ if $x < 0$. Consider some $\mathbf{x}_1, \ldots, \mathbf{x}_n \in \{-1, +1\}^m$. In associative memory, we have designed the synaptic weight matrix as $\mathbf{W} = \sum_{i=1}^{n} \mathbf{x}_i \mathbf{x}_i^T$. Suppose that there exists $\mathbf{z} \in \mathbb{R}^m$ that satisfies the following properties:

   - $\mathbf{z} = \phi(\mathbf{W}\mathbf{z})$.
   - For any $i \in \{1, \ldots, n\}$, we have $\mathbf{z} \neq \mathbf{x}_i$ and $\mathbf{z} \neq -\mathbf{x}_i$.

   In class, we have called $\mathbf{z}$ a spurious memory pattern. We never gave an example of $\mathbf{x}_1, \ldots, \mathbf{x}_n$ that results in the existence of a spurious pattern. Find such an example and show your work. You may use computer search.

2. **(60pts)** Consider the Hopfield network below. The activation function is, as usual, $\phi(x) = 1$ if $x \geq 0$, and $\phi(x) = -1$ if $x < 0$.




   Draw the state transition diagrams (with energy levels) for both synchronous and asynchronous update rules. Indicate the urstate(s) and the steady state(s) of the network. An urstate is a state with no predecessors (i.e., if $S$ is an urstate, then no other state, including $S$ itself, should be able to transition to $S$).

OLIVO JACOPO HW 8

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1] WITH THE HELP OF A MATLAB SCRIPT, WE CAN EXTRACT THE FOLLOWING VALUES:

$$x_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad x_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad x_3 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

WHICH RESULT IN

$$W = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

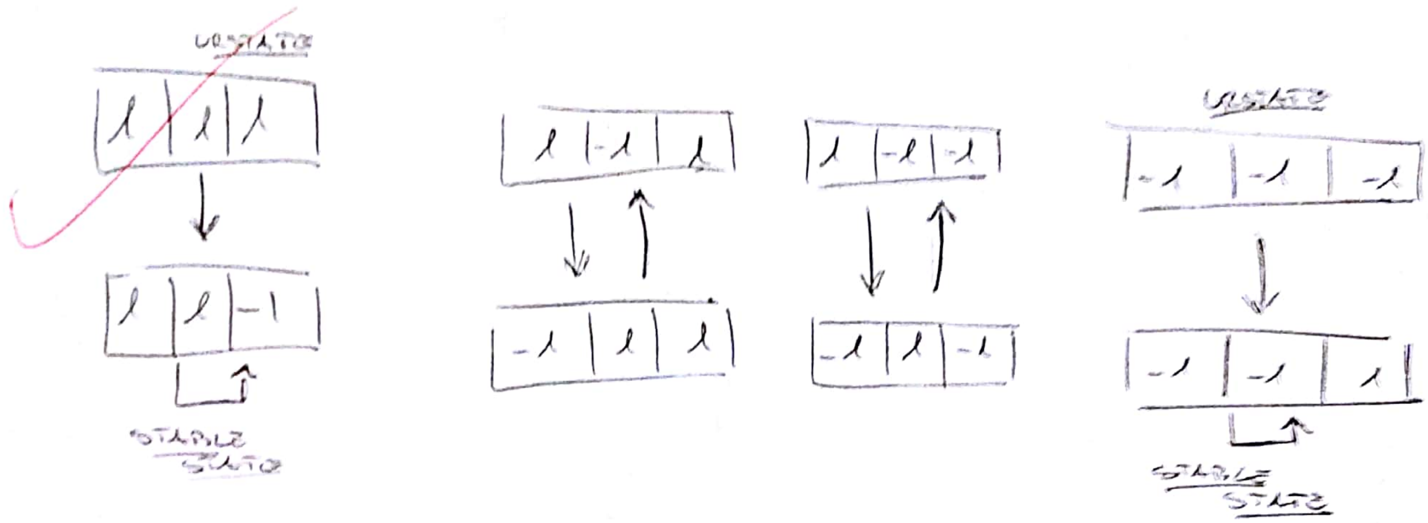
By choosing

$$z = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \qquad Wz = \text{sgn}\left( \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \right) = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

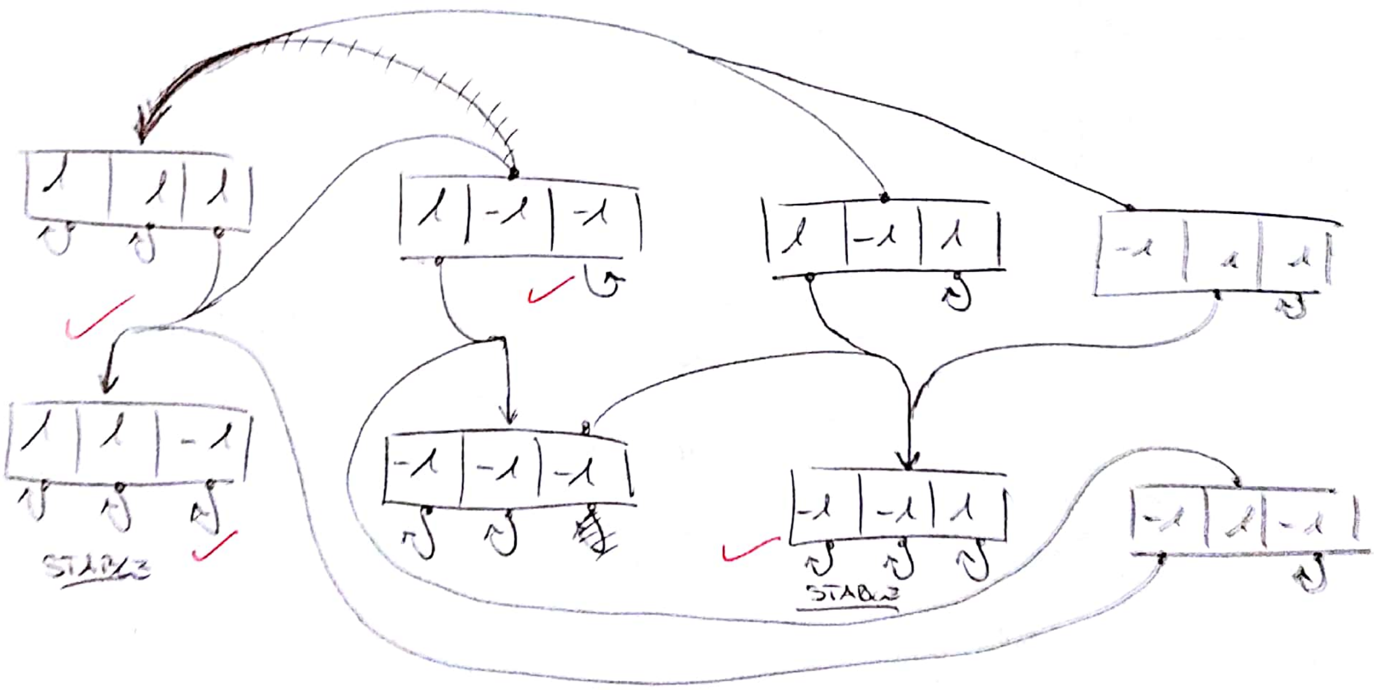
WHICH RESPECTS

$$z \neq x_i, \quad z \neq -x_i \quad \forall i$$

2] BY APPLYING SYNCHRONOUS UPDATE WE OBTAIN:




WHILE APPLYING ASYNCHRONOUS UPDATE:
(NOTE THE LACK OF UPSTATES)




THE ENERGY LEVELS FOR EACH STATE ARE:

1, -1, -1 -> 4

-1, 1, -1 -> 4

1, 1, -1 -> -10

-1, -1, 1 -> -8

1, -1, 1 -> 2

-1, 1, 1 -> 2

-1, -1, -1 -> 2

1, 1, 1 -> -4