Problem 3

1. Because

There are ten elements in therefore the is ten.

If we do explicit feature mapping, we would first calculate the mapped values and then use the mapped value to calculate the inner product. This feature transformation and inner product process may require a lot of computation. Because we have the input , we can directly evaluate the inner product – skipping the explicit mapping and hence saving the computation.

1. Positive semidefinite matrix

Let be any column vector

Therefore matrix is positive semidefinite.

Let be any column vector

Therefore matrix is strictly positive definite with non-zero .

Let be any column vector

is not guaranteed to be non-negative or non-positive

Therefore matrix is neither positive semidefinite or strictly positive definite.

1. Let and be psd matrices, then we have

Since

Therefore is also positive semidefinite.

A matrix is positive semidefinite if and only if all of its eigenvalues are non-negative. Because both and are positive semidefinite, their eigenvalues are all non-negative.

Let be the eigenvalues of matrix and be the eigenvalues of matrix . By the definition of positive semidefinite

There is corresponding eigenvectors for the Kronecker product and they are for all values of and . Because we have

Therefore, the Kronecker product of two positive semidefinite matrices is also positive semidefinite.

1. RBF kernel

Suppose that such that . Assume that

Using the Taylor series

That is, RBF kernel corresponds to the inner product in an infinite dimensional space