Prova 03 - PDS

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Disciplina: Processamento digital de sinais

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Turma: Engenharia da Computação 2018

Data: 24/08/21

1)

a) $X(z) = \frac{1 - \frac{1}{3}z^{-1}}{1 + \frac{17}{10}z^{-1} - 2z^{-2}} \cdot \frac{z^2}{z^2} = \frac{z^2 - \frac{1}{3}z}{z^2 + \frac{17}{10}z - 2}$

(Zeros)

$z^2 - \frac{1}{3}z = 0$

$\Delta = b^2 - 4 \cdot a \cdot c$

$\left(\frac{1}{3}\right)^2 - 4 \cdot 1 \cdot 0$

$\Delta = \frac{1}{9}$

$z = \frac{-b \pm \sqrt{\Delta}}{2a}$

$+: \frac{-\left(-\frac{1}{3}\right) + \sqrt{\frac{1}{9}}}{2 \cdot 1} = \frac{\frac{1}{3} + \frac{1}{3}}{2} = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$

$-: \frac{-\left(-\frac{1}{3}\right) - \sqrt{\frac{1}{9}}}{2 \cdot 1} = \frac{\frac{1}{3} - \frac{1}{3}}{2 \cdot 1} = \cancel{\frac{0}{2}} = 0$

(Pólos)

$z^2 + \frac{17}{10}z - 2 = 0$

$\Delta = b^2 - 4 \cdot a \cdot c$

$\left(\frac{17}{10}\right)^2 - 4 \cdot 1 \cdot (-2)$

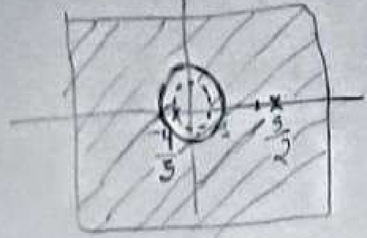
$\frac{289}{100} - (-8) \rightarrow \frac{289 + 800}{100} = \frac{1089}{100}$

$z = \frac{-b \pm \sqrt{\Delta}}{2 \cdot a}$

$+: \frac{-\left(\frac{17}{10}\right) + \sqrt{\frac{1089}{100}}}{2 \cdot 1} = \frac{+\frac{17}{10} + \frac{33}{10}}{2} = \frac{50}{10} \cdot \frac{1}{2} = \frac{5}{2}$

$-: \frac{-\left(\frac{17}{10}\right) - \frac{33}{10}}{2} = -\frac{16}{10} \cdot \frac{1}{2} = -\frac{8}{10} = -\frac{4}{5}$

Continuação letra (a) (ROC)



A transformada de fourier existe!
A ROC compreende o circulo unitário

b) $X(z) = \dfrac{1 - z^{-1}}{1 - \frac{5}{8}z^{-1} - \frac{3}{32}z^{-2}} \cdot \dfrac{z^2}{z^2} = \dfrac{z^2 - z}{z^2 - \frac{5}{8}z - \frac{3}{32}}$

(Zeros)

$z^2 - z = 0$

$\Delta = b^2 - 4 \cdot a \cdot c$

$\Delta = (-1)^2 - 4 \cdot 1 \cdot 0$

$\Delta = 1$

$x = \dfrac{-b \pm \sqrt{\Delta}}{2 \cdot a}$

$+ \rightarrow \dfrac{-(-1) + \sqrt{1}}{2 \cdot 1} = \dfrac{2}{2} = 1$

$- \rightarrow \dfrac{-(-1) - \sqrt{1}}{2 \cdot 1} = 0$

(Polos)

$z^2 - \frac{5}{8}z - \frac{3}{32} = 0$

$\Delta = \left(-\frac{5}{8}\right)^2 - 4 \cdot 1 \cdot \left(-\frac{3}{32}\right)$

$\Delta = \frac{25}{64} + \frac{12}{32} = \frac{49}{64}$

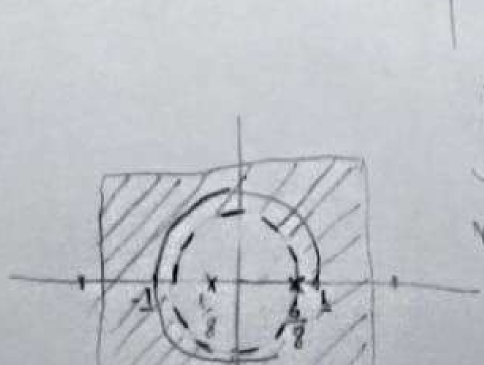
$x' \; + \rightarrow \dfrac{-\left(-\frac{5}{8}\right) + \sqrt{\frac{49}{64}}}{2 \cdot 1} = \dfrac{\frac{5}{8} + \frac{7}{8}}{2}$

$\frac{12}{8} \cdot \frac{1}{2} = \frac{6}{8}$

$- \rightarrow \dfrac{-\left(-\frac{5}{8}\right) - \sqrt{\frac{49}{64}}}{2 \cdot 1} = \dfrac{\frac{5}{8} - \frac{7}{8}}{2}$

$-\frac{2}{8} \cdot \frac{1}{2} = -\frac{1}{8}$

(ROC)



Para um sistema causal levamos em consideração o polo mais externo para determinar o ROC

2)

a) $X(z) = \dfrac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{4}z^{-2}}$, $|z| < \frac{1}{2}$

$$\frac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{4}z^{-2}} = \frac{\cancel{1 - \frac{1}{2}z^{-1}}}{\underbrace{\left(1 + \frac{1}{2}z^{-1}\right)\cancel{\left(1 - \frac{1}{2}z^{-1}\right)}}_{1 - \frac{1}{2}z^{-1} + \frac{1}{2}z^{-1} \ \frac{1}{4}z^{-2}}} = \frac{1}{1 + \left(\frac{1}{2}\right)z^{-1}}$$

$\hookrightarrow \alpha$ ou seja $|z| < |a|$

$$h[n] = -\alpha^n u[-n-1] = \underline{-\left(-\tfrac{1}{2}\right)^n u[-n-1] = Z^{-1}\{X(z)\}}$$

b) $X(z) = \dfrac{1 - \frac{1}{2}z^{-1}}{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$, $|z| > \frac{1}{2}$

$$\frac{1 - \frac{1}{2}z^{-1}}{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = \frac{1 - \frac{1}{2}z^{-1}}{\underbrace{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}_{1 + \frac{1}{4}z^{-1} + \frac{1}{2} + \frac{1}{8}z^{-2}}} = \frac{A}{1 + \frac{1}{2}z^{-1}} + \frac{B}{1 + \frac{1}{4}z^{-1}}$$

$$A = X(z) \cdot den(A)\Big|_{z=\frac{1}{2}} = \frac{1 - \frac{1}{2}z^{-1}}{\cancel{\left(1 + \frac{1}{2}z^{-1}\right)}\left(1 + \frac{1}{4}z^{-1}\right)} \cancel{\left(1 + \frac{1}{2}z^{-1}\right)} = \frac{1 - \frac{1}{2}z^{-1}}{1 + \frac{1}{4}z^{-1}}$$

$$A = \frac{\cancel{1 - \frac{1}{2} \cdot \frac{1}{2}^{-1}} \rightarrow 0}{1 + \frac{1}{4} \cdot \frac{1}{2}^{-1}} = \underline{0}$$

Continuação Letra b Questão 02

$$B = X(z)\cdot den(B)\Big|_{z=\frac{1}{4}} = \frac{1-\frac{1}{2}z^{-1}}{\left(1+\frac{1}{2}z^{-1}\right)\cancel{\left(1+\frac{1}{4}z^{-1}\right)}}\cancel{\left(1+\frac{1}{4}z^{-1}\right)} = \frac{1-\frac{1}{2}z^{-1}}{1+\frac{1}{2}z^{-1}}$$

$$B = \frac{1-\frac{1}{2}\cdot\frac{1}{4}^{-1}}{1+\frac{1}{2}\cdot\frac{1}{4}^{-1}} = \frac{1-\frac{4}{2}}{1+\frac{4}{2}} = \boxed{\frac{-1}{3}}$$

$$X(z) = -\frac{1}{3}\cdot\frac{1}{1+\frac{1}{4}z^{-1}}$$

α ou seja $|z| > |\alpha|$

$$\mathcal{Z}^{-1}\{X(z)\} = -\frac{1}{3}\,\alpha^n u[n] = \boxed{-\frac{1}{3}\cdot\left(\frac{1}{4}\right)^n u[n]}$$

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03)

a) $x[n] = u[-n-1] + \left(\frac{1}{4}\right)^n u[n]$

$$\boxed{\mathcal{Z}\{x[n]\} = -\frac{1}{1-z^{-1}} + \frac{1}{1-\frac{1}{4}z^{-1}}}$$

Questão 03 (Letra b): $1-\frac{1}{2}z^{-1}+z^{-1}-\frac{1}{2}z^{-2}$

Pólos $\begin{cases} -1 \\ +\frac{1}{2} \end{cases}$

$$Z\{y[n]\} = \frac{-\frac{1}{2}z^{-1}}{(1+z^{-1})(1-\frac{1}{2}z^{-1})} = \frac{A}{(1+z^{-1})} + \frac{B}{(1-\frac{1}{2}z^{-1})}$$

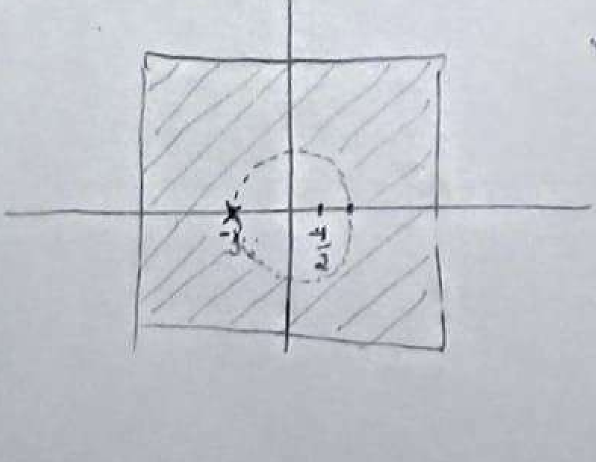
$$A = Z\{y[n]\}\cdot den(A)\Big|_{z=1} = \frac{-\frac{1}{2}(-1)^{-1}}{1-\frac{1}{2}(-1)^{-1}} = \frac{+\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$$

$$B = Z\{y[n]\}\cdot den(B)\Big|_{z=\frac{1}{2}} = \frac{-\frac{1}{2}\cdot\left(\frac{1}{2}\right)^{-1}}{1+\left(\frac{1}{2}\right)^{-1}} = -\frac{1}{3}$$

$$Z\{y[n]\} = \frac{1}{3}\cdot\frac{1}{1+z^{-1}} - \frac{1}{3}\cdot\frac{1}{1-\frac{1}{2}z^{-1}}$$

$$Z^{-1}\{Z\{y[n]\}\} = \frac{1}{3}U[n] - \frac{1}{3}\cdot\left(\frac{1}{2}\right)^{n}U[n]$$

(ROC)



não tem transformada de fourier