2° Lista de exercícios

04/08/21

DFT → Aulas 12 até 15
TZ → Aulas 16 até 20

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Disciplina: Processamento digital de sinais

1)

a)

DFT de $\underbrace{20 \text{ pontos}}$

em uma sequência de valores reais no domínio do tempo

Mínimo de amostras enviadas, para que se consiga
$X[m]$ analisar o espectro
complexas completamente?

$\frac{N}{2} = 10$ amostras

Resposta:

O número mínimo de amostras $X[m]$
deve ser igual (pelo critério da paridade de simetria) $= 0 \leqslant m \leqslant \frac{N}{2}$
para ser possível analisar o espectro completo
com o cálculo da DFT, magnitude, fase e as partes
reais e imaginárias de cada valor de m.

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b)

$f_s = 1000\ Hz$

Δf = espaçamento $= 45\ Hz$

$\Delta f = \frac{f_s}{N}$

$N = \frac{f_s}{\Delta f} = \frac{1000}{45} \simeq 22$ amostras

→ Porém pela paridade de simetria será necessário

$\frac{N}{2} = \underline{11 \text{ amostras}}$

2)

DFT com N pontos

1 segundo de duração do sinal

$f_s = 44,1 \text{ KHz}$

$\Delta f = 1 \text{ Hz}$

a) $N = \frac{f_s}{\Delta f} = \frac{44,1 \text{ KHz}}{1} = 44.100$ amostras

$\boxed{\frac{N}{2} = 22.050 \text{ amostras necessárias pela paridade de simetria}}$

b) Como foi dito o sinal do CD possui 1 segundo de duração total. Caso use apenas as necessárias a duração será 0,5 segundo

3) Letra a

$$X_1[0] = \sum_{n=0}^{N=7} x[n]\left[\cos\left(2\pi n \cdot \frac{m}{N}\right) - j\,\text{sen}\left(2\pi n \cdot \frac{m}{N}\right)\right]$$

$x[0]\left[\cos\left(2\pi 0 \cdot \frac{0}{8}\right) - j\,\text{sen}\left(2\pi 0 \cdot \frac{0}{8}\right)\right] + \;= 9$

$x[1][1-0]$ valores se repetem

...

$\boxed{X_1[0] = 72}$

$X_1[1] = x[0]\left[\cos\left(2\pi 0 \cdot \frac{1}{8}\right) - j\,\text{sen}\left(2\pi 0 \cdot \frac{1}{8}\right)\right] + \;= 9 + 0j$

$x[1]\left[\cos\left(2\pi 1 \cdot \frac{1}{8}\right) - j\,\text{sen}\left(2\pi 1 \cdot \frac{1}{8}\right)\right] + \;= 9,707 - 9,707j$

$x[2]\left[\cos\left(2\pi 2 \cdot \frac{1}{8}\right) - j\,\text{sen}\left(2\pi 2 \cdot \frac{1}{8}\right)\right] + \;= 0 + 9j$

$x[3]\left[\cos\left(2\pi 3 \cdot \frac{1}{8}\right) - j\,\text{sen}\left(2\pi 3 \cdot \frac{1}{8}\right)\right] + \;= -9,707 + 9,707j$

$x[4]\left[\cos\left(2\pi 4 \cdot \frac{1}{8}\right) - j\,\text{sen}\left(2\pi 4 \cdot \frac{1}{8}\right)\right] + \;= -9 + 0j$

$$X[1]\begin{cases} x[5][\cos(2\pi 5\cdot\frac{1}{8}) - sen(2\pi 5\cdot\frac{1}{8})] + = -9{,}707 + 9{,}707j \\ x[6][\cos(2\pi 6\cdot\frac{1}{8}) - sen(2\pi 6\cdot\frac{1}{8})] + = -9j \\ x[7][\cos(2\pi 7\cdot\frac{1}{8}) - sen(2\pi 7\cdot\frac{1}{8})] + = 9{,}707 + 9{,}707j \end{cases}$$

$$x_1[2]\begin{cases} x[0][\cos(2\pi 0\cdot\frac{2}{8}) - sen(2\pi 0\cdot\frac{2}{8}) + = 9 \\ x[1][\cos(2\pi 1\cdot\frac{2}{8}) - sen(2\pi 1\cdot\frac{2}{8})] + = 0 - 9j \\ x[2][\cos(2\pi 2\cdot\frac{2}{8}) - sen(2\pi 2\cdot\frac{2}{8})] + = -9 - 0j \\ x[3][\cos(2\pi 3\cdot\frac{2}{8}) - sen(2\pi 3\cdot\frac{2}{8})] + = 0 + 9j \\ x[4][\cos(2\pi 4\cdot\frac{2}{8}) - sen(2\pi 4\cdot\frac{2}{8})] + = 9 - 0j \\ x[5][\cos(2\pi 5\cdot\frac{2}{8}) - sen(2\pi 5\cdot\frac{2}{8})] + = 0 - 9j \\ x[6][\cos(2\pi 6\cdot\frac{2}{8}) - sen(2\pi 6\cdot\frac{2}{8}) + = -9 + 0 \\ x[7][\cos(2\pi 7\cdot\frac{2}{8}) - sen(2\pi 7\cdot\frac{2}{8}) + = 0 + 9j \end{cases} = 0$$

$$x_1[3]\begin{cases} x[0][\cos(2\pi 0\cdot\frac{3}{8}) - sen(2\pi 0\cdot\frac{3}{8}) + = 9 \\ x[1][\cos(2\pi 1\cdot\frac{3}{8}) - sen(2\pi 1\cdot\frac{3}{8}) + = -9{,}707 - 9{,}707j \\ x[2][\cos(2\pi 2\cdot\frac{3}{8}) - sen(2\pi 2\cdot\frac{3}{8}) + = 0 - 9j \\ x[3][\cos(2\pi 3\cdot\frac{3}{8}) - sen(2\pi 3\cdot\frac{3}{8}) + = 9{,}707 - 9{,}707j \\ x[4][\cos(2\pi 4\cdot\frac{3}{8}) - sen(2\pi 4\cdot\frac{3}{8}) + = -9 + 0j \\ x[5][\cos(2\pi 5\cdot\frac{3}{8}) - sen(2\pi 5\cdot\frac{3}{8}) + = 9{,}707 + 9{,}707j \\ x[6][\cos(2\pi 6\cdot\frac{3}{8}) - sen(2\pi 6\cdot\frac{3}{8}) + = 0 + 9j \\ x[7][\cos(2\pi 7\cdot\frac{3}{8}) - sen(2\pi 7\cdot\frac{3}{8}) + = -9{,}707 + 9{,}707j \end{cases} = 0$$

Pela regra da simetria todos os valores de $X[m]$ exceto $X[0]$ vão zerar pois $X_1[1] = X_1[7]$, $X_1[2] = X_1[6]$, $X_1[3] = X_1[5]$

Resultando $= X[m] = [72, 0, 0, 0, 0, 0, 0, 0]$

$$|X[0]| = \sqrt{72^2 + 0^2}$$
$$= 72$$

Magnitudes $= |X[m]| = [72, 0, 0, 0, 0, 0, 0, 0]$

03) (Letra B) $X_2[n] = [1, 0, 0, 0, 0, 0, 0, 0]$

$|X_2[m]| = [1, 1, 1, 1, 1, 1, 1, 1]$ ← magnitudes

descobrindo isso usando simetria e pelas amostras de $x_2[n]$ serem zero, exceto $x_2[0]$ o que resulta para todos os pontos de $x[m]$ o seguinte calculo

$$X_2[m] = \left\{ \overset{1}{\cancel{X[0]}}\left[\overset{1}{\cancel{\cos}}\left(2\pi 0 \cdot \frac{0}{8}\right) - \sin\left(\overset{0}{\cancel{2\pi 0 \cdot \frac{0}{8}}}\right)\right] \right. = 1 = |X_2[m]| = 1$$

(Letra C) Aplicando a relação de deslocamento para direita:

$$X_3[m] = e^{-j2\pi mK/N} X_2[m]$$

Como todos os valores são 1 de $X_2[m]$ da letra B temos

$X_3[0] = [\cos(2\pi 1 \cdot \frac{0}{8}) - sen(2\pi 1 \cdot \frac{0}{8})] = 1 + 0j$

$X_3[1] = [\cos(2\pi 1 \cdot \frac{1}{8}) - sen(2\pi 1 \cdot \frac{1}{8})] = 0{,}707 - 0{,}707j$

$X_3[2] = [\cos(2\pi 1 \cdot \frac{2}{8}) - sen(2\pi 1 \cdot \frac{2}{8})] = 0 + 1j$

$X_3[3] = [\cos(2\pi 1 \cdot \frac{3}{8}) - sen(2\pi 1 \cdot \frac{3}{8})] = -0{,}707 - 0{,}707j$

$X_3[4] = [\cos(2\pi 1 \cdot \frac{4}{8}) - sen(2\pi 1 \cdot \frac{4}{8})] = -1 + 0j$

$X_3[5]$ = usamos simetria $= X_3[3] = 0{,}707 - 0{,}707j$

$X_3[6] = [\cos(2\pi 1 \cdot \frac{6}{8}) - sen(2\pi 1 \cdot \frac{6}{8})] = 0 - 1j$

$X_3[7] = [\cos(2\pi 1 \cdot \frac{7}{8}) - sen(2\pi 1 \cdot \frac{7}{8})] = 0{,}707 - 0{,}707j$

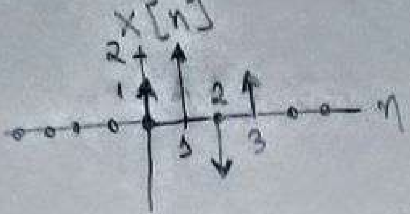
$n = K = 1$, $|X_3[m]| = [1, 1, 1, 1, 1, 1, 1, 1]$ ← magnitudes

$= \sqrt{0.707^2 + (-0.707)^2} = 1$

$= \sqrt{1 + 0} = 1$

$$4)\quad x[n] = \overset{n=0}{\delta[n]} + 2\overset{n=1}{\delta[n-1]} - \overset{n=2}{\delta[n-2]} + \overset{n=3}{\delta[n-3]}$$

$\boxed{\text{Letra a}}$



$$\text{Para } X[m=0] = \left\{\begin{aligned} & x[0][\cos(2\pi 0 \cdot \tfrac{0}{4}) - \text{sen}(2\pi 0 \cdot \tfrac{0}{4})] = 1 \\ & x[1][\cos(2\pi 1 \cdot \tfrac{0}{4}) - \text{sen}(2\pi 1 \cdot \tfrac{0}{4})] = 2 \\ & x[2][\cos(2\pi 2 \cdot \tfrac{0}{4}) - \text{sen}(2\pi 1 \cdot \tfrac{0}{4})] = -1 \\ & x[3][\cos(2\pi 3 \cdot \tfrac{0}{4}) - \text{sen}(2\pi 3 \cdot \tfrac{0}{4})] = 1 \end{aligned}\right\} = 3 - 0j$$

$$X[m=1] = \left\{\begin{aligned} & x[0][\cos(2\pi 0 \cdot \tfrac{1}{4}) - \text{sen}(2\pi 0 \cdot \tfrac{1}{4})] = 1 - 0j \\ & x[1][\cos(2\pi 1 \cdot \tfrac{1}{4}) - \text{sen}(2\pi 1 \cdot \tfrac{1}{4})] = 0 - 2j \\ & x[2][\cos(2\pi 2 \cdot \tfrac{1}{4}) - \text{sen}(2\pi 2 \cdot \tfrac{1}{4})] = +1 - 0j \\ & x[3][\cos(2\pi 3 \cdot \tfrac{1}{4}) - \text{sen}(2\pi 3 \cdot \tfrac{1}{4})] = 0 + 1j \end{aligned}\right\} 2 - 1j$$

$$X[m=2] = \left\{\begin{aligned} & x[0][\cos(2\pi 0 \cdot \tfrac{2}{4}) - \text{sen}(2\pi 0 \cdot \tfrac{2}{4})] = 1 \\ & x[1][\cos(2\pi 1 \cdot \tfrac{2}{4}) - \text{sen}(2\pi 1 \cdot \tfrac{2}{4})] = -2 - 0j \\ & x[2][\cos(2\pi 2 \cdot \tfrac{2}{4}) - \text{sen}(2\pi 2 \cdot \tfrac{2}{4})] = -1 - 0j \\ & x[3][\cos(2\pi 3 \cdot \tfrac{2}{4}) - \text{sen}(2\pi 3 \cdot \tfrac{2}{4})] = -1 - 0j \end{aligned}\right\} -3 - 0j$$

$X[m=3]$ = Por simetria = $X[m=1]$

$$\left\{\begin{aligned} & x[0][\cos(2\pi 0 \cdot \tfrac{3}{4}) - \text{sen}(2\pi 0 \cdot \tfrac{3}{4})] = 1 \\ & x[1][\cos(2\pi 1 \cdot \tfrac{3}{4}) - \text{sen}(2\pi 1 \cdot \tfrac{3}{4})] = 0 + 2j \\ & x[2][\cos(2\pi 2 \cdot \tfrac{3}{4}) - \text{sen}(2\pi 2 \cdot \tfrac{3}{4})] = +1 - 0j \\ & x[3][\cos(2\pi 3 \cdot \tfrac{3}{4}) - \text{sen}(2\pi 3\, \tfrac{3}{4})] = 0 - 1j \end{aligned}\right\} 2 + 1j$$

Letra b

$$x[n=0] = \frac{1}{6}\begin{cases} -1\left[\cos\left(\cancel{2\pi 1 \cdot \frac{0}{6}}\right) + sen\left(\cancel{2\pi 1 \cdot \frac{0}{6}}\right)\right] + \\ 1 \cdot \left[\cos\left(\cancel{2\pi 3 \cdot \frac{0}{6}}\right) + sen\left(\cancel{2\pi 3 \cdot \frac{0}{6}}\right)\right] \end{cases}$$

$$= \frac{1}{6}(-1+1)$$

$$= 0$$

$$x[n=1] = \frac{1}{6}\begin{cases} -1\left[\cos\left(2\pi 1 \cdot \frac{1}{6}\right) + sen\left(2\pi 1 \cdot \frac{1}{6}\right)\right] + = -0,5 - 0,866j \\ 1 \cdot \left[\cos\left(2\pi 3 \cdot \frac{1}{6}\right) + sen\left(2\pi 3 \cdot \frac{1}{6}\right)\right] + = -1 - 0j \end{cases} = -0,25 - 0,144j$$

$$x[n=2] = \frac{1}{6}\begin{cases} -1\left[\cos\left(2\pi 1 \cdot \frac{2}{6}\right) + sen\left(2\pi \frac{2}{6}\right)\right] + = +0,5 - 0,866j \\ 1\left[\cos\left(2\pi 3 \cdot \frac{2}{6}\right) + sen\left(2\pi 3 \cdot \frac{2}{6}\right)\right] + = 1 + 0j \end{cases} 0,25 - 0,144j$$

$$x[n=3] = \frac{1}{6}\begin{cases} -1\left[\cos\left(2\pi 1 \cdot \frac{3}{6}\right) + sen\left(2\pi \frac{3}{6}\right)\right] + = 1 - 0j \\ 1\left[\cos\left(2\pi 3 \cdot \frac{3}{6}\right) + sen\left(2\pi 3 \cdot \frac{3}{6}\right)\right] + = -1 + 0j \end{cases} 0 + 0j$$

$$x[n=4] = \frac{1}{6}\begin{cases} -1\left[\cos\left(2\pi 1 \cdot \frac{4}{6}\right) + sen\left(2\pi 1 \cdot \frac{4}{6}\right)\right] + = 0,5 + 0,866j \\ 1\left[\cos\left(2\pi 3 \cdot \frac{4}{6}\right) + sen\left(2\pi 3 \cdot \frac{4}{6}\right)\right] + = 1 + 0j \end{cases} 0,25 + 0,144j$$

$$x[n=5] = \frac{1}{6}\begin{cases} -1\left[\cos\left(2\pi 1 \cdot \frac{5}{6}\right) + sen\left(2\pi 1 \cdot \frac{5}{6}\right)\right] + = -0,5 + 0,866j \\ 1\left[\cos\left(2\pi 3 \cdot \frac{5}{6}\right) + sen\left(2\pi 3 \cdot \frac{5}{6}\right)\right] + = -1 + 0 \end{cases} -0,25 + 0,144j$$

$$x[n] = [0 \ -0,25 \ \ 0,25 \ \ 0 \ \ 0,25 \ -0,25]$$

$\boxed{\text{Letra C}}$ (04)

$$X[n=0] = \frac{1}{4}\left\{\begin{array}{l} -1[\cancelto{1}{\cos(2\pi 1 \cdot \frac{0}{4})} + \cancelto{0}{sen(2\pi 1 \cdot \frac{0}{4})}] + = -1 - 0j \\ 1[\cos(2\pi 3 \cdot \frac{0}{4}) + sen(2\pi 3 \cdot \frac{0}{4})] + = 1 + 0j \end{array}\right\} 0 + 0j$$

$$X[n=1] = \frac{1}{4}\left\{\begin{array}{l} -1[\cos(2\pi 1 \cdot \frac{1}{4}) + sen(2\pi 1 \cdot \frac{1}{4})] + = 0 - 1j \\ 1[\cos(2\pi 3 \cdot \frac{1}{4}) + sen(2\pi 3 \cdot \frac{1}{4})] + = 0 - 1j \end{array}\right\} 0 - 0{,}5j$$

$$X[n=2] = \frac{1}{4}\left\{\begin{array}{l} -1[\cos(2\pi 1 \cdot \frac{2}{4}) + sen(2\pi 1 \cdot \frac{2}{4})] + = +1 - 0j \\ 1[\cos(2\pi 3 \cdot \frac{2}{4}) + sen(2\pi 3 \cdot \frac{2}{4})] + = -1 + 0j \end{array}\right\} 0 + 0j$$

$$X[n=3] = \frac{1}{4}\left\{\begin{array}{l} -1[\cos(2\pi 1 \cdot \frac{3}{4}) + sen(2\pi 1 \cdot \frac{3}{4}) + = 0 + 1j \\ 1[\cos(2\pi 3 \cdot \frac{3}{4} + sen(2\pi 3 \cdot \frac{3}{4}) + = 0 + 1j \end{array}\right\} 0 + 0{,}5j$$

$$X[n] = [0\ 0\ 0\ 0]$$

(Questao 05)
$$X_1[m] = \sum_{n=0}^{2N-2} X_1[n]\left[\cos\left(2\pi n \frac{m}{2N-1}\right) - sen\left(2\pi n \frac{m}{2N-1}\right)\right]$$

$$X_2[m] = \sum_{n=0}^{2N-2} X_2[n]\left[\cos\left(2\pi n \frac{m}{2N-1}\right) - sen\left(2\pi n \frac{m}{2N-1}\right)\right]$$

$$X_3[K] = \frac{1}{2N-1}\left[\sum_{m=0}^{2N-2} [X_1[m] X_2[m]]\left[\cos\left(2\pi n \frac{K}{2N-1}\right) - sen\left(2\pi n \frac{K}{2N-1}\right)\right]\right]$$

Calculando DFT para o sinal X_1, terá $(2 \cdot 3) - 1 = 5$ amostras do 0 à 4

$$X_1[n=0] = \left\{\begin{array}{l} X_1[0][\cos(2\pi 0 \cdot \frac{0}{5}) - sen(2\pi 0 \cdot \frac{0}{5})] + = 1 - 0j \\ X_1[1][\cos(2\pi 1 \cdot \frac{0}{5}) - sen(2\pi 1 \cdot \frac{0}{5})] + = -1 - 0j \\ X_1[2][\cos(2\pi 2 \cdot \frac{0}{5}) - sen(2\pi 2 \cdot \frac{0}{5})] + = -0{,}5 - 0j \\ X_1[3][\cos(2\pi 3 \cdot \frac{0}{5}) - sen(2\pi 3 \cdot \frac{0}{5})] + = 0 \\ X_1[4][\cos(2\pi 4 \cdot \frac{0}{5}) - sen(2\pi 4 \cdot \frac{0}{5})] + = 0 \end{array}\right\} -0{,}5 - 0j$$

Esses dois últimos são sempre zero pois no sinal não tem valor para $X_1[3]$ e $X_1[4]$

$$X_1[m=1] = \begin{cases} x_1[0]\left[\cos\left(2\pi 0 \cdot \frac{1}{5}\right) - \text{sen}\left(2\pi 0 \cdot \frac{1}{5}\right)\right] + = 1 - 0j \\ x_1[1]\left[\cos\left(2\pi 1 \cdot \frac{1}{5}\right) - \text{sen}\left(2\pi 1 \cdot \frac{1}{5}\right)\right] + = -0,30 + 0,95j \\ x_1[2]\left[\cos\left(2\pi 2 \cdot \frac{1}{5}\right) - \text{sen}\left(2\pi 2 \cdot \frac{1}{5}\right)\right] + = +0,40 + 0,29j \end{cases} 1,1 + 1,24j$$

$$X_1[m=2] = \begin{cases} x_1[0]\left[\cos\left(2\pi 0 \cdot \frac{2}{5}\right) - \text{sen}\left(2\pi 0 \cdot \frac{2}{5}\right)\right] + = 1 - 0j \\ x_1[1]\left[\cos\left(2\pi 1 \cdot \frac{2}{5}\right) - \text{sen}\left(2\pi 1 \cdot \frac{2}{5}\right)\right] + = 0,80 + 0,58j \\ x_1[2]\left[\cos\left(2\pi 2 \cdot \frac{2}{5}\right) - \text{sen}\left(2\pi 2 \cdot \frac{2}{5}\right)\right] + = -0,15 - 0,47j \end{cases} 1,65 + 0,11j$$

$$X_1[m=3] = \begin{cases} x_1[0]\left[\cos\left(2\pi 0 \cdot \frac{3}{5}\right) - \text{sen}\left(2\pi 0 \cdot \frac{3}{5}\right)\right] + = 1 - 0j \\ x_1[1]\left[\cos\left(2\pi 1 \cdot \frac{3}{5}\right) - \text{sen}\left(2\pi 1 \cdot \frac{3}{5}\right)\right] + = +0,80 - 0,58j \\ x_1[2]\left[\cos\left(2\pi 2 \cdot \frac{3}{5}\right) - \text{sen}\left(2\pi 2 \cdot \frac{3}{5}\right)\right] + = -0,15 + 0,47j \end{cases} 1,65 - 0,11j$$

$$\boxed{x_1[0] = 1, x_1[1] = -1, x_1[2] = -0,5}$$

$$X_1[m=4] = \begin{cases} x_1[0]\left[\cos\left(2\pi 0 \cdot \frac{4}{5}\right) - \text{sen}\left(2\pi 0 \cdot \frac{4}{5}\right)\right] + = 1 - 0j \\ x_1[1]\left[\cos\left(2\pi 1 \cdot \frac{4}{5}\right) - \text{sen}\left(2\pi 1 \cdot \frac{4}{5}\right)\right] + = -0,30 - 0,95j \\ x_1[2]\left[\cos\left(2\pi 2 \cdot \frac{4}{5}\right) - \text{sen}\left(2\pi 2 \cdot \frac{4}{5}\right)\right] + = +0,40 - 0,29j \end{cases} 1,1 - 1,24j$$

Agora calculando DFT de $x_2[n]$: $\boxed{x_2[0] = 1, x_2[1] = -0,5, x_2[2] = -1}$

$$X_2[m=0] = \begin{cases} x_2[0]\left[\cos\left(2\pi 0 \cdot \frac{0}{5}\right) - \text{sen}\left(2\pi 0 \cdot \frac{0}{5}\right)\right] + = 1 - 0j \\ x_2[1]\left[\cos\left(2\pi 1 \cdot \frac{0}{5}\right) - \text{sen}\left(2\pi 1 \cdot \frac{0}{5}\right)\right] + = -1 - 0j \\ x_2[2]\left[\cos\left(2\pi 2 \cdot \frac{0}{5}\right) - \text{sen}\left(2\pi 2 \cdot \frac{0}{5}\right)\right] + = -0,5 - 0j \end{cases} -0,5 - 0j$$

$$X_2[m=1] = \begin{cases} x_2[0]\left[\cos\left(2\pi 0 \cdot \frac{1}{5}\right) - \text{sen}\left(2\pi 0 \cdot \frac{1}{5}\right)\right] + = 1 - 0j \\ x_2[1]\left[\cos\left(2\pi 1 \cdot \frac{1}{5}\right) - \text{sen}\left(2\pi 1 \cdot \frac{1}{5}\right)\right] + = -0,15 + 0,47j \\ x_2[2]\left[\cos\left(2\pi 2 \cdot \frac{1}{5}\right) - \text{sen}\left(2\pi 2 \cdot \frac{1}{5}\right)\right] + = 0,80 + 0,58j \end{cases} 1,65 + 1,05j$$

$$X_2[m=2] = \begin{cases} x_2[0]\left[\cos\left(2\pi 0 \cdot \frac{2}{5}\right) - \text{sen}\left(2\pi 0 \cdot \frac{2}{5}\right)\right] + = 1 - 0j \\ x_2[1]\left[\cos\left(2\pi 1 \cdot \frac{2}{5}\right) - \text{sen}\left(2\pi 1 \cdot \frac{2}{5}\right)\right] + = \\ x_2[2]\left[\cos\left(2\pi 2 \cdot \frac{2}{5}\right) - \text{sen}\left(2\pi 2 \cdot \frac{2}{5}\right)\right] + = \end{cases} 1,1 - 0,65$$

$$X_2[m=3] = \begin{cases} x_2[0][\cos(2\pi 0 \cdot \frac{3}{5}) - sen(2\pi 0 \cdot \frac{3}{5})] + = 1 \cdot 0j \\ x_2[1][\cos(2\pi 1 \cdot \frac{3}{5}) - sen(2\pi 1 \cdot \frac{3}{5})] + = \\ x_2[2][\cos(2\pi 2 \cdot \frac{3}{5}) - sen(2\pi 2 \cdot \frac{3}{5}) + = \end{cases} \Big\} 1,1 + 0,65$$

$$X_2[m=4] = \begin{cases} x_2[0][\cos(2\pi 0 \cdot \frac{4}{5}) - sen(2\pi 0 \cdot \frac{4}{5})] + = 1 \cdot 0j \\ x_2[1][\cos(2\pi 1 \cdot 4/5) - sen(2\pi 1 \cdot \frac{4}{5})] + = \\ x_2[2][\cos(2\pi 2 \cdot 4/5) - sen(2\pi 2 \cdot 4/5)] + = \end{cases} \Big\} 1,65 - 1,05$$

Agora calculando a convolução linear:

$$X_3[m=0] = \frac{1}{5} \begin{cases} [x_1[0]\, x_2[0]][\cos(2\pi 0 \cdot \frac{0}{5}) - sen(2\pi 0 \cdot \frac{0}{5})] + = 0,25 \\ [x_1[1]\, x_2[1]][\cos(2\pi 1 \cdot \frac{0}{5}) - sen(2\pi 1 \cdot \frac{0}{5})] + = \\ [x_1[2]\, x_2[2]][\cos(2\pi 2 \cdot \frac{0}{5}) - sen(2\pi 2 \cdot \frac{0}{5})] + = \\ [x_1[3]\, x_2[3]][\cos(2\pi 3 \cdot 0/5) - sen(2\pi 3 \cdot 0/5)] + = \\ [x_1[4]\, x_2[4]][\cos(2\pi 4 \cdot 0/5) - sen(2\pi 4 \cdot 0/5)] + = \end{cases} \Big\} 1$$

$X_3[m=0] = 1$

$$X_3[m=1] = \frac{1}{5} \begin{cases} [x_1[0]\, x_2[0]][\cos(2\pi 0 \cdot \frac{1}{5}) - sen(2\pi 0 \cdot \frac{1}{5})] + = \\ [x_1[1]\, x_2[1]][\cos(2\pi 1 \cdot \frac{1}{5}) - sen(2\pi 1 \cdot \frac{1}{5})] + = \\ [x_1[2]\, x_2[2]][\cos(2\pi 2 \cdot \frac{1}{5}) - sen(2\pi 2 \cdot \frac{1}{5})] + = \\ [x_1[3]\, x_2[3]][\cos(2\pi 3 \cdot \frac{1}{5}) - sen(2\pi 3 \cdot \frac{1}{5})] + = \\ [x_1[4]\, x_2[4]][\cos(2\pi 4 \cdot \frac{1}{5}) - sen(2\pi 4 \cdot \frac{1}{5})] + = \end{cases} \Big\} -1,5$$

$X_3[m=1] = -1,5$

$$X_3[m=2] = \frac{1}{5} \begin{cases} [x_1[0]\, x_2[0]][\cos(2\pi 0 \cdot \frac{2}{5}) - sen(2\pi 0 \cdot \frac{2}{5})] + = \\ [x_1[1]\, x_2[1]][\cos(2\pi 1 \cdot \frac{2}{5}) - sen(2\pi 1 \cdot \frac{2}{5})] + = \\ [x_1[2]\, x_2[2]][\cos(2\pi 2 \cdot \frac{2}{5}) - sen(2\pi 2 \cdot \frac{2}{5})] + = \\ [x_1[3]\, x_2[3]][\cos(2\pi 3 \cdot \frac{2}{5}) - sen(2\pi 3 \cdot \frac{2}{5})] + = \\ [x_1[4]\, x_2[4]][\cos(2\pi 4 \cdot \frac{2}{5}) - sen(2\pi 4 \cdot \frac{2}{5})] + = \end{cases} \Big\} -1$$

$X_3[m=2] = -1$

$$X_3[m=3] = \frac{1}{5}\left\{\begin{array}{l}[x_1[0]X_2[0]][\cos(2\pi 0\cdot\frac{3}{5}) - \text{sen}(2\pi 0\cdot\frac{3}{5})] + = \\ [x_1[1]X_2[1]][\cos(2\pi 1\cdot\frac{3}{5}) - \text{sen}(2\pi 1\cdot\frac{3}{5})] + = \\ [x_1[2]X_2[2]][\cos(2\pi 2\cdot\frac{3}{5}) - \text{sen}(2\pi 2\cdot\frac{3}{5}) + = \\ [x_1[3]X_2[3]][\cos(2\pi 3\cdot\frac{3}{5}) - \text{sen}(2\pi 3\cdot\frac{3}{5}) + = \\ [x_1[4]X_2[4]][\cos(2\pi 3\cdot\frac{4}{5}) - \text{sen}(2\pi 3\cdot\frac{4}{5})\end{array}\right\} 1{,}25$$

$$X_3[m=3] = 1{,}25$$

$$X_3[m=4] = \frac{1}{5}\left\{\begin{array}{l}[x_1[0]X_2[0]][\cos(2\pi 0\cdot\frac{4}{5}) - \text{sen}(2\pi 0\cdot\frac{4}{5})] + = \\ [x_1[1]X_2[1]][\cos(2\pi 1\cdot 4/5) - \text{sen}(2\pi 1\cdot 4/5)] + = \\ [x_1[2]X_2[2]][\cos(2\pi 2\cdot 4/5) - \text{sen}(2\pi 2\cdot 4/5)] + = \\ [x_1[3]X_2[3]][\cos(2\pi 3\cdot 4/5) - \text{sen}(2\pi 3\cdot 4/5)] + = \\ [x_1[4]X_2[4]][\cos(2\pi 4\cdot 4/5) - \text{sen}(2\pi 4\cdot 4/5)] + =\end{array}\right\} 0{,}5$$

$$x_1[0]X_2[0] = (0{,}5 - 0j)\cdot(-0{,}5 - 0j) = 0{,}25 - 0j$$

$$x_1[1]X_2[1] = (1{,}1 + 1{,}24j)\cdot(1{,}65 + 1{,}05j) = 0{,}48 + 3{,}22j$$

$$x_1[2]X_2[2] = (1{,}05 + 0{,}11j)\cdot(1{,}1 - 0{,}83j = 1{,}86 - 0{,}86j$$

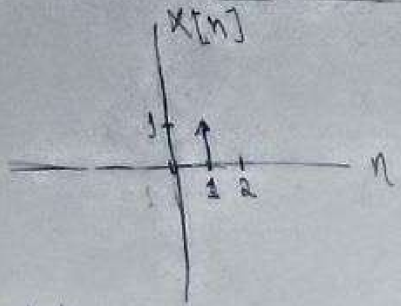
$$x_1[3]X_2[3] = (1{,}65 - 0{,}11j)\cdot(1{,}1 - 0{,}85j = 1{,}88 - 0{,}06j$$

$$x_1[4]X_2[4] = (1{,}1 - 1{,}24j)\cdot(1{,}65 - 1{,}05) = 0{,}48 - 3{,}22j$$

$$X_3[m] = [1\ \ -1{,}5\ \ -1\ \ 1{,}25\ \ 0{,}5]$$

" " "

(Questão 06) $\boxed{\text{Letra A}}$



$$X[m] = \sum_{n=0}^{N-1} x[n]\left[\cos\left(2\pi n \frac{m}{N}\right) - sen\left(2\pi n \frac{m}{N}\right)\right]$$

$$X[m=0] = \begin{cases} x[0]\left[\cos\left(2\pi 0 \frac{0}{3}\right) - sen\left(2\pi 0 \frac{0}{3}\right)\right] + = 0 \\ x[1]\left[\cos\left(2\pi 1 \cdot \frac{0}{3}\right) - sen\left(2\pi 1 \cdot \frac{0}{3}\right)\right] + = 1 - 0j \\ x[2]\left[\cos\left(2\pi 2 \cdot \frac{0}{3}\right) - sen\left(2\pi 2 \cdot \frac{0}{3}\right)\right] + = 0 \end{cases}$$

$$X[m=1] = \begin{cases} x[0]\left[\cos\left(2\pi 0 \frac{1}{3}\right) - sen\left(2\pi 0 \frac{1}{3}\right)\right] + = 0 \\ x[1]\left[\cos\left(2\pi 1 \cdot \frac{1}{3}\right) - sen\left(2\pi 1 \cdot \frac{1}{3}\right)\right] + = -0{,}5 - 0{,}86j \\ x[2]\left[\cos\left(2\pi 2 \cdot \frac{1}{3}\right) - sen\left(2\pi 2 \cdot \frac{1}{3}\right)\right] + = 0 \end{cases}$$

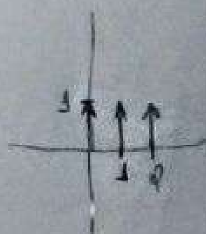
$$X[m=2] = \begin{cases} x[0]\left[\cos\left(2\pi 0 \cdot \frac{2}{3}\right) - sen\left(2\pi 0 \frac{2}{3}\right)\right] + = 0 \\ x[1]\left[\cos\left(2\pi 1 \cdot \frac{2}{3}\right) - sen\left(2\pi 1 \cdot \frac{2}{3}\right)\right] + = -0{,}5 + 0{,}86j \\ x[2]\left[\cos\left(2\pi 2 \cdot \frac{2}{3}\right) - sen\left(2\pi 2 \cdot \frac{2}{3}\right)\right] + = 0 \end{cases}$$

magnitudes:

$X[m=0] = \sqrt{1^2 + 0^2} = \underline{1}$

$X[m=1] = \sqrt{(-0{,}5)^2 + (-0{,}86)^2} = \sqrt{0{,}25 + 0{,}74} = 0{,}99 \simeq \underline{1}$

$X[m=2] \simeq \underline{1}$

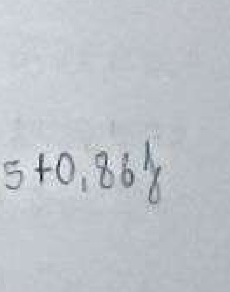


fases:

$X[m=0] = \arctan\left(\frac{0}{1}\right) = 0°$

$X[m=1] = \arctan\left(\frac{-0{,}86}{-0{,}5}\right) = 59{,}82 \simeq 60°$

$X[m=2] = \arctan\left(\frac{0{,}86}{-0{,}5}\right) \simeq -60°$



(Questão 06)(Letra B)

$x[n] = n$, para $0 \leq n \leq 3$

$$X[m=0] = \begin{cases} x[0]\left[\cos\left(2\pi 0 \cdot \frac{0}{4}\right) - sen\left(2\pi 0 \cdot \frac{0}{4}\right)\right] + = 0 \\ x[1]\left[\cos\left(2\pi 1 \cdot \frac{0}{4}\right) - sen\left(2\pi 1 \cdot \frac{0}{4}\right)\right] + = 1 \\ x[2]\left[\cos\left(2\pi 2 \cdot \frac{0}{4}\right) - sen\left(2\pi 2 \cdot \frac{0}{4}\right)\right] + = 2 \\ x[3]\left[\cos\left(2\pi 3 \cdot \frac{0}{4}\right) - sen\left(2\pi 3 \cdot \frac{0}{4}\right)\right] + = 3 \end{cases} \quad 6 + 0j$$

$$X[m=1] = \begin{cases} x[0]\left[\cos\left(2\pi 0 \cdot \frac{1}{4}\right) - sen\left(2\pi 0 \cdot \frac{1}{4}\right)\right] + = 0 + 0j \\ x[1]\left[\cos\left(2\pi 1 \cdot \frac{1}{4}\right) - sen\left(2\pi 1 \cdot \frac{1}{4}\right)\right] + = 0 - 1j \\ x[2]\left[\cos\left(2\pi 2 \cdot \frac{1}{4}\right) - sen\left(2\pi 2 \cdot \frac{1}{4}\right)\right] + = -2 + 0j \\ x[3]\left[\cos\left(2\pi 3 \cdot \frac{1}{4}\right) - sen\left(2\pi 3 \cdot \frac{1}{4}\right)\right] + = 0 + 3j \end{cases} \quad -2 + 2j$$

$$X[m=2] = \begin{cases} x[0]\left[\cos\left(2\pi 0 \cdot \frac{2}{4}\right) - sen\left(2\pi 0 \cdot \frac{2}{4}\right)\right] + = 0 \\ x[1]\left[\cos\left(2\pi 1 \cdot \frac{2}{4}\right) - sen\left(2\pi 1 \cdot \frac{2}{4}\right)\right] + = -1 + 0j \\ x[2]\left[\cos\left(2\pi 2 \cdot \frac{2}{4}\right) - sen\left(2\pi 2 \cdot \frac{2}{4}\right)\right] + = 2 + 0j \\ x[3]\left[\cos\left(2\pi 3 \cdot \frac{2}{4}\right) - sen\left(2\pi 3 \cdot \frac{2}{4}\right)\right] + = -3 + 0j \end{cases} \quad -2 + 0j$$

$$X[m=3] = \begin{cases} x[0]\left[\cos\left(2\pi \cdot 0 \cdot \frac{3}{4}\right) - sen\left(2\pi \cdot 0 \cdot \frac{3}{4}\right)\right] + = 0 \\ x[1]\left[\cos\left(2\pi \cdot 1 \cdot \frac{3}{4}\right) - sen\left(2\pi \cdot 1 \cdot \frac{3}{4}\right)\right] + = 0 + 1j \\ x[2]\left[\cos\left(2\pi 2 \cdot \frac{3}{4}\right) - sen\left(2\pi 2 \cdot \frac{3}{4}\right)\right] + = -2 + 0j \\ x[3]\left[\cos\left(2\pi 3 \cdot \frac{3}{4}\right) - sen\left(2\pi 3 \cdot \frac{3}{4}\right)\right] + = 0 - 3j \end{cases} \quad -2 - 2j$$

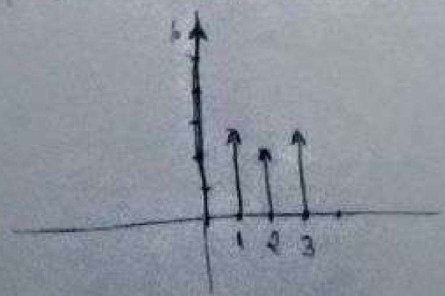
Magnitudes

$|X[m=0]| = \sqrt{6^2 + 0^2} = 6$

$|X[m=1]| = \sqrt{(-2)^2 + (2)^2} = \sqrt{8} = 2{,}82$

$|X[m=2]| = \sqrt{(-2)^2} = 2$

$|X[m=3]| = \sqrt{(-2)^2 + (2)^2} = \sqrt{8} = 2{,}82$



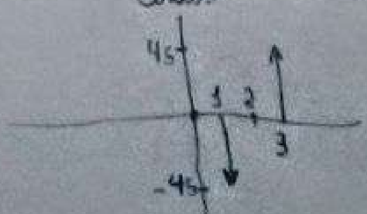
Fase

$X[m=0] = arctan\left(\frac{0}{6}\right) = 0$

$X[m=1] = arctan\left(\frac{2}{-2}\right) = -45$

$X[m=2] = arctan\left(\frac{-2}{0}\right) = 0$

$X[m=3] = arctan\left(\frac{2}{2}\right) = 45$



(Questão 07)

$x_1[n] = [1, 2.2, -4, 17, 21]$ $\qquad$ $x_2[n] = [1, 2.2, -4, 17, Q]$

$X_1[m] = [37.2, \quad]$ $\qquad$ $X_1[m=0] = 52.7$

$\uparrow$

$X_1[m=0]$

O primeiro valor da DFT corresponde a soma da parte real pois o sin(0) é zero, e como cos(0) = 1, basta soma todos os valores de n para achar o valor da DFT em m = 0.

$$X[m=0] = \begin{cases} X[0]\left[\cos\left(\cancel{2\pi 0 \cdot \tfrac{0}{5}}\right)^{1} - \cancel{sen\left(2\pi 0 \cdot \tfrac{0}{5}\right)}^{0}\right] \\ X[1]\left[\cos\left(\cancel{2\pi 1 \cdot \tfrac{0}{5}}\right) - \cancel{sen\left(2\pi 1 \cdot \tfrac{0}{5}\right)}\right] \\ X[2]\left[\cos\left(\cancel{2\pi 2 \cdot \tfrac{0}{5}}\right) - \cancel{sen\left(2\pi 2 \cdot \tfrac{0}{5}\right)}\right] \\ X[3]\left[\cos\left(\cancel{2\pi 3 \cdot \tfrac{0}{5}}\right) - \cancel{sen\left(2\pi 3 \cdot \tfrac{0}{5}\right)}\right] \\ X[4]\left[\cos\left(\cancel{2\pi 4 \cdot \tfrac{0}{5}}\right) - \cancel{sen\left(2\pi 4 \cdot \tfrac{0}{5}\right)}\right] \end{cases}$$

$$X[m=0] = X[0] + X[1] + X[2] + X[3] + X[4]$$

Achando o valor de Q a partir do $X_2[m=0]$

$$X_2[m=0] = 52,7 = 1 + 2,2 - 4 + 17 + Q$$

$$52,7 = 16,2 + Q$$

$$Q = 52,7 - 16,2$$

$$\boxed{Q = 36,5}$$ //

(Questão 08)

$X[0] = 3.1$

$X[2] = 2.5+4.6j$ → simetria com $X[7]$

$X[4] = -1.7+5.2j$ → simetria com $X[5]$

$X[6] = 9.3+6.3j$ → simetria com $X[3]$

$X[8] = 5.5-8.0j$ → simetria com $X[1]$

$X[m] = {}^*X[N-m]$ apenas o valor da parte imaginária inverte

$X[1] = X[9-1] = X[8]$

$X[2] = X[9-2] = X[7]$

$X[3] = X[9-3] = X[6]$

$X[4] = X[9-4] = X[5]$

$X[5] = X[9-5] = X[4]$

$X[1] = 5.5+8.0j$

$X[3] = 9.3-6.3j$

$X[5] = -1.7-5.2j$

$X[7] = 2.5-4.6j$