Listas - Trabalhos 26/05/21

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Disciplina: Processamento digital de Sinais
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1)

a) $y[n] = \cos(x[n])$

Linear:

$x_1[n] \longrightarrow y_1[n] = \cos(x_1[n])$

$x_2[n] \longrightarrow y_2[n] = \cos(x_2[n])$

$x_3[n] \longrightarrow y_3[n] = \cos(x_3[n])$

$x_3[n] = a\cos(x_1[n]) + b\cos(x_2[n])$

$y_3[n] = \cos(a\cos(x_1[n]) + b\cos(x_2[n]))$

é linear ✓

Inv. no tempo:

$x_1[n] \longrightarrow y_1[n] = \cos(x_1[n])$

$x_2 = x_1[n-a]$

$x_2[n] \longrightarrow y_2[n] = \cos(x_2[n]) = \cos(x_1[n-a])$

$y_1[n-a] = \cos(x_1[n-a])$

é invariante no tempo ✓

b) $y[n] = 2n^2 x[n] + n x[n+1]$

Linear:

$x_1[n] \rightarrow 2n^2 x_1[n] + n x_1[n+1]$

$x_2[n] \rightarrow 2n^2 x_2[n] + n x_2[n+1]$

$x_3[n] \rightarrow 2n^2 x_3[n] + n x_3[n+1]$

$x_3[n] = a x_1[n] + b x_2[n]$

$x_3[n] \rightarrow 2n^2 (a \cdot x_1[n] + b \cdot x_2[n]) + n (a \cdot x_1[n+1] + b x_2[n+1])$

$2n^2 (a x_1[n] + b x_2[n]) + n^2 (a \cdot x_1[n+1] + b x_2[n+1])$

$x_3[n] \rightarrow 2n^2 x_3[n] + n^2 x_3[n+1] \longrightarrow y_3[n] = a y_1[n] + b y_2[n]$

é linear

Inv. no tempo:

$x_1[n] \rightarrow 2n^2 x_1[n] + n x_1[n+1]$

$x_2[n] \rightarrow x_1[n-a]$

$x_2[n] \rightarrow 2n^2 x_2[n] + n x_2[n+1] = 2n^2 x_1[n-a] + n x_1[n+1-a]$

$y_1[n-a] = 2n^2 x_1[n-a] + n x_1[n+1-a]$

é invariante no tempo

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c) $y[n] = \begin{cases} x[n], & n \text{ é par} \\ x[n-1], & n \text{ é ímpar} \end{cases}$

$x_1[n] \rightarrow x_1[n]$

$x_2[n] \rightarrow x_2[n]$

$x_3[n] \rightarrow x_3[n]$

$x_3[n] = a x_1[n] + b x_2[n]$

$x_3[n] \rightarrow a x_1[n] + b x_2[n] \rightarrow y_3[n] = a \cdot y_1[n] + b \cdot y_2[n]$

} Primeira parte é linear

$x_1[n] \rightarrow x_1[n-1]$

$x_2[n] \rightarrow x_2[n-1]$

$x_3[n] \rightarrow x_3[n-1]$

$x_3[n] \rightarrow a \cdot x_1[n-1] + b \cdot x_2[n-1]$

$y_3[n] = a \cdot y_1[n-1] + b \cdot y_2[n-1]$

} segunda parte é linear

Inv no tempo:

$x_1[n] \longrightarrow x_1[n]$

$x_2[n] \longrightarrow x_1[n-a]$

$x_2[n] \longrightarrow x_2[n] = x_1[n-a]$ é invariante no tempo

$y_1[n-a] = x_1[n-a]$

d) $y[n] = x[n] + 2x[n-1] - 3x[n-2]$ $\quad x_3[n] \longrightarrow ax_1[n] + bx_2[n]$

Linear: $x_1[n] \longrightarrow x_1[n] + 2x_1[n-1] - 3x_1[n-2]$

$x_2[n] \longrightarrow x_2[n] + 2x_2[n-1] - 3x_2[n-2]$

$x_3[n] \longrightarrow x_3[n] + 2x_3[n-1] - 3x_3[n-2]$

$x_3[n] = ax_1[n] + bx_2[n] + 2(ax_1[n-1] + bx_2[n-1]) - 3(ax_1[n-2] + bx_2[n-2])$

$y_3[n] = a\,y_1[n] + b\,y_2[n]$

é linear

Inv no tempo:

$x_1[n] \longrightarrow x_1[n] + 2x_1[n-1] - 3x_1[n-2]$

$x_2[n] \longrightarrow x_1[n-a] + 2x_1[n-1-a] - 3x_1[n-2-a]$

$x_2[n] \longrightarrow x_2[n] + 2x_2[n-1] - 3x_2[n-2] =$

$y_1[n-a] = x_1[n-a] + 2x_1[n-1-a] - 3x_1[n-2-a]$

é invariante no tempo

e) $y[n] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k x[n-k]$

$x_1[n] \longrightarrow \sum_{0}^{\infty} \left(\frac{1}{2}\right)^k x_1[n-k]$

$x_2[n] \longrightarrow \sum_{0}^{\infty} \left(\frac{1}{2}\right)^k x_2[n-k]$

$x_3[n] \longrightarrow \sum_{0}^{\infty} \left(\frac{1}{2}\right)^k x_3[n-k] = \sum_{0}^{\infty} \left(\frac{1}{2}\right)^k \left(ax_1[n-k] + bx_2[n-k]\right)$

$y_3[n] = a\,y_1[n] + b\,y_2[n]$

é linear

Inv no tempo

$x_1[n] \rightarrow \sum_0^\infty \left(\frac{1}{2}\right)^k x_1[n-k]$

$x_2[n] \rightarrow x_1[n-a] = \sum_0^\infty \left(\frac{1}{2}\right)^k x_1[n-k-a]$

$y_1[n-a] = \sum_0^\infty \left(\frac{1}{2}\right)^k x_1[n-k-a]$

é invariante no tempo

f) $y[n] = x[2n]$

$x_1[n] \rightarrow x_1[2n]$

$x_2[n] \rightarrow x_2[2n]$

$x_3[n] \rightarrow x_3[2n] = a x_1[2n] + b x_2[2n]$

$y_3 = a y_1[2n] + b y_2[2n]$

é linear

$x_1[n] \rightarrow x_1[2n]$

$x_2[n] \rightarrow x_2[2n] = x_1[2n-a]$

$y_1[n-a] = x_1[2n-a]$

é inv. no tempo

g) $y[n] = 0,5x[2n] + 0,5x[2n-1]$

Linear:

$x_1[n] \rightarrow 0,5x_1[2n] + 0,5x_1[2n-1]$

$x_2[n] \rightarrow 0,5x_2[2n] + 0,5x_2[2n-1]$

$x_3[n] \rightarrow 0,5x_3[2n] + 0,5x_3[2n-1]$

$= 0,5(ax_1[2n] + bx_2[2n]) +$

$0,5(ax_1[2n-1] + bx_2[2n-1])$

$y_3[n] = a y_1[n] + b y_2[n]$

é linear

Inv no tempo:

$x_1[n] \rightarrow 0,5x_1[2n] + 0,5x_1[2n-1]$

$x_2[n] \rightarrow 0,5x_2[2n] + 0,5x_2[2n-1]$

$= x_1[n-a]$

$= 0,5x_1[2n-a] + 0,5x_1[2n-1-a]$

$y_1[n-a] = 0,5x_1[2n-a] + 0,5x_1[2n-1-a]$

é invariante no tempo

h) $y[n] = \begin{cases} x[n], n \text{ par} \\ -x[n], n \text{ ímpar} \end{cases}$

Linear:

$x_1[n] \rightarrow x_1[n]$

$x_2[n] \rightarrow x_2[n]$

$x_3[n] \rightarrow x_3[n] = a x_1[n] + b x_2[n]$

$y_3[n] = a y_1[n] + b y_2[n]$

é linear na primeira parte

$x_1[n] \rightarrow -x_1[n]$

$x_2[n] \rightarrow -x_2[n]$

$x_3[n] \rightarrow -x_3[n] = -(a x_1[n] + b x_2[n])$

$y_3[n] = -(a y_1[n] + b y_2[n])$

é linear na segunda parte

Inv. no tempo:

$x_1[n] \rightarrow x_1[n]$

$x_2[n] \rightarrow x_1[n-a] = x_2[n]$

$y_1[n-a] = x_1[n-a]$

é invariante no tempo na primeira parte.

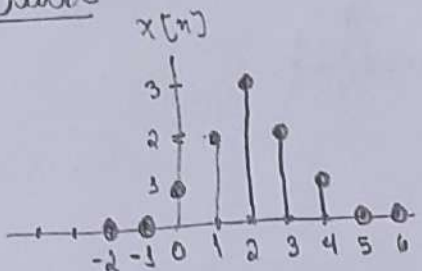
$x_1[n] \rightarrow -x_1[n]$

$x_2[n] \rightarrow -x_2[n] = -x_1[n-a]$

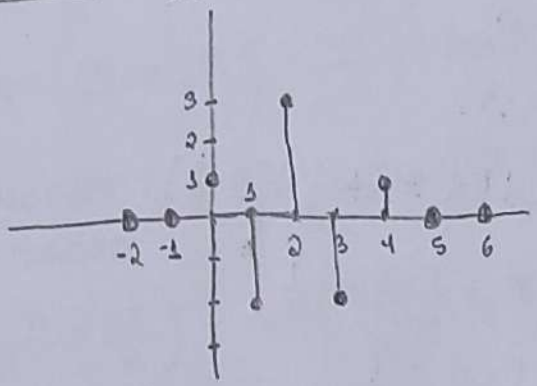
$y_1[n-a] = -x_1[n-a]$

é invariante no tempo na segunda parte

entrada



saída $y[n]$



i) $y[n] = (-1)^n x[n] + 2x[n-1]$

Linear:

$x_1[n] \rightarrow (-1)^n x_1[n] + 2x_1[n-1]$

$x_2[n] \rightarrow (-1)^n x_2[n] + 2x_2[n-1]$

$x_3[n] \rightarrow (-1)^n x_3[n] + 2x_3[n-1]$

$= (-1)^n (a x_1[n] + b x_2[n]) +$

$2(a x_1[n-1] + b x_2[n-1])$

$y_3[n] = a y_1[n] + b y_2[n]$

é linear

Inv. no tempo:

$x_1[n] \rightarrow (-1)^n x_1[n] + 2x_1[n-1]$

$x_2[n] \rightarrow x_1[n-a] = (-1)^n x_1[n-a] + 2x_1[n-a]$

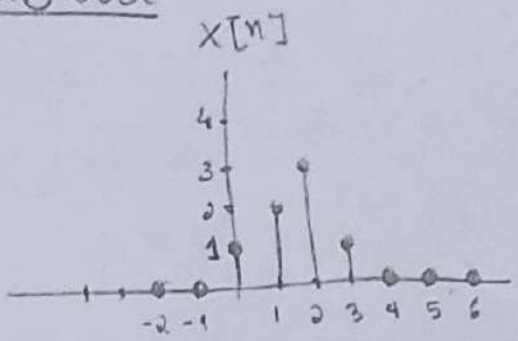
$y_1[n-1] = (-1)^n x_1[n-a] + 2x_1[n-a]$

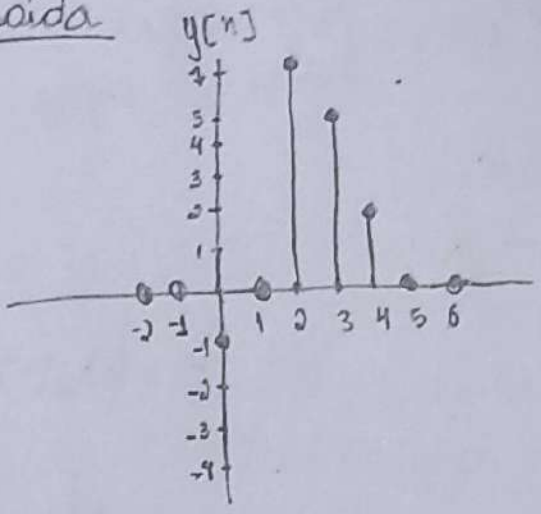
é inv. no tempo

entrada

$y[n] = (-1)^n x[n] + 2x[n-1]$

saída





$y[-2] = (-1)^{-2} \cdot 0 + 2 \cdot 0 = 0$

$y[-1] = (-1)^{-1} \cdot 0 + 2 \cdot 0 = 0$

$y[0] = (-1)^0 \cdot 1 + 2 \cdot 0 = -1$

$y[1] = (-1)^1 \cdot 2 + 2 \cdot 1 = 0$

$y[2] = (-1)^2 \cdot 3 + 2 \cdot 2 = 7$

$y[3] = (-1)^3 \cdot 1 + 2 \cdot 3 = 5$

$y[4] = (-1)^4 \cdot 0 + 2 \cdot 1 = 2$

$y[5] = (-1)^5 \cdot 0 + 2 \cdot 0 = 0$

$y[6] = 0$

Resposta ao impulso:

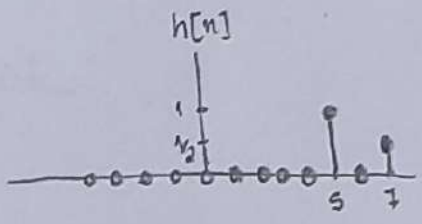
$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

a) $y[n] = x[n-5] + \frac{1}{2} x[n-7]$

$x[n] = \delta[n]$

$y[n] = h[n]$

$h[n] = \delta[n-5] + \frac{1}{2}\delta[n-7]$



$h[7] = 0 + \frac{1}{2} \cdot 1 = \frac{1}{2}$

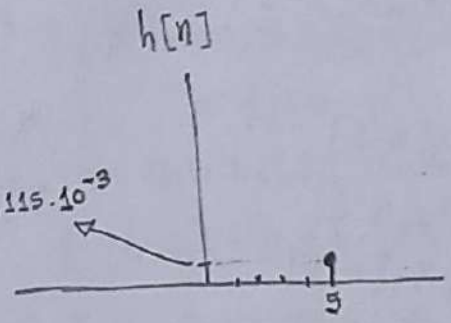
$h[5] = 1 + 0 = 1$

b) $y[n] = \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k x[n-k]$

$x[n] = \delta[n]$

$y[n] = h[n]$

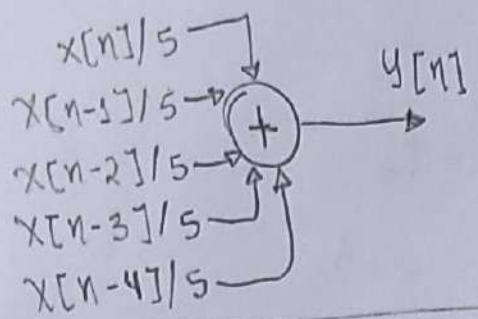
$h[n] = \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k \delta[n-k]$



$h[5] = \left(\frac{1}{3}\right)^5 \cdot 1 = 4{,}115 \cdot 10^{-3}$

3)

a) $y[n] = \frac{x[n]}{5} + \frac{x[n-1]}{5} + \frac{x[n-2]}{5} + \frac{x[n-3]}{5} + \frac{x[n-4]}{5} = \sum_{k=n-4}^{n} \frac{x[k]}{5}$



4) $x(t) = \text{sen}(14000\pi t + \frac{3\pi}{2}) + \text{sen}(2\pi 12000 t + \frac{\pi}{2})$

Amostra coletada a cada 20 μs → $20 \cdot 10^{-6}$ s → 0,00002

$x(t) = -\cos(14000\pi t) + \cos(24000\pi t)$ ← Aplicando identidades trigonométricas

$f_s = \frac{1}{0,00002} = 50\text{KHz}$

$x[n] = x(nt_s)$

$= \text{sen}(14000\pi \frac{n}{f_s} + \frac{3\pi}{2}) + \text{sen}(2\pi 12000 \frac{n}{f_s} + \frac{\pi}{2})$

$= \text{sen}(\frac{14000\pi n}{50000} + \frac{3\pi}{2}) + \text{sen}(\frac{2\pi 12000 n}{50000} + \frac{\pi}{2})$

$x[n] = \text{sen}(\frac{14\pi n}{50} + \frac{3\pi}{2}) + \text{sen}(\frac{12\pi n}{25} + \frac{\pi}{2})$

Seno 1:

$f_0 = \frac{2\pi}{14000\pi} = \frac{1}{7000} = 7000\text{Hz}$

Seno 2:

$f = \frac{2\pi}{24000\pi} = 12000\text{Hz}$

Aplicando MMC

84000 Hz = f_0 total

Como $f_s < 2 \cdot f_0$ irá ocorrer aliasing

$50\text{KHz} < 168\text{KHz}$

5) $a(t) = \cos(4000\pi t)$
$b(t) = \cos(200\pi t)$
$\Rightarrow x(t) = a(t) \cdot b(t)$

Calculando o período da função:

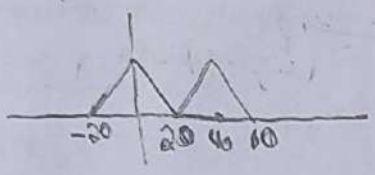
$a(t) = \frac{2\pi}{4000\pi} = \frac{1}{2000}$

$b(t) = \frac{2\pi}{200\pi} = \frac{1}{100}$

$\frac{\frac{1}{2000}}{\frac{1}{100}} = \frac{1}{20} = t_0$

$f_0 = 20\ Hz$

A frequência de amostragem deve ser maior que duas vezes a máxima frequência do sinal amostrado.



$f_s = 2f_0$ no mínimo

$f_s = 40\ Hz$

6)

a) $x(t) = \text{sen}(2\pi 250t)$
$x[n] = \text{sen}(4\pi n)$

Achar frequência de amostragem

$$x[n] = x(nt_s)$$

$$t_s = \frac{1}{f_s}$$

$$\text{sen}(4\pi n) = \text{sen}(2\pi 250 t)$$

$$= 4\pi n = 2\pi 250 n t_s$$

$$= 4\pi n = 2\pi 250 n \frac{1}{f_s}$$

$$4 = \frac{500}{f_s}$$

$$f_s = \frac{500}{4} = \underline{125\ Hz}$$

b) $x(t) = \text{sen}(17000\pi t + \frac{3\pi}{2}) \longrightarrow -\cos(17000\pi t)$
$x[n] = \cos(\pi(8{,}5n + 1))$

$$x[n] = x(nt_s)$$

$$\cos(8{,}5\pi n + \pi) = \text{sen}(17000\pi n t_s + \frac{3\pi}{2})$$

$$8{,}5\pi n = 17000\pi n t_s$$

$$8{,}5\pi n = 17000\pi n \frac{1}{f_s}$$

$$f_s = \frac{17000}{8{,}5} = \underline{2kHz} \leftarrow \text{Resposta}$$

$$\text{sen}(17000\pi t + \frac{3\pi}{2}) = \text{sen}(17000\pi t) \cdot \cos(\frac{3\pi}{2}) + \text{sen}(\frac{3\pi}{2}) \cdot \cos(17000\pi t)$$

$$-1 \quad 0 \quad 1$$

$$= -1$$

$$= -\cos(17000\pi t)$$

7) $x[n] = \text{sen}\left(\frac{n\pi}{4}\right)$ $\quad x(t) = \text{sen}(2\pi f_0 t)$

taxa de amostragem $= 160\,Hz = f_s$ $\quad \hookrightarrow f_s\,Hz$

$x[n] = x(nt_s)$

$\text{sen}\left(\frac{n\pi}{4}\right) = \text{sen}(2\pi f_0 t)$

$\frac{n\pi}{4} = 2\pi f_0 t_s n$

$\frac{n\pi}{4} = \frac{2\pi f_0 n}{f_s}$

$\frac{n\pi}{4} = \frac{2\pi f_0 n}{160}$

$\frac{1}{4} \cdot 80 = f_0$

$f_0 = 20\,Hz$

$f_a = f_0 + K f_s$ $\quad \begin{cases} f_0 = 20\,Hz \\ f_s = 160\,Hz \end{cases}$

$f_{o1} = 20 + 4 \cdot 160 = 660\,Hz$

$f_{o2} = 20 + 6 \cdot 160 = 980\,Hz$

$f_{o3} = 20 + 8 \cdot 160 = 1300\,Hz$