Aluno: Jogo Costa

Disciplina: Sistema de Controle

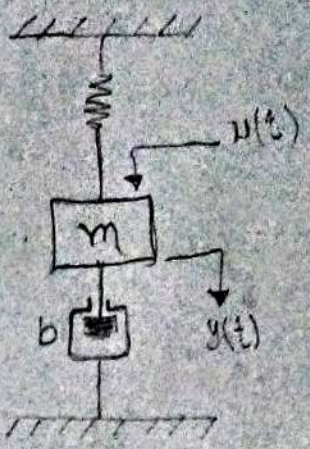
Professora: Dra Lesley

Modelagem no espaço de estados

07/10/21

Exemplo 01

$$m\ddot{y} + b\dot{y} + ky = u$$



$$\ddot{y} + a_1\dot{y} + a_2 y = b_0\ddot{u} + b_1\dot{u} + b_2 u$$

$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = 0\ddot{u} + 0\dot{u} + \frac{1}{m}u$$

$$x_1 = y - \beta_0 u^{0} = y$$

$$x_2 = \dot{y} - \beta_0 u - \beta_1 \dot{u} = x_1 - \beta_1 u$$

$$x_2 = \dot{x}_1 - \frac{b}{m}\dot{u}$$

$$x_3 = \ddot{y} - \beta_0\ddot{u} - \beta_1\dot{u} - \beta_2 u = \dot{x}_2 - \beta_2 u$$

$$\dot{x}_n = -a_n x_1 - a_{n-1}x_2 - \cdots - a_1 x_n + \beta_n u$$

$$x_n = \dot{x}_{n-1} - \beta_{n-1}u$$

$$\dot{x}_1 = x_2 + \beta_0 \dot{u}^{0} = x_2$$

$$\dot{x}_2 = x_3 + \beta_2 u = -a_2 x_1 - a_1 x_2 + \beta_2 u$$

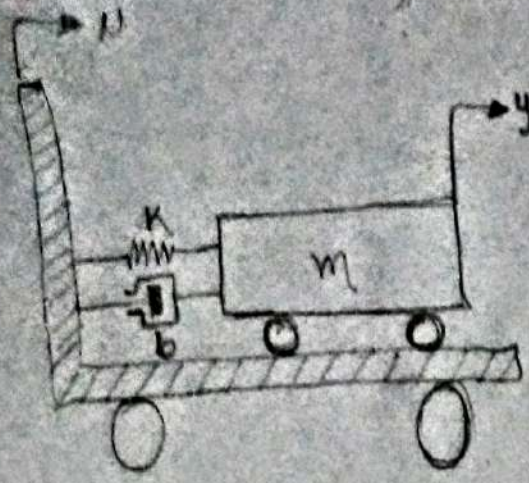
$$\dot{x}_2 = -\frac{k}{m}x_1 - \frac{b}{m}x_2 + \frac{1}{m}u$$

$$\beta_0 = b_0 = 0$$

$$\beta_1 = b_1 - a_1\beta_0 = 0$$

$$\beta_2 = b_2 - a_1\beta_1^{0} - a_2\beta_0^{0} = \frac{k}{m}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{K}{m} & -\frac{b}{m}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m}\end{bmatrix}[u]$$



$$ma = \sum F$$

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + Ky = b\frac{du}{dt} + Ku$$

$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{K}{m}y = \frac{b}{m}\dot{u} + \frac{K}{m}u$$

$$\ddot{y} + a_1\dot{y} + a_0 y = b_0\ddot{u} + b_1\dot{u} + b_0 u$$

$$\begin{cases} \beta_0 = b_0 = 0 \\ \beta_1 = b_1 - a_1\beta_0 = \frac{b}{m} \\ \beta_2 = b_2 - a_1\beta_1 - a_0\beta_0 = \frac{K}{m} - \frac{b}{m}\cdot\frac{b}{m} \end{cases}$$

Achar variáveis de estado e suas derivadas:

$$x_1 = y - \beta_0 u = y$$

$$x_2 = \dot{x}_1 - \beta_1 u = \dot{x}_1 - \frac{b}{m}u$$

$$\dot{x}_1 = x_2 + \beta_1 u = x_2 + \frac{b}{m}u$$

$$\dot{x}_2 = -a_2 x_1 - a_1 x_2 + \beta_2 u = -\frac{K}{m}x_1 - \frac{b}{m}x_2 + \left[\frac{K}{m} - \left(\frac{b}{m}\right)^2\right]u$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-\frac{b}{m} & 0\\ -\frac{K}{m} & -\frac{b}{m}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\frac{b}{m}\\ \frac{K}{m} - \left(\frac{b}{m}\right)^2\end{bmatrix}u \qquad y = [1\ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$