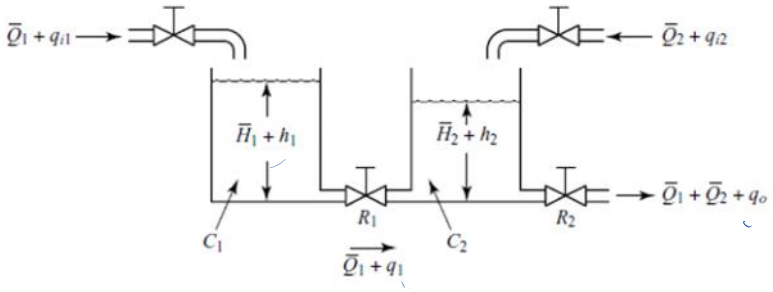

$$q_o = \frac{h}{R} \qquad C\,dh = (q_i - q_o)\,dt$$

$$C_1 dh_1 = (q_{i1} - q_1)\,dt$$

$$q_1 = \frac{h_1 - h_2}{R_1}$$

$$C_2 dh_2 = (q_{i2} + q_1 - q_o)\,dt$$

$$q_o = \frac{h_2}{R_2}$$

$$C_1 \frac{dh_1}{dt} = q_{i1} - q_1$$

$$\frac{dh_1}{dt} = \frac{q_{i1} - q_1}{C_1}$$

$$C_2 \frac{dh_2}{dt} = (q_{i2} + q_1 - q_o)$$

$$\frac{dh_2}{dt} = \frac{q_{i2} + q_1 - q_o}{C_2}$$

Variáveis de estado

$y_1 = h_1 \qquad y_2 = h_2$

$$\beta_0 = b_0$$
$$\beta_1 = b_1 - a_1\beta_0$$
$$\beta_2 = b_2 - a_1\beta_1 - a_2\beta_0$$
$$\beta_3 = b_3 - a_1\beta_2 - a_2\beta_1 - a_3\beta_0$$
$$\vdots$$
$$\beta_{n-1} = b_{n-1} - a_1\beta_{n-2} - \cdots - a_{n-2}\beta_1 - a_{n-1}\beta_0$$

$$\dot{y} + a_1\dot{y} + a_2 y = b_0\ddot{u} + b_1\dot{u} + b_2 u$$

$$\dot{h}_1 = \frac{q_{i1} - q_1}{C_1}$$

$$x_1 = y - \beta_0 u$$
$$x_2 = \dot{y} - \beta_0\dot{u} - \beta_1 u = \dot{x}_1 - \beta_1 u$$

$x_1 = y_1 = h_1 \qquad x_2 = \dot{h}_1$

$x_3 = y_2 = h_2 \qquad x_4 = \dot{h}_2$

$$\dot{x}_1 = x_2 + \beta_1 u$$

$\dot{x}_1 = \dot{h}_1$

$\dot{x}_2 = \dot{h}_2$

$$\dot{x}_1 = \frac{q_{i1} - q_1}{C} = \frac{q_{i1} - \frac{h_1 - h_2}{R_1}}{C}$$

$$= \frac{\frac{q_{i1}R_1 - (h_1 - h_2)}{R_1}}{C}$$

$q_{i1} = u_1$
$q_{i2} = u_2$

$$= \frac{q_{i1}R_1 - \frac{C_1}{(h_1 - h_2)}}{C_1 R_1}$$

$$= \frac{q_{i1}R_1 - (x_1 - x_3)}{C_1 R_1}$$

$$\dot{x_1} = \frac{u_1 R_1 - (x_1 - x_3)}{C_1 R_1}$$

$$\dot{x_1} = -\frac{1}{R_1 C_1}x_1 + \frac{1}{R_1 C_1}x_3 + \frac{1}{C_1}u_1$$

$$\frac{dh_2}{dt} = \frac{q_{i2} + q_1 - q_0}{C_2}$$

$$q_0 = \frac{h_2}{R_2} \qquad q_1 = \frac{h_1 - h_2}{R_1}$$

$$\dot{x_2} = \frac{u_2 + \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2}}{C_2}$$

$$\dot{x_2} = \frac{u_2 R_1 R_2 + R_2(x_1 - x_3) - R_1(x_3)}{C_2 R_1 R_2}$$

$$\dot{x_2} = \frac{1}{R_1 C_2}x_1 - \left(\frac{1}{R_1 C_2} + \frac{1}{R_2 C_2}\right)x_3 + \frac{1}{C_2}u_2$$