Math Booklet

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Contents

Algebra

1.1 Linear Algebra

1.1.1 Matrices

- Notation
$$A = [a_{ij}]$$
- Matrix Addition
$$[a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$$
- Scalar multiplication
$$c[a_{ij}] = [ca_{ij}]$$
- Transpose
$$(aT)_{ij} = a_{ji}$$
- Matrix Multiplication
$$c_{ij} = (\text{ith row of A})(\text{jth column of B}) = \sum_{k=1}^{n} a_{ik} b_{kj}$$

Geometry

2.1 Analytic Geometry

2.1.1 Coordinate systems

- Cartesian coordinates ($\mathbb{R}^2$ and $\mathbb{R}^3$)

$$(x, y) \qquad (x, y, z)$$

- Polar coordinates ($\mathbb{R}^2$)

$$(r, \theta)$$

  - Typical restrictions

$$r \geq 0$$
$$0 \leq \theta \leq 2\pi$$

  - Polar/rectangular conversions

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \qquad \begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = \frac{y}{x} \end{cases}$$

- Cylindrical coordinates ($\mathbb{R}^3$)

$$(r, \theta, z)$$

  - Typical restrictions

$$r \geq 0$$
$$0 \leq \theta \leq 2\pi$$

  - Cylindrical/rectangular conversions

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \qquad \begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = \frac{y}{x} \\ z = z \end{cases}$$

- Spherical coordinates ($\mathbb{R}^3$)

$$(\rho, \phi, \theta)$$

  - Typical restrictions

$$\rho \geq 0$$
$$0 \leq \phi \leq \pi$$
$$0 \leq \theta \leq 2\pi$$

  - Spherical/cylindrical conversions

$$\begin{cases} r = \rho\sin\phi \\ \theta = \theta \\ z = \rho\cos\phi \end{cases} \qquad \begin{cases} \rho^2 = r^2 + z^2 \\ \tan\phi = \frac{r}{z} \\ \theta = \theta \end{cases}$$

– Spherical/rectangular conversions

$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases} \qquad \begin{cases} \rho^2 = x^2 + y^2 + z^2 \\ \tan\phi = \frac{\sqrt{x^2+y^2}}{z} \\ \tan\theta = \frac{y}{x} \end{cases}$$

Calculus

3.1 Multivariable Calculus

3.1.1 Limits

$\mathbf{f}: X \subseteq \mathbb{R}^n \to \mathbb{R}^m$

$$\lim_{\mathbf{x} \to \mathbf{a}} \mathbf{f}(\mathbf{x}) = \mathbf{L}$$

- Rigorous definition

$$\text{if } \mathbf{x} \in X \text{ and } 0 < ||\mathbf{x} - \mathbf{a}|| \leq \delta, \text{ then } ||\mathbf{f}(\mathbf{x}) - \mathbf{L}|| < \varepsilon$$

  $\delta > 0$
  $\varepsilon > 0$

3.1.2 Partial Derivatives

$$\frac{\partial f}{\partial x_i} = \lim_{h \to 0} \frac{f(x_1, ..., x_i + h, ..., x_n) - f(x_1, ..., x_n)}{h}$$

- Gradient

$$\nabla f(\mathbf{a}) = (f_{x_1}(\mathbf{a}), ..., f_{x_n}(\mathbf{a}))$$

- Tangent plane

$$z = h(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$
$$h(\mathbf{x}) = f(\mathbf{a}) + f_{x_1}(\mathbf{a})(x_1 - a_1) + ... + f_{x_n}(\mathbf{a})(x_n - a_n)$$

  - Normal vector

$$\mathbf{n} = -f_x(a, b)\hat{i} - f_y(a, b)\hat{j} + \hat{k} = (-f_x(a, b), -f_y(a, b), 1)$$

- Differentiability

  1. All partial derivatives exist
  2. $$\lim_{\mathbf{x} \to \mathbf{a}} \frac{f(\mathbf{x}) - h(\mathbf{x})}{||\mathbf{x} - \mathbf{a}||} = 0$$