

# Math Booklet

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# Contents

<b>1</b>	<b>Algebra</b>	<b>3</b>
1.1	Linear Algebra . . . . .	3
1.1.1	Matrices . . . . .	3
<b>2</b>	<b>Geometry</b>	<b>4</b>
2.1	Analytic Geometry . . . . .	4
2.1.1	Coordinate systems . . . . .	4
<b>3</b>	<b>Calculus</b>	<b>6</b>
3.1	Multivariable Calculus . . . . .	6
3.1.1	Limits . . . . .	6
3.1.2	Partial Derivatives . . . . .	6
3.1.3	Higher-order Partial Derivatives . . . . .	6

# Algebra

## 1.1 Linear Algebra

### 1.1.1 Matrices

- Notation

$$A = [a_{ij}]$$

- Matrix Addition

$$[a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$$

- Scalar multiplication

$$c[a_{ij}] = [ca_{ij}]$$

- Transpose

$$(aT)_{ij} = a_{ji}$$

- Matrix Multiplication

$$c_{ij} = (\text{ith row of A})(\text{jth column of B}) = \sum_{k=1}^n a_{ik}b_{kj}$$

# Geometry

## 2.1 Analytic Geometry

### 2.1.1 Coordinate systems

- Cartesian coordinates ( $\mathbb{R}^2$  and  $\mathbb{R}^3$ )

$$(x, y) \quad (x, y, z)$$

- Polar coordinates ( $\mathbb{R}^2$ )

$$(r, \theta)$$

- Typical restrictions

$$\begin{aligned} r &\geq 0 \\ 0 &\leq \theta \leq 2\pi \end{aligned}$$

- Polar/rectangular conversions

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases} \quad \begin{cases} r^2 = x^2 + y^2 \\ \tan \theta = \frac{y}{x} \end{cases}$$

- Cylindrical coordinates ( $\mathbb{R}^3$ )

$$(r, \theta, z)$$

- Typical restrictions

$$\begin{aligned} r &\geq 0 \\ 0 &\leq \theta \leq 2\pi \end{aligned}$$

- Cylindrical/rectangular conversions

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = z \end{cases} \quad \begin{cases} r^2 = x^2 + y^2 \\ \tan \theta = \frac{y}{x} \\ z = z \end{cases}$$

- Spherical coordinates ( $\mathbb{R}^3$ )

$$(\rho, \phi, \theta)$$

- Typical restrictions

$$\begin{aligned} \rho &\geq 0 \\ 0 &\leq \phi \leq \pi \\ 0 &\leq \theta \leq 2\pi \end{aligned}$$

- Spherical/cylindrical conversions

$$\begin{cases} r = \rho \sin \phi \\ \theta = \theta \\ z = \rho \cos \phi \end{cases} \quad \begin{cases} \rho^2 = r^2 + z^2 \\ \tan \phi = \frac{r}{z} \\ \theta = \theta \end{cases}$$

– Spherical/rectangular conversions

$$\begin{cases} x = \rho \sin \phi \cos \theta \\ y = \rho \sin \phi \sin \theta \\ z = \rho \cos \phi \end{cases} \quad \begin{cases} \rho^2 = x^2 + y^2 + z^2 \\ \tan \phi = \frac{\sqrt{x^2 + y^2}}{z} \\ \tan \theta = \frac{y}{x} \end{cases}$$

# Calculus

## 3.1 Multivariable Calculus

$$\mathbf{f}: X \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^m$$

$$f: X \subseteq \mathbb{R}^n \rightarrow \mathbb{R}$$

### 3.1.1 Limits

$$\lim_{\mathbf{x} \rightarrow \mathbf{a}} \mathbf{f}(\mathbf{x}) = \mathbf{L}$$

- Rigorous definition

if  $\mathbf{x} \in X$  and  $0 < \|\mathbf{x} - \mathbf{a}\| \leq \delta$ , then  $\|\mathbf{f}(\mathbf{x}) - \mathbf{L}\| < \varepsilon$

$$\delta > 0$$

$$\varepsilon > 0$$

### 3.1.2 Partial Derivatives

$$\frac{\partial f}{\partial x_i} = \lim_{h \rightarrow 0} \frac{f(x_1, \dots, x_i + h, \dots, x_n) - f(x_1, \dots, x_n)}{h}$$

- Gradient

$$\nabla f(\mathbf{a}) = (f_{x_1}(\mathbf{a}), \dots, f_{x_n}(\mathbf{a}))$$

- Derivative matrix

$$D\mathbf{f}(\mathbf{a}) =$$

- Tangent plane

$$z = h(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

- Normal vector

$$\mathbf{n} = -f_x(a, b)\hat{i} - f_y(a, b)\hat{j} + \hat{k} = (-f_x(a, b), -f_y(a, b), 1)$$

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$$\mathbf{h}(\mathbf{x}) = \mathbf{f}(\mathbf{a}) + D\mathbf{f}(\mathbf{a})(\mathbf{x} - \mathbf{a})$$

- Differentiability

1.  $D\mathbf{f}(\mathbf{a})$  exists
- 2.

$$\lim_{\mathbf{x} \rightarrow \mathbf{a}} \frac{\mathbf{f}(\mathbf{x}) - \mathbf{h}(\mathbf{x})}{\|\mathbf{x} - \mathbf{a}\|} = 0$$

- Chain rule

$$\frac{df}{dt} = \nabla f(\mathbf{x}) \bullet \mathbf{x}'(t)$$

### 3.1.3 Higher-order Partial Derivatives