Math Booklet

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Contents

Algebra

1.1 Linear Algebra

1.1.1 Matrices

- Notation

$$A = [a_{ij}]$$

- Matrix Addition

$$[a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$$

- Scalar multiplication

$$c[a_{ij}] = [ca_{ij}]$$

- Transpose

$$(aT)_{ij} = a_{ji}$$

- Matrix Multiplication

$$c_{ij} = \text{(ith row of A)(jth column of B)} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

Geometry

2.1 Analytic Geometry

2.1.1 Coordinate systems

- Cartesian coordinates ($\mathbb{R}^2$ and $\mathbb{R}^3$)

$$(x, y) \qquad (x, y, z)$$

- Polar coordinates ($\mathbb{R}^2$)

$$(r, \theta)$$

  - Typical restrictions

$$r \geq 0$$
$$0 \leq \theta \leq 2\pi$$

  - Polar/rectangular conversions

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \qquad \begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = \frac{y}{x} \end{cases}$$

- Cylindrical coordinates ($\mathbb{R}^3$)

$$(r, \theta, z)$$

  - Typical restrictions

$$r \geq 0$$
$$0 \leq \theta \leq 2\pi$$

  - Cylindrical/rectangular conversions

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \qquad \begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = \frac{y}{x} \\ z = z \end{cases}$$

- Spherical coordinates ($\mathbb{R}^3$)

$$(\rho, \phi, \theta)$$

  - Typical restrictions

$$\rho \geq 0$$
$$0 \leq \phi \leq \pi$$
$$0 \leq \theta \leq 2\pi$$

  - Spherical/cylindrical conversions

$$\begin{cases} r = \rho\sin\phi \\ \theta = \theta \\ z = \rho\cos\phi \end{cases} \qquad \begin{cases} \rho^2 = r^2 + z^2 \\ \tan\phi = \frac{r}{z} \\ \theta = \theta \end{cases}$$

– Spherical/rectangular conversions

$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases} \qquad \begin{cases} \rho^2 = x^2 + y^2 + z^2 \\ \tan\phi = \frac{\sqrt{x^2+y^2}}{z} \\ \tan\theta = \frac{y}{x} \end{cases}$$

Calculus

3.1 Multivariable Calculus

3.1.1 Limits

$\overrightarrow{f} : X \subseteq \mathbb{R}^n \to \mathbb{R}^m$

$$\lim_{\overrightarrow{x} \to \overrightarrow{a}} \overrightarrow{f}(\overrightarrow{x}) = \overrightarrow{L}$$

- Rigorous definition

$$\text{if } \overrightarrow{x} \in X \text{ and } 0 < ||\overrightarrow{x} - \overrightarrow{a}|| \leq \delta, \text{ then } \left|\left|\overrightarrow{f}(\overrightarrow{x}) - \overrightarrow{L}\right|\right| < \varepsilon$$

  $\delta > 0$
  $\varepsilon > 0$