Measurement of the Muon Lifetime and the Fermi Coupling Constant

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Muons, produced by cosmic rays in the upper atmosphere, experience relativistic time dilation, which extends their lifetime and allows them to reach Earth's surface. In this experiment, we measured their lifetime by observing their decay in a plastic scintillator detector. By recording the muon decay times and fitting the data to an exponential decay model, we determined a charge-averaged lifetime of $\tau = 2.10(2)\ \mu s$, which lies within the literature values for the lifetime of positive and negative muons. From the measured lifetime, we estimated the reduced Fermi coupling constant, obtaining a value of $\bar{G}_F = 1.190(6) \times 10^{-5}\,\mathrm{GeV}^{-2}$, which shows some disagreement with the literature value due to the charge-averaging nature of the experiment. These results provide valuable insights into how precise measurements advance our understanding of particle physics and the standard model.

I. INTRODUCTION

Muons were first discovered in 1936 by Carl D. Anderson and Seth Neddermeyer while studying cosmic radiation [1]. They observed particles that curved differently from electrons and protons in a magnetic field, suggesting an intermediate mass. The existence of these particles was confirmed in the following year [2].

The role of muons in the study of the weak interaction was gradually established in the years after their discovery. Initially, muons were thought to be part of the meson family, which had been hypothesized by Yukawa to mediate the strong nuclear force [3]. However, the work of Conversi, Piccioni, and Pancini revealed that muons decay in a manner inconsistent with strong interaction mediators [4]. Instead, it was found that muons decay via the weak interaction, much like electrons, but with a longer lifetime. This led to the realization that muons were leptons, not hadrons, and their behavior was governed by the weak force. The theoretical framework for weak interactions had been proposed earlier by Enrico Fermi in 1933 in the context of beta decay, introducing the Fermi coupling constant G_F [5].

In the modern era, muon lifetime measurements such as the one reported here remain important for testing the foundations of the Standard Model. Precise measurements of the muon lifetime allow for increasingly accurate determinations of G_F [6]. By reproducing one of the key results in particle physics history, this experiment provides an accessible way to explore the weak force and extract a fundamental physical constant from direct observation.

In this experiment, we used a plastic scintillator detector to measure the lifetime of muons. With this measurement, we estimated the Fermi coupling constant. In Section II, we provide an overview of muon decay and its connection to the Fermi coupling constant. Section III describes the working principles of the scintillator detector and our experimental procedures. Section IV presents our results and analysis. Finally, we conclude in Section V with a summary of our findings and possible areas of improvement.

II. THEORY

Cosmic rays are high-energy particles originating from space, primarily composed of protons. When these cosmic rays strike nuclei in the upper atmosphere, they produce secondary particles. Among these, we find charged pions, which decay into muons through

$$\pi^+ \to \mu^+ + \nu_\mu \tag{1}$$

and

$$\pi^- \to \mu^- + \bar{\nu}_\mu. \tag{2}$$

These muons then continue traveling toward Earth's surface.

Given that muons have a lifetime on the order of $\sim 1\ \mu s$ and it takes about $\sim 1\ ms$ to travel from the upper atmosphere to sea level at relativistic speeds, one might question how we are able to detect muons in Earth's surface. The answer relies on time dilation [7], which extends their lifetime from our point of view as stationary observers on Earth.

Once slowed down in our detector, muons decay into electrons or positrons via

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu \tag{3}$$

or

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu, \tag{4}$$

respectively. By collecting the decay times of Eqs. (3)–(4) inside the detector, we can examine the distribution of muon lifetimes inside the detector.

If we were to account for the time it takes for muons to travel from the upper atmosphere to our detector, we would get a distribution that follows an exponential decay of the form

$$N(t) = N_0 e^{-t/\tau} \tag{5}$$

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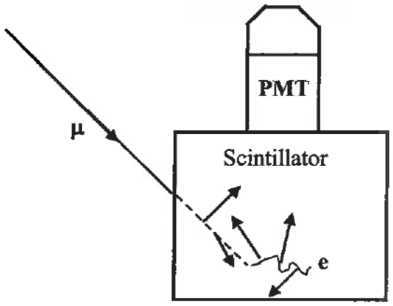

FIG. 1: Schematic showing the two pulses created in the scintillator and detected by a photomultipler tube (PMT). This figure is reproduced from [7].

[7], where $N(t)$ is the number of muons decaying after time t, N_0 is the starting number of muons, and τ is the muon lifetime. Fortunately, if we instead account only for the time that muons stay in our detector, we obtain the same exponential form $e^{-t/\tau}$, which is independent of N_0. Therefore, by fitting our histogram of measured decay times to a function like Eq. (5), we can extract an experimental value for τ.

The muon decays in Eqs. (3)–(4) occur via the weak interaction. The strength of this weak force is given by the Fermi coupling constant G_F, which is related to the particle's lifetime τ by

$$G_F = \sqrt{192\pi^3 \frac{\hbar^7}{\tau m^5 c^4}} \tag{6}$$

[7], where m is the particle's mass. Hence, by measuring the muon lifetime, we can estimate G_F.

III. METHODS

To measure the lifetime of the muon, we use a plastic scintillator detector (FIG. 1), which works by identifying muons that stop inside the scintillator and subsequently decay. When a muon enters the scintillator, it slows down and eventually stops, ionizing the scintillator material in the process. This ionization produces a pulse of scintillation light, which is detected and starts a timing clock [7].

After coming to rest, the muon decays according to Eq. (3) or Eq. (4). The resulting electron or positron emerges with significant kinetic energy and ionizes the scintillator material as it passes through, producing a second pulse of light and stopping the clock [7]. The time interval between the two pulses then corresponds to the muon's lifetime inside the detector.

The detector's ability to accurately and precisely measure decay times is limited by two main factors. First, its time resolution affects how well we can distinguish short lifetimes. This becomes important when fitting our data to an exponential decay model, as discussed in the next section. Second, extraneous single-event signals affect the start and stop triggers of our timing clock. These could be background signals – caused by cosmic rays or environmental radiation – or muons that do not decay inside the detector and instead pass through it.

To address such single-event effects, the detector includes an interval limit. Once the first scintillation pulse is recorded, the system waits up to 20,000 ns for a second pulse [7]. If no second pulse is detected in that period, the event is discarded. This helps ensure that only valid decay events are included in our data.

One might worry that overlapping muon decays will break our timing clock strategy, but this is not a concern given how often we have muon decays in our detector. Over the course of 20 seconds, we can estimate that 97 single events occur based on [8]. That is, we have a rate of ~ 5 single events per second. Over the course of 599,405 seconds, we recorded 12,608 muon decays. That is, we have a rate of ~ 0.02 muon decay per second. Therefore, at each second, the probability of a muon stopping is $\sim 0.4\%$. This means that it is very unlikely for more than one muon to decay at the same time, allowing us to treat each decay as an independent event.

IV. RESULTS

A histogram of our measured decay intervals is shown in FIG. 2. To find the best possible binning strategy of our data, we perform the following steps for each given number of bins N.

1. Calculate bin width.
2. Categorize data into bins.
3. Remove any empty bins.
4. Find bin with the maximum count and ignore any bins before it.
5. Calculate the uncertainty of each bin i with N_i counts as $\delta N_i = \sqrt{N_i}$.
6. Use `SciPy`'s `curve_fit` procedure [9] to fit our data as an exponential decay of the form

   $$N(t) = Ae^{-t/\tau} + C, \tag{7}$$

   where A is the amplitude, τ is the timescale, and C is a vertical offset.
7. Calculate reduced chi squared $\bar{\chi}^2$.

Note that step 4 is needed in order to address the time resolution limitation of the detector. By starting the fit at the bin with the maximum count, we ignore bins at under-resolved times.

We performed steps 1–7 for all $50 \leq N \leq 500$ (at incremental steps of 5). We found the best reduced chi squared value,

$$\bar{\chi}^2 = 1.007, \tag{8}$$

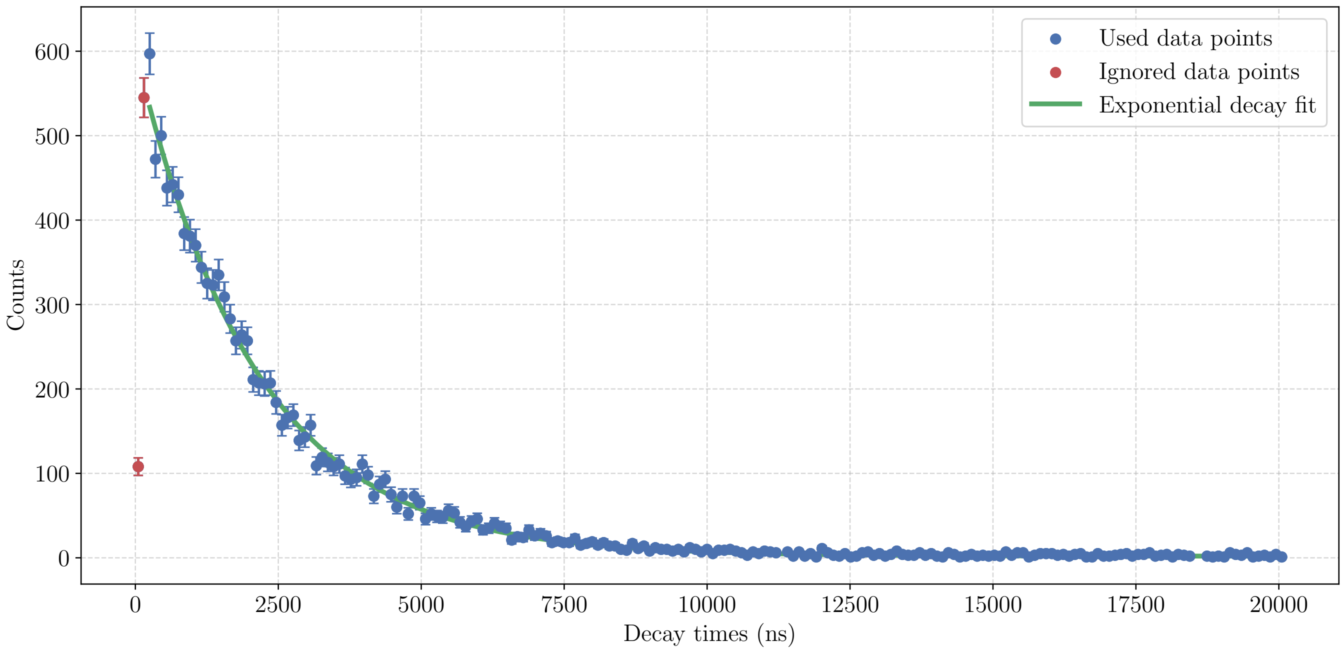

FIG. 2: Histogram of muon decay times.

with $N = 200$ bins (out of which 195 bins remain after steps 3 and 4). With this, our measurement for the lifetime of the muon is

$$\tau = 2.10(2)\ \mu\text{s}, \tag{9}$$

where the uncertainty $\delta\tau$ was taken directly from the fit standard deviation.

As mentioned in the previous section, both positive and negative muons interact with the atoms in the scintillator material, producing scintillation light upon entry. However, negative muons can also be captured by the protons in the nuclei, leading to an alternate decay pathway given by

$$\mu^- + p \to n + \nu_\mu \tag{10}$$

[7]. Because of this, μ^- has a lower lifetime compared to μ^+ [7]. Hence, Eq. (9) is a charge-averaged measurement of the lifetime of the muon.

From [7], we know that the literature lifetimes of μ^+ and μ^- are

$$\tau^+ = 2.19703(2)\ \mu s \tag{11}$$

and

$$\tau^- = 2.043(3)\ \mu s, \tag{12}$$

respectively. Based on [7], we know that we can find the ratio of the number of μ^+ and the number of μ^- with

$$\rho = -\frac{\tau^+}{\tau^-}\left(\frac{\tau^- - \tau}{\tau^+ - \tau}\right) \tag{13}$$

$$= 0.6(4). \tag{14}$$

Note that the error in (14) is very large. It was computed using the error propagation formula

$$\delta\rho = \frac{\tau^+}{\tau^-}\frac{(\tau^+ - \tau^-)}{(\tau^+ - \tau)^2}\delta\tau, \tag{15}$$

where we are ignoring the uncertainties $\delta\tau^+$ and $\delta\tau^-$ in Eqs. (11)–(12) because the uncertainty $\delta\tau$ in Eq. (9) dominates. Eq. (15) can be recast as

$$\frac{\delta\rho}{\rho} = \frac{\tau(\tau^+ - \tau^-)}{(\tau^+ - \tau)(\tau - \tau^-)}\frac{\delta\tau}{\tau}. \tag{16}$$

Using our experimental values, Eq. (16) becomes

$$\frac{\delta\rho}{\rho} = 58.5\ \frac{\delta\tau}{\tau}. \tag{17}$$

This tells us that the error in τ gets significantly amplifies the error in ρ. Then, to get more accurate value of ρ, we need a much more accurate measurement of τ.

Let's now determine the Fermi coupling constant, which is related to the muon lifetime by Eq. (6). In natural units, we define the reduced Fermi constant as

$$\bar{G}_F = \frac{G_F}{(\hbar c)^3} = \sqrt{192\pi^3\frac{\hbar}{\tau(mc^2)^5}}. \tag{18}$$

Using $\hbar = 6.5821 \times 10^{-16}\ eV \cdot s$ [10] and $mc^2 = 105.66\ MeV$ [11], we find

$$\bar{G}_F = 1.190(6) \times 10^{-5}\ GeV^{-2}, \tag{19}$$

where the uncertainty was calculated with

$$\frac{\delta\bar{G}_F}{\bar{G}_F} = \frac{1}{2}\frac{\delta\tau}{\tau}. \tag{20}$$

Compared to the literature value, $1.1663 \times 10{-}5\ GeV^{-2}$ [12], our measurement (19) is significantly larger. This is expected since we found that our τ is smaller than the literature value due to the interaction of μ^- with matter.

V. CONCLUSION

Using a plastic scintillator detector, we measured the charge-averaged muon lifetime and obtained a value of $\tau = 2.10(2)\ \mu s$, which is in reasonable agreement with literature values for positive and negative muons. From this, we determined the ratio of positive to negative muons to be $\rho = 0.6(4)$. Finally, we estimated the reduced Fermi coupling constant as $\bar{G}_F = 1.190(6) \times 10^{-5}\,\mathrm{GeV}^{-2}$, which – as expected – is larger than the accepted value due to the charge-averaging nature of our measurement. These results provide a direct observation of the weak interaction through muon decay, making the experiment not only a valuable exercise in particle detection techniques, but also a meaningful link to the historical development of particle physics.

Despite the success of the experiment, the large uncertainty in ρ highlights the need for improved accuracy. Future experiments could benefit from increased time resolution, longer data collection periods to build a larger sample size, and better suppression of background noise. An additional improvement would be to separate positive and negative muons in our detector, allowing us to measure their individual lifetimes. This would enable the estimation of the true Fermi coupling constant using only positive muons.

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[1] C. D. Anderson and S. H. Neddermeyer, Cloud chamber observations of cosmic rays at 4300 meters elevation and near sea-level, Phys. Rev. **50**, 263 (1936).

[2] J. C. Street and E. C. Stevenson, New evidence for the existence of a particle of mass intermediate between the proton and electron, Phys. Rev. **52**, 1003 (1937).

[3] H. Yukawa, On the interaction of elementary particles, Proceedings of the Physico-Mathematical Society of Japan **17**, 139 (1935).

[4] M. Conversi and O. Piccioni, On the mean life of slow mesons, Phys. Rev. **70**, 859 (1946).

[5] E. Fermi, Tentativo di una teoria dei raggi β, La Ricerca Scientifica **2**, 12 (1933).

[6] D. B. Chitwood, T. I. Banks, M. J. Barnes, *et al.* (MuLan Collaboration), Improved measurement of the positive-muon lifetime and determination of the fermi constant, Phys. Rev. Lett. **99**, 032001 (2007).

[7] T. E. Coan and J. Ye, *Muon Physics* (TeachSpin) v120806.0.

[8] S. Weisrock, Physics 418 notebook (2025), p. 68.

[9] P. Virtanen, R. Gommers, T. E. Oliphant, *et al.*, `SciPy` 1.0: Fundamental algorithms for scientific computing in python, Nature Methods **17**, 261 (2020).

[10] NIST, Codata value: reduced planck constant in ev s (2022), accessed: 2025-03-20.

[11] NIST, Codata value: muon mass (2022), accessed: 2025-03-20.

[12] NIST, Codata value: Fermi coupling constant (2022), accessed: 2025-03-20.