Testing Bell's inequality with entangled photons

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Introduction

Motivation

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  - Quantum mechanics (QM) is incomplete $\Rightarrow$ allows for "spooky action at a distance"
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- 1964: Bell's inequality
  - Experimentally testable way to distinguish between QM and HVT
- 1972: Freedman-Clauser experiment
  - First experimental test of Bell's inequality
  - Used entangled photons

Theory

Entangled photons

- Created in a pair of beta barium borate (BBO) crystals,
- Nonlinear optic properties

$$1 \text{ pump photon} \to 2 \text{ entangled photons}$$

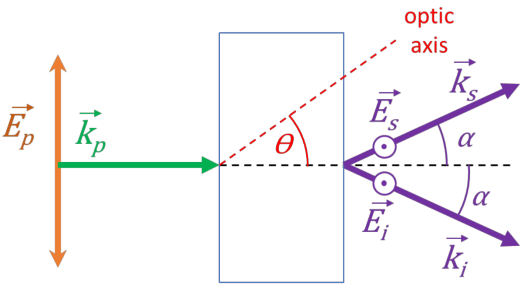

Figure 1: Reproduced from lab manual.

Entangled photons (continued)

- Entangled state:

$$|\psi\rangle = \cos\theta_l |HH\rangle + e^{i\phi}\sin\theta_l |VV\rangle \tag{1}$$

- Maximum entanglement: $\theta_l = 45°$, $\phi = 0°$

$$|\psi_{\max}\rangle = \frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right) \tag{2}$$

Detector polarizers

- Photon detections at angles α and β
- Probability of both being “vertical”:

$$P_{VV}(\alpha, \beta) = \sin^2\alpha\sin^2\beta\cos^2\theta_l + \cos^2\alpha\cos^2\beta\sin^2\theta_l + \frac{1}{4}\sin 2\alpha\sin 2\beta\sin 2\theta_l\cos\phi_m \quad (3)$$

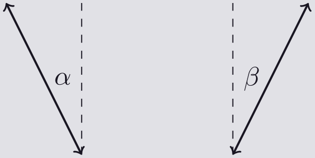

Figure 2

Bell's inequality

- John Bell (1964) found an inequality that must be true for any HVT

$$S \leq 2 \tag{4}$$

- Definitions:

$$S \equiv E(a,b) - E(a,b') + E(a',b) + E(a',b') \tag{5}$$

$$E(\alpha,\beta) \equiv P_{VV}(\alpha,\beta) + P_{VV}(\alpha_\perp,\beta_\perp) - P_{VV}(\alpha,\beta_\perp) - P_{VV}(\alpha_\perp,\beta) \tag{6}$$

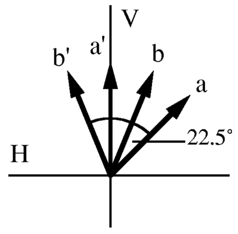

Figure 3: $a = -45°$, $b = -22.5°$, $a = 0°$, and $b' = 22.5°$. Reproduced from lab manual.

Violation of Bell's inequality

- In the **maximally entangled state**, quantum mechanics predicts

$$S > 2 \tag{7}$$

- Violation of Bell's inequality $\Rightarrow$ test of quantum mechanics
- Note: quantum mechanics still allows $S \leq 2$ for other states

Methods

Experimental setup

- Half-wave plate: rotates polarization to set $\theta_l \approx 45°$
- Quarter-wave plate: corrects phase shift $\phi_m \approx 0$

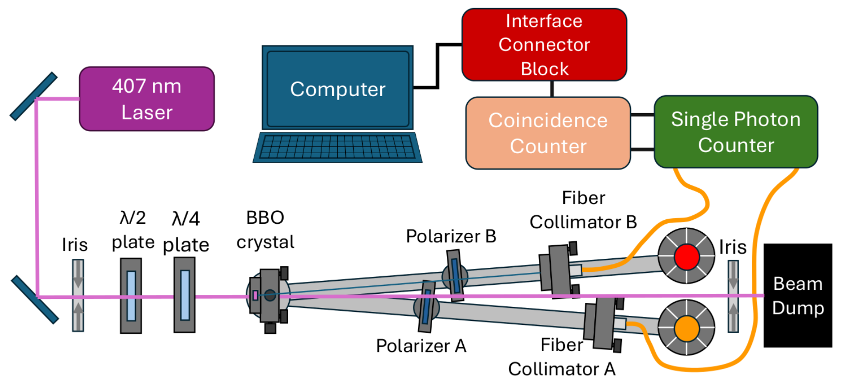

Figure 4: Reproduced from lab manual.

Results

Analysis of state

- Model for number of coincidences:

$$N(\alpha,\beta) = A(\sin^2\alpha\sin^2\beta\cos^2\theta_l + \cos^2\alpha\cos^2\beta\sin^2\theta_l + \frac{1}{4}\sin 2\alpha\sin 2\beta\sin 2\theta_l\cos\phi_m) + C \tag{8}$$

  4 parameters: A, C, θ_l, ϕ_m

- 2 approaches:
  1. Strategic measurements $\Rightarrow$ find parameters analytically
  2. More measurements $\Rightarrow$ fit model

Analytic check

- 5 measurements taken with $T = 60$ s (less accurate)

$$C = N(0^\circ, 90^\circ) \qquad \Rightarrow C = 4(2) \tag{9}$$

$$A = N(0^\circ, 0^\circ) + N(90^\circ, 90^\circ) - 2C \qquad \Rightarrow A = 19(6) \tag{10}$$

$$\tan^2\theta_l = \frac{N(90^\circ, 90^\circ) - C}{N(0^\circ, 0^\circ) - C} \qquad \Rightarrow \theta_l = 55(8)^\circ \tag{11}$$

$$\cos\phi_m = \frac{1}{\sin 2\theta_l}\left(4\frac{N(45^\circ, 45^\circ) - C}{A} - 1\right) \qquad \Rightarrow \cos\phi_m = 2(1) \tag{12}$$

Fit model

- 16 measurements taken with $T = 8$ min (more accurate)
- Used `SciPy`'s `least_squares` procedure to fit model to data

$$A = 200(20) \tag{13}$$

$$\theta_l = 25(4)^\circ \tag{14}$$

$$\phi_m = -0.003^\circ \pm 300000^\circ \tag{15}$$

$$C = 20(7) \tag{16}$$

State models

- Corrected model: keep everything from fit model, but set $\theta_l = 45°$

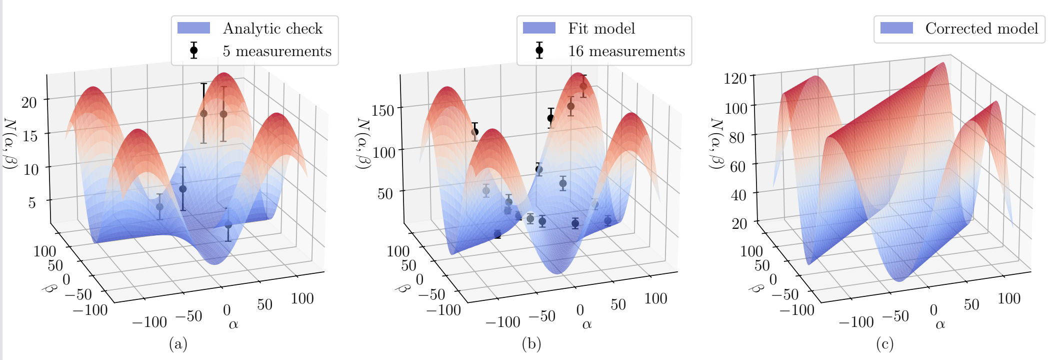

Figure 5

Bell's inequality

- Using 16 measurements:

$$S = 1.8(1) \tag{17}$$

$\Rightarrow$ not helpful (doesn't prove anything)

Bell's inequality

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- Using corrected model:

$$S = 2.0(1) \tag{18}$$

$\Rightarrow$ closer to the limit, but large uncertainty doesn't let us draw any conclusions

Conclusion

Summary

- Used entangled photons to test Bell's inequality
- Not able to draw significant conclusions due to...
    1. Photons not being in a maximally entangled state
    2. Large uncertainty in measurements

Summary

- Used entangled photons to test Bell's inequality
- Not able to draw significant conclusions due to...
  1. Photons not being in a maximally entangled state
  2. Large uncertainty in measurements
- Possible improvements:
  1. Preparation of state
     - Calibration and alignment of setup
     - New BBO crystal (it was cracked)
  2. Accuracy
     - More coincidences
     - Longer duration

The end

Thank you! Any questions?