Quantum Optics
Post-lab

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General to-do's for the complete lab report:

- Give a historical overview.
- Explain the quantum mechanical states better instead of simply mentioning the maximum entangled state in Eq. (7).
- Mention the alignment process.

I. BACKGROUND

In this experiment, we use non-linear optic properties of birefringent crystals to generate entangled photon pairs, which are then sent to a couple of detectors that are preceded by polarizers. If α and β represent the angles from the detector polarizers to the vertical, it can be shown that the probability that both detections come out as "vertical" is given by

$$\begin{aligned} P_{VV}(\alpha,\beta) = {} & \sin^2\alpha \sin^2\beta \cos^2\theta_l \\ & + \cos^2\alpha \cos^2\beta \sin^2\theta_l \\ & + \frac{1}{4}\sin 2\alpha \sin 2\beta \sin 2\theta_l \cos\phi_m \end{aligned} \tag{1}$$

[1], where θ_l represents the initial polarization of the laser beam and ϕ_m is an averaged accumulated phase shift.

In the maximally entangled state ($\theta_l = \pi/4$ and $\phi = 0$), Eq. (1) reduces to

$$P_{VV}^{\mathrm{QM}}(\alpha,\beta) = \frac{1}{2}\cos^2(\beta-\alpha) \tag{2}$$

[1]. However, if we consider a local realistic hidden variable theory (HVT), it can be shown that such maximally entangled state would lead to a probability of

$$P_{VV}^{\mathrm{HVT}}(\alpha,\beta) = \frac{1}{2} - \frac{|\beta-\alpha|}{\pi} \tag{3}$$

[1].

In studying the feasibility of a HVT, John Bell derived an inequality that must be true for any HVT [2]. In the context of Eq. (3), it was found in [3] that this inequality can be expressed as

$$|S| \leq 2 \tag{4}$$

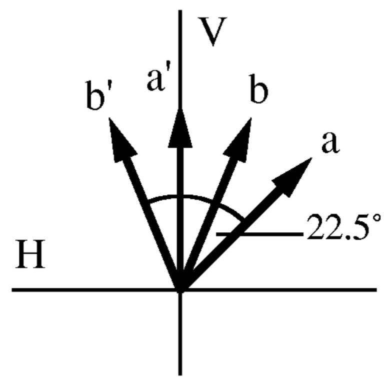

FIG. 1: Schematic showing the angles used in Eq. (5). Here, $a = -45°$, $b = -22.5°$, $a = 0°$, and $b' = 22.5°$. This figure is reproduced from [1].

[1], where

$$S \equiv E(a,b) - E(a,b') + E(a',b) + E(a',b') \tag{5}$$

$$\begin{aligned} E(\alpha,\beta) \equiv {} & P_{VV}(\alpha,\beta) + P_{VV}(\alpha_\perp,\beta_\perp) \\ & - P_{VV}(\alpha,\beta_\perp) - P_{VV}(\alpha_\perp,\beta), \end{aligned} \tag{6}$$

and the angles a, b, a', and b' are defined as in Fig. 1.

This means that if a HVT exists, then Eq. (4) must be satisfied. However, if the quantum mechanical prediction from Eq. (2) is correct, then the value of S will be greater than 2, violating Eq. (4).

II. PROCEDURE

It is important to note that Quantum Mechanics does not necessarily violate Eq. (4) in all cases. However, we have chosen the angles in Fig. 1 such that the quantum mechanical prediction for S is maximized (and greater than 2) in the maximally entangled state [1], thus violating Eq. (4).

Our experimental setup is shown in Fig. 2. A vertically polarized 407 nm laser beam is sent into a couple of wave plates. The half-wave plate (HWP) is used to rotate the polarization of the photons to be as close as possible to the desired angle $\theta_l = 45°$, while the quarter-wave plate (QWP) phase-shifts the photons so that we achieve the maximally entangled state. We will discuss how we have calibrated the angles of these wave plates later in the next section. If done correctly, the photons emerging from these wave plates will be in the maximally entangled

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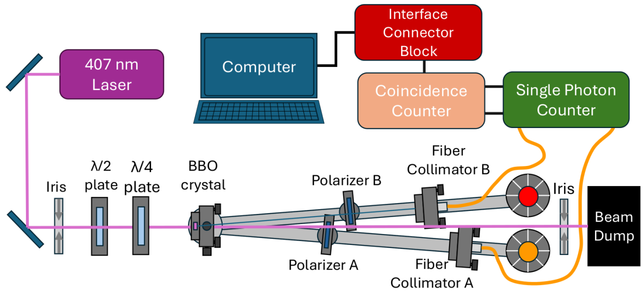

FIG. 2: Experimental setup. This figure is reproduced from [4].

state,

$$\frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle) \tag{7}$$

[4]. If not done correctly, the photons will not be in the maximally entangled state, and the value of S might not be help us in evaluating Bell's inequality.

The entangled photons are then sent to a pair of beta barium borate (BBO) crystals, where they undergo spontaneous parametric down-conversion, producing two orthogonally polarized beams [1]. The two beams are then sent to polarizers at angles α and β from the vertical. The photons are then detected by a couple of single-photon detectors. The detectors are connected to a coincidence counter, which records the number of times both detectors detect a photon at the same time in a coincidence window of $\tau = 25$ ns [4].

The coincidence counter gives us $N(\alpha, \beta)$, the number of coincidences for a given configuration of the polarizers. We can re-write Eq. (6) in terms of $N(\alpha, \beta)$ as

$$E(\alpha,\beta) \equiv \frac{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) - N(\alpha,\beta_\perp) - N(\alpha_\perp,\beta)}{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha,\beta_\perp) + N(\alpha_\perp,\beta)} \tag{8}$$

[1]. With this, we can calculate the value of S using Eq. (5).

It's important to note that we need to subtract background single-photon counts from our measurements. By blocking the beam path, we measured the background single-photon count rate in detector A $\dot{N}_A^{\text{background}} \approx 624$ Hz and in detector B $\dot{N}_B^{\text{background}} \approx 923$ Hz. After measuring the raw counts in both detectors (N_A^{raw} and N_B^{raw}), we define

$$N_A = N_A^{\text{raw}} - T\dot{N}_A^{\text{background}}, \tag{9}$$

$$N_B = N_B^{\text{raw}} - T\dot{N}_B^{\text{background}}, \tag{10}$$

where T is the duration of the measurement.

Additionally, we know from [4] that there is an expected number of accidental coincidences given by

$$N^{\text{ac}} = N_A N_B \frac{\tau}{T}. \tag{11}$$

After measuring the raw coincidences N^{raw}, we can then define the corrected coincidences as

$$N = N^{\text{raw}} - N^{\text{ac}}. \tag{12}$$

III. RESULTS

A. Calibration of wave plates

To calibrate the HWP, we varied its angle from the vertical λ_H and recorded $N(\alpha, \beta)$ in two configurations: $(\alpha, \beta) = (0°, 0°)$ and $(\alpha, \beta) = (90°, 90°)$. We then fitted the data to the expected shape of $\sin^2(\lambda_{\text{HWP}})$ using `SciPy`'s `curve_fit` procedure [5]. The results are shown in the left panel of Fig. 3. Seeking the state in Eq. (7), we want to equalize $N(0°, 0°)$ and $N(90°, 90°)$, which is satisfied when $\lambda_H \sim 62°$.

We calibrated the QWP similarly. We varied its angle from the vertical λ_Q and recorded $N(45°, 45°)$, which we want to maximize. The resulting data and fit are shown in the right panel of Fig. 3. From this, we see that the maximum of $N(45°, 45°)$ is satisfied when $\lambda_Q \sim 140°$.

All of these measurements had a duration of $T = 30$ s.

B. Analysis of state

Our first set of measurements (shown in Table I) was taken with $T = 60$ s in order to analyze our prepared state. Our second set of measurements (shown in Table II) was taken with $T = 8$ minutes to be used in to evaluate S.

From [4], we know that the expected number of coincidences can be modelled by

$$\begin{aligned} N(\alpha,\beta) = A(&\sin^2\alpha \sin^2\beta \cos^2\theta_l \\ &+ \cos^2\alpha\cos^2\beta\sin^2\theta_l \\ &+ \frac{1}{4}\sin 2\alpha \sin 2\beta \sin 2\theta_l \cos\phi_m) + C \end{aligned} \tag{13}$$

and that θ_l can be estimated with

$$\tan^2\theta_l = \frac{N(90°,90°) - N(0°,90°)}{N(0°,0°) - N(0°,90°)}. \tag{14}$$

Using the results from Table I in Eq. (14), we find

$$\theta_l \approx 56° \tag{15}$$

[To-do: find uncertainty]. This clearly deviates from the desired $\theta_l = 45°$, which is likely due to improper calibration of the wave plates or misalignment of the other optical components.

In order to get a better estimate of the prepared state used in the data of actual interest, we can fit the measurements from Table II into a model like Eq. (13). We performed such fit using `SciPy`'s `least_squares` procedure [5]. The resulting fit is shown in the left plot of Fig. 4. This fit gives us

$$A \approx 196, \tag{16}$$

$$\theta_l \approx 25°, \tag{17}$$

$$\phi_m \approx 0.003°, \tag{18}$$

$$C \approx 19. \tag{19}$$

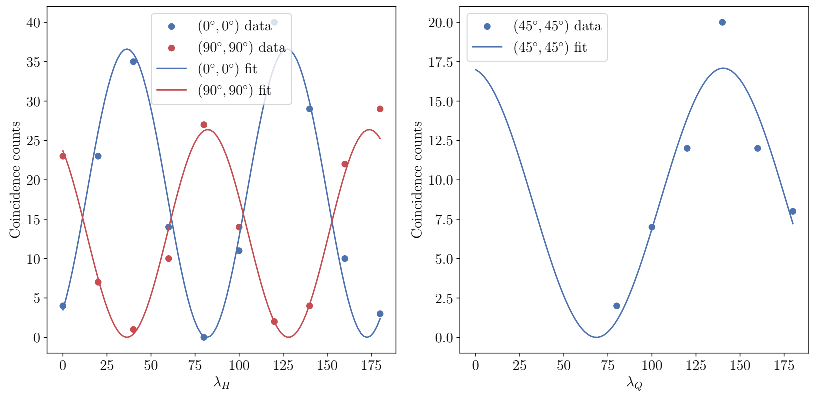

FIG. 3: Calibration of the HWP (left) and the QWP (right).

[To-do: get uncertainties] Again, this is disappointly far from the desired $\theta_l = 45°$. The significant difference between Eq. (15) and Eq. (17) is concerning and should be investigated further in future experiments.

C. Bell's inequality

Using the measurements in Table II, we can calculate the value of S using Eq. (5) and Eq. (8). The resulting value is

$$S \approx 1.8(1). \tag{20}$$

Unfortunately, we do not get the expected violation of Bell's inequality, as this value is less than 2. This is likely due to the fact that we did not properly prepare the photons in the maximally entangled state, as discussed in the previous section.

That said, we can try to estimate what out S value would be if we use our model from Eq. (13) with the parameters found in Eqs. (16)–(19), except that we enforce that $\theta_l = 45°$. By doing this and re-calculating S using Eq. (5) and Eq. (8), we find

$$S \approx 2.0(1). \tag{21}$$

While this brings us closer to the expected violation of Bell's inequality, it is still not enough to conclude that we have violated Eq. (4) because of our uncertainty.

ACKNOWLEDGMENTS

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[5] P. Virtanen, R. Gommers, T. E. Oliphant, *et al.*, `SciPy` 1.0: Fundamental algorithms for scientific computing in python, Nature Methods **17**, 261 (2020).

α	β	N_A^{raw}	N_B^{raw}	N^{raw}
$0°$	$0°$	66195	64087	10
$90°$	$90°$	76322	97093	17
$0°$	$90°$	64369	96829	4
$45°$	$45°$	74406	86618	19
$45°$	$-45°$	73555	76268	6

TABLE I: Set of 5 measurements taken with $T = 30$ s.

α	β	N_A^{raw}	N_B^{raw}	N^{raw}
$-45°$	$-22.5°$	548510	565065	76
$-45°$	$22.5°$	546175	626254	24
$-45°$	$67.5°$	546660	807829	63
$-45°$	$112.5°$	545895	738278	120
$0°$	$-22.5°$	522904	522904	48
$0°$	$22.5°$	520821	625289	37
$0°$	$67.5°$	519373	808650	27
$0°$	$112.5°$	522048	744202	19
$45°$	$-22.5°$	605777	578021	41
$45°$	$22.5°$	597014	632914	75
$45°$	$67.5°$	592238	810973	141
$45°$	$112.5°$	593777	745974	66
$90°$	$-22.5°$	623314	568441	39
$90°$	$22.5°$	617312	620594	45
$90°$	$67.5°$	619769	817720	175
$90°$	$112.5°$	621931	746304	139

TABLE II: Set of 16 measurements taken with $T = 8$ minutes.

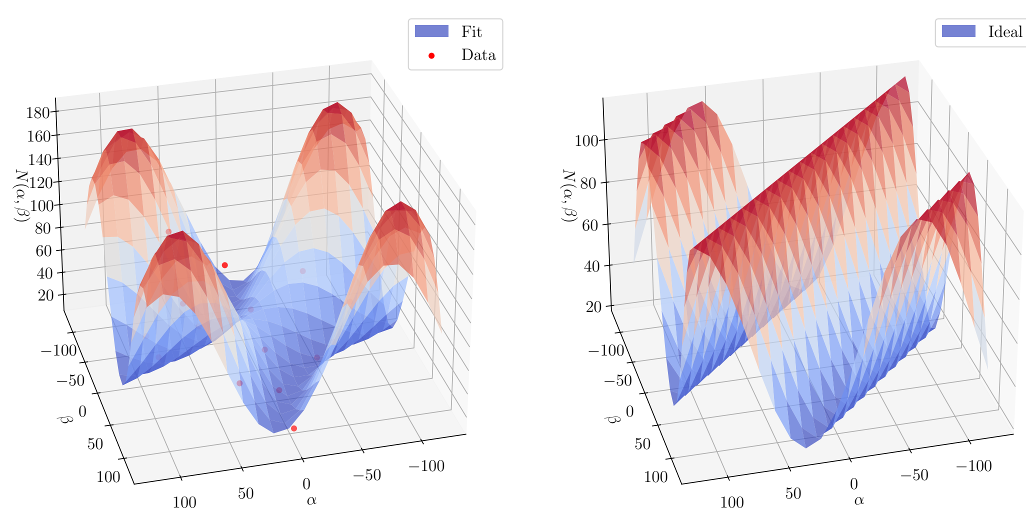

FIG. 4: Analysis of the state of the measurements in Table II.