Physics Booklet

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Contents

Classical Mechanics

1.1 Kinematics

1.1.1 Basic concepts

- Velocity
$$\overrightarrow{v}(t) = \frac{d\overrightarrow{r}}{dt}$$
  $\overrightarrow{r}(t)$: position
- Speed
$$v = |\overrightarrow{v}| = \left|\frac{d\overrightarrow{r}}{dt}\right|$$
- Acceleration
$$\overrightarrow{a}(t) = \frac{d\overrightarrow{v}}{dt}$$
- Other rates
  - Jerk
$$\overrightarrow{j}(t) = \frac{d\overrightarrow{a}}{dt}$$
  - Snap
$$\overrightarrow{s}(t) = \frac{d\overrightarrow{j}}{dt}$$
  - Crackle
$$\overrightarrow{c}(t) = \frac{d\overrightarrow{s}}{dt}$$
  - Pop
$$\overrightarrow{p}(t) = \frac{d\overrightarrow{c}}{dt}$$

1.1.2 Constant acceleration cases (theorems)

1. $v(t) = v_0 + at$
$$a = \frac{dv}{dt} \quad \therefore \quad \int_{t=t_0}^{t=t} a dt = \int_{t=t_0}^{t=t} \frac{dv}{dt} dt$$
$$\therefore \quad a \cdot (t - t_0) = [v(t)]_{t_0}^{t} \quad \therefore \quad v(t) = v_0 + at$$
2. $x(t) = x_0 + v_0 t + \frac{at^2}{2}$
$$v(t) = v_0 + at \quad \therefore \quad \int_{t=t_0}^{t=t} v(t) dt = \int_{t=t_0}^{t=t} (v_0 + at) dt$$
$$\therefore \quad x(t) = x_0 + v_0 t + \frac{at^2}{2}$$

3. $x(t) = x_0 + vt - \frac{at^2}{2}$

$$v = v_0 + at \quad \therefore \quad v_0 = v - at$$
$$\therefore \quad x(t) = x_0 + (v - at)t + \frac{at^2}{2} \quad \therefore \quad x(t) = x_0 + vt - \frac{at^2}{2}$$

4. $x(t) = x_0 + \frac{(v_0+v)t}{2}$

$$2x = x(t) + x(t) = \left(x_0 + v_0 t + \frac{at^2}{2}\right) + \left(x_0 + vt - \frac{at^2}{2}\right)$$
$$\therefore \quad 2x = 2x_0 + v_0 t + vt \quad \therefore \quad x(t) = x_0 + \frac{(v_0 + v)t}{2}$$

5. $v^2 = v_0^2 + 2a(x - x_0)$

$$\begin{cases} v(t) = v_0 + at \\ x(t) = x_0 + \frac{(v_0+v)t}{2} \end{cases} \quad \therefore \quad \begin{cases} v - v_0 = at \\ v + v_0 = \frac{2(x-x_0)}{t} \end{cases}$$
$$\therefore \quad v^2 = v_0^2 + 2a(x - x_0)$$

1.1.3 Uniform circular motion

- Angular velocity

$$\omega = \frac{d\theta}{dt}$$

- Position

$$\overrightarrow{r}(t) = R\cos(\omega t)\hat{i} + R\sin(\omega t)\hat{j}$$

- Velocity

$$\overrightarrow{v}(t) = -\omega R\sin(\omega t)\hat{i} + \omega R\cos(\omega t)\hat{j}$$

- Speed

$$v = \omega R$$

- Centripetal acceleration

$$\overrightarrow{a} = -\omega^2\overrightarrow{r}$$
$$a = \omega^2 R = \frac{v^2}{R}$$

1.2 Forces

1.2.1 Newton's Laws

1. Inertia

Every object moves in a straight line unless acted upon by a force.

2. $F = ma$

$$\overrightarrow{F}_{net} = \sum \overrightarrow{F} = m\overrightarrow{a}$$

3. Action and reaction

For every action, there is an equal and opposite reaction

1.2.2 Weight | Near-Earth gravitional force (W)

- Definition

$$\overrightarrow{W} = -mg\hat{k}$$

- Gravity

$$g \approx 9.81\,\frac{m}{s^2} \quad \text{(downward)}$$

1.2.3 Tension (T)

- Definition

Pulling force transmitted axially by the means of a rope to keep it from changing its length.

- Ideal rope
  - massless
  - doesn't stretch or break

1.2.4 Normal force (N)

- Definition

Contact force orthogonal to a surface that keeps two solid objects from passing through each other.

1.2.5 Friction

- Definition

Resistance to sliding at an interface.

- Static friction

$$\left|\overrightarrow{F_s}\right| \leq \mu_s N$$

μ_s: coefficient of static friction

- Kinectic friction

$$\left|\overrightarrow{F_k}\right| = \mu_k N$$

μ_k: coefficient of kinectic friction

- General relation between constants

$$\mu_s > \mu_k$$

1.2.6 Drag (D)

- Viscous force (linear drag)

$$D \propto v$$

- Air resistance (quadratic drag)

$$D = \frac{1}{2} C \rho A v^2$$

C: drag coefficient (associated with shape)
ρ: mass density of air
A: cross-section surface area

- Terminal speed

$$v_t = \sqrt{\frac{2mg}{C\rho A}}$$

1.2.7 Spring force

- Hooke's law

$$F = -kx$$

1.3 Energy

1.3.1 Basic concepts

- Work (W)

$$W = \int \overrightarrow{F} \bullet d\overrightarrow{r}$$

- Kinetic energy (K)

$$K = \frac{1}{2}mv^2$$

- Potential energy (U)

$$\Delta U = -\int \overrightarrow{F_{cons}} \bullet d\overrightarrow{r} = -\Delta W_{cons}$$

  - Spring

$$U = \frac{1}{2}kx^2$$

  - Weight

$$U = mgz$$

- Work-Energy theorem

$$W_{net} = \Delta K$$

  - Derivation

$$W_{net} = \int_i^f F_{net}\,dx = m\int_i^f \frac{dx}{dt}dv = m\int_i^f v\,dv$$
$$\therefore \quad W_{net} = \left[\frac{mv^2}{2}\right]_i^f = \Delta K$$

1.3.2 Convervation of energy

- Definition

$$E_T = K + U = \text{constant}$$
$$\Delta E_T = \Delta K + \Delta U = 0$$

  if there's no dissipative force

- Conservative forces requirements
  - the force has to depend only on the position of objects
  - work done by the force has to depend only on the initial and final states of a system (not on how it got from initial to final)

- Work done by dissipative forces

$$W_{diss} = \Delta E_T$$

  - Derivation

$$W_{net} = W_{cons} + W_{diss} = -\Delta U + W_{diss} = \Delta K$$
$$\therefore \quad W_{diss} = \Delta K + \Delta U = \Delta E_T$$

1.3.3 Work done by conservative forces

1. Spring force

$$W = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{k}{2}\left(x_i^2 - x_f^2\right)$$

2. Weight

$$W = -mg\Delta z$$

1.4 Gravitation

1.4.1 Newton's law of gravitation

$$\overrightarrow{F_g} = -\frac{Gm_1 m_2}{r^2}\hat{r} \qquad F_g = \frac{Gn_1 m_2}{r^2}$$

$\hat{r}$: radius direction $\left(\hat{r} = \frac{x}{r}\hat{\imath} + \frac{y}{r}\hat{\jmath} + \frac{z}{r}\hat{k}\right)$

- Gravitational constant

$$G \approx 6.67 \cdot 10^{-11}\,\frac{Nm^2}{kg^2}$$

- Gravitational acceleration

$$a_g = \frac{GM}{R^2}$$

1.4.2 Potential energy

$$U_g(r) = -\frac{GMm}{r}$$

- Derivation

$$W_g = \int_{t=t_0}^{t=t} \overrightarrow{F_g} \bullet d\overrightarrow{r} = -\int_{t=t_0}^{t=t} \frac{GMm}{r^2} dr = \Delta\left[\frac{GMm}{r}\right]$$
$$(W = -\Delta U)$$
$$\therefore \quad U_g = -\frac{GMm}{r}$$

- Escape velocity

$$v_e = \sqrt{\frac{2GM}{R}}$$

  – Derivation

$$E_t = K + U = 0 \quad \therefore \quad \frac{1}{2}mv_e^2 = \frac{GmM}{R}$$
$$\therefore \quad v_e = \sqrt{\frac{2GM}{R}}$$

1.4.3 Kepler's laws

1.5 Oscillations

1.5.1 Simple harmonic motion

System considered: mass attached to a massless spring, on a frictionless table.

- Position

$$x(t) = A\cos(\omega t + \phi_0)$$

  A: amplitude
  ϕ_0: initial phase
  ω: angular frequency $\left(\omega = \sqrt{\frac{k}{m}}\right)$

- Speed

$$v(t) \equiv \dot{x(t)} = -A\omega\sin(\omega t + \phi_0)$$

- Period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

- Frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

1.5.2 Damped harmonic motion

1.6 Momentum

1.7 Angular momentum

1.8 Lagrangian method

Relativistic Mechanics

Electromagnetism

Thermodynamics

Statistical mechanics

Quantum mechanics