Presentation Notes – Week 1

Private Reading: Quantum Computing
February 14, 2025
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1. Classical bits (Cbits)

- 2 possible states: $|0\rangle$ or $|1\rangle$
- Reversible operations
  - Definition: initial states can be retrieved with the knowledge of the final states and the operation
  - Example of irreversible operations: ERASE

$$|0\rangle \to |0\rangle \tag{1}$$
$$|1\rangle \to |0\rangle \tag{2}$$

  - For n Cbits, there are $(2^n)!$ reversible operations
    $\Rightarrow$ mapping each state to another one (permutation of states)
  - 1-Cbit operators:
    * Identity

$$\hat{1}|1\rangle = |1\rangle \tag{3}$$
$$\hat{1}|0\rangle = |0\rangle \tag{4}$$
$$\hat{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{5}$$

    * NOT

$$\hat{X}|1\rangle = |0\rangle \tag{6}$$
$$\hat{X}|0\rangle = |1\rangle \tag{7}$$
$$\hat{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{8}$$

  - Example of 2-Cbit operator:
    * cNOT ($\hat{C}_{ij}$): “controlled-NOT”
      $i \to$ “control Cbit” (determines when to apply NOT)

$j \to$ "target Cbit" (receives NOT)

$$\hat{C}_{01}|10\rangle = |11\rangle \tag{9}$$

$$\hat{C}_{10}|10\rangle = |10\rangle \tag{10}$$

$$\hat{C}_{00}|10\rangle = |00\rangle \tag{11}$$

$$\vdots$$

$$\hat{C}_{01} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{12}$$

$$\vdots$$

- "For a reversible classical computer, at least one 3-Cbit gate is needed to build up general logical operations"
  $\Rightarrow$ Toffoli gate

2. Quantum bits (Qbits)

- Infinite possible states:
  $$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle \tag{13}$$
  $\alpha_0, \alpha_1 \in \mathbb{C}$
- n-Qbit system:
  $$|\Psi\rangle = \sum_{x=0}^{2^n-1} \alpha_x |x\rangle_n \tag{14}$$
- Reversible operations: any unitary transformations!
  $\Rightarrow$ maintain normalized amplitudes
  $$\hat{u}\hat{u}^\dagger = \hat{1} \quad \text{(1-Qbit)} \tag{15}$$
  $$\hat{U}\hat{U}^\dagger = \hat{1} \quad (n\text{-Qbit}) \tag{16}$$
  - Hadamard ($\hat{H}$): definite state $\to$ superposition
    $$\hat{H}|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \tag{17}$$
    $$\hat{H}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \tag{18}$$
    $$\hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{19}$$
- For general logical operations, we only need 1-Qbit and 2-Qbit gates
  $\Rightarrow$ can construct a Toffoli gate (shown in the next chapter)

3. Measurement

- Only irreversible operation of quantum computers
  $\Rightarrow$ collapse of the state

- "Born rule": probability of an outcome

$$p(x) = |\alpha_x|^2 \tag{20}$$

- "Generalized Born rule": measurement of only one Qbit

$$|\Psi\rangle_{n+1} = \alpha_0|0\rangle|\Phi_0\rangle_n + \alpha_1|1\rangle|\Phi_1\rangle_n \tag{21}$$

  If applied multiple times, gives the usual Born probability
  $\Rightarrow$ can use only 1-Qbit measurement gates