Álgebra

· Boletín 1

1.-

a) Sí b) Sí c) No d) Sí e) No f) No g) Sí (integral definida)

2.-

A → Escalonada reducida B → No escalonada C → Escalonada reducida

D → No escalonada E → Escalonada (no reducida)

3.-

$$E^* = \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{array}\right) \rightarrow \boxed{\text{2 pivotes y no admite solución}}$$

4.-

a) $A = \begin{pmatrix} 2 & -1 & 3 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ b) $A^* = \left(\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ -7 & 0 & 7 & 0 \\ -1 & 1 & 0 & -1 \end{array}\right)$ c) $A_{31} = 0$ d) $A^*_{14} = 2$

e) $$S = \left(\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \end{array}\right) \xrightarrow{E_{32}(1)} \left(\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & -1 \end{array}\right) \xrightarrow{E_2(7)} \left(\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ -7 & 0 & 7 & 0 \\ -1 & 1 & 0 & -1 \end{array}\right) = S'$$

Son equivalentes porque se puede pasar de una a otra mediante transformaciones elementales.

5.-

$x_1, \dots, x_n$ → A matriz ampliada, escalonada con n pivotes

$\boxed{\text{Depende}}$, porque el último pivote puede estar en la última columna y por lo tanto sería incompatible.

6.-

$$\begin{pmatrix} 0 & 1 & -2 & 2 \\ 2 & -1 & 4 & -3 \\ 4 & -1 & 6 & -4 \\ -2 & 2 & -6 & 5 \end{pmatrix} \xrightarrow[E_{34}(2)]{E_{24}(1)} \begin{pmatrix} 0 & 1 & -2 & 2 \\ 0 & 1 & -2 & 2 \\ 0 & 3 & -6 & 6 \\ -2 & 2 & -6 & 5 \end{pmatrix} \xrightarrow{E_{14}} \begin{pmatrix} -2 & 2 & -6 & 5 \\ 0 & 1 & -2 & 2 \\ 0 & 3 & -6 & 6 \\ 0 & 1 & -2 & 2 \end{pmatrix} \xrightarrow{E_3(1/3)} \begin{pmatrix} -2 & 2 & -6 & 5 \\ 0 & 1 & -2 & 2 \\ 0 & 1 & -2 & 2 \\ 0 & 1 & -2 & 2 \end{pmatrix}$$

$$\xrightarrow[E_{32}(-1)]{E_{43}(-1)} \begin{pmatrix} -2 & 2 & -6 & 5 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{E_1(1/2)} \begin{pmatrix} 1 & -1 & 3 & -5/2 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{E_{12}(1)} \begin{pmatrix} 1 & 0 & 1 & -1/2 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

7.-

$$\left(\begin{array}{ccc|c} 6 & -3 & 2 & 11 \\ -3 & 2 & -1 & -4 \\ 5 & -3 & 2 & 9 \end{array}\right) \xrightarrow[E_3(6)]{E_{21}(1/2)} \left(\begin{array}{ccc|c} 6 & -3 & 2 & 11 \\ 0 & 1/2 & 0 & 3/2 \\ 30 & -18 & 12 & 54 \end{array}\right) \xrightarrow{E_{31}(5)} \left(\begin{array}{ccc|c} 6 & -3 & 2 & 11 \\ 0 & 1/2 & 0 & 3/2 \\ 0 & -3 & 2 & -1 \end{array}\right) \xrightarrow{E_2(6)} \left(\begin{array}{ccc|c} 6 & -3 & 2 & 11 \\ 0 & 3 & 0 & 9 \\ 0 & -3 & 2 & -1 \end{array}\right)$$

$$\xrightarrow{E_{32}(1)} \left(\begin{array}{ccc|c} 6 & -3 & 2 & 11 \\ 0 & 3 & 0 & 9 \\ 0 & 0 & 2 & 8 \end{array}\right)$$

$2x_3 = 8$; $\boxed{x_3 = 4}$ $\quad$ $3x_2 = 9$; $\boxed{x_2 = 3}$

$6x_1 - (3\cdot 3) + 2\cdot 4 = 11$; $6x_1 - 1 = 11$; $\boxed{x_1 = 2}$

$$\boxed{\text{Solución} \rightarrow x_1 = 2,\ x_2 = 3,\ x_3 = 4}$$

8.-

a)

$$\left(\begin{array}{cccc|c} -3 & 2 & 3 & -1 & -2 \\ 6 & -4 & -5 & 3 & 7 \\ 0 & 0 & 1 & 1 & 3 \\ -9 & 6 & 12 & 0 & 3 \end{array}\right)$$

b) $\rightarrow$

$$\xrightarrow[E_{41}(-3)]{E_{21}(2)} \left(\begin{array}{cccc|c} -3 & 2 & 3 & -1 & -2 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 3 & 3 & 9 \end{array}\right) \xrightarrow{E_4(1/3)} \left(\begin{array}{cccc|c} -3 & 2 & 3 & -1 & -2 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right) \xrightarrow[E_{32}(1)]{E_{43}(1)}$$

$$\rightarrow \left(\begin{array}{cccc|c} -3 & 2 & 3 & -1 & -2 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{E_1(1/3)} \left(\begin{array}{cccc|c} -1 & 2/3 & 1 & -1/3 & -2/3 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{E_{12}(-1)} \left(\begin{array}{cccc|c} -1 & 2/3 & 0 & -4/3 & -11/3 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \rightarrow$$

$$\xrightarrow{E_1(-1)} \left(\begin{array}{cccc|c} 1 & -2/3 & 0 & +4/3 & +11/3 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

c) Variables libres → y, t Variables directoras → x, z

d) $\boxed{y=\alpha}$ $\boxed{t=\beta}$

$z+\beta=3;\ \boxed{z=3-\beta}$ $x-2/3\alpha+4/3\beta=11/3;\ x=11/3-4/3\beta+2/3\alpha;$

$$\boxed{x=\frac{11-4\beta+2\alpha}{3}}$$

9.-

a) $\left(\begin{array}{cccc|c} 1 & -3 & 0 & 1 & 5 \\ -1 & 1 & 5 & 3 & 2 \\ 0 & 1 & -1 & -1 & 0 \end{array}\right) \xrightarrow{E_{21}(1)}$ b) $\left(\begin{array}{cccc|c} 1 & -3 & 0 & 1 & 5 \\ 0 & -2 & 5 & 4 & 7 \\ 0 & 1 & -1 & -1 & 0 \end{array}\right) \xrightarrow{E_{32}(1/2)} \left(\begin{array}{cccc|c} 1 & -3 & 0 & 1 & 5 \\ 0 & -2 & 5 & 4 & 7 \\ 0 & 0 & 2 & 1 & 7/2 \end{array}\right)$ * ~~$E_{2}(1/2)$~~

$\xrightarrow{E_3(1/2)} \left(\begin{array}{cccc|c} 1 & -3 & 0 & 1 & 5 \\ 0 & 1 & -5/2 & -2 & -7/2 \\ 0 & 0 & 1 & 1/2 & 7/4 \end{array}\right) \xrightarrow{E_{12}(3)} \left(\begin{array}{cccc|c} 1 & 0 & 15/2 & -5 & -11/2 \\ 0 & 1 & -5/2 & -2 & -7/2 \\ 0 & 0 & 1 & 1/2 & 7/4 \end{array}\right) \xrightarrow{E_{23}(5/2)} \left(\begin{array}{cccc|c} 1 & 0 & 15/2 & -5 & -11/2 \\ 0 & 1 & 0 & -3/4 & 7/8 \\ 0 & 0 & 1 & 1/2 & 7/4 \end{array}\right) \xrightarrow{E_{13}(-15/2)}$

$\rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & -35/4 & -149/8 \\ 0 & 1 & 0 & -3/4 & 7/8 \\ 0 & 0 & 1 & 1/2 & 7/4 \end{array}\right)$

c) La variable libre es t, las directoras x, y, z.

d) $z+1/2\beta=7/4;\ \boxed{z=7/4-1/2\beta}$

$\boxed{y=7/8-3/4\beta}$ $\boxed{x=-149/8+35/4\beta}$

MAL

* $\xrightarrow{E_{23}(-5/2)} \left(\begin{array}{cccc|c} 1 & -3 & 0 & 1 & 5 \\ 0 & -2 & 0 & 3/2 & -7/4 \\ 0 & 0 & 2 & 1 & 7/2 \end{array}\right) \xrightarrow{E_{12}(-3/2)} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -5/4 & 61/8 \\ 0 & -2 & 0 & 3/2 & -7/4 \\ 0 & 0 & 2 & 1 & 7/2 \end{array}\right) \xrightarrow[E_3(1/2)]{E_2(-1/2)} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -5/4 & 61/8 \\ 0 & 1 & 0 & -3/4 & 7/8 \\ 0 & 0 & 1 & 1/2 & 7/4 \end{array}\right)$

MAL

$\boxed{10.-}$

$\boxed{S_1} = \left(\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 2 & 3 & -2 & 5 \\ -1 & 0 & 6 & -2 \\ 1 & 6 & 14 & 4 \end{array}\right) \xrightarrow[\substack{E_{31}(1) \\ E_{41}(-1)}]{E_{21}(-2)} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & -1 & -2 & -1 \\ 0 & 2 & 6 & 1 \\ 0 & 4 & 14 & 1 \end{array}\right) \xrightarrow{E_{32}(2)} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & -1 & -2 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 4 & 14 & 1 \end{array}\right) \xrightarrow{E_{42}(4)} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & -1 & -2 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 6 & -3 \end{array}\right) \xrightarrow{E_4(1/3)}$

$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & -1 & -2 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 2 & -1 \end{array}\right) \xrightarrow{E_{43}(-1)} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & -1 & -2 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \longrightarrow$ 3 pivotes = nº variables, $\boxed{S.C.D}$

$\boxed{S_2} =$ ~~1 1 1 2 2 / 2 2 2 3 4~~ $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 3 & 2 & 0 \\ 2 & 4 & 3 & 2 \end{array}\right) \xrightarrow[E_{31}(-2)]{E_{21}(-1)} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & -1 \\ 0 & 2 & 1 & 0 \end{array}\right) \xrightarrow{E_{32}(-1)} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$

$\longrightarrow \boxed{S.I}$

$\boxed{S_3} = \left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 2 \\ 2 & 2 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 & 1 \\ 2 & 0 & 1 & 1 & -1 \end{array}\right) \xrightarrow[\substack{E_{31}(-3) \\ E_{41}(-2)}]{E_{24}(-1)} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 2 \\ 0 & 2 & 1 & 2 & 5 \\ 0 & -2 & -1 & -2 & -5 \\ 0 & -2 & -1 & -3 & -5 \end{array}\right) \xrightarrow[E_{42}(1)]{E_{32}(1)} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 2 \\ 0 & 2 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{array}\right) \xrightarrow{E_{34}}$

$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 2 \\ 0 & 2 & 1 & 2 & 5 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \longrightarrow$ S.C.I, variable libre $\rightarrow x_3$

$\boxed{S_4} = \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & -5 \\ 1 & 0 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -3 \\ 2 & 0 & 2 & 0 & -2 \end{array}\right) \xrightarrow[\substack{E_{31}(-1) \\ E_{41}(-2)}]{E_{21}(-1)} \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & -5 \\ 0 & 0 & -2 & 0 & 4 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & -2 & 8 \end{array}\right) \xrightarrow{E_{23}} \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & -5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 & 4 \\ 0 & 0 & 0 & -2 & 8 \end{array}\right) \xrightarrow[E_4(-1/2)]{E_3(-1/2)}$

$\longrightarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & -5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & -4 \end{array}\right) \xrightarrow{E_{13}(-1)} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & -3 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & -4 \end{array}\right) \xrightarrow{E_{14}(-1)} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & -4 \end{array}\right) \longrightarrow \boxed{SCD}$

11.-

$$\left(\begin{array}{cccc|c} 2 & 1 & 0 & 1 & -1 \\ 1 & 0 & -1 & 1 & 1 \\ 3 & 1 & -1 & 1 & 1 \\ -1 & 0 & 1 & -1 & -1 \end{array}\right) \xrightarrow[\substack{E_{34}(3) \\ E_4(2)}]{E_{24}(1)} \begin{pmatrix} 2 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -2 & -2 \\ 2 & 0 & 2 & -2 & -2 \end{pmatrix} \xrightarrow{E_{41}(1)} \begin{pmatrix} 2 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -2 & -2 \\ 0 & 1 & 2 & -1 & -3 \end{pmatrix} \xrightarrow[E_{24}]{E_{34}(-1)}$$

$$\to \begin{pmatrix} 2 & 1 & 0 & 1 & -1 \\ 0 & 1 & 2 & -1 & -3 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \longrightarrow \text{S.C.I , 2 variables libres, 3 directoras}$$

$\boxed{x_3 = \alpha}$ $\boxed{x_0 = \beta}$ $\quad -x_4 + \beta = 0; \boxed{x_4 = \beta} \quad x_2 + 2\alpha - \beta - 3\beta = 0;$

$\boxed{x_2 = 4\beta - 2\alpha}$

$2x_1 + (4\beta - 2\alpha) + \beta - \beta = 0;$

$x_1 = (2\alpha - 4\beta)/2 ; \boxed{x_1 = \alpha - 2\beta}$

$\boxed{\text{Solución: } \{ (\alpha - 2\beta, 4\beta - 2\alpha, \alpha, \beta, \beta) \}}$

12.- $a, b \in \mathbb{R}$

$$\left(\begin{array}{ccc|c} a & 0 & b & 2 \\ a & a & 4 & 4 \\ 0 & a & 2 & b \end{array}\right)$$

① Si $a=0$ y $b=0$ $\to \left(\begin{array}{ccc|c} 0 & 0 & 0 & 2 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 2 & 0 \end{array}\right) \to \boxed{\text{S.I}}$

② Si $a=1$ y $b=0$ $\to$ $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 1 & 1 & 4 & 4 \\ 0 & 1 & 2 & 0 \end{array}\right) \xrightarrow{E_{21}(-1)} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 4 & 2 \\ 0 & 1 & 2 & 0 \end{array}\right) \xrightarrow{E_{32}(-1)}$

(o $a \neq 0$)

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & -2 & -2 \end{array}\right) \xrightarrow{E_{23}(+2)} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -2 & -2 \end{array}\right) \xrightarrow{E_3(-\frac{1}{2})} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right) \longrightarrow \boxed{\text{S.C.D}}$$

③ Si $a=0$ y $b=1$ o $b \neq 0$; $\longrightarrow \left(\begin{array}{ccc|c} 0 & 0 & 1 & 2 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 2 & 1 \end{array}\right) \longrightarrow \boxed{\text{S.C.I}}$

④ Si $a=1$ y $b=1$ $\to \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 1 & 1 & 4 & 4 \\ 0 & 1 & 2 & 1 \end{array}\right) \longrightarrow \boxed{\text{S.C.D}}$

13.- $f(x) = a + bx + cx^2 \quad (-2,4), (1,1), (-1,-3)$

$$\left.\begin{array}{l} a - 2b + 4c = 4 \\ a + b + c = 1 \\ a - b + c = -3 \end{array}\right\} \left(\begin{array}{ccc|c} 1 & -2 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -3 \end{array}\right) \xrightarrow[E_{31}(-1)]{E_{21}(-1)} \left(\begin{array}{ccc|c} 1 & -2 & 4 & 4 \\ 0 & 3 & -3 & -3 \\ 0 & 1 & -3 & -7 \end{array}\right) \xrightarrow{E_{32}(-1/3)} \left(\begin{array}{ccc|c} 1 & -2 & 4 & 4 \\ 0 & 3 & -3 & -3 \\ 0 & 0 & -2 & -6 \end{array}\right)$$

$$\xrightarrow[E_3(-1/2)]{E_2(1/3)} \left(\begin{array}{ccc|c} 1 & -2 & 4 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -3 \end{array}\right) \xrightarrow{E_{12}(2)} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right) \xrightarrow{E_{13}(-2)} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right) \xrightarrow{E_{23}(1)} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$\boxed{a = -4 \quad b = 2 \quad c = 3} \to f(x) = -4 + 2x + 3x^2 ; \boxed{f(x) = 3x^2 + 2x - 4}$

$\boxed{14.-}$ $f(x) = a + bx + cx^2 + dx^3 \quad (-1,5), (1,1), (0,1), (-2,7)$

$$\left.\begin{array}{l} a-b+c-d=5 \\ a+b+c+d=1 \\ a+0+0+0=1 \\ a-2+4c-8d=7 \end{array}\right\} \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 5 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & -2 & 4 & -8 & 7 \end{array}\right) \xrightarrow[E_{41}(-1)]{E_{21}(-1)\; E_{31}(-1)} \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 5 \\ 0 & 2 & 0 & 2 & -4 \\ 0 & 1 & -1 & 1 & -4 \\ 0 & -1 & 3 & -7 & 2 \end{array}\right) \xrightarrow{E_{34}(1)}$$

$$\longrightarrow \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 5 \\ 0 & 2 & 0 & 2 & -4 \\ 0 & 0 & 2 & -6 & -2 \\ 0 & -1 & 3 & -7 & 2 \end{array}\right) \xrightarrow{E_{42}(1/2)} \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 5 \\ 0 & 2 & 0 & 2 & -4 \\ 0 & 0 & 2 & -6 & -2 \\ 0 & 0 & 3 & -6 & 0 \end{array}\right) \xrightarrow[E_4(1/3)]{E_3(1/2)} \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 5 \\ 0 & 2 & 0 & 2 & -4 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & -2 & 0 \end{array}\right) \xrightarrow{E_{43}(-1)}$$

$$\longrightarrow \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 5 \\ 0 & 2 & 0 & 2 & -4 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \Bigg\} \quad \boxed{d=1} \quad c-3=-1; \boxed{c=2} \quad 2b=-6; \boxed{b=-3}$$

$$\boxed{a=1} \longrightarrow \boxed{f(x) = x^3 + 2x^2 - 3x + 1}$$

Boletín 2

$\boxed{2.-}$

$$A = \begin{pmatrix} 0 & -2 \\ 4 & 3 \\ 1 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -4 \\ 8 & 7 \\ 0 & 3 \end{pmatrix}$$

$$(\lambda A + B)^t = \begin{pmatrix} 1 & -2\lambda-4 \\ 4\lambda+8 & 3\lambda+7 \\ \lambda & 2\lambda+3 \end{pmatrix}^t = \begin{pmatrix} 1 & 4\lambda+8 & \lambda \\ -2\lambda-4 & 3\lambda+7 & 2\lambda+3 \end{pmatrix} \quad \Bigg\| \quad \text{Si } \lambda=-2: \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \end{pmatrix}$$

- Por lo tanto, Existe $\lambda \in \mathbb{R}$ tal que $(\lambda A + B)^t$ es escalonada reducida.

$\boxed{3.-}$ $XA = XA$ para todo A

La matriz nula y la identidad forman parte de la solución, pero no son exclusivamente esas. λI_2 también porque son modificaciones de la identidad así que da lo mismo. Las matrices diagonales incluyen a todas las anteriores, por lo que son esas, $\boxed{\text{opción C}}$.

4 $A, B \in M_{n\times n}(\mathbb{R}), n \geq 2$ tales que $AB = 0$

$AB = 0$ no implica $BA = 0 \longrightarrow A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$$\left.\begin{aligned} A \cdot B &= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ B \cdot A &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \end{aligned}\right\} \neq$$

5.- Es falso, si C es la matriz nula las matrices A y B pueden ser distintas y el resultado será el mismo, $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

$A = \begin{pmatrix} 7 & 4 \\ 2 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 1 \\ 2 & 6 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$A \cdot C = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad BC = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad A \neq B, \ AB = BC$

6.- A_1, A_2 son invertibles.

Su suma será invertible también siempre que esa matriz, mediante operaciones elementales, se pueda transformar en I_n. Pero no es posible siempre.

$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad A_2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad A_1 + A_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \longrightarrow A_1 + A_2$ no tiene inversa porque $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ no se puede transformar en la matriz I_2 $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ mediante operaciones elementales.

7 $A \in M_{n\times n}(\mathbb{R}), n \geq 2 \quad k > 2 \quad A^k \quad \boxed{A^k = I_n}$

Si A^k produce la matriz Id_n, entonces A debe tener inversa porque es transformable o equivalente por filas a la matriz identidad.

8: $A \in M_{3\times 3} \quad A \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \longleftrightarrow A = \begin{pmatrix} 0 & \lambda & \lambda \\ 0 & \lambda & \lambda \\ 0 & \lambda & \lambda \end{pmatrix}$

a) No tiene inversa porque no es equivalente por filas a la matriz I_n

b) No lo es porque al ser equivalente por filas a A, no lo será a la matriz I_n.

9.- $A=\begin{pmatrix} 0 & 0 & \lambda \\ 0 & 0 & \lambda \\ 0 & 0 & \lambda \end{pmatrix}$ a y b → mismas respuestas que en el 8.

10.- $A_1 \to$ Sí lo es $A_2 \to$ No $\to E_{12}\cdot E_{21}(2)$ $A_3 \to$ Sí $A_4 \to$ No

$A_5 \to$ Sí $A_6 \to$ No $\to E_{13}(2)\cdot E_{23}$ $A_7 \to$ No $\to E_1(5)\cdot E_2(5)\cdot E_3(5)$ $A_8 \to$ No $\to E_{13}\cdot E_{23}$

11.-

$$A_1=\left(\begin{array}{cc|cc} 3 & -2 & 1 & 0 \\ 5 & -3 & 0 & 1 \end{array}\right) \xrightarrow{E_{21}(-1)} \left(\begin{array}{cc|cc} 3 & -2 & 1 & 0 \\ 2 & -1 & -1 & 1 \end{array}\right) \xrightarrow{E_{12}(-1)} \left(\begin{array}{cc|cc} 1 & -1 & 2 & -1 \\ 2 & -1 & -1 & 1 \end{array}\right) \xrightarrow{E_{21}(-2)} \left(\begin{array}{cc|cc} 1 & -1 & 2 & -1 \\ 0 & 1 & -5 & 3 \end{array}\right)$$

$$\xrightarrow{E_{12}(1)} \left(\begin{array}{cc|cc} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 3 \end{array}\right) \longrightarrow \boxed{A_1^{-1}=\begin{pmatrix} -3 & 2 \\ -5 & 3 \end{pmatrix}}$$

$$A_2=\left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ -4 & -1 & -3 & 0 & 1 & 0 \\ 3 & 0 & 3 & 0 & 0 & 1 \end{array}\right) \xrightarrow[E_{31}(-3)]{E_{21}(4)} \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 7 & -7 & 4 & 1 & 0 \\ 0 & -6 & 6 & -3 & 0 & 1 \end{array}\right) \xrightarrow[E_3(1/6)]{E_2(1/7)} \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 4/7 & 1/7 & 0 \\ 0 & -1 & 1 & -3/6 & 0 & 1/6 \end{array}\right) \xrightarrow{E_{32}(1)}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 4/7 & 1/7 & 0 \\ 0 & 0 & 0 & 1/14 & 1/7 & 1/6 \end{array}\right) \longrightarrow$$ **No es invertible**, tiene una fila nula.

$$A_3=\left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 1 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[E_{31}(-1)]{E_{21}(-1)} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 1 \end{array}\right) \xrightarrow{E_{23}} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right) \xrightarrow{E_{23}(-2)}$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & +1 & -2 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right) \xrightarrow[E_{13}(1)]{E_2(-1)} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right) \xrightarrow{E_{12}(1)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 3 & -1 \\ 0 & 1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right) \longrightarrow$$

$$\longrightarrow \boxed{A_3^{-1}=\begin{pmatrix} -1 & 3 & -1 \\ -1 & 2 & -1 \\ -1 & 1 & 0 \end{pmatrix}}$$

(está bien el result. pero la matriz está mal copiada, así que la inv. es otra)

12.-

$A_1 X=\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ $X = A^{-1}\begin{pmatrix} 3 \\ -1 \end{pmatrix}=\begin{pmatrix} -3 & 2 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \end{pmatrix}$; $\boxed{X=\begin{pmatrix} -11 \\ -18 \end{pmatrix}}$

$A_2 X=\begin{pmatrix} 2 \\ 6 \\ -6 \end{pmatrix}$ $X=$ **No tiene sol.**

Escaneado con CamScanner

$A_3X = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \\ 2 \end{pmatrix} \longrightarrow \boxed{X = \begin{pmatrix} 7 \\ 3 \\ 2 \end{pmatrix}}$

13.- $AX = B \quad A = \begin{pmatrix} 2 & 1 & -1 \\ -4 & 3 & 3 \\ 6 & 8 & -3 \end{pmatrix} \quad B = \begin{pmatrix} 3 \\ -13 \\ 4 \end{pmatrix}$

$$\begin{pmatrix} 2 & 1 & -1 \\ -4 & 3 & 3 \\ 6 & 8 & -3 \end{pmatrix} \xrightarrow{E_{21}(2)} \begin{pmatrix} 2 & 1 & -1 \\ 0 & 5 & 1 \\ 6 & 8 & -3 \end{pmatrix} \xrightarrow{E_{31}(-3)} \begin{pmatrix} 2 & 1 & -1 \\ 0 & 5 & 1 \\ 0 & 5 & 0 \end{pmatrix} \xrightarrow{E_{32}(-1)} \begin{pmatrix} 2 & 1 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & -1 \end{pmatrix} = U$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{E_{21}(-2)} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{E_{31}(3)} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} \xrightarrow{E_{32}(1)} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & 1 & 1 \end{pmatrix} = L$$

- $Ly = B \longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & 1 & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 3 \\ -13 \\ 4 \end{pmatrix} \longrightarrow \boxed{y_1 = 3}$
  $-2\cdot 3 + y_2 = -13 \,;\, \boxed{y_2 = -7}$
  $3\cdot 3 - 7 + y_3 = 4 \,;\, \boxed{y_3 = 2}$
  $\Big\}\; y = \begin{pmatrix} 3 \\ -7 \\ 2 \end{pmatrix}$

- $UX = y \longrightarrow \begin{pmatrix} 2 & 1 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ -7 \\ 2 \end{pmatrix} \longrightarrow -x_3 = 2 \,;\, \boxed{x_3 = -2}$
  $5x_2 - 2 = -7 \,;\, \boxed{x_2 = -1}$
  $2x_1 - 1 + 2 = 3 \,;\, \boxed{x_1 = 1}$
  $\Big\{\; \boxed{X = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}}$ ✓

14.- $AX = \begin{pmatrix} 2 \\ -6 \\ -2 \end{pmatrix} \quad U = \begin{pmatrix} 2 & 0 & 1 & 3 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

- Hallar L:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{E_{21}(-4)} \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{E_{31}(2)} \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \xrightarrow{E_{32}(-3)} \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 2 & -3 & 1 \end{pmatrix} = L$$

- $Ly = B \longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 2 & -3 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \\ -2 \end{pmatrix} \longrightarrow \boxed{y_1 = 2}$
  $-4\cdot 2 + y_2 = -6 \,;\, \boxed{y_2 = 2}$
  $2\cdot 2 - 3\cdot 2 + y_3 = -2 \,;\, \boxed{y_3 = 0}$
  $\Big\}\; y = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$

- $UX = y \longrightarrow \begin{pmatrix} 2 & 0 & 1 & 3 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$