Boletín 3

1.-

a) $(0,0) \in \{(x_1,x_2) \mid x_1 = 0\}$

Suma $\rightarrow (x_1, x_2) + (y_1, y_2) = (x_1+y_1, x_2+y_2) = (0, x_2 + y_2)$, $(0,0) \in$ Suma.

Producto por escalar $\rightarrow \lambda(x_1, x_2) = (\lambda x_1, \lambda x_2) = (\lambda \cdot 0, \lambda \cdot x_2) = (0, \lambda \cdot x_2)$, $(0,0) \in$ producto

b), c) ... $\rightarrow$ hacer lo mismo

2.-

NULLA_i, $i = 1,2$

- $$A_1 = \begin{pmatrix} 1 & 2 & -1 & 2 \\ -4 & -1 & -3 & 6 \\ 3 & 0 & 3 & -6 \end{pmatrix} \xrightarrow[E_{31}(-3)]{E_{21}(4)} \begin{pmatrix} 1 & 2 & -1 & 2 \\ 0 & 7 & -7 & 14 \\ 0 & -6 & 6 & -12 \end{pmatrix} \xrightarrow[E_3(1/6)]{E_2(1/7)} \begin{pmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & 2 \\ 0 & -1 & 1 & -2 \end{pmatrix} \xrightarrow{E_{32}(1)} \begin{pmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\boxed{z = \alpha}$ $\boxed{t = \beta}$ $y - \alpha + 2\beta = 0$; $\boxed{y = \alpha - 2\beta}$ $x + 2\alpha - 4\beta - \alpha + 2\beta = 0$; $\boxed{x = 2\beta - \alpha}$

$$\boxed{X = \begin{pmatrix} 2\beta - \alpha \\ \alpha - 2\beta \\ \alpha \\ \beta \end{pmatrix}}$$

- $$A_2 = \begin{pmatrix} 1 & -1 & -1 \\ 1 & -1 & 0 \\ 1 & -2 & -1 \end{pmatrix} \xrightarrow[E_{31}(-1)]{E_{21}(-1)} \begin{pmatrix} 1 & -1 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \xrightarrow{E_{23}} \begin{pmatrix} 1 & -1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow[E_{13}(1)]{E_2(-1)} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{E_{12}(1)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\boxed{z = 0}$ $\boxed{y = 0}$ $\boxed{x = 0}$

$$\boxed{X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}$$

3.-

$(1,2,3)$: $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix} \xrightarrow{E_{31}(-1)} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{pmatrix} \xrightarrow{E_{32}(-1)} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow (1,2,3)$ sí es combinación lineal de los otros 2.

$(1,1,1)$: $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix} \xrightarrow{E_{31}(-1)} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 0 \end{pmatrix} \longrightarrow (1,1,1)$ no es combinación lineal.

4.-

$$\begin{pmatrix} 1 & 2 & 0 & 3 \\ 2 & 1 & 4 & 3 \\ 0 & 3 & -4 & 3 \\ 1 & 5 & -4 & 6 \end{pmatrix} \xrightarrow[E_{41}(-1)]{E_{21}(-2)} \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & -3 & 4 & -3 \\ 0 & 3 & -4 & 3 \\ 0 & 3 & -4 & 3 \end{pmatrix} \xrightarrow[E_{42}(1)]{E_{32}(1)} \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & -3 & 4 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Así queda demostrado. Hay más formas porque ~~podría haber hecho 0 con la 3^a fila en vez de con la segunda.~~ hay dos filas sin pivote y por lo tanto, variables libres.

5.- $(1,4,a,b) \in \mathbb{R}^4$

$$\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 4 \\ -1 & 2 & a \\ -2 & 1 & b \end{pmatrix} \xrightarrow[E_{31}(1),\ E_{42}(1)]{E_{21}(-2)} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & a+1 \\ 0 & 2 & b+4 \end{pmatrix} \xrightarrow[E_{42}(-2)]{E_{32}(-2)} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & a-3 \\ 0 & 0 & b \end{pmatrix}$$

, para que pertenezca, $\boxed{a=3}$ $\boxed{b=0}$

6.- $S = \{(1,2,-1,3), (2,0,1,1), (1,-6,5,-7)\} \in \mathbb{R}^4$

a) $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 0 & 1 & 1 \\ 1 & -6 & 5 & -7 \end{pmatrix} \xrightarrow[E_{31}(-1)]{E_{21}(-2)} \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & -4 & 3 & -5 \\ 0 & -8 & 6 & -10 \end{pmatrix} \xrightarrow{E_{32}(-2)} \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & -4 & 3 & -5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, por lo que es dependiente

b) Para que sea base:

- B es conjunto generador.
- B es libre.

sacado de ahí

$\boxed{B = \{(1,2,-1,3), (2,0,1,1)\}}$

$\boxed{\text{DIM } W = 2}$

7.- $S = \left\{ \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 3 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 3 & -3 \end{pmatrix} \right\}$

a) $\begin{pmatrix} 0 & 1 \\ 1 & 3 \end{pmatrix} = 2 \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} - 1 \cdot \begin{pmatrix} 2 & 3 \\ 3 & -3 \end{pmatrix}$

b) $\boxed{B = \left\{ \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 3 & -3 \end{pmatrix} \right\}}$ $\boxed{\text{Dim } W = 2}$

8.- $S = \{v_1 = (1,1,3,-1), v_2 = (1,0,1,1), v_3 = (2,1,4,0), v_4 = (2,2,6,2)\}$

Con la matriz B miramos qué columnas son pivote y por lo tanto qué vectores componen la base. Son v_1, v_2 y v_4, por lo que:

$\boxed{B = \{(1,1,3,-1), (1,0,1,1), (2,2,6,2)\}}$ $\boxed{\text{Dim } W = 3}$ → 3 pivotes, 3 vectores que forman la base.

9.-

a) $\begin{pmatrix} 1 & 2 & 1 & 1 \\ 1 & 2 & 0 & -3 \\ -1 & -2 & 1 & 7 \end{pmatrix} \xrightarrow[E_{31}(1)]{E_{21}(-1)} \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & 2 & 8 \end{pmatrix} \xrightarrow{E_{32}(2)} \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

~~$B = \{(1,2,1,1), (1,2,0,-3)\}$~~

$\boxed{x_2 = s}$ $\boxed{x_4 = t}$ $\boxed{x_3 = -4t}$ $\boxed{x_1 = -2s + 3t}$

~~b)~~ $\cancel{\begin{array}{cc} v_1 & v_2 \end{array} \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & 2 & -3 \\ 1 & 0 & 8 \\ 1 & -3 & -2 \end{array}\right)}$

$W = \{(-2s+3t, s, -4t, t)\} = \{(-2s, s, 0, 0) + (3t, 0, -4t, t)\} = \boxed{\{s \cdot (-2,1,0,0) + t \cdot (3,0,-4,1)\}}$

→ $\boxed{B = \{(-2,1,0,0), (3,0,-4,1)\}}$

b)
$$\begin{array}{cc} b_1 & b_2 \\ \downarrow & \downarrow \end{array}$$
$$\left(\begin{array}{cc|c} -2 & 3 & 0 \\ 1 & 0 & -3 \\ 0 & -4 & 8 \\ 0 & 1 & -2 \end{array}\right) \xrightarrow{E_{34}(4)} \left(\begin{array}{cc|c} -2 & 3 & 0 \\ 1 & 0 & -3 \\ 0 & 0 & 0 \\ 0 & 1 & -2 \end{array}\right)$$

↳ de aquí ya sacamos: $(0,-3,8,-2) = b_1 \cdot (-3) + b_2 \cdot (-2)$

10.- $M_{2x2}(\mathbb{R})$

A triangular superior $\rightarrow \begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix} \rightarrow \boxed{A_{21}=0}$

$A_{11} - 2\cdot A_{12} + 3\cdot A_{21} + 4\cdot A_{22} = 0 \rightarrow A_{11} - 2\cdot A_{12} + 0 + 4\cdot A_{22} = 0 \rightarrow \boxed{A_{11} - 2\cdot A_{12} + 4\cdot A_{22} = 0}$

$A_{11} = 2\cdot A_{12} - 4\cdot A_{22} \rightarrow \begin{pmatrix} 2\cdot A_{12} - 4A_{22} & A_{12} \\ 0 & A_{22} \end{pmatrix} \rightarrow \begin{pmatrix} 2\cdot A_{12} & A_{12} \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} -4\cdot A_{22} & 0 \\ 0 & A_{22} \end{pmatrix} =$

$= \boxed{A_{12} \cdot \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix} + A_{22} \begin{pmatrix} -4 & 0 \\ 0 & 1 \end{pmatrix}} \rightarrow \boxed{B = \left\{ \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} -4 & 0 \\ 0 & 1 \end{pmatrix} \right\}} \quad \boxed{\text{Dim } W = 2}$

11.- $\begin{cases} x_1 + x_2 + 2x_3 = 0 \\ x_2 + x_3 + 2x_4 = 0 \end{cases}$ $\quad W = \left\{ \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} \in M_{2x2}(\mathbb{R}) \mid x_1 + x_2 + 2x_3 = 0,\ x_2 + x_3 + 2x_4 = 0 \right\}$

$x_1 = -x_2 - 2x_3 \qquad \boxed{x_2 = -x_3 - 2x_4}$

$x_1 = -(-x_3 - 2x_4) - 2x_3 \rightarrow x_1 = x_3 + 2x_4 - 2x_3 \rightarrow \boxed{x_1 = 2x_4 - x_3}$

$\begin{pmatrix} -x_3 + 2x_4 & -x_3 - 2x_4 \\ x_3 & x_4 \end{pmatrix} \rightarrow x_3 \cdot \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix} + x_4 \begin{pmatrix} 2 & -2 \\ 0 & 1 \end{pmatrix} \rightarrow \boxed{B = \left\{ \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 2 & -2 \\ 0 & 1 \end{pmatrix} \right\}}$

$\boxed{\text{Dim } W = 2}$

12.-

$A = \begin{pmatrix} 1 & 1 & 2 & 1 & 2 \\ 1 & 0 & 1 & 0 & 2 \\ 3 & 1 & 4 & 1 & 6 \\ -1 & 1 & 0 & 1 & 2 \end{pmatrix} \in M_{4x5}(\mathbb{R})$

a) $\xrightarrow[E_{31}(-3),\ E_{41}(1)]{E_{21}(-1)} \begin{pmatrix} 1 & 1 & 2 & 1 & 2 \\ 0 & -1 & -1 & -1 & 0 \\ 0 & -2 & -2 & -2 & 0 \\ 0 & 2 & 2 & 2 & 4 \end{pmatrix} \xrightarrow[E_{42}(2)]{E_{32}(-1)} \begin{pmatrix} 1 & 1 & 2 & 1 & 2 \\ 0 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix} \rightarrow$

$\xrightarrow[E_{34}]{E_2(-1)} \begin{pmatrix} 1 & 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \boxed{Rg\ A = 3}$

b) $\cancel{B = \{(1,1,2,1,2),(1,0,1,0,2),(3,1,4,1,6)\}}$ $\quad B = \{(1,1,2,1,2),(0,-1,-1,-1,0),(0,0,0,0,4)\}$

c) NULL H $= \tilde{A} \cdot X = 0$

$$\begin{pmatrix} 1 & 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = 0 \rightarrow \begin{array}{l} x_1+x_2+2x_3+x_4+2x_5=0 \\ x_2+x_3+x_4=0 \\ 4x_5=0;\ \boxed{x_5=0} \end{array} \quad \boxed{x_3=s} \quad \boxed{x_4=t}$$

$\boxed{x_2=-s-t}$ $\quad x_1 -s\cancel{-t}+2s\cancel{+t}+0=0;$

$\boxed{x_1=-s}$

$$\boxed{X = \begin{pmatrix} -s \\ -s-t \\ s \\ t \\ 0 \end{pmatrix}}$$

d) $B' = \{(1,1,2,1,2),(0,-1,-1,-1,0),(0,0,0,0,4),(1,0,0,0,0),(0,0,0,1,0)\}$

$Rg\, B' = 5$ ✓ • Buscar los vectores con 1s en las posiciones de la matriz donde no hay pivote. IMPORTANTE

$\boxed{13.-}$

a) $S=\{(1,2,0)\} \subset \mathbb{R}^3$ $\quad Rg\, S = 1 \quad B=\{(1,2,0)\}$

$$(1\ 2\ 0)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \rightarrow x_1+2x_2=0;\ \boxed{x_2=-x_1/2}$$

$\boxed{x_3=0}$

b) $S=\{(1,0,1),(-1,2,4),(1,3,3),(2,1,0)\} \subset \mathbb{R}^3$

$$\begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 4 \\ 1 & 3 & 3 \\ 2 & 1 & 0 \end{pmatrix} \xrightarrow[E_{31}(-1),\ E_{41}(-2)]{E_{21}(1)} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 5 \\ 0 & 3 & 2 \\ 0 & 1 & -2 \end{pmatrix} \xrightarrow[E_{42}(-1/2)]{E_{32}(-3/2)} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & -11/2 \\ 0 & 0 & -9/2 \end{pmatrix} \xrightarrow{E_4(-11)} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & -11/2 \\ 0 & 0 & 99/2 \end{pmatrix} \rightarrow$$

$$\xrightarrow{E_{43}(9)} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & -11/2 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \boxed{Rg\, S = 3} \qquad \boxed{B=\{(1,0,1),(-1,2,4)(1,3,3)\}}$$

$$A \cdot x = 0 \quad \cancel{\begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 4 \\ 1 & 3 & 3 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}} \quad \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 3 \\ 1 & 4 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0;$$

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 3 \\ 1 & 4 & 3 \end{pmatrix} \xrightarrow{E_{31}(-1)} \begin{matrix} 1 & -1 & 1 \\ 0 & 2 & 3 \\ 0 & 5 & 4 \end{matrix}$$

$\boxed{14.-}$

$$A=\begin{pmatrix}1&1&1&2&2\\2&2&2&3&4\\3&1&2&4&1\\2&0&1&1&-1\end{pmatrix}\in M_{4\times5}(\mathbb{R})$$

a) $$A^t=\begin{pmatrix}1&2&3&2\\1&2&1&0\\1&2&2&1\\2&3&4&1\\2&4&1&-1\end{pmatrix}\xrightarrow[\substack{E_{31}(-1)\\E_{41}(-2)\\E_{51}(-2)}]{E_{21}(-1)}\begin{pmatrix}1&2&3&2\\0&0&-2&-2\\0&0&-1&-1\\0&-1&-2&-3\\0&0&-5&-5\end{pmatrix}\xrightarrow{E_{24}}\begin{pmatrix}1&2&3&2\\0&-1&-2&-3\\0&0&-1&-1\\0&0&-2&-2\\0&0&-5&-5\end{pmatrix}\xrightarrow[E_{43}(-2)]{E_{53}(-5)}$$

$$\longrightarrow\begin{pmatrix}1&2&3&2\\0&-1&-2&-3\\0&0&-1&-1\\0&0&0&0\\0&0&0&0\end{pmatrix}\longrightarrow\boxed{\text{rg por columnas } A=3}$$

b) $Col\ A = NULL\ H$

$$\begin{pmatrix}1&2&3&2\\0&-1&-2&-3\\0&0&-1&-1\\\cancel{0}&\cancel{0}&\cancel{0}&\cancel{0}\\\cancel{0}&\cancel{0}&\cancel{0}&\cancel{0}\\x_1&x_2&x_3&x_4\end{pmatrix}\xrightarrow{E_{12}(+2)}\begin{pmatrix}1&0&-1&-4\\0&-1&-2&-3\\0&0&-1&-1\\x_1&x_2&x_3&x_4\end{pmatrix}\xrightarrow[E_{23}(-2)]{E_{13}(-1)}\begin{pmatrix}1&0&0&-3\\0&-1&0&-1\\0&0&-1&-1\\x_1&x_2&x_3&x_4\end{pmatrix}\xrightarrow{E_{41}(-x_1)}$$

$$\longrightarrow\begin{pmatrix}1&0&0&-3\\0&-1&0&-1\\0&0&-1&-1\\0&x_2&x_3&x_4+3x_1\end{pmatrix}\xrightarrow{E_{42}(+x_2)}\begin{pmatrix}1&0&0&-3\\0&-1&0&-1\\0&0&-1&-1\\0&0&x_3&x_4+3x_1-x_2\end{pmatrix}\xrightarrow{E_{43}(x_3)}\begin{pmatrix}1&0&0&-3\\0&-1&0&-1\\0&0&-1&-1\\0&0&0&\underbrace{x_4+3x_1-x_2-x_3}_{\text{tiene que ser }=0}\end{pmatrix}$$

$$\}\ 3x_1-x_2-x_3+x_4=0\ \{\Rightarrow\underbrace{\begin{pmatrix}3&-1&-1&1\end{pmatrix}}_{H}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}=0$$

c) $V=\mathcal{L}\left((1,2,3,2),(1,2,1,0),(1,2,2,1)\right)\subset\mathbb{R}^4$

$$\begin{pmatrix}1&2&3&2\\1&2&1&0\\1&2&2&1\end{pmatrix}\xrightarrow[E_{31}(-1)]{E_{21}(-1)}\begin{pmatrix}1&2&3&2\\0&0&-2&-2\\0&0&-1&-1\end{pmatrix}\xrightarrow{E_{32}(-1)}\begin{pmatrix}1&2&3&2\\0&0&-2&-2\\0&0&0&0\end{pmatrix}\ //\ \boxed{B=\{(1,2,3,2),(1,2,1,0)\}}$$

$[(2,3,4,1)]_B$ $[(2,4,1,-1)]_B$

① ②

①

$$\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 2 & 3 \\ 3 & 1 & 4 \\ 2 & 0 & 1 \end{array}\right) \xrightarrow[E_{41}(-2)]{E_{21}(-2),\ E_{31}(-3)} \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & -1 \\ 0 & -2 & -2 \\ 0 & -2 & -3 \end{array}\right) \longrightarrow$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 1 & 2 & 1 & 0 \\ 2 & 3 & 4 & 1 \end{array}\right) \xrightarrow[E_{31}(-2)]{E_{21}(-1)} \left(\begin{array}{cccc} 1 & 2 & 3 & 2 \\ 0 & 0 & -2 & -2 \\ 0 & -1 & -2 & -3 \end{array}\right) \xrightarrow{E_{23}} \left(\begin{array}{cccc} 1 & 2 & 3 & 2 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & -2 & -2 \end{array}\right) \longrightarrow$$ El rango es 3,

por lo que el vector no puede obtenerse como combinación lineal de los vectores de la base y, por lo tanto, no hay coordenadas existentes.

②

$$\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 3 & 1 & 1 \\ 2 & 0 & -1 \end{array}\right) \xrightarrow[E_{31}(-3),\ E_{41}(-2)]{E_{21}(-2)} \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & -2 & -5 \\ 0 & -2 & -5 \end{array}\right) \xrightarrow{E_{43}(-1)} \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & -2 & -5 \\ 0 & 0 & 0 \end{array}\right) \xrightarrow{E_{23}} \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$-2x_2 = -5$; $\boxed{x_2 = 5/2}$ $x_1 + 5/2 = 2$; $\boxed{x_1 = -1/2}$

$$\boxed{\text{Coordenadas} = \begin{pmatrix} -1/2 \\ 5/2 \end{pmatrix}}$$

$\boxed{15.}$ $v = (1,-6,5,-7) \in \mathcal{L}(B)$ $B = \{(1,2,-1,3),(2,0,1,1)\}$

$$\left(\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 0 & -6 \\ -1 & 1 & 5 \\ 3 & 1 & -7 \end{array}\right) \xrightarrow[E_{31}(1),\ E_{41}(-3)]{E_{21}(-2)} \left(\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -4 & -8 \\ 0 & 3 & 6 \\ 0 & -5 & -10 \end{array}\right) \xrightarrow[E_3(1/3),\ E_4(-1/5)]{E_2(-1/4)} \left(\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{array}\right) \xrightarrow[E_{42}(-1)]{E_{32}(-1)} \left(\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right) \longrightarrow$$

$\longrightarrow \begin{cases} x_1 + 2x_2 = 1 \longrightarrow x_1 + 4 = 1;\ \boxed{x_1 = -3} \\ \boxed{x_2 = 2} \end{cases}$

Escaneado con CamScanner

16.- $A=\begin{pmatrix}0 & -4\\ 2 & 1\end{pmatrix}$ $[A]_B=?$

La base que tenía: $B=\left\{\begin{pmatrix}-1 & 1\\ 1 & 0\end{pmatrix},\begin{pmatrix}2 & -2\\ 0 & 1\end{pmatrix}\right\}$

Para calcular las coordenadas expreso las matrices como vectores:

$A=(0,-4,2,1)$ $B=\{(-1,1,1,0),(2,-2,0,1)\}$

$$\left(\begin{array}{cc|c}-1 & 2 & 0\\ 1 & -2 & -4\\ 1 & 0 & 2\\ 0 & 1 & 1\end{array}\right)\xrightarrow[E_{31}(1)]{E_{21}(1)}\left(\begin{array}{cc|c}-1 & 2 & 0\\ 0 & 0 & -4\\ 0 & 2 & 2\\ 0 & 1 & 1\end{array}\right)$$

$\longrightarrow$ No tiene solución, por lo que no hay coordenadas para A respecto de B. $\boxed{\nexists [A]_B}$

17.- $W=\{A\in M_{2\times 2}(\mathbb{R})\mid A \text{ triangular inferior } a_{11}+a_{21}=0\}$ $A=\begin{pmatrix}-3 & 0\\ 3 & 2\end{pmatrix}\in W$

$C=\begin{pmatrix}a_{11} & 0\\ a_{21} & a_{22}\end{pmatrix}$ $a_{11}+a_{21}=0;\ a_{21}=-a_{11}$ $\begin{pmatrix}a_{11} & 0\\ -a_{11} & a_{22}\end{pmatrix}=a_{11}\begin{pmatrix}1 & 0\\ -1 & 0\end{pmatrix}+a_{22}\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$

$$\boxed{B=\left\{\begin{pmatrix}1 & 0\\ -1 & 0\end{pmatrix},\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\right\}}$$

$A=(-3,0,3,2)$

$B=\{(1,0,-1,0),(0,0,0,1)\}$

$$\left(\begin{array}{cc|c}1 & 0 & -3\\ 0 & 0 & 0\\ -1 & 0 & 3\\ 0 & 1 & 2\end{array}\right)\xrightarrow{E_{31}(1)}\left(\begin{array}{cc|c}1 & 0 & -3\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 1 & 2\end{array}\right)\xrightarrow{E_{24}}\left(\begin{array}{cc|c}1 & 0 & -3\\ 0 & 1 & 2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{array}\right)\longrightarrow\begin{cases}\boxed{x_1=-3}\\ \boxed{x_2=2}\end{cases}$$

$$\boxed{[A]_B=\begin{pmatrix}-3\\ 2\end{pmatrix}}$$

18.- $B=\{1,x,x^2\}$ $p(x)=3+2x^2$ $[p(x)]_B=?$

$(\alpha_1\cdot 1)+(\alpha_2\cdot x)+(\alpha_3\cdot x^2)=3+2x^2\rightarrow \underline{\alpha_1}+\underline{\alpha_2}x+\underline{\alpha_3}x^2=\underbrace{3}_{\alpha_1}\cdot 1+\underbrace{0}_{\alpha_2}\cdot x+\underbrace{2}_{\alpha_3}\cdot x^2$

$$\boxed{[p(x)]_B=\begin{pmatrix}3\\ 0\\ 2\end{pmatrix}}$$

$\boxed{19.-}$ $B = \{ w_1 = (1,1,0), w_2 = (0,1,1), w_3 = (1,0,1) \}$ $\mathbb{R}^3$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \xrightarrow{E_{31}(-1)} \begin{pmatrix} 1 & 1 & 0 \\ 0 & +1 & 1 \\ 0 & -1 & 1 \end{pmatrix} \xrightarrow{E_{32}(1)} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix} \xrightarrow{E_{23}(-1/2)} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \xrightarrow[E_3(1/2)]{E_{12}(-1)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \longrightarrow \boxed{Rg\ A = 3}, \text{ por lo que es base de } \mathbb{R}^3.$$

$$[v]_B = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \longrightarrow v = 2(1,1,0) + -1(0,1,1) + 3(1,0,1) = (2,2,0) + (0,-1,-1) + (3,0,3) =$$

$$= (2+0+3, 2-1+0, 0-1+3) = (5,1,2) \longrightarrow \boxed{v = (5,1,2)}$$

$\boxed{20.-}$ $B = \{ w_1 = (1,1), w_2 = (1,-1) \}$ $B' = \{ u_1 = (1,2), u_2 = (4,-1) \}$ $\mathbb{R}^2$

$[v]_B = \begin{pmatrix} 6 \\ a \end{pmatrix}$ $[v]_{B'} = \begin{pmatrix} 1 \\ b \end{pmatrix}$ $a, b \in \mathbb{R}$

$$\left.\begin{aligned} v &= 6 \cdot (1,1) + a(1,-1) = (6+a, 6-a) \\ v &= 1 \cdot (1,2) + b \cdot (4,-1) = (1+4b, 2-b) \end{aligned}\right\} \begin{aligned} &6+a = 1+4b \rightarrow \boxed{a = 4b-5} \rightarrow 12-5 \rightarrow \boxed{a=7} \\ &6-a = 2-b \rightarrow 6-(4b-5) = 2-b; \\ &;\ -4b+11 = 2-b;\ 11-2 = -b+4b; \\ &;\ 9 = 3b \rightarrow \boxed{b=3} \end{aligned}$$

$V = (6+7, 6-7);$

$\boxed{V = (13, -1)}$

$\boxed{21.-}$ $\mathbb{R}^2$ $C_2 = \{ e_1 = (1,0), e_2 = (0,1) \}$ $B = \{ v_1 = (1,-3), v_2 = (2,0) \}$

a) $\boxed{[e_1]_B = \begin{pmatrix} 0 \\ 1/2 \end{pmatrix}}$

$$\left(\begin{array}{cc|c} 1 & 2 & 1 \\ -3 & 0 & 0 \end{array}\right) \xrightarrow{E_{21}(3)} \left(\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 6 & 3 \end{array}\right) \longrightarrow \begin{cases} 6x_2 = 3;\ \boxed{x_2 = 1/2} \\ x_1 + 1 = 1;\ ;\boxed{x_1 = 0} \end{cases}$$

$$[e_2]_B = \left(\begin{array}{cc|c} 1 & 2 & 0 \\ -3 & 0 & 1 \end{array}\right) \xrightarrow{E_{21}(3)} \left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 6 & 1 \end{array}\right) \longrightarrow \begin{cases} \boxed{x_2 = 1/6} \\ x_1 + 1/3 = 0;\ \boxed{x_1 = -1/3} \end{cases}$$

$$\boxed{[e_2]_B = \begin{pmatrix} -1/3 \\ 1/6 \end{pmatrix}}$$

• $\boxed{[v_1]_{C_2} = \begin{pmatrix}1\\-3\end{pmatrix}}$ • $\boxed{[v_2]_{C_2} = \begin{pmatrix}2\\0\end{pmatrix}}$

$\left(\begin{array}{cc|c}1&0&1\\0&1&-3\end{array}\right) \longrightarrow \begin{cases}\boxed{x_1=1}\\ \boxed{x_2=-3}\end{cases}$ $\left(\begin{array}{cc|c}1&0&2\\0&1&0\end{array}\right) \longrightarrow \begin{cases}\boxed{x_1=2}\\ \boxed{x_2=0}\end{cases}$

b) $[w]_B = \begin{pmatrix}-1\\1\end{pmatrix}$ $[w]_{C_2}$ $w = -1(1,-3)+1(2,0) = (-1+2\ ,\ 3) = (1,3)$

$\boxed{[w]_{C_2} = \begin{pmatrix}1\\3\end{pmatrix}}$

$\left(\begin{array}{cc|c}1&0&1\\0&1&3\end{array}\right) \longrightarrow \begin{cases}\boxed{x_1=1}\\ \boxed{x_2=3}\end{cases}$

c) $v=(4,6)$ $[v]_B = ?$

$\left(\begin{array}{cc|c}1&2&4\\-3&0&6\end{array}\right) \xrightarrow{E_{21}(3)} \left(\begin{array}{cc|c}1&2&4\\0&6&18\end{array}\right) \longrightarrow \begin{cases}\boxed{x_2=3}\\ \boxed{x_1=-2}\end{cases}$ $\boxed{[v]_B = \begin{pmatrix}-2\\3\end{pmatrix}}$

22.- $B=\{v_1,v_2\}$ $B'=\{w_1,w_2\}$ $\mathbb{R}^2$

$u = 6w_2 - v_1 \rightarrow u = 6w_2 - 4w_1 + 2w_2 \rightarrow \boxed{u = 8w_2 - 4w_1}$

$\boxed{[u]_{B'} = \begin{pmatrix}-4\\8\end{pmatrix}}$

$u = 6w_2 - v_1 = 6\cdot\left(\frac{1}{2}v_1 + \frac{2}{3}v_2\right) - v_1 = 3v_1 + 4v_2 - v_1 \rightarrow \boxed{u = 2v_1 + 4v_2}$

$\boxed{[u]_B = \begin{pmatrix}2\\4\end{pmatrix}}$