1.-

a) $U_1 = \{(x,y,z,t) \mid x=t, y=z=0\}$

$U_1 = \{(x,y,z,t) \mid (t,0,0,t)\}$

$(t,0,0,t) = t\cdot(1,0,0,1) \Rightarrow \boxed{\begin{array}{l} B=\{(1,0,0,1)\} \\ \mathrm{Dim}(U_1)=\mathrm{Dim}\,B=1\end{array}}$

b) $U_2 = \{(x,y,z,t) \mid x+y+t=0=y+z\}$

$x+y+t=0;\ \boxed{y=-x-t}$

$y+z=0;\ -x-t+z=0;\ \boxed{z=x+t}$

$(x,-x-t,x+t,t) = (x,-x,x,0)+(0,-t,t,t) =$

$= x\cdot(1,-1,1,0)+t(0,-1,1,1)$

$\boxed{\begin{array}{l} B=\{(1,-1,1,0),(0,-1,1,1)\} \\ \mathrm{Dim}\,B=2\end{array}}$

2.-

a) $U_1=\{A\in M_{2\times2}(\mathbb{R}) \mid AB=BA\}$, donde $B=\begin{pmatrix}0&1\\1&1\end{pmatrix}$

Si $AB=BA$, $A=I_2$ o $A=B$.

$\left.\begin{array}{l}\cdot\ I\cdot B=B\cdot I\ \checkmark \\ \cdot\ B\cdot B=B\cdot B\ \checkmark\end{array}\right\}\ \boxed{B=\left\{\begin{pmatrix}1&0\\0&1\end{pmatrix},\begin{pmatrix}0&1\\1&1\end{pmatrix}\right\},\ \dim B=2}$

b) $U_2=\{A\in M_{2\times2}(\mathbb{R}) \mid MA$ es simétrica$\}$, donde $M=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$

$A=\begin{pmatrix}a&b\\c&d\end{pmatrix}$ $\quad M\cdot A=\begin{pmatrix}0&1\\-1&0\end{pmatrix}\cdot\begin{pmatrix}a&b\\c&d\end{pmatrix}=\begin{pmatrix}c&d\\-a&-b\end{pmatrix}$

· Para que $M\cdot A$ sea simétrica: $\boxed{d=-a}$

$M\cdot A=\begin{pmatrix}c&d\\d&-b\end{pmatrix} \Rightarrow b\cdot\begin{pmatrix}0&0\\0&-1\end{pmatrix}+c\cdot\begin{pmatrix}1&0\\0&0\end{pmatrix}+d\cdot\begin{pmatrix}0&1\\1&0\end{pmatrix}$

$\boxed{B=\left\{\begin{pmatrix}0&0\\0&-1\end{pmatrix},\begin{pmatrix}1&0\\0&0\end{pmatrix},\begin{pmatrix}0&1\\1&0\end{pmatrix}\right\},\ \mathrm{Dim}\,B=3}$

b) $U_2 = \{ A \in M_{2x2}(\mathbb{R}) \mid MA$ es simétrica $\}$, donde $M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$$M \cdot A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} c & d \\ -a & -b \end{pmatrix}$$

· Para que M·A sea simétrica $\Rightarrow M \cdot A = (MA)^t$:

$$\begin{pmatrix} c & d \\ -a & -b \end{pmatrix} = \begin{pmatrix} c & -a \\ d & -b \end{pmatrix} \longrightarrow \boxed{-a = d}$$

$$A = \begin{pmatrix} a & b \\ c & -a \end{pmatrix} \longrightarrow a \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + b \cdot \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\boxed{B = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}, \ \dim B = 3}$$

3.-

a) $U_1 = \{ A \in M_{nxn}(\mathbb{R}) \mid tr(A) = 0 \}$

~~si $|A|^t = 0$, entonces $A = 0$, así que $\dim U_1 = 0$~~

$tr(A) = \sum_{i=1}^{n} a_{ii}$ · Sea $A \in U_1$

4.- $B = \{ (1,1,1), (1,1,0) (1,0,0) \}$

a) $$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \xrightarrow{E_{32}(-1)} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{E_{21}(-1)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

· Para que sea base:

- Debe ser un conjunto generador
- Debe ser libre

Cumple las dos porque es equiv por filas a I_3, cuyas filas corresponden a los vectores de la base canónica C_3 así que es conjunto generador y libre porque no hay ninguno que se pueda expresar como comb. lineal de los otros.

Escaneado con CamScanner

b) P_{CB} , $C = C_3$ (base canónica de $\mathbb{R}^3$)

$C_3 = \{ \overset{e_1}{(1,0,0)}, \overset{e_2}{(0,1,0)}, \overset{e_3}{(0,0,1)} \}$

$B = \{ \overset{v_1}{(1,1,1)}, \overset{v_2}{(1,1,0)}, \overset{v_3}{(1,0,0)} \}$

$P_{CB} = \underbrace{(P_{BC})^{-1}}$ Más fácil de conseguir

$$P_{BC} = \begin{array}{c} \begin{matrix} v_1 & v_2 & v_3 \end{matrix} \\ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \end{array} \qquad P_{CB} = (P_{BC})^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}^{-1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow{E_{31}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{array}\right) \xrightarrow{E_{32}(-1)}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array}\right) \xrightarrow{E_{21}(-1)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array}\right)$$

$\underbrace{\hphantom{xxxx}}_{P_{CB}}$

$$\boxed{P_{CB} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{pmatrix}}$$

c) ¿$[v]_B$? $v = (2,0,-3)$

$$\begin{array}{c} \begin{matrix} v_1 & v_2 & v_3 & \end{matrix} \\ \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & -3 \end{array}\right) \end{array} \Big\} \quad \boxed{x_1 = -3} \quad \boxed{x_2 = 3} \quad \boxed{x_1 = 2}$$

$$\boxed{[v]_B = (-3,3,2)}$$

d) $P_{B'B} = P_{C_3B'} \cdot P_{BC_3}$ $\qquad B' = \{(1,-1,1), (1,1,-1), (-1,1,1)\}$

$$P_{C_3B'} = (P_{B'C})^{-1} = \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{E_{32}(1)} \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 & 1 \end{array}\right)$$

$$\xrightarrow{E_{21}(1)} \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 & 1 \end{array}\right) \xrightarrow[E_3(1/2)]{E_2(1/2)} \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & 1/2 \end{array}\right) \xrightarrow[E_{13}(1)]{E_{12}(-1)}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 1 & 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & 1/2 \end{array}\right) \qquad \boxed{P_{C_3B'} = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{pmatrix}}$$

$\boxed{5.-}$ $B = \{w_1, w_2\}$ $P_{CB} = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$

a) $P_{BC} = (P_{CB})^{-1}$

$$\left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{E_{21}(-1)} \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right) \xrightarrow{E_{12}(1)} \left(\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 \end{array}\right)$$

$$\boxed{P_{BC} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}}$$

b) $$P_{BC} = \begin{array}{c} \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \\ \begin{array}{cc} \uparrow & \uparrow \\ w_1 & w_2 \end{array} \end{array}$$

$w = (0,-1) + (1,1) \Rightarrow \boxed{w = (1,0)}$