1.-

$$A=\begin{pmatrix} 0 & 0 & 0 & 1 & -2 \\ 0 & 2 & 4 & -2 & 6 \\ -1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & -2 \end{pmatrix} \xrightarrow[E_2(1/2)]{E_{43}(1)} \begin{pmatrix} 0 & 0 & 0 & 1 & -2 \\ 0 & 1 & 2 & -1 & 3 \\ -1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & -1 \end{pmatrix} \xrightarrow{E_{13}}$$

$$\begin{pmatrix} -1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & -1 & 3 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 1 & 2 & 1 & -1 \end{pmatrix} \xrightarrow{E_{42}(-1)} \begin{pmatrix} -1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & -1 & 3 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 2 & -4 \end{pmatrix} \xrightarrow{E_{43}(-2)} \begin{pmatrix} -1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & -1 & 3 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow[E_1(-1)]{E_{23}(1)} \begin{pmatrix} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \checkmark$$

2.-

$$A=\begin{pmatrix} \alpha & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{pmatrix} \xrightarrow{E_{13}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ \alpha & 2 & 3 \end{pmatrix} \xrightarrow{E_{31}(-\alpha)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{pmatrix} \xrightarrow{E_{32}(-2)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix} \Rightarrow \boxed{rg(A)=3}$$

$$\cancel{E_{13}\cdot E_{31}(\alpha)\cdot E_{32}(-2)\cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}}$$

Transformación de I_3:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{E_{13}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \xrightarrow{E_{31}(-\alpha)} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -\alpha \end{pmatrix} \xrightarrow{E_{32}(-2)} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & -2 & -\alpha \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & -2 & -\alpha \end{array}\right) \xrightarrow{E_{23}(+2)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 & -3 & -2\alpha \\ 0 & 0 & -1 & 1 & -2 & -\alpha \end{array}\right) \xrightarrow{E_3(-1)}$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 & -3 & -2\alpha \\ 0 & 0 & 1 & -1 & 2 & \alpha \end{array}\right) \quad \boxed{A^{-1}=\begin{pmatrix} 0 & 0 & 1 \\ 2 & -3 & -2\alpha \\ -1 & 2 & \alpha \end{pmatrix}} \checkmark$$

Escaneado con CamScanner

3.-

$$A=\begin{pmatrix} 0 & -1 & 0 & -1 \\ 2 & -1 & 3 & 2 \\ -1 & 0 & -2 & -2 \\ 3 & -1 & 5 & 5 \end{pmatrix}$$

$$|A| = -(-1)\cdot\begin{vmatrix} 2 & 3 & 2 \\ -1 & -2 & -2 \\ 3 & 5 & 5 \end{vmatrix} - (-1)\cdot\begin{vmatrix} 2 & -1 & 3 \\ -1 & 0 & -2 \\ 3 & -1 & 5 \end{vmatrix} = 1\cdot\overset{(1)}{\begin{vmatrix} 2 & 3 & 2 \\ -1 & -2 & -2 \\ 3 & 5 & 5 \end{vmatrix}} +$$

$$1\cdot\overset{(2)}{\begin{vmatrix} 2 & -1 & 3 \\ -1 & 0 & -2 \\ 3 & -1 & 5 \end{vmatrix}} = -1 + 0 = \boxed{-1}$$

(2) $0+6+3-0-5-4 = 9-9 = \boxed{0}$

(1) $\cancel{-20} - 18 - 10 - (-12) - (-15) \cancel{-(-20)} = -28 + 27 = \boxed{-1}$

4.- $\alpha, \beta \in \mathbb{R}$ $\quad A=(a_{ij}) \in M_{3\times 3}(\mathbb{R})$

$$a_{ij}\begin{cases} \alpha & \text{si } i=j \\ \beta & \text{si } i\neq j \end{cases} \qquad A=\begin{pmatrix} \alpha & \beta & \beta \\ \beta & \alpha & \beta \\ \beta & \beta & \alpha \end{pmatrix}$$

$$|A| = \alpha\cdot\begin{vmatrix} \alpha & \beta \\ \beta & \alpha \end{vmatrix} - \beta\cdot\begin{vmatrix} \beta & \beta \\ \beta & \alpha \end{vmatrix} + \beta\cdot\begin{vmatrix} \beta & \alpha \\ \beta & \beta \end{vmatrix} =$$

$$= \alpha\cdot(\alpha^2-\beta^2) - \beta\cdot(\alpha\cdot\beta - \beta^2) + \beta\cdot(\beta^2 - \alpha\beta) =$$

$$= \alpha^3 - \alpha\beta^2 - \alpha\cdot\beta^2 + \beta^3 + \beta^3 - \alpha\cdot\beta^2 = \boxed{\alpha^3 - 3\alpha\beta^2 + 2\beta^3}$$

5.- $\alpha \in \mathbb{R}$ $\quad T=\{(\alpha,1,1),(1,\alpha,1),(1,1,\alpha)\}$

$$\begin{pmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{pmatrix} \xrightarrow[E_{31}(-1/\alpha)]{E_{21}(-1/\alpha)} \begin{pmatrix} \alpha & 1 & 1 \\ 0 & \frac{-1+\alpha^2}{\alpha} & \frac{-1+\alpha}{\alpha} \\ 0 & \frac{-1+\alpha}{\alpha} & \frac{-1+\alpha^2}{\alpha} \end{pmatrix} \xrightarrow{E_{32}(-1/\alpha+1)} \begin{pmatrix} \alpha & 1 & 1 \\ 0 & \frac{-1+\alpha^2}{\alpha} & \frac{-1+\alpha}{\alpha} \\ 0 & 0 & \frac{\alpha^2+\alpha-2}{\alpha+1} \end{pmatrix}$$

- Para que sea indpte.: $\frac{\alpha^2+\alpha-2}{\alpha+1} \neq 0$

$$\frac{\alpha^2+\alpha-2}{\alpha+1} = 0;\ \alpha^2+\alpha-2=0;\ \alpha = \frac{-1\pm\sqrt{1-4\cdot1\cdot(-2)}}{2\cdot 1} =$$

$$= \frac{-1\pm\sqrt{9}}{2} \begin{cases} \alpha_1 = \frac{-1+3}{2} = \frac{2}{2} = \boxed{1} \\ \alpha_2 = \frac{-1-3}{2} = \frac{-4}{2} = \boxed{-2} \end{cases}$$

Escaneado con CamScanner

9.-

a) $\begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 1 & 3 \\ 3 & 6 & 1 & 4 \end{pmatrix} \xrightarrow[E_{31}(-3)]{E_{21}(-2)} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -2 & -2 \end{pmatrix} \xrightarrow{E_{32}(-1)} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\boxed{x_4 = t}$ $\quad -x_3 - t = 0;\ \boxed{x_3 = -t}$

$\boxed{x_2 = s}$

$x_1 + 2s + x_3 + 2t = 0;\ x_1 + 2s - t + 2t = 0;\ \boxed{x_1 = -2s - t}$

$\boxed{(x, y, z, t) = (-2s - t, s, -t, t)} \longrightarrow s \cdot (-2, 1, 0, 0) + t \cdot (-1, 0, -1, 1)$

$\boxed{S = \{(-2, 1, 0, 0), (-1, 0, -1, 1)\}}$ ✓

b) $\begin{pmatrix} 2 & 2 & 2 \\ 2 & 5 & 7 \\ 3 & 6 & 6 \end{pmatrix} \xrightarrow[E_3(1/3)]{E_{21}(-1)} \begin{pmatrix} 2 & 2 & 2 \\ 0 & 3 & 5 \\ 1 & 2 & 2 \end{pmatrix} \xrightarrow{E_1(1/2)} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 5 \\ 1 & 2 & 2 \end{pmatrix} \xrightarrow{E_{31}(-1)} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 5 \\ 0 & 1 & 1 \end{pmatrix}$

$\xrightarrow{E_{32}(-1/3)} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & -2/3 \end{pmatrix} \longrightarrow -2/3\, x_3 = 0 \longrightarrow \boxed{x_3 = 0}$

$3x_2 + 5 \cdot 0 = 0;\ \boxed{x_2 = 0}$

$x_1 + 0 + 0 = 0;\ \boxed{x_1 = 0}$

$\boxed{S(x, y, z) = \{(0, 0, 0)\}}$ ✓

10.-

a) $\left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 2 & 4 & 1 & 3 & 4 \\ 3 & 6 & 1 & 4 & 5 \end{array}\right) \xrightarrow[E_{31}(-3)]{E_{21}(-2)} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & -1 & -1 & -2 \\ 0 & 0 & -2 & -2 & -4 \end{array}\right) \xrightarrow{E_{32}(-2)}$

$\longrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & -1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$ $\quad \boxed{t = \beta}$ $\quad \boxed{y = \alpha}$

$+z + \beta = 2;\ \boxed{z = 2 - \beta}$

$x + 2\alpha + (2 - \beta) + 2\beta = 3;$

$x + 2\alpha + \beta + 2 = 3;\ \boxed{x = 1 - 2\alpha - \beta}$

Escaneado con CamScanner

$$(1-2\alpha-\beta,\ \alpha,\ 2-\beta,\ \beta) = (1,0,2,0) + \alpha(-2,1,0,0) + \beta(-1,0,-1,1)$$

$$\boxed{S = (1,0,2,0) + \{(-2,1,0,0),\ (-1,0,-1,1)\}}$$

Escaneado con CamScanner

b)

$$\left(\begin{array}{ccccc|c} 1 & 1 & 2 & 2 & 1 & 1 \\ 2 & 2 & 4 & 4 & 3 & 1 \\ 2 & 2 & 4 & 4 & 2 & 2 \\ 3 & 5 & 8 & 6 & 5 & 3 \end{array}\right) \xrightarrow[E_{41}(-3)]{E_{21}(-2),\ E_{31}(-2)} \left(\begin{array}{ccccc|c} 1 & 1 & 2 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 2 & 0 \end{array}\right) \xrightarrow{E_{24}} \left(\begin{array}{ccccc|c} 1 & 1 & 2 & 2 & 1 & 1 \\ 0 & 2 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{array}\right)$$

$$\xrightarrow{E_{34}} \left(\begin{array}{ccccc|c} 1 & 1 & 2 & 2 & 1 & 1 \\ 0 & 2 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$\boxed{u=-1}$ $\quad 2y+2s-2=0;$

$\boxed{t=t}$ $\quad ; \boxed{y = 1-s}$

$\boxed{z=s}$

$x \cancel{+1} - s + 2s + 2t \cancel{-1} = 1;$

$\cancel{x + 1(1-2s) + 2s + 2t - 1 = 1;\ x + 2t + 1 = 1;}$ $\quad \boxed{x = 1 - s - 2t}$

$\cancel{\boxed{x = -2t+1}}$

$(1-s-2t,\ 1-s,\ s,\ t,\ -1) \Rightarrow \boxed{S = (1, 1, 0, 0, -1) + \{(-2, 0, 0, 1, 0), (-1, -1, 1, 0, 0)}$

$\boxed{11.-}$ $(3, 1, \lambda)$ $A \in M_{3\times3}(\mathbb{R})$ $\cancel{Ax=b}$ $Ax=b$

$\mathrm{Ker}(A) = \langle \{(1,1,0), (0,1,-1)\} \rangle$

$$A \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = b \qquad \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = v$$

$$(3, 1, \gamma) = (2,1,2) + \lambda(1,1,0) + \mu(0,1,-1)$$

$$3 = 2+\lambda \; ; \; \boxed{\lambda = 1}$$

$$1 = 1 + \lambda + \mu \Rightarrow \boxed{\mu = -1}$$

$$\gamma = 2 - \mu \Rightarrow \boxed{\gamma = 3} \longrightarrow \boxed{v = (3, 1, 3)}$$