

## A Note on Density-Dependent Natural Mortality in Catch Limit Algorithm Trials

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### ABSTRACT

The relationships between MSYL and MSYR and the parameters that determine the extent of density-dependence when density-dependence acts on natural mortality are explored. The relationship between MSYR and the parameters that determine the extent of density-dependence is non-linear while the value for MSYL is determined primarily by the parameter that determines the extent of compensation. There are combinations of MSYR and MSYL, particularly when MSYR and MSYL are defined in terms of the 1+ component of the population, which cannot be achieved. The set of MSYR and MSYL values compatible with density-dependence on natural mortality will be accounted for in the trials used to evaluate the performance of variants of the Catch Limit Algorithm.

### INTRODUCTION

The trials used to evaluate the performance of the IWC's Catch Limit Algorithm (CLA) are based on the assumption that density-dependence impacts fecundity (i.e. the birth rate), i.e.:

$$b_t = BN_t \{1 + A(1 - (\tilde{N}_t / \tilde{K})^z)\} \quad (1)$$

where  $b_t$  is the birth rate in year  $t$ ,  $B$  is the fecundity at carrying capacity,  $N_t$  is the number of mature females at the start of year  $t$ ,  $\tilde{N}_t$  is the number of animals for the population component on which density-dependence acts at the start of year  $t$ ,  $A$  is the resilience parameter,  $z$  is the degree of compensation, and  $\tilde{K}$  is carrying capacity (in terms of the population component on which density-dependence acts).

Punt (2015) introduced a way to implement density-dependence on the rate of natural mortality, i.e.:

$$M_t = M_\infty \frac{1 + A(\tilde{N}_t / \tilde{K})^z}{1 + A} \quad (2)$$

where  $M_t$  is the rate of natural mortality during year  $t$  and  $M_\infty$  is the rate of natural mortality at carrying capacity<sup>1</sup>.

Punt (2015) provided yield curves when MSYR and MSYL pertain to the mature female and 1+ components (MSYR<sub>mat</sub> and MSYR<sub>1+</sub> and MSYL<sub>mat</sub> and MSYL<sub>1+</sub> respectively) when density-dependence acts on fecundity and natural mortality. The population component that drives the extent of density-dependence was set to that corresponding to MSYR and MSYL. Punt (2015) found the relationship between MSYR<sub>mat</sub>/MSYL<sub>1+</sub> and MSYR<sub>mat</sub> was linear, but depended on whether density-dependence acted on fecundity or on natural mortality.

We have developed code to find the values for  $A$  and  $z$  when density-dependence acts on natural mortality. The paper extends the work of Punt (2015) by showing the relationships

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<sup>1</sup> Note that  $A$  and  $z$  in Equations 1 and 2 have different meanings.

between  $A$  and  $z$  and  $MSYR$  and  $MSYL$  for density-dependent natural mortality as well as for the ratio of  $MSYR_{mat}/MSYR_{1+}$  and  $MSYR_{mat}$ . The results of these analyses are interpreted in the context of the trials that will be used to evaluate the Norwegian tuning of the CLA.

## RESULTS AND DISCUSSION

Figure 1 shows the relationship between  $A$  and  $z$  and  $MSYR_{mat}$  and  $MSYL_{mat}$  and that between  $A$  and  $z$  and  $MSYR_{1+}$  and  $MSYL_{1+}$ . This plot highlights the non-linearity between  $MSYR$  and  $A/z$  and how  $MSYL$  is determined primarily by  $z$ . The maximum  $MSYL$  that can be obtained given the model for the relationship between density dependence and natural mortality is 0.7 while the lowest value is less than 0.1, but only for low values for  $MSYR$  (1% or lower). In addition, the maximum  $MSYR_{mat}$  is approximately 0.07, while values for  $MSYR_{1+}$  larger than 0.045 appear to be incompatible with the model assumed for density-dependent natural mortality.

Figure 2 shows  $MSYR_{mat}/MSYR_{1+}$  as a function of  $A$  and  $z$  and as a function of  $MSYR_{mat}$  and  $MSYL_{mat}$ . This plot confirms that there are combinations of  $MSYR_{mat}$  and  $MSYL_{mat}$  that are not comparable with Equation 2. It also shows that the relationship between  $MSYR_{1+}$  and  $MSYL_{mat}$  is linear for most of the range of  $MSYR_{mat}$  ( $MSYR_{1+} \sim 1.45 MSYR_{mat}$ ).

The results of this paper indicate that unlike when density-dependence acts on fecundity, there are combinations of  $MSYR$  and  $MSYL$  that are not compatible with the model for when density dependence on natural mortality. This reduces the set of scenarios that can be considered when evaluating the Norwegian proposal for a revision to the CLA.

## ACKNOWLEDGEMENTS

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## REFERENCES

Punt, A.E. 2015. A Quick Investigation of the Relationship between  $MSYR_{mat}$  and  $MSYR_{1+}$  based on a non-individual-based model. *J. Cetacean Res. Manage. (Suppl.)* 16:00-00.

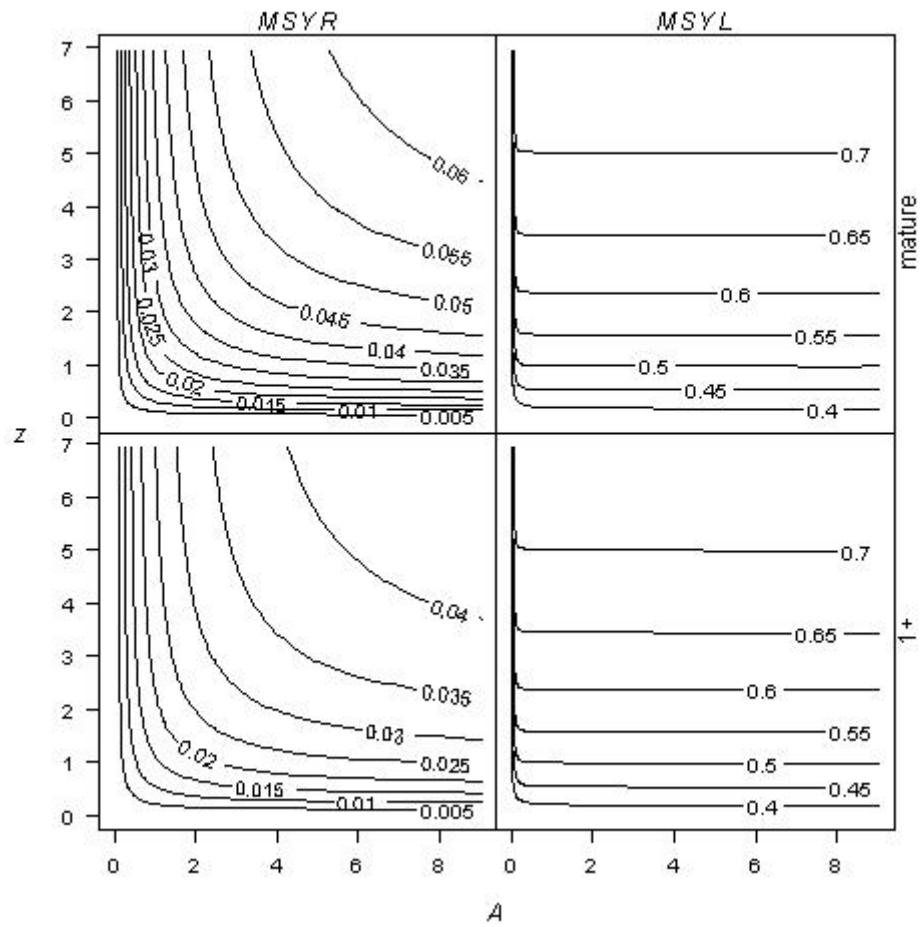


Figure 1. Relationships between  $MSYR$  and  $MSYL$  and  $A$  and  $z$  when density dependence acts on natural mortality. Results are shown when  $MSYR$  is defined in terms of the mature female component of the population (upper panels) and when it is defined in terms of the 1+ component of the population (lower panels).

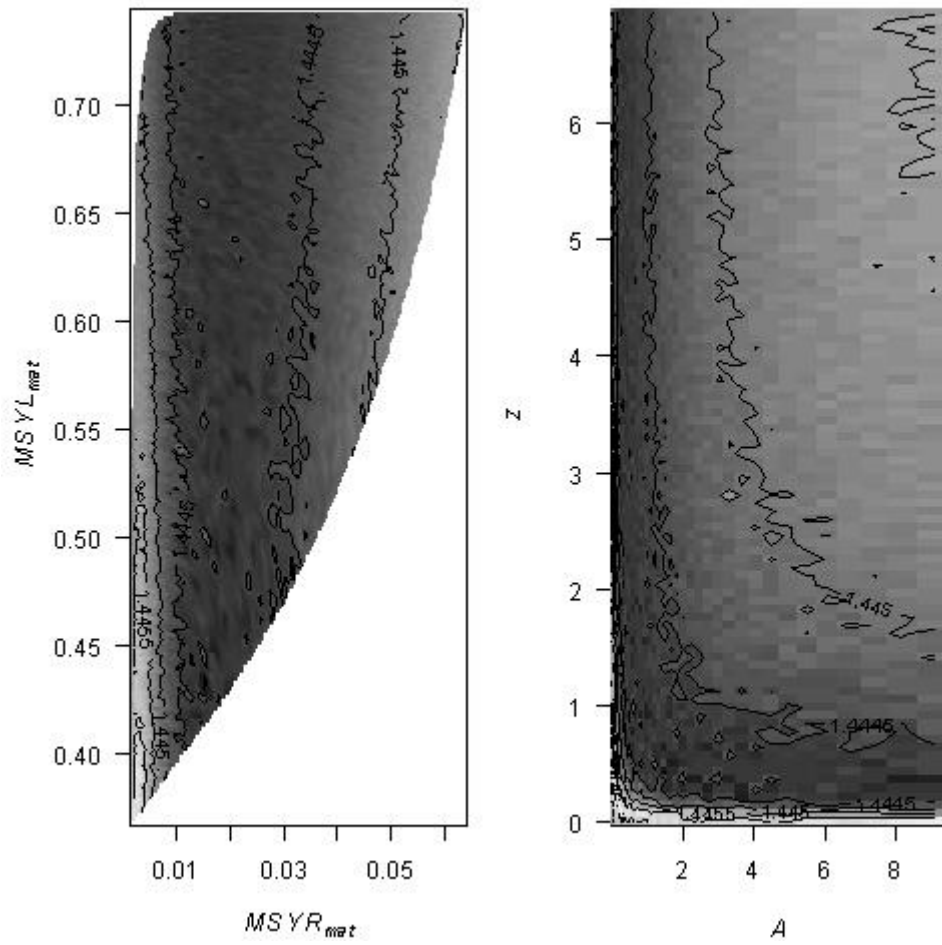


Figure 2. Contour plot of  $MSYR_{mat}/MSYR_{1+}$  as a function of  $MSYR_{mat}$  and  $MSYL_{mat}$  (left panel) and  $A$  and  $z$  (right panel) when density-dependence acts on natural mortality.