

# Competitive Programming Library

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# 1 Algorithms

## 1.1 Count inversions

**Description:** Count the number of inversions when transforming the vector  $l$  in the vector  $r$ , which is also equivalent to the minimum number of swaps required.

**Usage:** If no  $r$  vector is provided it considers  $r$  as the sorted vector, if there is no such way to turn  $l$  into  $r$  using swaps then  $-1$  is returned

**Time:**  $O(N \log N)$

```
#pragma once
#include "../Contest/template.cpp"
template <typename T>
ll countInversions(vector<T> l, vector<T> r = {}) {
    if (!len(r)) r = l, sort(all(r));
    int n = len(l);
    vi v(n), bit(n);
    vector<pair<T, int>> w;
    rep(i, 0, n) w.emplace(r[i], i + 1);
    sort(all(w));
    rep(i, 0, n) {
        auto it = lower_bound(all(w), make_pair(l[i], 0));
        if (it == w.end() or it->first != l[i]) return -1; // impossible
        v[i] = it->second;
        it->second = -1;
    }
    ll ans = 0;
    rrep(i, n - 1, 0 - 1) {
        for (int j = v[i] - 1; j; j -= j & -j) ans += bit[j];
        for (int j = v[i]; j < n; j += j & -j) bit[j]++;
    }
    return ans;
}
```

## 1.2 Ternary search (integer)

**Description:** Given a unimodal function  $f$  defined between the integers  $l$  and  $r$  finds an  $x$  such that  $f(x)$  is maximum/minimum.

**Usage:** Just pass the range  $l, r$  of the function you are interested, the function that receives an integer and if you want the maximum value use the `cmp = greater<ll>()`, otherwise `less<ll>()`.

**Time:**  $O(\log r - l + 1)$

**Memory:**  $O(1)$

```
#include "../Contest/template.cpp"
template <auto cmp = greater<ll>()>
ll ternary_search(ll l, ll r, function<ll(ll)> f) {
    static const ll eps = 3;
    while (r - l >= eps) {
        ll m1 = l + (r - l) / 3;
        ll m2 = r - (r - l) / 3;
        if (cmp(f(m1), f(m2)))
            r = m2;
        else
```

```
            l = m1;
    }
    rep(i, l, r + 1) if (cmp(f(i), f(l))) l = i;
    return l;
}
```

## 1.3 Ternary search (real)

```
#include "../Contest/template.cpp"
template <auto cmp = greater<ld>()>
ld ternarySearch(ld l, ld r, function<ld(ld)> f, const ld eps = 1e-9) {
    while (r - l >= eps) {
        ld m1 = l + (r - l) / 3;
        ld m2 = r - (r - l) / 3;
        if (cmp(f(m1), f(m2)))
            r = m2;
        else
            l = m1;
    }
    return l;
}
```

# 2 Combinatorics

## 2.1 Process all partitions of a set

**Description:** generate every distinct group of a set that contains elements from 0 to  $N-1$ , and pass it to the given function "process". If  $N$  is 4 the sets generated would be :

$\{\{0,1,2,3\}\}$   $\{\{0,1,2\},\{3\}\}$   $\{\{0,1,3\},\{2\}\}$   $\{\{0,1\},\{2,3\}\}$   $\{\{0,1\},\{2\},\{3\}\}$   $\{\{0,2,3\},\{1\}\}$   
 $\{\{0,2\},\{1,3\}\}$   $\{\{0,2\},\{1\},\{3\}\}$   $\{\{0,3\},\{1,2\}\}$   $\{\{0\},\{1,2,3\}\}$   $\{\{0\},\{1,2\},\{3\}\}$   
 $\{\{0,3\},\{1\},\{2\}\}$   $\{\{0\},\{1,3\},\{2\}\}$   $\{\{0\},\{1\},\{2,3\}\}$   $\{\{0\},\{1\},\{2\},\{3\}\}$

**Time:**  $O(B(N))$ , Bell Number of  $N$

**Memory:**  $O(N)$

```
#include "../Contest/template.cpp"
void process_all_partitions_of_a_set(
    const int N, const function<void(const vi2d &)> process) {
    vi2d groups;
    groups.reserve(N);
    function<void(int)> _dfs = [&](int idx) {
        if (idx == N) {
            process(groups);
            return;
        }
        rep(i, 0, len(groups)) {
            groups[i].emplace_back(idx);
            _dfs(idx + 1);
            groups[i].pop_back();
        }
        groups.emplace_back({idx});
        _dfs(idx + 1);
    };
    _dfs(0);
}
```

```

    groups.ppb();
};
_dfs(0);
}

```

## 3 Contest

### 3.1 bash config

---

```

#copy first argument to clipborad ! ONLY WORK ON XORG !
alias clip="xclip -sel clip"
# compile the $1 parameter, if a $2 is provided
# the name will be the the binary output, if
# none is provided the binary name will be
# 'a.out'
comp() {
    echo ">> COMPILING $1 <<" 1>&2
    if [ $# -gt 1 ]; then
        outfile="$2"
    else
        outfile="a.out"
    fi
    time g++ -std=c++23 \
        -O2 \
        -g3 \
        -Wall \
        -fsanitize=address,undefined \
        -fno-sanitize-recover \
        -D LOCAL \
        -o "${outfile}" \
        "$1"
    if [ $? -ne 0 ]; then
        echo ">> FAILED <<" 1>&2
        return 1
    fi
    echo ">> DONE << " 1>&2
}
# run the binary given in $1, if none is
# given it will try to run the 'a.out'
# binary
run() {
    to_run=./a.out
    if [ -n "$1" ]; then
        to_run="$1"
    fi
    time $to_run
}
# just comp and run your cpp file
# accpets <inl >out and everything else
comprun() {
    comp "$1" "a" && run ./a ${@:2}
}
testall() {

```

```

comp "$1" generator
comp "$2" brute
comp "$3" main
input_counter=1
while true; do
    echo "$input_counter"
    run ./generator >input
    run ./main <input >main_output.txt
    run ./brute <input >brute_output.txt
    diff brute_output.txt main_output.txt
    if [ $? -ne 0 ]; then
        echo "Outputs differ at input $input_counter"
        echo "Brute file output:"
        cat brute_output.txt
        echo "Main file output:"
        cat main_output.txt
        echo "input used: "
        cat input
        break
    fi
    ((input_counter++))
done
}
touch_macro() {
    cat "$1"/template.cpp >"$2"
    cat "$1"/run.cpp >>"$2"
    cp "$1"/debug.cpp .
}
# Creates a contest with hame $2
# Copies the macro and debug file from $1
# Already creates files a...z .cpp and .py
prepare_contest() {
    mkdir "$2"
    cd "$2"
    for i in {a..z}; do
        touch_macro $1 $i.cpp
    done
}
get_file_hash() {
    local hash=$(cpp -dD -P -fpreprocessed "$1" | tr -d '[:space:]' |
md5sum | cut -c-6)
    echo "$hash"
}

```

### 3.2 debug

---

```

template <typename T>
concept Printable = requires(T t) {
    { std::cout << t } -> std::same_as<std::ostream &>;
};
template <Printable T>
void __print(const T &x) {

```

```

    cerr << x;
}
template <size_t T>
void __print(const bitset<T> &x) {
    cerr << x;
}
template <typename A, typename B>
void __print(const pair<A, B> &p);
template <typename... A>
void __print(const tuple<A...> &t);
template <typename T>
void __print(stack<T> s);
template <typename T>
void __print(queue<T> q);
template <typename T, typename... U>
void __print(priority_queue<T, U...> q);
template <typename A>
void __print(const A &x) {
    bool first = true;
    cerr << '{';
    for (const auto &i : x) {
        cerr << (first ? "" : ","), __print(i);
        first = false;
    }
    cerr << '}';
}
template <typename A, typename B>
void __print(const pair<A, B> &p) {
    cerr << '(';
    __print(p.first);
    cerr << ',';
    __print(p.second);
    cerr << ')';
}
template <typename... A>
void __print(const tuple<A...> &t) {
    bool first = true;
    cerr << '(';
    apply(
        [&first](const auto &...args) {
            ((cerr << (first ? "" : ","), __print(args), first = false),
            ...);
        },
        t);
    cerr << ')';
}
template <typename T>
void __print(stack<T> s) {
    vector<T> debugVector;
    while (!s.empty()) {
        T t = s.top();
        debugVector.push_back(t);
        s.pop();
    }
    reverse(debugVector.begin(), debugVector.end());
    __print(debugVector);
}

```

```

}
template <typename T>
void __print(queue<T> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.front();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}
template <typename T, typename... U>
void __print(priority_queue<T, U...> q) {
    vector<T> debugVector;
    while (!q.empty()) {
        T t = q.top();
        debugVector.push_back(t);
        q.pop();
    }
    __print(debugVector);
}
void __print() { cerr << "]\n"; }
template <typename Head, typename... Tail>
void __print(const Head &H, const Tail &...T) {
    __print(H);
    if (sizeof...(T)) cerr << ", ";
    __print(T...);
}
#define dbg(x...) \
    cerr << "[" << #x << "]" = ["; \
    __print(x)

```

### 3.3 run

```

void __init();
void __run();
void __pre_run();
int32_t main() {
#ifdef LOCAL
    fastio;
#endif
    __init();
    int T = 1;
    cin >> T;
    rep(t, 0, T) {
        dbg(t);
        __pre_run();
        __run();
    }
}
/*
*
*

```

```

*
*
* */
void __init() {}
void __pre_run() {}
void __run() {}

```

### 3.4 short-template

```

#include <bits/stdc++.h>
using namespace std;
#define fastio
    ios_base::sync_with_stdio(0); \
    cin.tie(0);
void run() {}
int32_t main(void) {
    fastio;
    int t;
    t = 1;
    // cin >> t;
    while (t-->) run();
}

```

### 3.5 simple-template

```

#include <bits/stdc++.h>
using namespace std;

```

### 3.6 template

```

#pragma once
#include <bits/stdc++.h>
using namespace std;
#ifdef LOCAL
#include "debug.cpp"
#else
#define dbg(...)
#endif
#define fastio
    ios_base::sync_with_stdio(0); \
    cin.tie(0);
#define all(j) j.begin(), j.end()
#define rall(j) j.rbegin(), j.rend()
#define len(j) (int)j.size()
#define rep(i, a, b) \
    for (common_type_t<decltype(a), decltype(b)> i = (a); i < (b); i++)
#define rrep(i, a, b) \
    for (common_type_t<decltype(a), decltype(b)> i = (a); i > (b); i--)
#define repn(i, b) for (auto i = 0; i < (b); i++)

```

```

#define trav(xi, xs) for (auto &xi : xs)
#define rtrav(xi, xs) for (auto &xi : ranges::views::reverse(xs))
#define loop while (1)
using ll = long long;
#define endl '\n'
#define pb push_back
#define pf push_front
#define ppb pop_back
#define ppf pop_front
#define eb emplace_back
#define ef emplace_back
#define lb lower_bound
#define ub upper_bound
#define fi first
#define se second
#define emp emplace
#define ins insert
#define divc(a, b) ((a) + (b) - 1ll) / (b)
using str = string;
using ull = unsigned long long;
using ld = long double;
using vll = vector<ll>;
using pll = pair<ll, ll>;
using vll2d = vector<vll>;
using vi = vector<int>;
using vi2d = vector<vi>;
using pii = pair<int, int>;
using vpil = vector<pii>;
using vc = vector<char>;
using vs = vector<str>;
#define TT template <typename T>
#define TTU template <typename T, typename U>
TTU using umap = unordered_map<T, U>;
TT using pqmn = priority_queue<T, vector<T>, greater<T>>;
TT using pqmx = priority_queue<T, vector<T>>;
TTU inline bool chmax(T &a, U const &b) { return (a < b ? a = b, 1 : 0); }
TTU inline bool chmin(T &a, U const &b) { return (a > b ? a = b, 1 : 0); }
// read vector
// TODO: abstract this to any container.
TT std::istream &operator>>(std::istream &is, std::vector<T> &vec) {
    for (auto &element : vec) {
        is >> element;
    }
    return is;
}
// print vector
// TODO: abstract this to any container.
TT ostream &operator<<(ostream &os, vector<T> &xs) {
    rep(i, os.iword(0), xs.size()) os << xs[i] << (i == xs.size() ? "" : "
");
    os.iword(0) = 0;
    return os;
}
// print multiset

```



```
// TODO: abstract this to any container.
TT ostream &operator<<(ostream &os, multiset<T> &xs) {
    for (auto it = xs.begin(); it != xs.end(); it++)
        os << *it << (next(it) == end(xs) ? "" : " ");
    return os;
}
// sum a vector, using the default constructor as initial value
// TODO: abstract this to any container.
TT T sumList(vector<T> &xs) { return accumulate(all(xs), T()); }
/*
 * verify if a `predicate` is valid for
 * values in range [l, r]
 *
 * TODO: put a type in predicate ??
 */
// bool all_of_in_range(T l, T r, auto predicate) {
//     return ranges::all_of(views::iota(l, r + 1),
//                             [&](T v) { return predicate(v); });
// }
// checks if a value v is the interval [l, r]
TT bool inline between(T v, T l, T r) { return clamp(v, l, r) == v; }
```

### 3.7 vim config

```
set sta nu rnu sc cindent noswapfile
set ts=2 sw=2
set bg=dark ruler clipboard=unnamed,unnamedplus, timeoutlen=100
colorscheme default
syntax on

" Takes the hash of the selected text and put
" in the vim clipboard
function! HashSelectedText()
    " Yank the selected text to the unnamed register
    normal! gvy
    " Use the system() function to call sha256sum with the yanked text
    let l:hash = system('echo ' . shellescape(@@) . ' | sha256sum')
    " Yank the hash into Vim's unnamed register
    let @" = l:hash
endfunction
```

## 4 Data Structures

### 4.1 2D Segment Tree

#### 4.1.1 Point update query sum

```
#include "../Contest/template.cpp"
template <typename T, auto op>
struct SegmentTree2D {
    int h, w;
    vector<vector<T>> t;
```

```
SegmentTree2D(const vector<vector<T>> &a)
    : h(a.size()), w(a.back().size()), t(h * 4, vector<T>(w * 4)) {
    build_x(1, 0, h - 1, a);
}
void build_y(int vx, int lx, int rx, int vy, int ly, int ry,
             const vector<vector<T>> &a) {
    if (ly == ry) {
        if (lx == rx)
            t[vx][vy] = a[lx][ly];
        else
            t[vx][vy] = op(t[vx * 2][vy], t[vx * 2 + 1][vy]);
    } else {
        int my = (ly + ry) / 2;
        build_y(vx, lx, rx, vy * 2, ly, my, a);
        build_y(vx, lx, rx, vy * 2 + 1, my + 1, ry, a);
        t[vx][vy] = op(t[vx][vy * 2], t[vx][vy * 2 + 1]);
    }
}
void build_x(int vx, int lx, int rx, const vector<vector<T>> &a) {
    if (lx != rx) {
        int mx = (lx + rx) / 2;
        build_x(vx * 2, lx, mx, a);
        build_x(vx * 2 + 1, mx + 1, rx, a);
    }
    build_y(vx, lx, rx, 1, 0, w - 1, a);
}
T query_y(int vx, int vy, int tly, int try_, int ly, int ry) {
    if (ly > ry) return 0;
    if (ly == tly && try_ == ry) return t[vx][vy];
    int tmy = (tly + try_) / 2;
    return op(query_y(vx, vy * 2, tly, tmy, ly, min(ry, tmy)),
              query_y(vx, vy * 2 + 1, tmy + 1, try_, max(ly, tmy + 1),
                      ry));
}
T query_x(int vx, int tlx, int trx, int lx, int rx, int ly, int ry) {
    if (lx > rx) return 0;
    if (lx == tlx && trx == rx) return query_y(vx, 1, 0, w - 1, ly, ry);
}
int tmx = (tlx + trx) / 2;
return op(
    query_x(vx * 2, tlx, tmx, lx, min(rx, tmx), ly, ry),
    query_x(vx * 2 + 1, tmx + 1, trx, max(lx, tmx + 1), rx, ly, ry)
));
}
void update_y(int vx, int lx, int rx, int vy, int ly, int ry, int x,
              int y,
              int new_val) {
    if (ly == ry) {
        if (lx == rx)
            t[vx][vy] = new_val;
        else
            t[vx][vy] = op(t[vx * 2][vy], t[vx * 2 + 1][vy]);
    } else {
        int my = (ly + ry) / 2;
```

```

        if (y <= my)
            update_y(vx, lx, rx, vy * 2, ly, my, x, y, new_val);
        else
            update_y(vx, lx, rx, vy * 2 + 1, my + 1, ry, x, y, new_val);
    };
    t[vx][vy] = op(t[vx][vy * 2], t[vx][vy * 2 + 1]);
}

void update_x(int vx, int lx, int rx, int x, int y, T new_val) {
    if (lx != rx) {
        int mx = (lx + rx) / 2;
        if (x <= mx)
            update_x(vx * 2, lx, mx, x, y, new_val);
        else
            update_x(vx * 2 + 1, mx + 1, rx, x, y, new_val);
    }
    update_y(vx, lx, rx, 1, 0, w - 1, x, y, new_val);
}

T query(int lx, int rx, int ly, int ry) {
    return query_x(1, 0, h - 1, lx, rx, ly, ry);
}
};

```

## 4.2 SQRT decomposition

### 4.2.1 two-sequence-queries

```

using ll = long long;
const ll MOD = 998244353;
inline ll sum(const ll a, const ll b) { return (a + b) % MOD; }
ll sub(const ll a, const ll b) { return (a - b + MOD) % MOD; }
inline ll mul(const ll a, const ll b) { return (a * b) % MOD; }
struct SqrtDecomposition {
    struct t_sqrt {
        int l, r;
        ll x, y;
        ll prod;
        ll sum_as, sum_bs;
        t_sqrt() {
            l = numeric_limits<int>::max();
            r = numeric_limits<int>::min();
            x = y = prod = sum_as = sum_bs = 0;
        }
    };
};

int sqrtLen;
vector<t_sqrt> blocks;
vector<ll> as, bs;
SqrtDecomposition(const vector<ll> &as_, const vector<ll> &bs_) {
    int n = as_.size();
    sqrtLen = (int)sqrt(n + .0) + 1;
    blocks.resize(sqrtLen + 6.66);
    as = as_;
}

```

```

    bs = bs_;
    for (int i = 0; i < n; i++) {
        auto &bi = blocks[i / sqrtLen];
        bi.l = min(bi.l, i);
        bi.r = max(bi.r, i);
        bi.sum_as = sum(bi.sum_as, as[i]);
        bi.sum_bs = sum(bi.sum_bs, bs[i]);
        bi.prod = sum(bi.prod, mul(as[i], bs[i]));
    }
}

// adds x to a[i], and y to b[i], in range [l, r]
void update(int l, int r, ll x, ll y) {
    auto apply1 = [&](int idx, ll x, ll y) -> void {
        auto &block = blocks[idx / sqrtLen];
        block.prod = sub(block.prod, mul(as[idx], bs[idx]));
        block.sum_as = sub(block.sum_as, as[idx]);
        block.sum_bs = sub(block.sum_bs, bs[idx]);
        as[idx] = sum(as[idx], x);
        bs[idx] = sum(bs[idx], y);
        block.prod = sum(block.prod, as[idx] * bs[idx]);
        block.sum_as = sum(block.sum_as, as[idx]);
        block.sum_bs = sum(block.sum_bs, bs[idx]);
    };
    auto apply2 = [&](int idx, ll x, ll y) -> void {
        blocks[idx].x = sum(blocks[idx].x, x);
        blocks[idx].y = sum(blocks[idx].y, y);
    };
    int cl = l / sqrtLen, cr = r / sqrtLen;
    if (cl == cr) {
        for (int i = l; i <= r; i++) {
            apply1(i, x, y);
        }
    } else {
        for (int i = l; i <= (cl + 1) * sqrtLen - 1; i++) {
            apply1(i, x, y);
        }
        for (int i = cl + 1; i <= cr - 1; i++) {
            apply2(i, x, y);
        }
        for (int i = cr * sqrtLen; i <= r; i++) {
            apply1(i, x, y);
        }
    }
}

// sum of a[i]*b[i] in range [l, r]
ll query(int l, int r) {
    auto eval1 = [&](int idx) -> ll {
        auto &block = blocks[idx / sqrtLen];
        return mul(sum(as[idx], +block.x), sum(bs[idx], block.y));
    };
}

```

```

    auto eval2 = [&](int idx) -> ll {
        auto &block = blocks[idx];
        ll ret = 0;
        ret = sum(
            ret, mul(mul(block.x, block.y), sum(sub(block.r, block.l),
1)))));
        ret = sum(ret, block.prod);
        ret = sum(ret, block.y * block.sum_as);
        ret = sum(ret, block.x * block.sum_bs);
        return ret;
    };
    ll ret = 0;
    int cl = l / sqrtLen, cr = r / sqrtLen;
    if (cl == cr) {
        for (int i = l; i <= r; i++) {
            ret = sum(ret, eval1(i));
        }
    } else {
        for (int i = l; i <= (cl + 1) * sqrtLen - 1; i++) {
            ret = sum(eval1(i), ret);
        }
        for (int i = cl + 1; i <= cr - 1; i++) {
            ret = sum(ret, eval2(i));
        }
        for (int i = cr * sqrtLen; i <= r; i++) {
            ret = sum(ret, eval1(i));
        }
    }
    return ret;
};
}

```

## 4.3 Segment Tree Point Update Range Query (bottom-up)

### 4.3.1 Query GCD

```

using ll = long long;
struct Node {
    ll value;
    bool undef;
    Node() : value(1), undef(1) {}; // Neutral element
    Node(ll v) : value(v), undef(0) {};
};
inline Node combine(const Node &nl, const Node &nr) {
    if (nl.undef) return nr;
    if (nr.undef) return nl;
    Node m;
    m.value = gcd(nl.value, nr.value);
    m.undef = false;
    return m;
}
template <typename T = Node, auto F = combine>
struct SegTree {

```

```

    int n;
    vector<T> st;
    SegTree(int _n) : n(_n), st(n << 1) {}
    void assign(int p, const T &k) {
        for (st[p += n] = k; p >= 1; p >>= 1) st[p] = F(st[p << 1], st[p << 1 |
1]);
    }
    T query(int l, int r) {
        T ansL, ansR;
        for (l += n, r += n + 1; l < r; l >>= 1, r >>= 1) {
            if (l & 1) ansL = F(ansL, st[l++]);
            if (r & 1) ansR = F(st[--r], ansR);
        }
        return F(ansL, ansR);
    };
}

```

### 4.3.2 Query Max Subarray Sum

```

#pragma once
#include "../Contest/template.cpp"
#include "../Struct.cpp"
const ll _oo = 1e9;
struct Node {
    ll tot, suf, pref, best;
    // Neutral element
    Node() : tot(-_oo), suf(-_oo), pref(-_oo), best(-_oo) {} // Neutral
    element
    // for assign
    Node(ll x) { tot = x, suf = x, pref = x, best = max(0ll, x); }
};
Node combine(Node &nl, Node &nr) {
    if (nl.tot == -_oo) return nr;
    if (nr.tot == -_oo) return nl;
    Node m;
    m.tot = nl.tot + nr.tot;
    m.pref = max({nl.pref, nl.tot + nr.pref});
    m.suf = max({nr.suf, nr.tot + nl.suf});
    m.best = max({nl.best, nr.best, nl.suf + nr.pref});
    return m;
}
using SegTreeMaxSubarraySum = SegTreeBottomUp<Node, Node(), combine>;

```

### 4.3.3 Query min

```

#pragma once
#include "../Contest/template.cpp"
#include "../Struct.cpp"
template <typename T>
using SegTreeBottomUpMinQuery =
    SegTreeBottomUp<T, numeric_limits<T>::max(),
        [] (T a, T b) { return min(a, b); }>;

```

#### 4.3.4 Query sum

```
#pragma once
#include "../Contest/template.cpp"
#include "./Struct.cpp"
template <typename T>
using SegTreeBottomUpSumQuery =
    SegTreeBottomUp<T, T(0), [](T a, T b) { return a + b; }>;
```

#### 4.3.5 Struct

```
/*
 * @Description:
 *     merge should be function<T(T,T)>, that
 *     makes the necessary operation between two
 *     nodes in the segment tree
 * */
#pragma once
#include "../Contest/template.cpp"
template <typename T, T identity, auto merge>
struct SegTreeBottomUp {
    int size;
    vector<T> arr;
    SegTreeBottomUp(int n) {
        for (size = 1; size < n; size <= 1);
        arr.resize(size <= 1);
    }
    void assign(int pos, const T &val) {
        for (arr[pos += size] = val; pos >= 1; pos >= 1)
            arr[pos] = merge(arr[pos <= 1], arr[pos <= 1 | 1]);
    }
    T query(int l, int r) {
        T ans_l = identity, ans_r = identity;
        for (l += size, r += size + 1; l < r; l >= 1, r >= 1) {
            if (l & 1) ans_l = merge(ans_l, arr[l++]);
            if (r & 1) ans_r = merge(arr[--r], ans_r);
        }
        return merge(ans_l, ans_r);
    }
    SegTreeBottomUp(const vector<T> &vec) : SegTreeBottomUp(len(vec)) {
        copy(all(vec), begin(arr) + size);
        rrep(i, size - 1, 0) arr[i] = merge(arr[i <= 1], arr[i <= 1 | 1]);
    }
};
```

### 4.4 Segment tree (dynamic)

#### 4.4.1 Range Max Query Point Max Assignment

**Description:** Answers range queries in ranges until  $10^9$  (maybe more)

**Time:** Query and update  $O(n \cdot \log n)$

```
struct node;
node *newNode();
struct node {
    node *left, *right;
    int lv, rv;
    ll val;
    node() : left(NULL), right(NULL), val(-oo) {}
    inline void init(int l, int r) {
        lv = l;
        rv = r;
    }
    inline void extend() {
        if (!left) {
            int m = (lv + rv) / 2;
            left = newNode();
            right = newNode();
            left->init(lv, m);
            right->init(m + 1, rv);
        }
    }
    ll query(int l, int r) {
        if (r < lv || rv < l) {
            return 0;
        }
        if (l <= lv && rv <= r) {
            return val;
        }
        extend();
        return max(left->query(l, r), right->query(l, r));
    }
    void update(int p, ll newVal) {
        if (lv == rv) {
            val = max(val, newVal);
            return;
        }
        extend();
        (p <= left->rv ? left : right)->update(p, newVal);
        val = max(left->val, right->val);
    }
};

const int BUFFSZ(1e7);
node *newNode() {
    static int bufSize = BUFFSZ;
    static node buf[(int)BUFFSZ];
    assert(bufSize);
    return &buf[--bufSize];
}

struct SegTree {
    int n;
    node *root;
    SegTree(int _n) : n(_n) {
        root = newNode();
        root->init(0, n);
    }
};
```

```

    }
    ll query(int l, int r) { return root->query(l, r); }
    void update(int p, ll v) { root->update(p, v); }
};

```

#### 4.4.2 Range Sum Query Point Sum Update

**Description:** Answers range queries in ranges until  $10^9$  (maybe more)

**Time:** Query and update in  $O(n \cdot \log n)$

```

struct node;
node *newNode();
struct node {
    node *left, *right;
    int lv, rv;
    ll val;
    node() : left(NULL), right(NULL), val(0) {}
    inline void init(int l, int r) {
        lv = l;
        rv = r;
    }
    inline void extend() {
        if (!left) {
            int m = (rv - lv) / 2 + lv;
            left = newNode();
            right = newNode();
            left->init(lv, m);
            right->init(m + 1, rv);
        }
    }
    ll query(int l, int r) {
        if (r < lv || rv < l) {
            return 0;
        }
        if (l <= lv && rv <= r) {
            return val;
        }
        extend();
        return left->query(l, r) + right->query(l, r);
    }
    void update(int p, ll newVal) {
        if (lv == rv) {
            val += newVal;
            return;
        }
        extend();
        (p <= left->rv ? left : right)->update(p, newVal);
        val = left->val + right->val;
    }
};

const int BUFFSZ(1.3e7);
node *newNode() {
    static int bufSize = BUFFSZ;

```

```

    static node buf[(int)BUFFSZ];
    // assert(bufSize);
    return &buf[--bufSize];
}

struct SegTree {
    int n;
    node *root;
    SegTree(int _n) : n(_n) {
        root = newNode();
        root->init(0, n);
    }
    ll query(int l, int r) { return root->query(l, r); }
    void update(int p, ll v) { root->update(p, v); }
};

```

#### 4.5 Segment tree point update range query (top-down)

##### 4.5.1 Query hash (top down)

```

#include "../Contest/template.cpp"
const ll MOD = 1'000'000'009;
const ll P = 31;
const int MAXN = 2'000'000;
ll pows[MAXN + 1];
void computepows() {
    pows[0] = 1;
    for (int i = 1; i <= MAXN; i++) {
        pows[i] = (pows[i - 1] * P) % MOD;
    }
}

struct Node {
    ll hash;
    Node() : hash(-1) {}; // Neutral element
    Node(ll v) : hash(v) {};
};

inline Node combine(Node &vl, Node &vr, int nl, int nr, int ql, int qr) {
    if (vl.hash == -1) return vr;
    if (vr.hash == -1) return vl;
    Node vm;
    int nm = midpoint(nl, nr);
    int lsize = min(nm, qr) - max(nl, ql) + 1;
    vm.hash = (vl.hash + ((vr.hash * pows[lsize]) % MOD)) % MOD;
    return vm;
}

template <typename T = Node, auto F = combine>
struct SegTree {
    int n;
    vector<T> st;
    SegTree(int n) : n(n), st(n << 2) {}
    void assign(int p, const T &v) { assign(1, 0, n - 1, p, v); }
    void assign(int node, int l, int r, int p, const T &v) {
        if (l == r) {
            st[node] = v;

```

```

    return;
}
int m = midpoint(l, r);
if (p <= m)
    assign(node << 1, l, m, p, v);
else
    assign(node << 1 | 1, m + 1, r, p, v);
st[node] = F(st[node << 1], st[node << 1 | 1], l, r, l, r);
}
inline T query(int l, int r) { return query(1, 0, n - 1, l, r); }
inline T query(int node, int nl, int nr, int l, int r) const {
    if (r < nl or nr < l) return T();
    if (l <= nl and nr <= r) return st[node];
    int m = midpoint(nl, nr);
    auto a = query(node << 1, nl, m, l, r);
    auto b = query(node << 1 | 1, m + 1, nr, l, r);
    return F(a, b, nl, nr, l, r);
}
};

```

## 4.6 Segment tree range update range query

### 4.6.1 Arithmetic progression sum update, query sum

**Description:** Makes arithmetic progression updates in range and sum queries.

**Usage:** Considering  $PA(A, R) = [A + R, A + 2R, A + 3R, \dots]$

- **update\_set(l, r, A, R):** sets  $[l, r]$  to  $PA(A, R)$
- **update\_add(l, r, A, R):** sum  $PA(A, R)$  in  $[l, r]$
- **query(l, r):** sum in range  $[l, r]$

**Time:** build  $O(N)$ , updates and queries  $O(\log N)$

```

const ll oo = 1e18;
struct SegTree {
    struct Data {
        ll sum;
        ll set_a, set_r, add_a, add_r;
        Data(): sum(0), set_a(oo), set_r(0), add_a(0), add_r(0) {}
    };
    int n;
    vector<Data> seg;
    SegTree(int n_) : n(n_), seg(vector<Data>(4 * n)) {}
    void prop(int p, int l, int r) {
        int sz = r - l + 1;
        ll &sum = seg[p].sum, &set_a = seg[p].set_a, &set_r = seg[p].set_r,
        &add_a = seg[p].add_a, &add_r = seg[p].add_r;
        if (set_a != oo) {
            set_a += add_a, set_r += add_r;
            sum = set_a * sz + set_r * sz * (sz + 1) / 2;
            if (l != r) {
                int m = (l + r) / 2;
                seg[2 * p].set_a = set_a;

```

```

                seg[2 * p].set_r = set_r;
                seg[2 * p].add_a = seg[2 * p].add_r = 0;
                seg[2 * p + 1].set_a = set_a + set_r * (m - l + 1);
                seg[2 * p + 1].set_r = set_r;
                seg[2 * p + 1].add_a = seg[2 * p + 1].add_r = 0;
            }
            set_a = oo, set_r = 0;
            add_a = add_r = 0;
        } else if (add_a or add_r) {
            sum += add_a * sz + add_r * sz * (sz + 1) / 2;
            if (l != r) {
                int m = (l + r) / 2;
                seg[2 * p].add_a += add_a;
                seg[2 * p].add_r += add_r;
                seg[2 * p + 1].add_a += add_a + add_r * (m - l + 1);
                seg[2 * p + 1].add_r += add_r;
            }
            add_a = add_r = 0;
        }
    }
    int inter(pii a, pii b) {
        if (a.first > b.first) swap(a, b);
        return max(0, min(a.second, b.second) - b.first + 1);
    }
    ll set(int a, int b, ll aa, ll rr, int p, int l, int r) {
        prop(p, l, r);
        if (b < l or r < a) return seg[p].sum;
        if (a <= l and r <= b) {
            seg[p].set_a = aa;
            seg[p].set_r = rr;
            prop(p, l, r);
            return seg[p].sum;
        }
        int m = (l + r) / 2;
        int tam_l = inter({l, m}, {a, b});
        return seg[p].sum = set(a, b, aa, rr, 2 * p, l, m) +
            set(a, b, aa + rr * tam_l, rr, 2 * p + 1, m +
1, r);
    }
    void update_set(int l, int r, ll aa, ll rr) {
        set(l, r, aa, rr, 1, 0, n - 1);
    }
    ll add(int a, int b, ll aa, ll rr, int p, int l, int r) {
        prop(p, l, r);
        if (b < l or r < a) return seg[p].sum;
        if (a <= l and r <= b) {
            seg[p].add_a += aa;
            seg[p].add_r += rr;
            prop(p, l, r);
            return seg[p].sum;
        }
        int m = (l + r) / 2;
        int tam_l = inter({l, m}, {a, b});
        return seg[p].sum = add(a, b, aa, rr, 2 * p, l, m) +

```

```

        add(a, b, aa + rr * tam_l, rr, 2 * p + 1, m +
1, r);
    }
    void update_add(int l, int r, ll aa, ll rr) {
        add(l, r, aa, rr, 1, 0, n - 1);
    }
    ll query(int a, int b, int p, int l, int r) {
        prop(p, l, r);
        if (b < l or r < a) return 0;
        if (a <= l and r <= b) return seg[p].sum;
        int m = (l + r) / 2;
        return query(a, b, 2 * p, l, m) + query(a, b, 2 * p + 1, m + 1, r)
    };
    ll query(int l, int r) { return query(l, r, 1, 0, n - 1); }
};

```

#### 4.6.2 Increment update query min & max (bottom up)

```

using SegT = ll;
struct QueryT {
    SegT mx, mn;
    QueryT()
        : mx(numeric_limits<SegT>::min()), mn(numeric_limits<SegT>::max())
    {}
    QueryT(SegT _v) : mx(_v), mn(_v) {}
};
inline QueryT combine(QueryT ln, QueryT rn, pii lr1, pii lr2) {
    chmax(ln.mx, rn.mx);
    chmin(ln.mn, rn.mn);
    return ln;
}
using LazyT = SegT;
inline QueryT applyLazyInQuery(QueryT q, LazyT l, pii lr) {
    if (q.mx == QueryT().mx) q.mx = SegT();
    if (q.mn == QueryT().mn) q.mn = SegT();
    q.mx += l, q.mn += l;
    return q;
}
inline LazyT applyLazyInLazy(LazyT a, LazyT b) { return a + b; }
using UpdateT = SegT;
inline QueryT applyUpdateInQuery(QueryT q, UpdateT u, pii lr) {
    if (q.mx == QueryT().mx) q.mx = SegT();
    if (q.mn == QueryT().mn) q.mn = SegT();
    q.mx += u, q.mn += u;
    return q;
}
inline LazyT applyUpdateInLazy(LazyT l, UpdateT u, pii lr) { return l + u;
}
template <typename Qt = QueryT, typename Lt = LazyT, typename Ut = UpdateT
,
    auto C = combine, auto ALQ = applyLazyInQuery,
    auto ALL = applyLazyInLazy, auto AUQ = applyUpdateInQuery,

```

```

    auto AUL = applyUpdateInLazy>
struct LazySegmentTree {
    int n, h;
    vector<Qt> ts;
    vector<Lt> ds;
    vector<pii> lrs;
    LazySegmentTree(int _n)
        : n(_n),
          h(sizeof(int) * 8 - __builtin_clz(n)),
          ts(n << 1),
          ds(n),
          lrs(n << 1) {
        rep(i, 0, n) lrs[i + n] = {i, i};
        rrep(i, n - 1, 0) {
            lrs[i] = {lrs[i << 1].first, lrs[i << 1 | 1].second};
        }
    }
    LazySegmentTree(const vector<Qt> &xs) : LazySegmentTree(len(xs)) {
        copy(all(xs), ts.begin() + n);
        rep(i, 0, n) lrs[i + n] = {i, i};
        rrep(i, n - 1, 0) {
            ts[i] = C(ts[i << 1], ts[i << 1 | 1], lrs[i << 1], lrs[i << 1
| 1]);
        }
    }
    void set(int p, Qt v) {
        ts[p + n] = v;
        build(p + n);
    }
    void upd(int l, int r, Ut v) {
        l += n, r += n + 1;
        int l0 = l, r0 = r;
        for (; l < r; l >=> 1, r >=> 1) {
            if (l & 1) apply(l++, v);
            if (r & 1) apply(--r, v);
        }
        build(l0), build(r0 - 1);
    }
    Qt qry(int l, int r) {
        l += n, r += n + 1;
        push(l), push(r - 1);
        Qt resl = Qt(), resr = Qt();
        pii lr1 = {l, l}, lr2 = {r, r};
        for (; l < r; l >=> 1, r >=> 1) {
            if (l & 1) resl = C(resl, ts[l], lr1, lrs[l]), l++;
            if (r & 1) r--, resr = C(ts[r], resr, lrs[r], lr2);
        }
        return C(resl, resr, lr1, lr2);
    }
    void build(int p) {
        while (p > 1) {
            p >>= 1;
            ts[p] =

```

```

        ALQ(C(ts[p << 1], ts[p << 1 | 1], lrs[p << 1], lrs[p << 1
| 1])),
        ds[p], lrs[p]);
    }
}
void push(int p) {
    rrep(s, h, 0) {
        int i = p >> s;
        if (ds[i] != Lt()) {
            apply(i << 1, ds[i]), apply(i << 1 | 1, ds[i]);
            ds[i] = Lt();
        }
    }
}
inline void apply(int p, Ut v) {
    ts[p] = AUQ(ts[p], v, lrs[p]);
    if (p < n) ds[p] = AUL(ds[p], v, lrs[p]);
}
};

```

#### 4.6.3 Increment update sum query (top down)

```

struct Lnode {
    ll v;
    bool assign;
    Lnode() : v(), assign() {} // Neutral element
    Lnode(ll _v, bool a = 0) : v(_v), assign(a) {};
};
using Qnode = ll;
using Unode = Lnode;
struct LSegTree {
    int n, ql, qr;
    vector<Qnode> st;
    vector<Lnode> lz;
    /*-----*/
    Qnode merge(Qnode lv, Qnode rv, int nl, int nr) { return lv + rv; }
    void prop(int i, int l, int r) {
        if (lz[i].assign) {
            st[i] = lz[i].v * (r - l + 1);
            if (l != r) lz[tor(i)] = lz[i];
        } else {
            st[i] += lz[i].v * (r - l + 1);
            if (l != r) lz[tor(i)].v += lz[i].v, lz[tor(i)].v += lz[i].v;
        }
        lz[i] = Lnode();
    }
}
void applyV(int i, Unode v) {
    if (v.assign) {
        lz[i] = v;
    } else {
        lz[i].v += v.v;
    }
}

```

```

/*-----*/
LSegTree() {}
LSegTree(int _n) : n(_n), st(_n << 2), lz(_n << 2) {}
bool disjoint(int l, int r) { return qr < l or r < ql; }
bool contains(int l, int r) { return ql <= l and r <= qr; }
int tol(int i) { return i << 1; }
int tor(int i) { return i << 1 | 1; }
void build(vector<Qnode> &v) { build(v, 1, 0, n - 1); }
void build(vector<Qnode> &v, int i, int l, int r) {
    if (l == r) {
        st[i] = v[l];
        return;
    }
    int m = midpoint(l, r);
    build(v, tol(i), l, m);
    build(v, tor(i), m + 1, r);
    st[i] = merge(st[tor(i)], st[tor(i)], l, r);
}
void upd(int l, int r, Unode v) {
    ql = l, qr = r;
    upd(1, 0, n - 1, v);
}
void upd(int i, int l, int r, Unode v) {
    prop(i, l, r);
    if (disjoint(l, r)) return;
    if (contains(l, r)) {
        applyV(i, v);
        prop(i, l, r);
        return;
    }
    int m = midpoint(l, r);
    upd(tol(i), l, m, v);
    upd(tor(i), m + 1, r, v);
    st[i] = merge(st[tor(i)], st[tor(i)], l, r);
}
Qnode qry(int l, int r) {
    ql = l, qr = r;
    return qry(1, 0, n - 1);
}
Qnode qry(int i, int l, int r) {
    prop(i, l, r);
    if (disjoint(l, r)) return Qnode();
    if (contains(l, r)) return st[i];
    int m = midpoint(l, r);
    return merge(qry(tol(i), l, m), qry(tor(i), m + 1, r), l, r);
}
};

```

#### 4.7 2D Sparse Table

```

const int N = 1001;
ll matrix[N][N];
ll M[1001][1001][10][10];
ll op(ll a, ll b) { return gcd(a, b); }

```



```

void SparseMatrix(int n, int m) {
    int i, j, x, y;
    for (i = 0; (1 << i) <= n; i++) {
        for (j = 0; (1 << j) <= m; j++) {
            for (x = 0; (x + (1 << i) - 1) < n; x++) {
                for (y = 0; (y + (1 << j) - 1) < m; y++) {
                    if (i == 0 && j == 0)
                        M[x][y][i][j] = matrix[x][y];
                    else if (i == 0)
                        M[x][y][i][j] = op(M[x][y][i][j - 1],
                                             M[x][y + (1 << (j - 1))][i][j - 1]);
                    else if (j == 0)
                        M[x][y][i][j] = op(M[x][y][i - 1][j],
                                             M[x + (1 << (i - 1))][y][i - 1][j]);
                    else {
                        int tempa = op(M[x + (1 << (i - 1))][y][i - 1][j - 1],
                                       M[x][y + (1 << (j - 1))][i - 1][j - 1]);
                        int tempb = op(M[x][y][i - 1][j - 1],
                                       M[x + (1 << (i - 1))][y + (1 << (j - 1))][i - 1][j - 1]);
                        M[x][y][i][j] = op(tempa, tempb);
                    }
                }
            }
        }
    }
    return;
}

int lg2(int x) { return sizeof(int) * 8 - __builtin_clz(x) - 1; }

ll query2d(int x, int y, int x1, int y1) {
    int k = lg2(x1 - x + 1);
    int l = lg2(y1 - y + 1);
    int tempa = op(M[x][y][k][l], M[x1 - (1 << k) + 1][y][k][l]);
    int tempb = op(M[x][y1 - (1 << l) + 1][k][l],
                  M[x1 - (1 << k) + 1][y1 - (1 << l) + 1][k][l]);
    return op(tempa, tempb);
}

```

#### 4.8 Bitree 2D

**Description:** Given a 2D array you can increment an arbitrary position, and also query the subsum of a subgrid

**Time:** Update and query in  $O(\log N^2)$

```

struct Bit2d {
    int n;
    vll2d bit;
    Bit2d(int ni) : n(ni), bit(n + 1, vll(n + 1)) {}
    Bit2d(int ni, vll2d &xs) : n(ni), bit(n + 1, vll(n + 1)) {
        for (int i = 1; i <= n; i++) {

```

```

            for (int j = 1; j <= n; j++) {
                update(i, j, xs[i][j]);
            }
        }
    }

void update(int x, int y, ll val) {
    for (; x <= n; x += (x & (-x))) {
        for (int i = y; i <= n; i += (i & (-i))) {
            bit[x][i] += val;
        }
    }
}

ll sum(int x, int y) {
    ll ans = 0;
    for (int i = x; i; i -= (i & (-i))) {
        for (int j = y; j; j -= (j & (-j))) {
            ans += bit[i][j];
        }
    }
    return ans;
}

ll query(int x1, int y1, int x2, int y2) {
    return sum(x2, y2) - sum(x2, y1 - 1) - sum(x1 - 1, y2) +
           sum(x1 - 1, y1 - 1);
}
};

```

#### 4.9 Convex Hull Trick / Line Container

**Description:** Container where you can add lines of the form  $mx + b$ , and query the maximum value at point  $x$ .

**Usage:** `insert_line(m, b)` inserts the line  $m \cdot x + b$  in the container.

`eval(x)` find the highest value among all lines in the point  $x$ .

**Time:** Eval and insert in  $O(\log N)$

```

const ll LLINF = 1e18;
const ll is_query = -LLINF;
struct Line {
    ll m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line &rhs) const {
        if (rhs.b != is_query) return m < rhs.m;
        const Line *s = succ();
        if (!s) return 0;
        ll x = rhs.m;
        return b - s->b < (s->m - m) * x;
    }
};

struct Cht : public multiset<Line> { // maintain
    // max m*x+b

    bool bad(iterator y) {
        auto z = next(y);
        if (y == begin()) {
            if (z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
    }
};

```

```

    }
    auto x = prev(y);
    if (z == end()) return y->m == x->m && y->b <= x->b;
    return (ld)(x->b - y->b) * (z->m - y->m) >=
           (ld)(y->b - z->b) * (y->m - x->m);
}
void insert_line(ll m,
                ll b) { // min -> insert (-m,-b) -> -eval()
    auto y = insert({m, b});
    y->succ = [=] { return next(y) == end() ? 0 : &*next(y); };
    if (bad(y)) {
        erase(y);
        return;
    }
    while (next(y) != end() && bad(next(y))) erase(next(y));
    while (y != begin() && bad(prev(y))) erase(prev(y));
}
ll eval(ll x) {
    auto l = *lower_bound((Line){x, is_query});
    return l.m * x + l.b;
}
};

```

#### 4.10 DSU (with rollback)

**Description:** Performs every operation a regular DSU does, but you can roll back to a specific time.

**Usage:** `int t = uf.time(); ...; uf.rollback(t); T`

**Time:**  $O(\log(N))$

```

struct RollbackUF {
    vi e;
    vector<pii> st;
    RollbackUF(int n) : e(n, -1) {}
    int size(int x) { return -e[find(x)]; }
    int find(int x) { return e[x] < 0 ? x : find(e[x]); }
    int time() { return len(st); }
    void rollback(int t) {
        for (int i = time(); i-- > t; ) e[st[i].first] = st[i].second;
        st.resize(t);
    }
    bool join(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
        if (e[a] > e[b]) swap(a, b);
        st.push_back({a, e[a]});
        st.push_back({b, e[b]});
        e[a] += e[b];
        e[b] = a;
        return true;
    }
};

```

#### 4.11 DSU / UFDS

**Usage:** You may uncomment the commented parts to find online which nodes belong to each set, it makes the `union_set` method cost  $O(\log^2)$  instead  $O(A)$

```

struct DSU {
    vector<int> ps, sz;
    // vector<unordered_set<int>> sts;
    DSU(int N)
        : ps(N + 1),
          sz(N, 1) /*, sts(N) */
    {
        iota(ps.begin(), ps.end(), 0);
        // for (int i = 0; i < N; i++)
        // sts[i].insert(i);
    }
    int find_set(int x) { return ps[x] == x ? x : ps[x] = find_set(ps[x]); }
    int size(int u) { return sz[find_set(u)]; }
    bool same_set(int x, int y) { return find_set(x) == find_set(y); }
    void union_set(int x, int y) {
        if (same_set(x, y)) return;
        int px = find_set(x);
        int py = find_set(y);
        if (sz[px] < sz[py]) swap(px, py);
        ps[py] = px;
        sz[px] += sz[py];
        // sts[px].merge(sts[py]);
    }
};

```

#### 4.12 Lichao Tree (dynamic)

**Description:** Lichao Tree that creates the nodes dynamically, allowing to query and update from range  $[MAXL, MAXR]$

**Usage:**

- `query(x)` : find the highest point among all lines in the structure
- `add(a, b)` : add a line of form  $y = ax + b$  in the structure
- `addSegment(a, b, l, r)` : add a line segment of form  $y = ax + b$  which covers from range  $[l, r]$

**Time:**  $O(\log N)$

```

template <typename T = ll, T MAXL = 0, T MAXR = 1'000'000'001>
struct LiChaoTree {
    static const T inf = -numeric_limits<T>::max() / 2;
    bool first_best(T a, T b) { return a > b; }
    T get_best(T a, T b) { return first_best(a, b) ? a : b; }
    struct line {
        T m, b;
        T operator()(T x) { return m * x + b; }
    };
    struct node {
        line li;

```

```

    node *left, *right;
    node(line _li = {0, inf}) : li(_li), left(nullptr), right(nullptr)
    {}
    ~node() {
        delete left;
        delete right;
    }
};
node *root;
LiChaoTree(line li = {0, inf}) : root(new node(li)) {}
~LiChaoTree() { delete root; }
T query(T x, node *cur, T l, T r) {
    if (cur == nullptr) return inf;
    if (x < l or x > r) return inf;
    T mid = midpoint(l, r);
    T ans = cur->li(x);
    ans = get_best(ans, query(x, cur->left, l, mid));
    ans = get_best(ans, query(x, cur->right, mid + 1, r));
    return ans;
}
T query(T x) { return query(x, root, MAXL, MAXR); }
void add(line li, node *&cur, T l, T r) {
    if (cur == nullptr) {
        cur = new node(li);
        return;
    }
    T mid = midpoint(l, r);
    if (first_best(li(mid), cur->li(mid))) swap(li, cur->li);
    if (first_best(li(l), cur->li(l))) add(li, cur->left, l, mid);
    if (first_best(li(r), cur->li(r))) add(li, cur->right, mid + 1, r);
}
void add(T m, T b) { add({m, b}, root, MAXL, MAXR); }
void addSegment(line li, node *&cur, T l, T r, T lseg, T rseg) {
    if (r < lseg || l > rseg) return;
    if (cur == nullptr) cur = new node;
    if (lseg <= l && r <= rseg) {
        add(li, cur, l, r);
        return;
    }
    T mid = midpoint(l, r);
    if (l != r) {
        addSegment(li, cur->left, l, mid, lseg, rseg);
        addSegment(li, cur->right, mid + 1, r, lseg, rseg);
    }
}
void addSegment(T a, T b, T l, T r) {
    addSegment({a, b}, root, MAXL, MAXR, l, r);
}
};

```

### 4.13 Merge sort tree

**Description:** Like a segment tree but each node stores the ordered subsegment it represents.

**Usage:**

- `inrange(l, r, a, b)` : counts the number of positions  $i$ ,  $l \leq i \leq r$  such that  $a \leq x_i \leq b$ .

**Time:** Build  $O(N \log N^2)$ , `inrange`  $O(\log N^2)$   
**Memory:**  $O(n \log N)$

```

template <class T>
struct MergeSortTree {
    int n;
    vector<vector<T>> st;
    MergeSortTree(vector<T> &xs) : n(len(xs)), st(n << 1) {
        rep(i, 0, n) st[i + n] = vector<T>({xs[i]});
        rrep(i, n - 1, 0) {
            st[i].resize(len(st[i << 1]) + len(st[i << 1 | 1]));
            merge(all(st[i << 1]), all(st[i << 1 | 1]), st[i].begin());
        }
    }
    int count(int i, T a, T b) {
        return upper_bound(all(st[i]), b) - lower_bound(all(st[i]), a);
    }
    int inrange(int l, int r, T a, T b) {
        int ans = 0;
        for (l += n, r += n + 1; l < r; l >>= 1, r >>= 1) {
            if (l & 1) ans += count(l++, a, b);
            if (r & 1) ans += count(--r, a, b);
        }
        return ans;
    }
};

```

### 4.14 Mex with update

**Description:** This DS allows you to maintain an array of elements, insert, and remove, and query the MEX at any time.

**Usage:**

- `Mex(mxsz)`: Initialize the DS, `mxsz` must be the maximum number of elements that the structure may have.
- `add(x)`: just adds one copy of  $x$ .
- `rmv(x)`: just remove a copy of  $x$ .
- `operator()`: returns the MEX.

**Time:**

- `Mex(mxsz)`:  $O(\log mxsz)$
- `add(x)`:  $O(\log mxsz)$
- `rmv(x)`:  $O(\log mxsz)$
- `operator()`:  $O(1)$

```

struct Mex {
    int mx_sz;
    vi hs;
    set<int> st;
    Mex(int _mx_sz) : mx_sz(_mx_sz), hs(mx_sz + 1) {
        auto it = st.begin();
        rep(i, 0, mx_sz + 1) it = st.insert(it, i);
    }
};

```

```

void add(int x) {
    if (x > mx_sz) return;
    if (!hs[x]++) st.erase(x);
}
void rmv(int x) {
    if (x > mx_sz) return;
    if (!--hs[x]) st.emplace(x);
}
int operator()() const { return *st.begin(); }
/*
Optional, you can just create with size
len(xs) add N elements :D
*/
Mex(const vi &xs, int _mx_sz = -1) : Mex(~_mx_sz ? _mx_sz : len(xs)) {
    for (auto xi : xs) add(xi);
}
};

```

## 4.15 Orderd Set (GNU PBDS)

**Usage:** If you need an ordered **multi** set you may add an id to each value. Using `greater_equal`, or `less_equal` is considered undefined behavior.

- **order\_of\_key(k)**: Number of items strictly smaller/greater than `k`.
- **find\_by\_order(k)**: `K`-th element in a set (counting from zero).

**Time:** Both  $O(\log N)$

**Warning:** Is 2 or 3 times slower then a regular set/map.

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <typename T>
using ordered_set =
    tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

```

## 4.16 Prefix Sum 2D

**Description:** Given an 2D array with  $N$  lines and  $M$  columns, find the sum of the subarray that have the left upper corner at  $(x1, y1)$  and right bottom corner at  $(x2, y2)$ .

**Time:** Build  $O(N \cdot M)$ , Query  $O(1)$ .

```

template <typename T>
struct psum2d {
    vector<vector<T>> s;
    vector<vector<T>> psum;
    psum2d(vector<vector<T>> &grid, int n, int m)
        : s(n + 1, vector<T>(m + 1)), psum(n + 1, vector<T>(m + 1)) {
        for (int i = 1; i <= n; i++)
            for (int j = 1; j <= m; j++) {
                s[i][j] = s[i][j - 1] + grid[i - 1][j - 1];
                psum[i][j] = psum[i - 1][j] + s[i][j];
            }
    }
};

```

```

}
T query(int x1, int y1, int x2, int y2) {
    T ans = psum[x2 + 1][y2 + 1] + psum[x1][y1];
    ans -= psum[x2 + 1][y1] + psum[x1][y2 + 1];
    return ans;
}
};

```

## 4.17 Segment Tree Update Range Query (bottom-up)

```

/*
 * @Description:
 * merge should be function<T(T,T)>, that
 * makes the necessary operation between two
 * nodes in the segment tree
 * */
#include "../Contest/template.cpp"
template <typename T, T identity, auto merge>
struct SegTreeBottomUp {
    int size;
    vector<T> arr;
    SegTreeBottomUp(int n) {
        for (size = 1; size < n; size <= 1);
        arr.resize(size < 1);
    }
    void assign(int pos, const T &val) {
        for (arr[pos += size] = val; pos >= 1;)
            arr[pos] = merge(arr[pos < 1], arr[pos < 1 | 1]);
    }
    T query(int l, int r) {
        T ans_l = identity, ans_r = identity;
        for (l += size, r += size + 1; l < r; l >= 1, r >= 1) {
            if (l & 1) ans_l = merge(ans_l, arr[l++]);
            if (r & 1) ans_r = merge(arr[--r], ans_r);
        }
        return merge(ans_l, ans_r);
    }
    SegTreeBottomUp(const vector<T> &vec) : SegTreeBottomUp(len(vec)) {
        copy(all(vec), begin(arr) + size);
        rrep(i, size - 1, 0) arr[i] = merge(arr[i < 1], arr[i < 1 | 1]);
    }
};
using SegTreeBottomUpSumQuery =
    SegTreeBottomUp<ll, 0ll, [](ll a, ll b) { return a + b; }>;

```

## 4.18 Sparse table

```

template <typename T = ll,
auto cmp = [](T &src1, T &src2, T &dst) { dst = min(src1, src2); }>

```

```

class SparseTable {
private:
    int sz;
    vi logs;
    vector<vector<T>> st;
public:
    SparseTable(const vector<T> &v) : sz(len(v)), logs(sz + 1) {
        rep(i, 2, sz + 1) logs[i] = logs[i >> 1] + 1;
        st.resize(logs[sz] + 1, vector<T>(sz));
        rep(i, 0, sz) st[0][i] = v[i];
        for (int k = 1; (1 << k) <= sz; k++) {
            for (int i = 0; i + (1 << k) <= sz; i++) {
                cmp(st[k - 1][i], st[k - 1][i + (1 << (k - 1))], st[k][i])
            }
        }
    }
    T query(int l, int r) {
        r++;
        const int k = logs[r - l];
        T ret;
        cmp(st[k][l], st[k][r - (1 << k)], ret);
        return ret;
    }
};

```

#### 4.19 Static range queries

```

template <typename T = ll,
        auto op = [] (const T &src1, const T &src2,
                      T &dst) { dst = src1 + src2; },
        auto invop = [] (const T &src1, const T &src2,
                        T &dst) { dst = src1 - src2; }>
struct StaticRangeQueries {
    vector<T> acc;
    StaticRangeQueries(const vector<T> &XS) : acc(len(XS)) {
        acc[0] = XS[0];
        rep(i, 1, len(XS)) { op(acc[i - 1], XS[i], acc[i]); }
    }
    T operator()(int l, int r) {
        T lv = (l ? acc[l - 1] : T());
        T ret;
        invop(acc[r], lv, ret);
        return ret;
    }
};

```

#### 4.20 Venice Set

**Description:** A container that you can insert  $q$  copies of element  $e$ , increment every element in the container by  $x$ , query which is the best element and its quantity and also remove  $k$  copies of the greatest element.

**Time:**

- add element  $O(\log N)$

- remove  $O(\log N)$
- update:  $O(1)$
- query  $O(1)$

```

template <typename T = ll>
struct VeniceSet {
    using T2 = pair<T, ll>;
    priority_queue<T2, vector<T2>, greater<T2>> pq;
    T acc;
    VeniceSet() : acc() {}
    void add_element(const T &e, const ll q) { pq.emplace(e - acc, q); }
    void update_all(const T &x) { acc += x; }
    T2 best() {
        auto ret = pq.top();
        ret.first += acc;
        return ret;
    }
    void pop() { pq.pop(); }
    void pop_k(int k) {
        auto [e, q] = pq.top();
        pq.pop();
        q -= k;
        if (q) pq.emplace(e, q);
    }
};

```

#### 4.21 Venice Set (complete)

**Description:** A container which you can insert elements update all at once and also make a few queries

**Usage:**

- *add\_element(e, q)*: adds  $q$  copies of  $e$ , if no  $q$  is provided adds a single one
- *update\_all(x)*: increment every value by  $x$
- *erase(e)*: removes every copy of  $e$ , and returns how much was removed.
- *count(e)*: returns the number of  $e$  in the container
- *high()/low()*: returns the highest/lowest element, and its quantity
- *pop\_low(q)/pop\_high(q)*: removes  $q$  copies of the lowest/highest elements if no  $q$  is provided removes all copies of the lowest/highest element.

You may answer which is the  $K - th$  value and its quantity using an *ordered\_set*.

Probably works with other operations

**Time:** Considering  $N$  the number of distinct numbers in the container

- *add\_element(e, q)*:  $O(\log(N))$
- *update\_all(x)*:  $O(1)$
- *erase(e)*:  $O(\log(N))$
- *count(e)*:  $O(\log(N))$
- *high()/low()*:  $O(1)$
- *pop\_low(q)/pop\_high(q)*: worst case is  $O(N \cdot \log(N))$  if you remove all elements and so on...

**Warning:** There is no error handling if you try to *pop* more elements than exists or related stuff

```

struct VeniceSet {
    set<pll> st;
    ll acc;
    VeniceSet() : acc() {}
    ll add_element(ll e, ll q = 1) {
        q += erase(e);
        e -= acc;
        st.emplace(e, q);
        return q;
    }
    void update_all(ll x) { acc += x; }
    ll erase(ll e) {
        e -= acc;
        auto it = st.lb({e, LLONG_MIN});
        if (it == end(st) || (*it).first != e) return 0;
        ll ret = (*it).second;
        st.erase(it);
        return ret;
    }
    ll count(ll x) {
        x -= acc;
        auto it = st.lb({x, LLONG_MIN});
        if (it == end(st) || (*it).first != x) return 0;
        return (*it).second;
    }
    pll high() { return *rbegin(st); }
    pll low() { return *begin(st); }
    void pop_high(ll q = -1) {
        if (q == -1) q = high().second;
        while (q) {
            auto [e, eq] = high();
            st.erase(prev(end(st)));
            if (eq > q) add_element(e, eq - q);
            q = max(0ll, q - eq);
        }
    }
    void pop_low(ll q = -1) {
        if (q == -1) q = low().second;
        while (q) {
            auto [e, eq] = low();
            st.erase(st.begin());
            if (eq > q) add_element(e, eq - q);
            q = max(0ll, q - eq);
        }
    }
};

```

## 4.22 Wavelet tree

```

using ll = long long;
template <typename T>
struct WaveletTree {

```

```

    struct Node {
        T lo, hi;
        int left_child, right_child;
        vector<int> pcnt;
        vector<ll> psum;
        Node(int lo_, int hi_)
            : lo(lo_), hi(hi_), left_child(0), right_child(0), pcnt(),
              psum() {}
    };
    vector<Node> nodes;
    WaveletTree(vector<T> v) {
        nodes.reserve(2 * v.size());
        auto [mn, mx] = minmax_element(v.begin(), v.end());
        auto build = [&](auto &&self, Node &node, auto from, auto to) {
            if (node.lo == node.hi or from >= to) return;
            auto mid = midpoint(node.lo, node.hi);
            auto f = [&mid](T x) { return x <= mid; };
            node.pcnt.reserve(to - from + 1);
            node.pcnt.push_back(0);
            node.psum.reserve(to - from + 1);
            node.psum.push_back(0);
            T left_upper = node.lo, right_lower = node.hi;
            for (auto it = from; it != to; it++) {
                auto value = f(*it);
                node.pcnt.push_back(node.pcnt.back() + value);
                node.psum.push_back(node.psum.back() + *it);
                if (value)
                    left_upper = max(left_upper, *it);
                else
                    right_lower = min(right_lower, *it);
            }
            auto pivot = stable_partition(from, to, f);
            node.left_child = make_node(node.lo, left_upper);
            self(self, nodes[node.left_child], from, pivot);
            node.right_child = make_node(right_lower, node.hi);
            self(self, nodes[node.right_child], pivot, to);
        };
        build(build, nodes[make_node(*mn, *mx)], v.begin(), v.end());
    }
    T kth_element(int L, int R, int K) const {
        auto f = [&](auto &&self, const Node &node, int l, int r, int k)
        -> T {
            if (l > r) return 0;
            if (node.lo == node.hi) return node.lo;
            int lb = node.pcnt[l], rb = node.pcnt[r + 1], left_size = rb - lb;
            return (left_size > k
                    ? self(self, nodes[node.left_child], lb, rb - 1, k)
                    : self(self, nodes[node.right_child], l - lb, r -
                        k - left_size));
        };
        return f(f, nodes[0], L, R, K);
    }
};

```

```

}
pair<int, ll> count_and_sum_in_range(int L, int R, T a, T b) const {
    auto f = [&](auto &&self, const Node &node, int l,
                int r) -> pair<int, ll> {
        if (l > r or node.lo > b or node.hi < a) return {0, 0};
        if (a <= node.lo and node.hi <= b)
            return {r - l + 1,
                    (node.lo == node.hi ? (r - l + 1ll) * node.lo
                     : node.psum[r + 1] - node.psum
                      [l])};
        int lb = node.pcnt[l], rb = node.pcnt[r + 1];
        auto [left_cnt, left_sum] =
            self(self, nodes[node.left_child], lb, rb - 1);
        auto [right_cnt, right_sum] =
            self(self, nodes[node.right_child], l - lb, r - rb);
        return {left_cnt + right_cnt, left_sum + right_sum};
    };
    return f(f, nodes[0], L, R);
}

inline int count_in_range(int L, int R, T a, T b) const {
    return count_and_sum_in_range(L, R, a, b).first;
}

inline ll sum_in_range(int L, int R, T a, T b) const {
    return count_and_sum_in_range(L, R, a, b).second;
}

private:
int make_node(T lo, T hi) {
    int id = (int)nodes.size();
    nodes.emplace_back(lo, hi);
    return id;
}
};

```

## 5 Dynamic Programming

### 5.1 Binary Knapsack (bottom up)

**Description:** Given the points each element have, and it respective cost, computes the maximum points we can get if we can ignore/choose an element, in such way that the sum of costs don't exceed the maximum cost allowed.

**Time:**  $O(N * W)$

**Warning:** The vectors  $VS$  and  $WS$  starts at one, so it need an empty value at index 0.

```

const int MAXN(1'000), MAXCOST(1'000 * 20);
ll dp[MAXN + 1][MAXCOST + 1];
bool ps[MAXN + 1][MAXCOST + 1];
pair<ll, vi> knapsack(const vll &points, const vi &costs, int maxCost) {
    int n = len(points) - 1; // ELEMENTS START AT INDEX 1 !
    for (int m = 0; m <= maxCost; m++) {
        dp[0][m] = 0;
    }
    for (int i = 1; i <= n; i++) {

```

```

        dp[i][0] = dp[i - 1][0] + (costs[i] == 0) * points[i];
        ps[i][0] = costs[i] == 0;
    }
    for (int i = 1; i <= n; i++) {
        for (int m = 1; m <= maxCost; m++) {
            dp[i][m] = dp[i - 1][m], ps[i][m] = 0;
            int w = costs[i];
            ll v = points[i];
            if (w <= m and dp[i - 1][m - w] + v > dp[i][m]) {
                dp[i][m] = dp[i - 1][m - w] + v, ps[i][m] = 1;
            }
        }
    }
    vi is;
    for (int i = n, m = maxCost; i >= 1; --i) {
        if (ps[i][m]) {
            is.emplace_back(i);
            m -= costs[i];
        }
    }
    return {dp[n][maxCost], is};
}

```

### 5.2 Edit Distance

**Time:**  $O(N * M)$

```

#include "../Contest/template.cpp"
ll edit_distance(const string &a, const string &b) {
    int n = a.size();
    int m = b.size();
    vll2d dp(n + 1, vi(m + 1, 0));
    const ll ADD = 1, DEL = 1, CHG = 1;
    for (int i = 0; i <= n; ++i) {
        dp[i][0] = i * DEL;
    }
    for (int i = 1; i <= m; ++i) {
        dp[0][i] = ADD * i;
    }
    for (int i = 1; i <= n; ++i) {
        for (int j = 1; j <= m; ++j) {
            int add = dp[i][j - 1] + ADD;
            int del = dp[i - 1][j] + DEL;
            int chg = dp[i - 1][j - 1] + (a[i - 1] != b[j - 1]) * CHG;
            dp[i][j] = min({add, del, chg});
        }
    }
    return dp[n][m];
}

```



## 5.3 Knapsack

**Description:** Finds the maximum score you can achieve, given that you have  $N$  items, each item has a *cost*, a *point* and a *quantity*, you can spent at most *maxcost* and buy each item the maximum quantity it has.

**Time:**  $O(n \cdot \text{maxcost} \cdot \log \text{maxqtd})$

**Memory:**  $O(\text{maxcost})$ .

```
ll knapsack(const vi &weight, const vll &value, const vi &qtd, int maxCost)
{
    vi costs;
    vll values;
    for (int i = 0; i < len(weight); i++) {
        ll q = qtd[i];
        for (ll x = 1; x <= q; q -= x, x <= 1) {
            costs.eb(x * weight[i]);
            values.eb(x * value[i]);
        }
        if (q) {
            costs.eb(q * weight[i]);
            values.eb(q * value[i]);
        }
    }
    vll dp(maxCost + 1);
    for (int i = 0; i < len(values); i++) {
        for (int j = maxCost; j > 0; j--) {
            if (j >= costs[i]) dp[j] = max(dp[j], values[i] + dp[j - costs[i]]);
        }
    }
    return dp[maxCost];
}
```

## 5.4 Lis-pro-max

```
/*
@Description: Just a regular lis, but very quick to code
**/
```

```
multiset<int> s;
for (int i = 0; i < n; i++) {
    auto it = s.upper_bound(a[i]);
    if (it != s.end()) s.erase(it);
    s.insert(a[i]);
}
lis = len(s);
```

## 5.5 Longest Increasing Subsequence

**Description:** Find the pair  $(sz, psx)$  where  $sz$  is the size of the longest subsequence and  $psx$  is a vector where  $psx_i$  tells the size of the longest increase subsequence that ends at position  $i$ . *get\_idx* just tells which indices could be in the longest increasing subsequence. If you want the "Longest Non Decreasing Subsequence" you can just change the *lower\_bound* to an *upper\_bound*

**Time:**  $O(n \log n)$

```
#include "../Contest/template.cpp"
template <typename T>
pair<int, vi> lis(const vector<T> &xs, int n) {
    vector<T> dp(n + 1, numeric_limits<T>::max());
    dp[0] = numeric_limits<T>::min();
    int sz = 0;
    vi psx(n);
    rep(i, 0, n) {
        int pos = lower_bound(all(dp), xs[i]) - dp.begin();
        sz = max(sz, pos);
        dp[pos] = xs[i];
        psx[i] = pos;
    }
    return {sz, psx};
}
template <typename T>
vi get_idx(vector<T> xs) {
    int n = xs.size();
    auto [sz1, psx1] = lis(xs, n);
    transform(rall(xs), xs.begin(), [](T x) { return -x; });
    auto [sz2, psx2] = lis(xs, n);
    vi ans;
    rep(i, 0, n) {
        int l = psx1[i];
        int r = psx2[n - i - 1];
        if (l + r - 1 == sz1) ans.eb(i);
    }
    return ans;
}
```

## 5.6 Monery sum

**Description:** Find every possible sum using the given values only once.

```
set<int> money_sum(const vi &xs) {
    using vc = vector<char>;
    using vvc = vector<vc>;
    int _m = accumulate(all(xs), 0);
    int _n = xs.size();
    vvc _dp(_n + 1, vc(_m + 1, 0));
    set<int> _ans;
    _dp[0][xs[0]] = 1;
    for (int i = 1; i < _n; ++i) {
        for (int j = 0; j <= _m; ++j) {
            if (j == 0 or _dp[i - 1][j]) {
                _dp[i][j + xs[i]] = 1;
                _dp[i][j] = 1;
            }
        }
    }
}
```



```

    }
}
for (int i = 0; i < _n; ++i)
    for (int j = 0; j <= _m; ++j)
        if (_dp[i][j]) _ans.insert(j);
return _ans;
}

```

## 5.7 Steiner tree

```

template <typename T>
T steinerCost(const vector<vector<T>> &adj, const vi ks,
              T inf = numeric_limits<T>::max()) {
    int k = len(ks), n = len(adj);
    vector<vector<T>> dp(n, vector<T>(1 << k, inf));
    vi inks(n);
    trav(ki, ks) inks[ki] = 1;
    trav(ki, ks) {
        rep(j, 0, n) {
            if (count(all(ks), j) == 0) {
                dp[j][1 << ki] = adj[ki][j];
            }
        }
    }
    rep(mask, 2, (1 << k)) {
        rep(i, 0, n) {
            if (inks[i]) continue;
            for (int mask2 = (mask - 1) & mask; mask2 >= 1;
                mask2 = (mask2 - 1) & mask) {
                int mask3 = mask ^ mask2;
                chmin(dp[i][mask], dp[i][mask2] + dp[i][mask3]);
            }
            rep(j, 0, n) {
                if (inks[j]) continue;
                chmin(dp[j][mask], dp[i][mask] + adj[i][j]);
            }
        }
    }
    T ans = inf;
    rep(i, 0, n) chmin(ans, dp[i][(1 << k) - 1]);
    return ans;
}

```

## 5.8 Sum of Subsets

**Description:** Allows you to find if some mask  $X$  is a super mask of any of the given masks  
**Usage:** Call *build* with the *masks* then it returns a vector of bool  $V$  where  $V_X$  says if  $X$  is a super mask of any of the initial masks

You can change it to count how many submasks of each mask exists, by changing the bitwise or by a plus sign...

**Time:**  $O(LOG \cdot 2^{LOG})$

**Memory:**  $O(LOG^2 \cdot 2^{LOG})$

**Warning:** Remember to set *LOG* with the highest bit possible

```

const int LOG = 20;
vc build(const vi &masks) {
    vc ret(1 << LOG);
    trav(mi, masks) ret[mi] = 1;
    rep(b, 0, LOG) {
        rep(mask, 0, (1 << LOG)) {
            if (mask & (1 << b)) ret[mask] |= ret[mask ^ (1 << b)];
        }
    }
    return ret;
}

```

## 5.9 Travelling Salesman Problem

**Time:**  $O(N^2 \cdot 2^N)$

**Memory:**  $O(N^2 \cdot 2^N)$

```

vll2d dist;
vll memo;
int tsp(int i, int mask, int N) {
    if (mask == (1 << N) - 1) return dist[i][0];
    if (memo[i][mask] != -1) return memo[i][mask];
    int ans = INT_MAX << 1;
    for (int j = 0; j < N; ++j) {
        if (mask & (1 << j)) continue;
        auto t = tsp(j, mask | (1 << j), N) + dist[i][j];
        ans = min(ans, t);
    }
    return memo[i][mask] = ans;
}

```

## 6 Extras

### 6.1 Binary to gray

```

string binToGray(string bin) {
    string gray(bin.size(), '0');
    int n = bin.size() - 1;
    gray[0] = bin[0];
    for (int i = 1; i <= n; i++) {
        gray[i] = '0' + (bin[i - 1] == '1') ^ (bin[i] == '1');
    }
    return gray;
}

```

### 6.2 Get permutation cycles

**Description:** Receives a permutation  $[0, n-1]$  and return a vector 2D with each cycle.

```

vll2d getPermutationCicles(const vll &ps) {
    ll n = len(ps);
    vector<char> visited(n);
    vector<vll> cicles;
    rep(i, 0, n) {
        if (visited[i]) continue;
        vll cicle;
        ll pos = i;
        while (!visited[pos]) {
            cicle.pb(pos);
            visited[pos] = true;
            pos = ps[pos];
        }
        cicles.push_back(vll(all(cicle)));
    }
    return cicles;
}

```

### 6.3 Max & Min Check

**Description:** Returns the min/max value in range  $[l, r]$  that satisfies the lambda function check, if there is no such value the 'nullopt' is returned.

**Usage:** check must be a function that receives an integer and return a boolean.

**Time:**  $O(\log r - l + 1)$

```

template <typename T>
optional<T> maxCheck(T l, T r, auto check) {
    optional<T> ret;
    while (l <= r) {
        T m = midpoint(l, r);
        if (check(m))
            ret ? chmax(ret, m) : ret = m, l = m + 1;
        else
            r = m - 1;
    }
    return ret;
}

template <typename T>
optional<T> minCheck(T l, T r, auto check) {
    optional<T> ret;
    while (l <= r) {
        T m = midpoint(l, r);
        if (check(m))
            ret ? chmin(ret, m) : ret = m, r = m - 1;
        else
            l = m + 1;
    }
    return ret;
}

```

### 6.4 Merge Intervals

**Time:**  $(N \log N)$

**Warning:** It destroys the original array

```

#include "../Contest/template.cpp"
template <typename T>
vector<pair<T, T>> merge_intervals(vector<pair<T, T>> &intervals) {
    if (!len(intervals)) return {};
    using Pt = pair<T, T>;
    sort(all(intervals));
    vector<Pt> ret{intervals.front()};
    rep(i, 1, len(ret)) {
        auto &[pl, pr] = ret.back();
        auto &[l, r] = intervals[i];
        if (l <= pr)
            chmax(pr, r);
        else
            ret.eb(l, r);
    }
    return ret;
}

```

### 6.5 Mo's algorithm

```

template <typename T, typename Tans>
struct Mo {
    struct Query {
        int l, r, idx, block;
        Query(int l, int r, int idx, int block)
            : l(l), r(r), idx(idx), block(block) {}
        bool operator<(const Query &q) const {
            if (block != q.block) return block < q.block;
            return (block & 1 ? (r < q.r) : (r > q.r));
        }
    };
    vector<T> vs;
    vector<Query> qs;
    const int block_size;
    Mo(const vector<T> &a) : vs(a), block_size((int)ceil(sqrt(a.size()))) {}
    void add_query(int l, int r) {
        qs.emplace_back(l, r, qs.size(), l / block_size);
    }
    auto solve() {
        // get answer return type
        vector<Tans> answers(qs.size());
        sort(all(qs));
        int cur_l = 0, cur_r = -1;
        for (auto q : qs) {
            while (cur_l > q.l) add(--cur_l);
            while (cur_r < q.r) add(++cur_r);
            while (cur_l < q.l) remove(cur_l++);
            while (cur_r > q.r) remove(cur_r--);
        }
    }
};

```

```

        answers[q.idx] = get_answer();
    }
    return answers;
}
private:
// add value at idx from data structure
inline void add(int idx) {}
// remove value at idx from data structure
inline void remove(int idx) {}
// extract current answer of the data structure
inline Tans get_answer() {}
};

```

## 6.6 \_\_int128 stream

```

void print(__int128 x) {
    if (x < 0) {
        cout << '-';
        x = -x;
    }
    if (x > 9) print(x / 10);
    cout << (char)((x % 10) + '0');
}

__int128 read() {
    string s;
    cin >> s;
    __int128 x = 0;
    for (auto c : s) {
        if (c != '-') x += c - '0';
        x *= 10;
    }
    x /= 10;
    if (s[0] == '-') x = -x;
    return x;
}

```

## 7 Geometry

### 7.1 All i know about 2D stuff

Time:  $O(N)$

```

#include <iterator>
#include "../Contest/template.cpp"
/*
=====
*/
const double EPS{1e-4};
const ld PI = acos(-1);
enum PointPosition { IN, ON, OUT };
template <class Point>
vector<Point> segInter(Point a, Point b, Point c, Point d);
template <typename T>

```

```

bool equals(T a, T b) {
    if (std::is_floating_point<T>::value)
        return fabs(a - b) < EPS;
    else
        return a == b;
}
/*
=====
*/
template <class T>
struct Point {
    typedef Point P;
    T x, y;
    explicit Point(T x = 0, T y = 0) : x(x), y(y) {}
    bool operator<(P p) { return tie(x, y) < tie(p.x, p.y); }
    bool operator>(P& rhs) { return rhs < *this; }
    bool operator==(P p) { return tie(x, y) == tie(p.x, p.y); }
    P operator+(P p) { return P(x + p.x, y + p.y); }
    P operator-(P p) { return P(x - p.x, y - p.y); }
    P operator*(T d) { return P(x * d, y * d); }
    P operator/(T d) { return P(x / d, y / d); }
    T dot(P p) { return x * p.x + y * p.y; }
    T cross(P p) { return x * p.y - y * p.x; }
    T cross(P a, P b) { return (a - *this).cross(b - *this); }
    T dist2() { return x * x + y * y; }
    double dist() { return sqrt((double)dist2()); }
    // angle to x-axis in interval [-pi, pi]
    double angle() { return atan2(y, x); }
    P unit() { return *this / dist(); } // makes dist()=1
    P perp() { return P(-y, x); } // rotates +90 degrees
    P normal() { return perp().unit(); }
    // returns point rotated 'a' radians ccw around
    // the origin
    P rotate(double a) {
        return P(x * cos(a) - y * sin(a), x * sin(a) + y * cos(a));
    }
}

pair<T, T> slope(Point<T>& o) {
    auto a = o.x - x;
    auto b = o.y - y;
    if (!is_floating_point<T>::value) {
        auto g = __gcd(a, b);
        if (g) a /= g, b /= g;
    }
    return {b, a};
}

friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << ", " << p.y << ")";
}

double distanceTo(Point<T>& other) {
    return hypot(other.x - x, other.y - y);
}
};
/*
=====
*/

```

```

template <typename T>
struct Line {
    T a, b, c;
    Point<T> p1, p2;
    Line(T a = 0, T b = 0, T c = 0) : a(a), b(b), c(c) {
        if (a != 0) {
            double x = 0;
            double y = (-c) / b;
            p1 = Point<T>(x, y);
        }
        if (b != 0) {
            double y = 0;
            double x = (-c) / a;
            p2 = Point<T>(x, y);
        }
    }
    Line(Point<T>& p, Point<T>& q) {
        a = p.y - q.y;
        b = q.x - p.x;
        c = p.cross(q);
        p1 = p, p2 = q;
    }
    bool operator==(Line<T>& other) {
        return tie(a, b, c) == tie(other.a, other.b, other.c);
    }
    // Less-than operator
    bool operator<(Line& rhs) {
        return tie(a, b, c) < tie(rhs.a, rhs.b, rhs.c);
    }
    bool operator>(Line& rhs) { return rhs < *this; }
    Line<T> norm() {
        T d = a == 0 ? b : a;
        return Line(a / d, b / d, c / d);
    }
    bool contains(Point<T>& p) { return equals(a * p.x + b * p.y + c, (T)0); }
    bool parallel(Line<T>& r) {
        auto det = a * r.b - b * r.a;
        return equals(det, 0) and !(*this == r);
    }
    bool orthogonal(Line<T>& r) { return equals(a * r.a + b * r.b, 0); }
    T direction(Point<T>& p3) { return p1.cross(p2, p3); }
    friend ostream& operator<<(ostream& os, Line l) {
        return os << fixed << setprecision(6) << "(" << l.a << "," << l.b
        << "," << l.c << ")";
    }
    double distance(Point<T>& p) {
        return (a * p.x + b * p.y + c) / hypot(a, b);
    }
    Point<T> closest(Point<T>& p) {
        auto den = (a * a + b * b);
        auto x = (b * (b * p.x - a * p.y) - a * c) / den;
        auto y = (a * (-b * p.x + a * p.y) - b * c) / den;
        return Point<T>{x, y};
    }
}

```

```

};
/*
=====
*/
template <typename T>
struct LineSegment {
    Point<T> p1, p2;
    LineSegment(Point<T> p, Point<T> q) { p1 = p, p2 = q; }
    LineSegment(T a, T b, T c, T d)
        : LineSegment(Point<T>(a, b), Point<T>(c, d)) {}
    bool operator==(LineSegment<T>& other) {
        return tie(p1, p2) == tie(other.p1, other.p2);
    }
    // Less-than operator
    bool operator<(LineSegment& rhs) {
        return tie(p1, p2) < tie(rhs.p1, rhs.p2);
    }
    bool operator>(LineSegment& rhs) { return rhs < *this; }
    T direction(Point<T>& p3) { return p1.cross(p2, p3); }
    friend ostream& operator<<(ostream& os, LineSegment l) {
        return os << "(" << l.p1 << "," << l.p2 << ")";
    }
    vector<Point<T>> intersection(LineSegment<T>& other) {
        return segInter(p1, p2, other.p1, other.p2);
    }
}
// Verifica se o ponto P da reta r que contém A e B pertence ao segmento
bool contains(Point<T>& P) {
    return equals(p1.x, p2.x)
        ? min(p1.y, p2.y) <= P.y and P.y <= max(p1.y, p2.y)
        : min(p1.x, p2.x) <= P.x and P.x <= max(p1.x, p2.x);
}
// Ponto mais próximo de P no segmento AB
Point<T> closest(Point<T>& P) {
    Line<T> r(p1, p2);
    auto Q = r.closest(P);
    if (this->contains(Q)) return Q;
    auto distp1 = P.distanceTo(p1);
    auto distp2 = P.distanceTo(p2);
    if (distp1 <= distp2)
        return p1;
    else
        return p2;
}
};
/*
=====
*/
template <typename T>
struct Circle {
    Point<T> c;
    T r;
    Circle(Point<T> _c, T _r) : c(_c), r(_r) {}
    Circle(T _r) : Circle(Point<T>(0, 0), _r) {}
}

```

```

ld area() const { return PI * r * r; }
ld perimeter() const { return 2.0 * PI * r; }
ld arc(ld theta) const { return theta * r; }
ld chord(ld theta) const { return 2.0 * r * sin(theta / 2.0); }
ld sector(ld theta) const { return (theta * r * r) / 2.0; }
ld segment(ld theta) const { return ((theta - sin(theta)) * r * r) /
2.0; }
PointPosition position(const Point<T>& p) const {
    auto d = c.dist(p);
    return equals(d, r) ? ON : (d < r ? IN : OUT);
}
};
/*
=====
*/
template <typename T>
struct Rectangle {
    Point<T> P, Q;
    T b, h;
    Rectangle(const Point<T>& p, const Point<T>& q) : P(p), Q(q) {
        assert(P != Q);
        b = max(P.x, Q.x) - min(P.x, Q.x);
        h = max(P.y, Q.y) - min(P.y, Q.y);
    }
    Rectangle(T base, T height)
        : P(0, 0), Q(base, height), b(base), h(height) {}
    T perimeter() const { return 2 * b + 2 * h; }
    T area() const { return b * h; }
    optional<Rectangle> intersection(const Rectangle& r) const {
        using pt = pair<T, T>;
        auto i = pt(min(P.x, Q.x), max(P.x, Q.x));
        auto u = pt(min(r.P.x, r.Q.x), max(r.P.x, r.Q.x));
        auto a = max(i.first, u.first);
        auto b = min(i.second, u.second);
        i = pt(min(P.y, Q.y), max(P.y, Q.y));
        u = pt(min(r.P.y, r.Q.y), max(r.P.y, r.Q.y));
        auto c = max(i.first, u.first);
        auto d = max(i.second, u.first);
        if (d < c or b < a) return nullopt;
        return {{a, c}, {b, d}};
    }
};
/*
=====
*/
template <typename T>
struct Trapezium {
    T B, b, h;
    T area() const { return ((b + B) * h) / 2; }
};
/*
=====
*/
template <typename T>
struct Triangle {

```

```

Point<T> A, B, C;
enum SidesClass { EQUILATERAL, ISOCELES, SCALENE };
SidesClass classification_by_sides() const {
    auto a = A.distanceTo(B);
    auto b = B.distanceTo(C);
    auto c = C.distanceTo(A);
    if (equals(a, b) && equals(b, c)) return EQUILATERAL;
    if (equals(a, b) or equals(a, c) or equals(b, c)) return ISOCELES;
    return SCALENE;
}
enum AnglesClass { RIGHT, ACUTE, OBTUSE };
AnglesClass classification_by_angles() const {
    auto a = dist(A, B);
    auto b = dist(B, C);
    auto c = dist(C, A);
    auto alpha = acos((a * a - b * b - c * c) / (-2 * b * c));
    auto beta = acos((b * b - a * a - c * c) / (-2 * a * c));
    auto gamma = acos((c * c - a * a - b * b) / (-2 * a * b));
    auto right = PI / 2.0;
    if (equals(alpha, right) || equals(beta, right) || equals(gamma,
right))
        return RIGHT;
    if (alpha > right || beta > right || gamma > right) return OBTUSE;
    return ACUTE;
}
double perimeter() const {
    auto a = dist(A, B), b = dist(B, C), c = dist(C, A);
    return a + b + c;
}
double area() const {
    Line<T> r(A, B);
    auto b = dist(A, B);
    auto h = r.distance(C);
    return (b * h) / 2;
}
};
template <typename T>
Point<T> triangleBarycenter(const Point<T>& a, const Point<T>& b,
const Point<T>& c) {
    return Point<T>((a.x + b.x + c.x) / 3.0, (a.y + b.y + c.y) / 3.0);
}
template <typename T>
Point<T> triangleOrthocenter(const Point<T>& a, const Point<T>& b,
const Point<T>& c) {
    Line<T> r(a, b), s(a, c);
    Line<T> u{r.b, -r.a, -(c.x * r.b - c.y * r.a)};
    Line<T> v{s.b, -s.a, -(b.x * s.b - b.y * s.a)};
    auto det = u.a * v.b - u.b * v.a;
    auto x = (-u.c * v.b + v.c * u.b) / det;
    auto y = (-v.c * u.a + u.c * v.a) / det;
    return {x, y};
}

```

```

template <typename T>
Point<double> triangleIncenter(const Point<T>& a, const Point<T>& b,
                               const Point<T>& c) {
    auto dab = distance(a, b);
    auto dbc = distance(b, c);
    auto dca = distance(c, a);
    auto p = dab + dbc + dca;
    auto x = (a.x * dab + b.x * dbc + c.x * dca) / (p);
    auto y = (a.y * dab + b.y * dbc + c.y * dca) / (p);
    return Point<double>(x, y);
}

template <typename T>
Point<T> triangleCircumcenter(const Point<T>& A, const Point<T>& B,
                              const Point<T>& C) {
    auto D = 2 * (A.x * (B.y - C.y) + B.x * (C.y - A.y) + C.x * (A.y - B.y));
    auto A2 = A.x * A.x + A.y * A.y;
    auto B2 = B.x * B.x + B.y * B.y;
    auto C2 = C.x * C.x + C.y * C.y;
    auto x = (A2 * (B.y - C.y) + B2 * (C.y - A.y) + C2 * (A.y - B.y)) / D;
    auto y = (A2 * (C.x - B.x) + B2 * (A.x - C.x) + C2 * (B.x - A.x)) / D;
    return {x, y};
}

template <typename T>
Point<T> triangleCircumradius(const Point<T>& a, const Point<T>& b,
                              const Point<T>& c) {
    auto dab = distance(a, b);
    auto dbc = distance(b, c);
    auto dca = distance(c, a);
    return (dab + dbc + dca) / triangleArea(a, b, c);
}

/*
=====
*/

template <class Point>
vector<Point> segInter(Point a, Point b, Point c, Point d) {
    auto oa = c.cross(d, a), ob = c.cross(d, b), oc = a.cross(b, c),
        od = a.cross(b, d);
    // Checks if intersection is single non-endpoint
    // point.
    if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
        return {(a * ob - b * oa) / (ob - oa)};
    set<Point> s;
    if (onSegment(c, d, a)) s.insert(a);
    if (onSegment(c, d, b)) s.insert(b);
    if (onSegment(a, b, c)) s.insert(c);
    if (onSegment(a, b, d)) s.insert(d);
    return {all(s)};
}

/*
=====
*/

template <typename T>
double angle(const Point<T>& P, const Point<T>& Q, const Point<T>& R,
             const Point<T>& S) {

```

```

    auto ux = P.x - Q.x;
    auto uy = P.y - Q.y;
    auto vx = R.x - S.x;
    auto vy = R.y - S.y;
    auto num = ux * vx + uy * vy;
    auto den = hypot(ux, uy) * hypot(vx, vy);
    // Caso especial: se den == 0, algum dos vetores é degenerado: os dois
    // pontos são iguais. Neste caso, o ângulo não está definido
    return acos(num / den);
}

/*
=====
*/

struct pt {
    double x, y;
    int id;
};

int orientation(pt a, pt b, pt c) {
    double v = a.x * (b.y - c.y) + b.x * (c.y - a.y) + c.x * (a.y - b.y);
    if (v < 0) return -1; // clockwise
    if (v > 0) return +1; // counter-clockwise
    return 0;
}

bool cw(pt a, pt b, pt c, bool include_collinear) {
    int o = orientation(a, b, c);
    return o < 0 || (include_collinear && o == 0);
}

bool collinear(pt a, pt b, pt c) { return orientation(a, b, c) == 0; }

void convex_hull(vector<pt>& pts, bool include_collinear = false) {
    pt p0 = *min_element(all(pts), [](pt a, pt b) {
        return make_pair(a.y, a.x) < make_pair(b.y, b.x);
    });
    sort(all(pts), [&p0](const pt& a, const pt& b) {
        int o = orientation(p0, a, b);
        if (o == 0)
            return (p0.x - a.x) * (p0.x - a.x) + (p0.y - a.y) * (p0.y - a.y) <
                (p0.x - b.x) * (p0.x - b.x) + (p0.y - b.y) * (p0.y - b.y);
        return o < 0;
    });
    if (include_collinear) {
        int i = len(pts) - 1;
        while (i >= 0 && collinear(p0, pts[i], pts.back())) i--;
        reverse(pts.begin() + i + 1, pts.end());
    }
    vector<pt> st;
    for (int i = 0; i < len(pts); i++) {
        while (st.size() > 1 && !cw(st[len(st) - 2], st.back(), pts[i], include_collinear))
            st.pop_back();
        st.push_back(pts[i]);
    }
}

```

```

    pts = st;
}
/*
=====
*/
template <typename T>
double ccRadius(const Point<T>& A, const Point<T>& B, const Point<T>& C) {
    return (B - A).dist() * (C - B).dist() * (A - C).dist() /
        abs((B - A).cross(C - A)) / 2;
}

template <typename T>
Point<T> ccCenter(const Point<T>& A, const Point<T>& B, const Point<T>& C)
{
    Point<T> b = C - A, c = B - A;
    return A + (b * c.dist2() - c * b.dist2()).perp() / b.cross(c) / 2;
}

template <typename T>
pair<Point<T>, double> mec(vector<Point<T>> ps) {
    shuffle(all(ps), mt19937(time(0)));
    Point<T> o = ps[0];
    double r = 0, EPS = 1 + 1e-8;
    rep(i, 0, len(ps)) if ((o - ps[i]).dist() > r * EPS) {
        o = ps[i], r = 0;
        rep(j, 0, i) if ((o - ps[j]).dist() > r * EPS) {
            o = (ps[i] + ps[j]) / 2;
            r = (o - ps[i]).dist();
            rep(k, 0, j) if ((o - ps[k]).dist() > r * EPS) {
                o = ccCenter(ps[i], ps[j], ps[k]);
                r = (o - ps[i]).dist();
            }
        }
    }
    return {o, r};
}

/*
=====
*/
template <typename T>
Line<T> perpendicular_bisector(const Point<T>& P, const Point<T>& Q) {
    auto a = 2 * (Q.x - P.x);
    auto b = 2 * (Q.y - P.y);
    auto c = (P.x * P.x + P.y * P.y) - (Q.x * Q.x + Q.y * Q.y);
    return {a, b, c};
}

/*
=====
*/
ll cross(ll x1, ll y1, ll x2, ll y2) { return x1 * y2 - x2 * y1; }
ll polygonArea(vector<pll>& pts) {
    ll ats = 0;
    for (int i = 2; i < len(pts); i++)
        ats += cross(pts[i].first - pts[0].first, pts[i].second - pts[0].second,
            pts[i - 1].first - pts[0].first,
            pts[i - 1].second - pts[0].second);
}

```

```

    return abs(ats / 2ll);
}

ll boundary(vector<pll>& pts) {
    ll ats = pts.size();
    for (int i = 0; i < len(pts); i++) {
        ll deltax = (pts[i].first - pts[(i + 1) % pts.size()].first);
        ll deltax = (pts[i].second - pts[(i + 1) % pts.size()].second);
        ats += abs(__gcd(deltax, deltax)) - 1;
    }
    return ats;
}

pll latticePoints(vector<pll>& pts) {
    ll bounds = boundary(pts);
    ll area = polygonArea(pts);
    ll inside = area + 1ll - bounds / 2ll;
    return {inside, bounds};
}

/*
=====
*/
template <typename T>
bool contains(const Point<T>& A, const Point<T>& B, const Point<T>& P) {
    // Verifica se P é est na região retangular
    auto xmin = min(A.x, B.x);
    auto xmax = max(A.x, B.x);
    auto ymin = min(A.y, B.y);
    auto ymax = max(A.y, B.y);
    if (P.x < xmin || P.x > xmax || P.y < ymin || P.y > ymax) return false;
    // Verifica relação de semelhança no ângulo
    return equals((P.y - A.y) * (B.x - A.x), (P.x - A.x) * (B.y - A.y));
}

/*
=====
*/
// the polygon area of a intersection between a circle and a ccw polygon
template <typename T>
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(Point<T> c, double r, vector<Point<T>> ps) {
    auto tri = [&](Point<T> p, Point<T> q) {
        auto r2 = r * r / 2;
        Point<T> d = q - p;
        auto a = d.dot(p) / d.dist2(), b = (p.dist2() - r * r) / d.dist2();
        auto det = a * a - b;
        if (det <= 0) return arg(p, q) * r2;
        auto s = max(0., -a - sqrt(det)), t = min(1., -a + sqrt(det));
        if (t < 0 || 1 <= s) return arg(p, q) * r2;
        Point<T> u = p + d * s, v = p + d * t;
        return arg(p, u) * r2 + u.cross(v) / 2 + arg(v, q) * r2;
    };
    auto sum = 0.0;
    rep(i, 0, len(ps)) sum += tri(ps[i] - c, ps[(i + 1) % len(ps)] - c);
    return sum;
}

```



```

/* =====
*/
bool checkIfPolygonIsConvex(vector < Point<T> > {
    if (n < 3) return false;
}

```

## 7.2 Angle between three points

**Description:** Computes the angle apb in radians

**Warning:**  $a$  is equal to  $b$  then the angle isn't defined.

```

#include "template.cpp"
template <typename T>
ld angle(const Point<T>& p, const Point<T>& a, const Point<T>& b) {
    auto ux = p.x - a.x;
    auto uy = p.y - a.y;
    auto vx = p.x - b.x;
    auto vy = p.y - b.y;
    auto num = ux * vx + uy * vy;
    auto den = hypot(ux, uy) * hypot(vx, vy);
    return acos(num / den);
}

```

## 7.3 Area of union of rectangles

```

using SegT = ll;
const SegT eSeg = 1e9;
struct QueryT {
    SegT q, v;
    QueryT() : q(0), v(eSeg) {}
    QueryT(SegT _v) : q(1), v(_v) {}
};
inline QueryT combine(QueryT ln, QueryT rn, pii lr1, pii lr2) {
    QueryT ret;
    if (ln.v < rn.v) ret = ln;
    if (rn.v < ln.v) ret = rn;
    if (rn.v == ln.v) {
        ret.v = ln.v;
        ret.q = ln.q + rn.q;
    }
    return ret;
}
using LazyT = SegT;
inline QueryT applyLazyInQuery(QueryT q, LazyT l, pii lr) {
    if (l == LazyT()) return q;
    if (q.v == eSeg) q.v = 0, q.q = 1;
    q.v += l;
    return q;
}
inline LazyT applyLazyInLazy(LazyT a, LazyT b) { return a + b; }

```

```

using UpdateT = SegT;
inline QueryT applyUpdateInQuery(QueryT q, UpdateT u, pii lr) {
    return applyLazyInQuery(q, u, lr);
}
inline LazyT applyUpdateInLazy(LazyT l, UpdateT u, pii lr) { return l + u; }
template <typename Qt = QueryT, typename Lt = LazyT, typename Ut = UpdateT>
    auto C = combine, auto ALQ = applyLazyInQuery,
    auto ALL = applyLazyInLazy, auto AUQ = applyUpdateInQuery,
    auto AUL = applyUpdateInLazy>
struct LazySegmentTree {
    int n, h;
    vector<Qt> ts;
    vector<Lt> ds;
    vector<pii> lrs;
    LazySegmentTree(int _n)
        : n(_n),
          h(sizeof(int) * 8 - __builtin_clz(n)),
          ts(n << 1),
          ds(n),
          lrs(n << 1) {
        rep(i, 0, n) lrs[i + n] = {i, i};
        rrep(i, n - 1, 0) {
            lrs[i] = {lrs[i << 1].first, lrs[i << 1 | 1].second};
        }
    }
    LazySegmentTree(const vector<Qt> &xs) : LazySegmentTree(len(xs)) {
        copy(all(xs), ts.begin() + n);
        rep(i, 0, n) lrs[i + n] = {i, i};
        rrep(i, n - 1, 0) {
            ts[i] = C(ts[i << 1], ts[i << 1 | 1], lrs[i << 1], lrs[i << 1
| 1]);
        }
    }
    void set(int p, Qt v) {
        ts[p + n] = v;
        build(p + n);
    }
    void upd(int l, int r, Ut v) {
        l += n, r += n + 1;
        int l0 = l, r0 = r;
        for (; l < r; l >>= 1, r >>= 1) {
            if (l & 1) apply(l++, v);
            if (r & 1) apply(--r, v);
        }
        build(l0), build(r0 - 1);
    }
    Qt qry(int l, int r) {
        l += n, r += n + 1;
        push(l), push(r - 1);
        Qt resl = Qt(), resr = Qt();
        pii lr1 = {l, l}, lr2 = {r, r};
        for (; l < r; l >>= 1, r >>= 1) {

```



```

        if (l & 1) resl = C(resl, ts[l], lr1, lrs[l]), l++;
        if (r & 1) resr = C(ts[r], resr, lrs[r], lr2);
    }
    return C(resl, resr, lr1, lr2);
}

void build(int p) {
    while (p > 1) {
        p >>= 1;
        ts[p] =
            ALQ(C(ts[p << 1], ts[p << 1 | 1], lrs[p << 1], lrs[p << 1
| 1]),
                ds[p], lrs[p]);
    }
}

void push(int p) {
    rrep(s, h, 0) {
        int i = p >> s;
        if (ds[i] != Lt()) {
            apply(i << 1, ds[i]), apply(i << 1 | 1, ds[i]);
            ds[i] = Lt();
        }
    }
}

inline void apply(int p, Ut v) {
    ts[p] = AUQ(ts[p], v, lrs[p]);
    if (p < n) ds[p] = AUL(ds[p], v, lrs[p]);
}

};

ll areaOfRectanglesUnion(
    const vector<pair<Point<int>, Point<int>>> &rectangles) {
    if (!size(rectangles)) return 0;
    int maxy = INT_MIN;
    for (auto &[p1, p2] : rectangles) {
        assert(p1.x < p2.x && p1.y < p2.y);
        maxy = max({maxy, p1.y, p2.y});
    }
    vector<array<int, 4>> sl;
    sl.reserve(size(rectangles) * 2);
    for (auto &[p1, p2] : rectangles) {
        sl.push_back({p1.x, p1.y, p2.y - 1, 1});
        sl.push_back({p2.x, p1.y, p2.y - 1, -1});
    }
    sort(sl.begin(), sl.end());
    vector<QueryT> aux(maxy, QueryT(0));
    LazySegmentTree seg(aux);
    // memset(seg_vec, 0, sizeof(ll) * maxy);
    // seg::build(maxy, seg_vec);
    int prevx = get<0>(sl.front());
    ll ans = 0;
    for (auto &[curx, ys, yf, inc] : sl) {
        auto [q, v] = seg.qry(0, maxy - 1);
        // auto [q, v] = seg::query(0, maxy - 1);
        ans += (ll)(curx - prevx) * (v ? maxy : maxy - q);
        seg.upd(ys, yf, inc);
    }
}

```

```

        prevx = curx;
    }
    return ans;
}

```

## 7.4 Area: polygon

```

#include "./template.cpp"
template <typename T>
ld area(const vector<Point<T>>& pts) {
    ld a = 0.0;
    int n = size(pts);
    for (int i = 0; i < n; i++) {
        a += pts[i].x * pts[(i + 1) % n].y;
        a -= pts[i].y * pts[(i + 1) % n].x;
    }
    return fabs(a) / (ld)2;
}

```

## 7.5 Check if point belongs to line

```

#pragma once
#include "./Define line from two points.cpp"
#include "./template.cpp"
template <typename T>
bool lineContainsPoint(const Point<T>& r, const Point<T>& p,
    const Point<T>& q) {
    auto [a, b, c] = defineLine(p, q);
    return equals((T)0, a * r.x + b * r.y + c);
}

```

## 7.6 Check if point belongs to segment

```

#include "./template.cpp"
template <class P>
bool segmentContainsPoint(const P& p, const P& a, const P& b) {
    auto xmin = min(a.x, b.x);
    auto xmax = max(a.x, b.x);
    auto ymin = min(a.y, b.y);
    auto ymax = max(a.y, b.y);
    if (p.x < xmin or p.x > xmax or p.y < ymin or p.y > ymax) return false;
    return equals((p.y - a.y) * (b.x - a.x), (p.x - a.x) * (b.y - a.y));
}

```

## 7.7 Check if point is inside polygon

**Description:** checks if the point  $p$  is inside the polygon with vertices in  $pts$ , works for both convex and concave polygons.

**Warning:** If not working for integers points try to use long double.

```
#pragma once
#include "../Angle between three points.cpp"
#include "../Check if point belongs to segment.cpp"
#include "../Determinant.cpp"
#include "../template.cpp"

template <typename T>
bool contains(const vector<Point<T>>& pts, const Point<T>& p) {
    int n = size(pts);
    if (n < 3) return false; // may treat it appart
    T sum = 0.0;
    for (int i = 0; i < n; i++) {
        auto d = determinant(p, pts[i], pts[(i + 1) % n]);
        auto a = angle(p, pts[i], pts[(i + 1) % n]);
        sum += d > 0 ? a : (d < 0 ? -a : 0);
    }
    return equals(fabs(sum), 2 * PI);
}

// 0: outside, 1: inside, 2: boundary
template <class P>
int pointInPolygon(const vector<P>& pts, const P& p) {
    if (contains(pts, p)) return 1;
    int n = size(pts);
    for (int i = 0; i < n; i++) {
        if (segmentContainsPoint(p, pts[i], pts[(i + 1) % n])) {
            return 2;
        }
    }
    return 0;
}
```

## 7.8 Convex hull

```
#include "../Contest/template.cpp"
#include "../Determinant.cpp"
#include "../template.cpp"

template <typename T>
vector<Point<T>> convexHull(vector<Point<T>> pts) {
    if (len(pts) <= 1) return pts;
    sort(all(pts));
    vector<Point<T>> h(len(pts) + 1);
    int s = 0, t = 0;
    for (int it = 2; it--; s = --t, reverse(all(pts)))
        for (Point<T> p : pts) {
            while (t >= s + 2 && determinant(h[t - 2], h[t - 1], p) <= 0)
                t--;
            h[t++] = p;
        }
    return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

```
}

template <typename T>
vector<Point<T>> convexHull2(vector<Point<T>> pts) {
    int n = len(pts);
    sort(pts.begin(), pts.end());
    vector<Point<T>> l, u;
    for (int i = 0; i < n; i++) {
        while (len(l) >= 2 &&
            determinant(l[len(l) - 1], l[len(l) - 2], pts[i]) < 0) {
            l.pop_back();
        }
        l.push_back(pts[i]);
    }
    for (int i = n - 1; ~i; --i) {
        while (len(u) >= 2 &&
            determinant(u[len(u) - 1], u[len(u) - 2], pts[i]) < 0) {
            u.pop_back();
        }
        u.push_back(pts[i]);
    }
    u.pop_back(), l.pop_back();
    u.reserve(len(u) + len(l));
    u.insert(u.end(), all(l));
    return u;
}
```

## 7.9 Cross product between points

```
#pragma once
#include "../template.cpp"

template <typename T>
T cross(const Point<T>& p, const Point<T>& q) {
    return p.x * q.y - p.y * q.x;
}
```

## 7.10 Define line from two points

```
#pragma once
#include "../template.cpp"

template <typename T>
inline tuple<T, T, T> defineLine(const Point<T>& p, const Point<T>& q) {
    return {p.y - q.y, q.x - p.x, cross(p, q)};
}
```

## 7.11 Determinant

```
#pragma once
#include "../template.cpp"

template <typename T>
T determinant(const Point<T>& p, const Point<T>& q, const Point<T>& r) {
    return cross(q - p, r - p);
}
```

```

    return (p.x * q.y + p.y * r.x + q.x * r.y) -
           (r.x * q.y + r.y * p.x + q.x * p.y);
}

```

## 7.12 Distance: point to point

```

#include "../template.cpp"
template <typename T>
T distance(const Point<T>& p, const Point<T>& q) {
    return hypot(p.x - q.x, p.y - q.y);
}

```

## 7.13 Halfplane intersection

```

#pragma once
#include "../Point.cpp"
#include "../template.cpp"
// Basic half-plane struct.
struct Halfplane {
    // 'p' is a passing point of the line and 'pq' is the direction vector
    // of the line.
    Point<ld> p, pq;
    long double angle;
    Halfplane() {}
    Halfplane(const Point<ld>& a, const Point<ld>& b) : p(a), pq(b - a) {
        angle = atan2l(pq.y, pq.x);
    }
    // Check if point 'r' is outside this half-plane.
    // Every half-plane allows the region to the LEFT of its line.
    bool out(const Point<ld>& r) { return cross(pq, r - p) < -EPS; }
    // Intersection point of the lines of two half-planes. It is assumed
    // they're
    // never parallel.
    friend Point<ld> inter(const Halfplane& s, const Halfplane& t) {
        long double alpha = cross((t.p - s.p), t.pq) / cross(s.pq, t.pq);
        return s.p + (s.pq * alpha);
    }
};
// Actual algorithm
// receive it by reference if don't care messing with it
vector<Point<ld>> hp_intersect(vector<Halfplane> H) {
    const ld inf = 2e6;
    Point<ld> box[4] = { // Bounding box in CCW order
        Point<ld>(inf, inf), Point<ld>(-inf, inf),
        Point<ld>(-inf, -inf), Point<ld>(inf, -inf) };
    for (int i = 0; i < 4; i++) { // Add bounding box half-planes.
        Halfplane aux(box[i], box[(i + 1) % 4]);
        H.push_back(aux);
    }
}

```

```

// Sort by angle and start algorithm
sort(H.begin(), H.end(), [&](const Halfplane& a, const Halfplane& b) {
    return a.angle < b.angle;
});
deque<Halfplane> dq;
int len = 0;
for (int i = 0; i < int(H.size()); i++) {
    // Remove from the back of the deque while last half-plane is
    // redundant
    while (len > 1 && H[i].out(inter(dq[len - 1], dq[len - 2]))) {
        dq.pop_back();
        --len;
    }
    // Remove from the front of the deque while first half-plane is
    // redundant
    while (len > 1 && H[i].out(inter(dq[0], dq[1]))) {
        dq.pop_front();
        --len;
    }
    // Special case check: Parallel half-planes
    if (len > 0 && fabsl(cross(H[i].pq, dq[len - 1].pq)) < EPS) {
        // Opposite parallel half-planes that ended up checked against
        // each
        // other.
        if (dot(H[i].pq, dq[len - 1].pq) < 0.0) return vector<Point<ld>>();
        // Same direction half-plane: keep only the leftmost half-
        // plane.
        if (H[i].out(dq[len - 1].p)) {
            dq.pop_back();
            --len;
        } else
            continue;
    }
    // Add new half-plane
    dq.push_back(H[i]);
    ++len;
}
// Final cleanup: Check half-planes at the front against the back and
// vice-versa
while (len > 2 && dq[0].out(inter(dq[len - 1], dq[len - 2]))) {
    dq.pop_back();
    --len;
}
while (len > 2 && dq[len - 1].out(inter(dq[0], dq[1]))) {
    dq.pop_front();
    --len;
}
// Report empty intersection if necessary
if (len < 3) return vector<Point<ld>>();
// Reconstruct the convex polygon from the remaining half-planes.
vector<Point<ld>> ret(len);
for (int i = 0; i + 1 < len; i++) {

```

```

    ret[i] = inter(dq[i], dq[i + 1]);
}
ret.back() = inter(dq[len - 1], dq[0]);
return ret;
}

```

## 7.14 Lattice points

```

#pragma once
#include "../Contest/template.cpp"
#include "../Area: polygon.cpp"
#include "../template.cpp"
template <typename T>
pair<ll, ll> latticePoints(const vector<Point<T>> &pts) {
    ll bounds = pts.size();
    int n = pts.size();
    for (int i = 0; i < n; i++) {
        ll deltax = (pts[i].x - pts[(i + 1) % n].x);
        ll deltax = (pts[i].y - pts[(i + 1) % n].y);
        bounds += abs(__gcd(deltax, deltax)) - 1;
    }
    ll a = area(pts);
    ll inside = a + 1 - bounds / 2ll;
    return {inside, bounds};
}

```

## 7.15 Left of polygon cut

**Warning:** if some vertex lies exactly on the line A B, these vertex will be included in the answer

```

#include "../Determinant.cpp"
#include "../template.cpp"
template <typename T>
vector<Point<T>> leftOfPolygonCut(const vector<Point<T>>& vs, const Point<T>& A,
                                const Point<T>& B) {
    //   Interse   entre a reta AB e o segmento de reta PQ
    auto intersection = [&](const Point<T>& P, const Point<T>& Q,
                           const Point<T>& A, const Point<T>& B) -> Point<T> {
        auto a = B.y - A.y;
        auto b = A.x - B.x;
        auto c = B.x * A.y - A.x * B.y;
        auto u = fabs(a * P.x + b * P.y + c);
        auto v = fabs(a * Q.x + b * Q.y + c);
        //   M  ia ponderada pelas dist  ncias de P e Q    a reta AB
        return {(P.x * v + Q.x * u) / (u + v), (P.y * v + Q.y * u) / (u + v)};
    };
    vector<Point<T>> points;
    int n = size(vs);

```

```

    for (int i = 0; i < n; ++i) {
        auto d1 = determinant(A, B, vs[i]);
        auto d2 = determinant(A, B, vs[(i + 1) % n]);
        //   V  rtice    esquerda da reta
        if (d1 > -EPS) points.push_back(vs[i]);
        // A aresta cruza a reta
        if (d1 * d2 < -EPS)
            points.push_back(intersection(vs[i], vs[(i + 1) % n], A, B));
    }
    return points;
}

```

## 7.16 Perimeter: polygon

```

#include "../Distance: point to point.cpp"
#include "../template.cpp"
template <typename T>
T perimeter(const vector<Point<T>>& pts) {
    T p = 0.0;
    int n = size(pts);
    for (int i = 0; i < n; i++) {
        p += distance(pts[i], pts[(i + 1) % n]);
    }
    return p;
}

```

## 7.17 Point

```

// Basic point/vector struct.
template <typename T>
struct Point {
    T x, y;
    Point(T x = 0, T y = 0) : x(x), y(y) {}
    // Addition, subtraction, multiply by constant, dot product, cross product.
    friend Point<T> operator+(const Point<T>& p, const Point<T>& q) {
        return Point<T>(p.x + q.x, p.y + q.y);
    }
    friend Point<T> operator-(const Point<T>& p, const Point<T>& q) {
        return Point<T>(p.x - q.x, p.y - q.y);
    }
    template <typename T2>
    friend Point<T> operator*(const Point<T>& p, T2 k) {
        return Point<T>(p.x * k, p.y * k);
    }
    friend T dot(const Point<T>& p, const Point<T>& q) {
        return p.x * q.x + p.y * q.y;
    }
    friend T cross(const Point<T>& p, const Point<T>& q) {
        return p.x * q.y - p.y * q.x;
    }
}

```

```
};
}
```

## 7.18 Polygon (regular): apothem

```
#include "../Distance: point to point.cpp"
#include "../template.cpp"
template <typename T>
ld apothem(const vector<Point<T>>& pts) {
    auto s = distance(pts[0], pts[1]);
    int n = size(pts);
    return (s / 2.0) * (1.0 / tan(PI / n));
}
```

## 7.19 Polygon (regular): circumradius

```
#include "../Distance: point to point.cpp"
#include "../template.cpp"
template <typename T>
ld circumradius(const vector<Point<T>>& pts) {
    auto s = distance(pts[0], pts[1]);
    int n = size(pts);
    return (s / 2.0) * (1.0 / sin(PI / (ld)n));
}
```

## 7.20 Polygon: check if is convex

```
#include "../Determinant.cpp"
#include "../template.cpp"
template <typename T>
bool checkIfPolygonIsConvex(vector<Point<T>>& pts) {
    int n = size(pts);
    if (n < 3) return false;
    int l, g, e;
    l = g = e = 0;
    for (int i = 0; i < n; i++) {
        auto d = determinant(pts[i], pts[(i + 1) % n], pts[(i + 2) % n]);
        d ? (d > 0 ? g++ : l++) : e++;
    }
    return l == n or g == n;
}
```

## 7.21 Rectangle intersection

```
/*
Assumes that the points P, Q that define
a rectangle are the bottom-left and top-right
corner, and also that the sides are parallel to the axis.
*/
```

```
*/
#pragma once
#include "../Contest/template.cpp"
#include "../Point.cpp"
template <typename T>
optional<pair<Point<T>, Point<T>>> rectangleIntersection(
    const pair<Point<T>, Point<T>> &r1, const pair<Point<T>, Point<T>> &r2
) {
    assert(r1.first.x < r1.second.x && r1.first.y < r1.second.y);
    assert(r2.first.x < r2.second.x && r2.first.y < r2.second.y);
    T x1 = max(r1.first.x, r2.first.x);
    T x2 = min(r1.second.x, r2.second.x);
    T y1 = max(r1.first.y, r2.first.y);
    T y2 = min(r1.second.y, r2.second.y);
    if (x1 >= x2 or y1 >= y2) return nullopt;
    return pair<Point<T>, Point<T>>{{x1, y1}, {x2, y2}};
}
```

## 7.22 point-rotation

```
/*
rotate a point in the counter-clockwise
*/
#include "../template.cpp"
template <typename T>
Point<T> rotate(const Point<T>& P, T angle) {
    auto x = cos(angle) * P.x - sin(angle) * P.y;
    auto y = sin(angle) * P.x + cos(angle) * P.y;
    return {x, y};
}
```

## 7.23 template

```
#pragma once
#include <bits/stdc++.h>
using namespace std;
using ld = long double;
template <typename T>
using Point = pair<T, T>;
#define x first
#define y second
const double EPS{1e-6};
const ld PI = acos(-1);
template <typename T>
bool equals(T a, T b) {
    if (std::is_floating_point<T>::value)
        return fabs(a - b) < EPS;
    else
        return a == b;
}
template <typename T>
```

```
bool equals(Point<T> a, Point<T> b) {
    if (std::is_floating_point<T>::value)
        return fabs(a.x - b.x) < EPS && fabs(a.y - b.y) < EPS;
    else
        return a == b;
}
```

## 8 Graphs

### 8.1 Heavy-Light Decomposition (point update)

#### 8.1.1 Maximum number on path

```
struct Node {
    ll value;
    Node()
        : value(numeric_limits<ll>::min()) {}; // Neutral
                                                // element
    Node(ll v) : value(v) {};
};

Node combine(Node l, Node r) {
    Node m;
    m.value = max(l.value, r.value);
    return m;
}

template <typename T = Node, auto F = combine>
struct SegTree {
    int n;
    vector<T> st;
    SegTree(int _n) : n(_n), st(n << 1) {}
    void set(int p, const T &k) {
        for (st[p += n] = k; p >= 1; p >>= 1) st[p] = F(st[p << 1], st[p << 1 |
1]);
    }
    T query(int l, int r) {
        T ansL, ansR;
        for (l += n, r += n + 1; l < r; l >>= 1, r >>= 1) {
            if (l & 1) ansL = F(ansL, st[l++]);
            if (r & 1) ansR = F(st[--r], ansR);
        }
        return F(ansL, ansR);
    }
};

template <typename SegT = Node, auto SegOp = combine>
struct HeavyLightDecomposition {
    int n;
    vi ps, ds, sz, heavy, head, pos;
    SegTree<SegT, SegOp> seg;
    HeavyLightDecomposition(const vi2d &g, const vector<SegT> &v, int root
= 0)
        : n(len(g)), seg(n) {
        ps = ds = sz = heavy = head = pos = vi(n, -1);
```

```
auto dfs = [&](auto &&self, int u) -> void {
    sz[u] = 1;
    int mx = 0;
    for (auto x : g[u])
        if (x != ps[u]) {
            ps[x] = u;
            ds[x] = ds[u] + 1;
            self(self, x);
            sz[u] += sz[x];
            if (sz[x] > mx) mx = sz[x], heavy[u] = x;
        }
};
dfs(dfs, root);
for (int i = 0, cur = 0; i < n; i++) {
    if (ps[i] == -1 or heavy[ps[i]] != i)
        for (int j = i; j != -1; j = heavy[j]) {
            head[j] = i;
            pos[j] = cur++;
        }
    rep(i, 0, n) seg.set(pos[i], v[i]);
}

vector<pii> disjoint_ranges(int u, int v) {
    vector<pii> ret;
    for (; head[u] != head[v]; v = ps[head[v]]) {
        if (ds[head[u]] > ds[head[v]]) swap(u, v);
        ret.eb(pos[head[v]], pos[v]);
    }
    if (ds[u] > ds[v]) swap(u, v);
    ret.eb(pos[u], pos[v]);
    return ret;
}

SegT query_path(int u, int v) {
    SegT res;
    for (auto [l, r] : disjoint_ranges(u, v)) {
        res = SegOp(res, seg.query(l, r));
    }
    return res;
}

SegT query_subtree(int u) const {
    return seg.query(pos[u], pos[u] + sz[u] - 1);
}

void set(int u, SegT x) { seg.set(pos[u], x); }
```

### 8.2 2-SAT

**Description:** Calculates a valid assignment to boolean variables  $a, b, c, \dots$  to a 2-SAT problem, so that an expression of the type  $(a||b)\&\&(!a|c)\&\&(d||!b)\&\&\dots$  becomes true, or reports that it is unsatisfiable.

**Usage:** Negated variables are represented by bit-inversions ( $\tilde{x}$ ).

Returns true iff it is solvable ts.values[0..N-1] holds the assigned values to the vars.

**Time:**  $O(N + E)$ , where  $N$  is the number of boolean variables, and  $E$  is the number of clauses.

```

/
struct TwoSat {
    int N;
    vector<vi> gr;
    vi values; // 0 = false, 1 = true
    TwoSat(int n = 0) : N(n), gr(2 * n) {}
    int addVar() { // (optional)
        gr.eb();
        gr.eb();
        return N++;
    }
    void add_or(int x, int y) {
        x = max(2 * x, -1 - 2 * x);
        y = max(2 * y, -1 - 2 * y);
        gr[x].pb(y ^ 1);
        gr[y].pb(x ^ 1);
    }
    void add_true(int x) { add_or(x, x); }
    void add_impl(int f, int j) { add_or(~f, j); } // (optional)
    void add_xor(int x, int y) { // (optional) not tested yet
        add_or(x, y), add_or(~x, ~y);
    }
    void add_eq(int x, int y) { // (optional) not tested yet
        add_xor(~x, y);
    }
    void at_most_one(const vi &li) { // (optional)
        if (len(li) <= 1) return;
        int cur = ~li[0];
        rep(i, 2, len(li)) {
            int next = addVar();
            add_or(cur, ~li[i]);
            add_or(cur, next);
            add_or(~li[i], next);
            cur = ~next;
        }
        add_or(cur, ~li[1]);
    }
}
vi val, comp, z;
int time = 0;
int dfs(int i) {
    int low = val[i] = ++time, x;
    z.pb(i);
    for (int e : gr[i])
        if (!comp[e]) low = min(low, val[e] ? dfs(e));
    if (low == val[i]) do {
        x = z.back();
        z.ppb();
        comp[x] = low;
        if (values[x >> 1] == -1) values[x >> 1] = x & 1;
    } while (x != i);
}

```

```

        return val[i] = low;
    }
    bool solve() {
        values.assign(N, -1);
        val.assign(2 * N, 0);
        comp = val;
        rep(i, 0, 2 * N) if (!comp[i]) dfs(i);
        rep(i, 0, N) if (comp[2 * i] == comp[2 * i + 1]) return 0;
        return 1;
    }
};

```

### 8.3 BFS-01

**Description:** Similar to a Dijkstra given a weighted graph finds the distance from source  $s$  to every other node.

**Time:**  $O(V + E)$

**Warning:** Applicable only when the weight of the edges  $\in \{0, x\}$

```

vector<pair<ll, int>> adj[maxn];
ll dists[maxn];
int s, n;
void bfs_01() {
    fill(dists, dists + n, oo);
    dist[s] = 0;
    deque<int> q;
    q.emplace_back(s);
    while (not q.empty()) {
        auto u = q.front();
        q.pop_front();
        for (auto [v, w] : adj[u]) {
            if (dist[v] <= dist[u] + w) continue;
            dist[v] = dist[u] + w;
            w ? q.emplace_back(v) : q.emplace_front(v);
        }
    }
}

```

### 8.4 Bellman ford

**Description:** Find shortest path from a single source to all other nodes. Can detect negative cycles.

**Time:**  $O(V \cdot E)$

```

bool bellman_ford(const vector<vector<pair<int, ll>>> &g, int s,
                 vector<ll> &dist) {
    int n = (int)g.size();
    dist.assign(n, LLONG_MAX);
    vector<int> count(n);
    vector<char> in_queue(n);
    queue<int> q;
    dist[s] = 0;
    q.push(s);
}

```



```

in_queue[s] = true;
while (not q.empty()) {
    int cur = q.front();
    q.pop();
    in_queue[cur] = false;
    for (auto [to, w] : g[cur]) {
        if (dist[cur] + w < dist[to]) {
            dist[to] = dist[cur] + w;
            if (not in_queue[to]) {
                q.push(to);
                in_queue[to] = true;
                count[to]++;
                if (count[to] > n) return false;
            }
        }
    }
}
return true;
}

```

## 8.5 Bellman-Ford (find negative cycle)

**Description:** Given a directed graph find a negative cycle by running  $n$  iterations, and if the last one produces a relaxation than there is a cycle.

**Time:**  $O(V \cdot E)$

```

const ll oo = 2500 * 1e9;
using graph = vector<vector<pair<int, ll>>>;
vi negative_cycle(graph &g, int n) {
    vll d(n, oo);
    vi p(n, -1);
    int x = -1;
    d[0] = 0;
    for (int i = 0; i < n; i++) {
        x = -1;
        for (int u = 0; u < n; u++) {
            for (auto &[v, l] : g[u]) {
                if (d[u] + l < d[v]) {
                    d[v] = d[u] + l;
                    p[v] = u;
                    x = v;
                }
            }
        }
    }
    if (x == -1)
        return {};
    else {
        for (int i = 0; i < n; i++) x = p[x];
        vi cycle;
        for (int v = x;; v = p[v]) {
            cycle.eb(v);
            if (v == x and len(cycle) > 1) break;
        }
    }
}

```

```

        reverse(all(cycle));
        return cycle;
    }
}

```

## 8.6 Biconnected Components

**Description:** Build a vector of vectors, where the  $i$ -th vector correspond to the nodes of the  $i$ -th, biconnected component, a biconnected component is a subset of nodes and edges in which there is no cut point, also exist at least two distinct routes in vertex between any two vertex in the same biconnected component.

**Time:**  $O(N + M)$

```

const int maxn(5'000'000);
int tin[maxn], stck[maxn], bcc_cnt, n, top = 0, timer = 1;
vector<int> g[maxn], nodes[maxn];
int tarjan(int u, int p = -1) {
    int lowu = tin[u] = timer++;
    int son_cnt = 0;
    stck[++top] = u;
    for (auto v : g[u]) {
        if (!tin[v]) {
            son_cnt++;
            int lowx = tarjan(v, u);
            lowu = min(lowu, lowx);
            if (lowx >= tin[u]) {
                while (top != -1 && stck[top + 1] != v)
                    nodes[bcc_cnt].emplace_back(stck[top--]);
                nodes[bcc_cnt++].emplace_back(u);
            }
        } else {
            lowu = min(lowu, tin[v]);
        }
    }
    if (p == -1 && son_cnt == 0) {
        nodes[bcc_cnt++].emplace_back(u);
    }
    return lowu;
}
void build_bccs() {
    timer = 1;
    top = -1;
    memset(tin, 0, sizeof(int) * n);
    for (int i = 0; i < n; i++) nodes[i] = {};
    bcc_cnt = 0;
    for (int u = 0; u < n; u++)
        if (!tin[u]) tarjan(u);
}

```

## 8.7 Binary Lifting/Jumping

**Description:** Given a function/successor graph answers queries of the form which is the node after  $k$  moves starting from  $u$ .



**Time:** Build  $O(N \cdot \text{MAXLOG2})$ , Query  $O(\text{MAXLOG2})$ .

```
const int MAXN(2e5), MAXLOG2(30);
int bl[MAXN][MAXLOG2 + 1];
int N;

int jump(int u, ll k) {
    for (int i = 0; i <= MAXLOG2; i++) {
        if (k & (1ll << i)) u = bl[u][i];
    }
    return u;
}

void build() {
    for (int i = 1; i <= MAXLOG2; i++) {
        for (int j = 0; j < N; j++) {
            bl[j][i] = bl[bl[j][i - 1]][i - 1];
        }
    }
}
```

## 8.8 Bipartite Graph

**Description:** Given a graph, find the 'left' and 'right' side if is a bipartite graph, if is not then a empty vi2d is returned

**Time:**  $O(N + M)$

```
vi2d bipartite_graph(vi2d &adj) {
    int n = len(adj);
    vi side(n, -1);
    vi2d ret(2);
    rep(u, 0, n) {
        if (side[u] == -1) {
            queue<int> q;
            q.emp(u);
            side[u] = 0;
            ret[0].eb(u);
            while (len(q)) {
                int u = q.front();
                q.pop();
                for (auto v : adj[u]) {
                    if (side[v] == -1) {
                        side[v] = side[u] ^ 1;
                        ret[side[v]].eb(v);
                        q.push(v);
                    } else if (side[u] == side[v])
                        return {};
                }
            }
        }
    }
    return ret;
}
```

## 8.9 Block-Cut Tree \* \*

**Description:** Builds the Block-Cut of a undirected graph. \* \*

**Usage:** `isGraphCutpoint[u]` answers how many connected components \* are created when the node u is removed from the graph, if \* `isGraphCutpoint[u]` is greater than 1, it means that u is a \* cutpoint. \* \*

**Time:**  $O(N + M)$  \* \*

**Memory:**  $O(N)$  \* \*

**Warning:** Always careful with disconnected graphs ! you may end up having \* multiple trees. \* \*

```
#pragma once
#include "../Contest/template.cpp"

struct BlockCutTree {
    int n;
    vi idOnTree, tin, low, stk, isGraphCutpoint, isTreeCutpoint;
    vi2d comps, treeAdj;
    BlockCutTree(vi2d &g)
        : n(len(g)), idOnTree(n), tin(n), low(n), isGraphCutpoint(n) {
        rep(i, 0, n) {
            if (!tin[i]) {
                int timer = 0;
                dfs(i, -1, timer, g);
            }
        }
        buildTree();
    }

    void buildTree() {
        int node_id = 0;
        rep(u, 0, n) {
            if (isGraphCutpoint[u]) {
                idOnTree[u] = node_id++;
                isTreeCutpoint.eb(true);
                treeAdj.pb({});
            }
        }
        for (auto &comp : comps) {
            int node = node_id++;
            treeAdj.pb({});
            isTreeCutpoint.eb(false);
            for (int u : comp) {
                if (!isGraphCutpoint[u]) {
                    idOnTree[u] = node;
                } else {
                    treeAdj[node].eb(idOnTree[u]),
                    treeAdj[idOnTree[u]].eb(node);
                }
            }
        }
    }

    void dfs(int u, int p, int &timer, vi2d &g) {
        tin[u] = low[u] = ++timer;
        stk.eb(u);
        for (auto v : g[u]) {
            if (v == p) continue;
            if (!tin[v]) {
```

```

    dfs(v, u, timer, g);
    chmin(low[u], low[v]);
    if (low[v] >= tin[u]) {
        isGraphCutpoint[u] += (tin[u] > 1 or tin[v] > 2);
        comps.pb({u});
        while (comps.back().back() != v) {
            comps.back().eb(stk.back());
            stk.ppb();
        }
    } else
        low[u] = min(low[u], tin[v]);
}
}
int countMandatoryNodesOnPath(int startNode, int endNode);
};

```

## 8.10 Centroid Decomposition

**Description:** Builds a vector *fat* where *fat<sub>i</sub>* is who is the father of the node *i* in the centroid decomposed tree.

```

#pragma once
#include "../Contest/template.cpp"
vi centroidDecomposition(const vi2d &g) {
    int n = len(g);
    vi fat(n, -1), szt(n), tk(n);
    function<int(int, int)> calcsz = [&](int x, int f) {
        szt[x] = 1;
        for (auto y : g[x])
            if (y != f && !tk[y]) szt[x] += calcsz(y, x);
        return szt[x];
    };
    function<void(int, int, int)> cdfs = [&](int x, int f, int sz) {
        if (sz < 0) sz = calcsz(x, -1);
        for (auto y : g[x])
            if (!tk[y] && szt[y] * 2 >= sz) {
                szt[x] = 0;
                cdfs(y, f, sz);
                return;
            }
        tk[x] = true;
        fat[x] = f;
        for (auto y : g[x])
            if (!tk[y]) cdfs(y, x, -1);
    };
    cdfs(0, -1, -1);
    return fat;
}

```

## 8.11 Count mandatory nodes on a single path \* \*

**Description:** Given a *startNode* and an *endNode*, count the mandatory nodes \* in the path from *startNode* to *endNode*, that is the number of nodes such \* that are present in

every possible such path. \* \*

**Time:**  $O(N + M)$  \* \*

**Memory:**  $O(N)$  \* \*

**Warning:** The *startNode* and *endNode* is always included in the counting, \* ajust your final answer depending on the problem. Be careful with a \* **disconnected graph** where the path may not exist, treat it appart !. \* \*

```

#pragma once
#include "../Contest/template.cpp"
#include "../Block-Cut tree.cpp"
int BlockCutTree::countMandatoryNodesOnPath(int startNode, int endNode) {
    startNode = idOnTree[startNode], endNode = idOnTree[endNode];
    int ans = !isTreeCutpoint[startNode] + !isTreeCutpoint[endNode];
    int artPoints = 0;
    function<void(int, int)> dfsCount = [&](int u, int p) {
        artPoints += isTreeCutpoint[u];
        if (u == endNode) ans += artPoints;
        for (auto v : treeAdj[u]) {
            if (v != p) {
                dfsCount(v, u);
            }
        }
        artPoints -= isTreeCutpoint[u];
    };
    dfsCount(startNode, -1);
    return ans;
}

```

## 8.12 DSU query

```

struct DSU {
    V<ii> p;
    V<int> s;
    int sum = 0;
    DSU(int n) : p(n, {-1, -1}), s(n, 1) {}
    int find(int x) {
        if (p[x].ff < 0) return x;
        return find(p[x].ff);
    }
    void join(int x, int y, int w) {
        x = find(x);
        y = find(y);
        if (x == y) return;
        sum += w;
        if (s[x] < s[y]) swap(x, y);
        s[x] += s[y];
        p[y] = mp(x, w);
    }
    int query(int x, int y) {
        int r = 0;
        while (x != y) {

```

```

        if (s[x] < s[y])
            r = max(r, p[x].ss), x = p[x].ff;
        else
            r = max(r, p[y].ss), y = p[y].ff;
    }
    return r;
};

```

### 8.13 D'Escopo-Pape

**Description:** Is a single source shortest path that works faster than Dijkstra's algorithm and the Bellman-Ford algorithm in most cases, and will also work for negative edges. However not for negative cycles. There exists cases where it runs in exponential time.

**Usage:** Returns a pair containing two vectors, the first one with the distance from  $s$  to every other node, and another one with the ancestor of each node, note that the ancestor of  $s$  is  $-1$

```

using Edge = pair<ll, int>;
using Adj = vector<vector<Edge>>;
pair<vll, vi> desopo_pape(int s, int n, const Adj &adj) {
    vll ds(n, LLONG_MAX), ps(n, -1);
    ds[s] = 0;
    vi ms(n, 2);
    deque<int> q;
    q.eb(s);
    while (len(q)) {
        int u = q.front();
        q.pop_front();
        ms[u] = 0;
        for (auto [w, v] : adj[u]) {
            if (chmin(ds[v], w + ds[u])) {
                ps[v] = u;
                if (ms[v] == 2)
                    ms[v] = 1, q.pb(v);
                else if (ms[v] == 0)
                    ms[v] = 1, q.pf(v);
            }
        }
    }
    return {ds, ps};
}

```

### 8.14 Dijkstra

```

const int MAXN = 1'000'000;
const ll MAXW = 1'000'000ll;
constexpr ll OO = MAXW * MAXN + 1;
using Edge = pair<ll, int>; // { weight, node}
using Adj = vector<vector<Edge>>;
template <typename T>
using min_heap = priority_queue<T, vector<T>, greater<T>>;
pair<vll, vi> dijkstra(const Adj &g, int s) {

```

```

    int n = len(g);
    min_heap<Edge> pq;
    vll ds(n, 00);
    vi ps(n, -1);
    pq.emp(0, s);
    ds[s] = 0;
    while (len(pq)) {
        auto [du, u] = pq.top();
        pq.pop();
        if (ds[u] < du) continue;
        for (auto [w, v] : g[u]) {
            ll ndv = du + w;
            if (chmin(ds[v], ndv)) {
                ps[v] = u;
                pq.emp(ndv, v);
            }
        }
        return {ds, ps};
    }
    // optional !
    vi recover_path(int source, int ending, const vi &ps) {
        if (ps[ending] == -1) return {};
        int cur = ending;
        vi ans;
        while (cur != -1) {
            ans.eb(cur);
            cur = ps[cur];
        }
        reverse(all(ans));
        return ans;
    }
}

```

### 8.15 Dijkstra (K-shortest paths)

```

const ll oo = 1e9 * 1e5 + 1;
using adj = vector<vector<pll>>;
vector<priority_queue<ll>> dijkstra(const vector<vector<pll>> &g, int n,
    int s,
                                int k) {
    priority_queue<pll, vector<pll>, greater<pll>> pq;
    vector<priority_queue<ll>> dist(n);
    dist[0].emplace(0);
    pq.emplace(0, s);
    while (!pq.empty()) {
        auto [d1, v] = pq.top();
        pq.pop();
        if (not dist[v].empty() and dist[v].top() < d1) continue;
        for (auto [d2, u] : g[v]) {
            if (len(dist[u]) < k) {
                pq.emplace(d2 + d1, u);
                dist[u].emplace(d2 + d1);
            } else {

```

```

        if (dist[u].top() > d1 + d2) {
            dist[u].pop();
            dist[u].emplace(d1 + d2);
            pq.emplace(d2 + d1, u);
        }
    }
}
return dist;
}

```

## 8.16 Extra Edges to Make Digraph Fully Strongly Connected

**Description:** Given a directed graph  $G$  find the necessary edges to add to make the graph a single strongly connected component.

**Time:**  $O(N + M)$

**Memory:**  $O(N)$

```

struct SCC {
    int n, num_sccs;
    vi2d adj;
    vi scc_id;
    SCC(int _n) : n(_n), num_sccs(0), adj(n), scc_id(n, -1) {}
    SCC(const vi2d &adj) : SCC(len(adj)) {
        adj = _adj;
        find_sccs();
    }
    void add_edge(int u, int v) { adj[u].eb(v); }
    void find_sccs() {
        int timer = 1;
        vi tin(n), st;
        st.reserve(n);
        function<int(int)> dfs = [&](int u) -> int {
            int low = tin[u] = timer++; siz = len(st);
            st.eb(u);
            for (int v : adj[u])
                if (scc_id[v] < 0) low = min(low, tin[v] ? tin[v] : dfs(v));
            if (tin[u] == low) {
                rep(i, siz, len(st)) scc_id[st[i]] = num_sccs;
                st.resize(siz);
                num_sccs++;
            }
            return low;
        };
        for (int i = 0; i < n; i++)
            if (!tin[i]) dfs(i);
    };
};

vector<array<int, 2>> extra_edges(const vi2d &adj) {
    SCC scc(adj);
    auto scc_id = scc.scc_id;
    auto num_sccs = scc.num_sccs;

```

```

    if (num_sccs == 1) return {};
    int n = len(adj);
    vi2d scc_adj(num_sccs);
    vi zero_in(num_sccs, 1);
    rep(u, 0, n) {
        for (int v : adj[u]) {
            if (scc_id[u] == scc_id[v]) continue;
            scc_adj[scc_id[u]].eb(scc_id[v]);
            zero_in[scc_id[v]] = 0;
        }
    }
    int random_source = max_element(all(zero_in)) - zero_in.begin();
    vi vis(num_sccs);
    function<int(int)> dfs = [&](int u) {
        if (empty(scc_adj[u])) return u;
        for (int v : scc_adj[u])
            if (!vis[v]) {
                vis[v] = 1;
                int zero_out = dfs(v);
                if (zero_out != -1) return zero_out;
            }
        return (int)-1;
    };
    vector<array<int, 2>> edges;
    vi in_unused;
    rep(i, 0, num_sccs) {
        if (zero_in[i]) {
            vis[i] = 1;
            int zero_out = dfs(i);
            if (zero_out != -1)
                edges.push_back({zero_out, i});
            else
                in_unused.push_back(i);
        }
    }
    rep(i, 1, len(edges)) { swap(edges[i][0], edges[i - 1][0]); }
    rep(i, 0, num_sccs) {
        if (scc_adj[i].empty() && !vis[i]) {
            if (!in_unused.empty()) {
                edges.push_back({i, in_unused.back()});
                in_unused.pop_back();
            } else {
                edges.push_back({i, random_source});
            }
        }
    }
    for (int u : in_unused) edges.push_back({0, u});
    vi to_node(num_sccs);
    rep(i, 0, n) to_node[scc_id[i]] = i;
    for (auto &[u, v] : edges) u = to_node[u], v = to_node[v];
    return edges;
}

```

## 8.17 Find Articulation/Cut Points

**Description:** Given an **undirected** graph find it's articulation points.

**Time:**  $O(N + M)$

**Warning:** A vertex  $u$  can be an articulation point if and only if has at least 2 adjacent vertex

```
const int MAXN(100);
int N;
vi2d G;
int timer;
int tin[MAXN], low[MAXN];
set<int> cpoints;

int dfs(int u, int p = -1) {
    int cnt = 0;
    low[u] = tin[u] = timer++;
    for (auto v : G[u]) {
        if (not tin[v]) {
            cnt++;
            dfs(v, u);
            if (low[v] >= tin[u]) cpoints.insert(u);
            low[u] = min(low[u], low[v]);
        } else if (v != p)
            low[u] = min(low[u], tin[v]);
    }
    return cnt;
}

void getCutPoints() {
    memset(low, 0, sizeof(low));
    memset(tin, 0, sizeof(tin));
    cpoints.clear();
    timer = 1;
    for (int i = 0; i < N; i++) {
        if (tin[i]) continue;
        int cnt = dfs(i);
        if (cnt == 1) cpoints.erase(i);
    }
}
```

## 8.18 Find Bridge-Tree components

**Usage:** *label2CC(u, p)* finds the 2-edge connected component of every node.

**Time:**  $O(n + m)$

```
const int maxn(3 '00' 000);
int tin[maxn], compId[maxn], qtdComps;
vi g[maxn], stck;
int n;
int dfs(int u, int p = -1) {
    int low = tin[u] = len(stck);
    stck.emplace_back(u);
    bool multEdge = false;
    for (auto v : g[u]) {
        if (v == p and !multEdge) {
```

```
            multEdge = 1;
            continue;
        }
        low = min(low, tin[v] == -1 ? dfs(v, u) : tin[v]);
    }
    if (low == tin[u]) {
        for (int i = tin[u]; i < len(stck); i++) compId[stck[i]] =
            qtdComps;
        stck.resize(tin[u]);
        qtdComps++;
    }
    return low;
}

void label2CC() {
    memset(compId, -1, sizeof(int) * n);
    memset(tin, -1, sizeof(int) * n);
    stck.reserve(n);
    for (int i = 0; i < n; i++) {
        if (tin[i] == -1) dfs(i);
    }
}
```

## 8.19 Find Bridges

**Description:** Find every bridge in a **undirected** connected graph.

**Warning:** Remember to read the graph as pair where the second is the id of the edge !

**@Time :**  $O(N + M)$  const int MAXN(10000), MAXM(100000);

```
int N, M, clk, tin[MAXN], low[MAXN], isBridge[MAXM];
vector<pii> G[MAXN];

void dfs(int u, int p = -1) {
    tin[u] = low[u] = clk++;
    for (auto [v, i] : G[u]) {
        if (v == p) continue;
        if (tin[v]) {
            low[u] = min(low[u], tin[v]);
        } else {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] > tin[u]) {
                isBridge[i] = 1;
            }
        }
    }
}

void findBridges() {
    fill(tin, tin + N, 0);
    fill(low, low + N, 0);
    fill(isBridge, isBridge + M, 0);
    clk = 1;
    for (int i = 0; i < N; i++) {
        if (!tin[i]) dfs(i);
    }
}
```

## 8.20 Find Centroid

**Description:** Given a tree (don't forget to make it 'undirected'), find it's centroids.

@Time :  $O(V)$

```
#pragma once
#include "../Contest/template.cpp"
void dfs(int u, int p, int n, vi2d &g, vi &sz, vi &centroid) {
    sz[u] = 1;
    bool iscentroid = true;
    for (auto v : g[u])
        if (v != p) {
            dfs(v, u, n, g, sz, centroid);
            if (sz[v] > n / 2) iscentroid = false;
            sz[u] += sz[v];
        }
    if (n - sz[u] > n / 2) iscentroid = false;
    if (iscentroid) centroid.eb(u);
}

vi getCentroid(vi2d &g, int n) {
    vi centroid;
    vi sz(n);
    dfs(0, -1, n, g, sz, centroid);
    return centroid;
}
```

## 8.21 Find bridges (online)

```
// O((n+m)*log(n))
struct BridgeFinder {
    // 2ecc = 2 edge connected component
    // cc = connected component
    vector<int> parent, dsu_2ecc, dsu_cc, dsu_cc_size;
    int bridges, lca_iteration;
    vector<int> last_visit;
    BridgeFinder(int n)
        : parent(n, -1),
          dsu_2ecc(n),
          dsu_cc(n),
          dsu_cc_size(n, 1),
          bridges(0),
          lca_iteration(0),
          last_visit(n) {
        for (int i = 0; i < n; i++) {
            dsu_2ecc[i] = i;
            dsu_cc[i] = i;
        }
    }

    int find_2ecc(int v) {
        if (v == -1) return -1;
        return dsu_2ecc[v] == v ? v : dsu_2ecc[v] = find_2ecc(dsu_2ecc[v]);
    }
};
```

```
int find_cc(int v) {
    v = find_2ecc(v);
    return dsu_cc[v] == v ? v : dsu_cc[v] = find_cc(dsu_cc[v]);
}

void make_root(int v) {
    v = find_2ecc(v);
    int root = v;
    int child = -1;
    while (v != -1) {
        int p = find_2ecc(parent[v]);
        parent[v] = child;
        dsu_cc[v] = root;
        child = v;
        v = p;
    }
    dsu_cc_size[root] = dsu_cc_size[child];
}

void merge_path(int a, int b) {
    ++lca_iteration;
    vector<int> path_a, path_b;
    int lca = -1;
    while (lca == -1) {
        if (a != -1) {
            a = find_2ecc(a);
            path_a.push_back(a);
            if (last_visit[a] == lca_iteration) {
                lca = a;
                break;
            }
            last_visit[a] = lca_iteration;
            a = parent[a];
        }
        if (b != -1) {
            b = find_2ecc(b);
            path_b.push_back(b);
            if (last_visit[b] == lca_iteration) {
                lca = b;
                break;
            }
            last_visit[b] = lca_iteration;
            b = parent[b];
        }
    }

    for (auto v : path_a) {
        dsu_2ecc[v] = lca;
        if (v == lca) break;
        --bridges;
    }
    for (auto v : path_b) {
        dsu_2ecc[v] = lca;
        if (v == lca) break;
        --bridges;
    }
}
```

```

void add_edge(int a, int b) {
    a = find_2ecc(a);
    b = find_2ecc(b);
    if (a == b) return;
    int ca = find_cc(a);
    int cb = find_cc(b);
    if (ca != cb) {
        ++bridges;
        if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
            swap(a, b);
            swap(ca, cb);
        }
        make_root(a);
        parent[a] = dsu_cc[a] = b;
        dsu_cc_size[cb] += dsu_cc_size[a];
    } else {
        merge_path(a, b);
    }
}
};

```

## 8.22 Floyd Warshall

**Description:** Simply finds the minimal distance for each node to every other node.  $O(V^3)$

```

vector<vll> floyd_warshall(const vector<vll> &adj, ll n) {
    auto dist = adj;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            for (int k = 0; k < n; ++k) {
                dist[j][k] = min(dist[j][k], dist[j][i] + dist[i][k]);
            }
        }
    }
    return dist;
}

```

## 8.23 Functional/Successor Graph

**Description:** Given a functional graph find the vertice after  $k$  moves starting at  $u$  and also the distance between  $u$  and  $v$ , if it's impossible to reach  $v$  starting at  $u$  returns -1.

**Time:** build  $O(N \cdot \text{MAXLOG2})$ , kth  $O(\text{MAXLOG2})$ , dist  $O(\text{MAXLOG2})$

```

const int MAXN(2'000'000), MAXLOG2(24);
int N;
vi2d succ(MAXN, vi(MAXLOG2 + 1));
vi dst(MAXN, 0);
int vis[MAXN];
void dfsbuild(int u) {
    if (vis[u]) return;
    vis[u] = 1;
    int v = succ[u][0];
    dfsbuild(v);
}

```

```

    dst[u] = dst[v] + 1;
}
void build() {
    for (int i = 0; i < N; i++) {
        if (not vis[i]) dfsbuild(i);
    }
    for (int k = 1; k <= MAXLOG2; k++) {
        for (int i = 0; i < N; i++) {
            succ[i][k] = succ[succ[i][k - 1]][k - 1];
        }
    }
}
int kth(int u, ll k) {
    if (k <= 0) return u;
    for (int i = 0; i <= MAXLOG2; i++)
        if ((1ll <= i) & k) u = succ[u][i];
    return u;
}
int dist(int u, int v) {
    int cu = kth(u, dst[u]);
    if (kth(u, dst[u] - dst[v]) == v)
        return dst[u] - dst[v];
    else if (kth(cu, dst[cu] - dst[v]) == v)
        return dst[u] + (dst[cu] - dst[v]);
    else
        return -1;
}

```

## 8.24 Graph Diameter (General Undirected Graph)

**Description:** Computes the diameter of a connected undirected graph — defined as the longest shortest path between any pair of nodes — and returns both the diameter length and the two endpoints that realize it.

**Time:**  $O(n + m)$ , where  $n$  is the number of nodes and  $m$  is the number of edges

**Memory:**  $O(n)$

**Warning:** This implementation assumes the graph is connected and undirected.

```

#pragma once
#include "../Contest/template.cpp"
pair<int, pair<int, int>> uug_diameter(const vector<vector<int>>& adj) {
    int n = adj.size();
    auto bfs = [&](int start) -> pair<int, int> {
        vector<int> dist(n, -1);
        queue<int> q;
        q.push(start);
        dist[start] = 0;
        int farthest = start;
        while (!q.empty()) {
            int v = q.front();
            q.pop();
            for (int u : adj[v]) {
                if (dist[u] == -1) {
                    dist[u] = dist[v] + 1;

```

```

        q.push(u);
        if (dist[u] > dist[farthest]) {
            farthest = u;
        }
    }
}
return {farthest, dist[farthest]};
};
int u = bfs(0).first;
auto [v, diameter] = bfs(u);
return {diameter, {u, v}};
}

```

## 8.25 Heavy light decomposition (supreme)

```

struct HLD {
    int V;
    int id;
    int nb_heavy_path;
    std::vector<std::vector<int>> g;
    std::vector<pair<int, int>> edges;
    std::vector<int> par;

    std::vector<int> depth;
    root // depth[i] = distance between
    // and vertex i
    std::vector<int> subtree_sz;
    // subtree_sz[i] = size of
    // subtree whose root is i
    std::vector<int> heavy_child;
    // heavy_child[i] = child of
    // vertex i on heavy path
    // (Default: -1)
    std::vector<int> tree_id;
    // tree_id[i] = id of tree vertex
    // i belongs to
    std::vector<int> aligned_id,
    aligned_id_inv;
    // aligned_id[i] = aligned
    // id for vertex i
    // (consecutive on heavy
    // edges)
    std::vector<int> head;
    // head[i] = id of vertex on heavy
    // path of vertex i, nearest to root
    std::vector<int> head_ids;
    // consist of head vertex id's
    std::vector<int> heavy_path_id;
    // heavy_path_id[i] =
    // heavy_path_id for vertex
    // [i]

    HLD(const std::vector<std::vector<int>> &e, vector<int> roots = {0})
    : HLD((int)e.size()) {
        g = e;
        build(roots);
    }
    HLD(int sz = 0)
    : V(sz),
      id(0),
      nb_heavy_path(0),

```

```

      g(sz),
      par(sz),
      depth(sz),
      subtree_sz(sz),
      heavy_child(sz),
      tree_id(sz, -1),
      aligned_id(sz),
      aligned_id_inv(sz),
      head(sz),
      heavy_path_id(sz, -1) {}
    void add_edge(int u, int v) {
        edges.emplace_back(u, v);
        g[u].emplace_back(v);
        g[v].emplace_back(u);
    }
    void build_dfs(int root) {
        std::stack<std::pair<int, int>> st;
        par[root] = -1;
        depth[root] = 0;
        st.emplace(root, 0);
        while (!st.empty()) {
            int now = st.top().first;
            int &i = st.top().second;
            if (i < (int)g[now].size()) {
                int nxt = g[now][i++];
                if (nxt == par[now]) continue;
                par[nxt] = now;
                depth[nxt] = depth[now] + 1;
                st.emplace(nxt, 0);
            } else {
                st.pop();
                int max_sub_sz = 0;
                subtree_sz[now] = 1;
                heavy_child[now] = -1;
                for (auto nxt : g[now]) {
                    if (nxt == par[now]) continue;
                    subtree_sz[now] += subtree_sz[nxt];
                    if (max_sub_sz < subtree_sz[nxt])
                        max_sub_sz = subtree_sz[nxt], heavy_child[now] =
nxt;
                }
            }
        }
    }
    void build_bfs(int root, int tree_id_now) {
        std::queue<int> q({root});
        while (!q.empty()) {
            int h = q.front();
            q.pop();
            head_ids.emplace_back(h);
            for (int now = h; now != -1; now = heavy_child[now]) {
                tree_id[now] = tree_id_now;
                aligned_id[now] = id++;
                aligned_id_inv[aligned_id[now]] = now;
            }
        }
    }

```



```

        heavy_path_id[now] = nb_heavy_path;
        head[now] = h;
        for (int nxt : g[now])
            if (nxt != par[now] and nxt != heavy_child[now])
                q.push(nxt);
    }
    nb_heavy_path++;
}

void build(std::vector<int> roots = {0}) {
    int tree_id_now = 0;
    for (auto r : roots) _build_dfs(r), _build_bfs(r, tree_id_now++);
}

// data[i] = value of vertex i
template <class T>
std::vector<T> segtree_rearrange(const std::vector<T> &data) const {
    assert(int(data.size()) == V);
    std::vector<T> ret;
    ret.reserve(V);
    for (int i = 0; i < V; i++) ret.emplace_back(data[aligned_id_inv[i]]);
    return ret;
}

// data[i] = weight of edge[i]
template <class T>
std::vector<T> segtree_rearrange_weighted(
    const std::vector<T> &data) const {
    assert(data.size() == edges.size());
    vector<T> ret(V);
    for (int i = 0; i < (int)edges.size(); i++) {
        auto [u, v] = edges[i];
        if (depth[u] > depth[v]) swap(u, v);
        ret[aligned_id[v]] = data[i];
    }
    return ret;
}

int segtree_edge_index(int i) const {
    auto [u, v] = edges[i];
    if (depth[u] > depth[v]) swap(u, v);
    return aligned_id[v];
}

// query for vertices on path [u, v] (INCLUSIVE)
void for_each_vertex(int u, int v, const auto &f) const {
    static_assert(std::is_invocable_r_v<void, decltype(f), int, int>);
    assert(tree_id[u] == tree_id[v] and tree_id[u] >= 0);
    while (true) {
        if (aligned_id[u] > aligned_id[v]) std::swap(u, v);
        f(std::max(aligned_id[head[v]], aligned_id[u]), aligned_id[v]);
        if (head[u] == head[v]) break;
        v = par[head[v]];
    }
}

void for_each_vertex_noncommutative(int from, int to, const auto &fup,
    const auto &fdown) const {

```

```

        static_assert(std::is_invocable_r_v<void, decltype(fup), int, int
>);
        static_assert(std::is_invocable_r_v<void, decltype(fdown), int,
int>);
        assert(tree_id[from] == tree_id[to] and tree_id[from] >= 0);
        int u = from, v = to;
        const int lca = lowest_common_ancestor(u, v), dlca = depth[lca];
        while (u >= 0 and depth[u] > dlca) {
            const int p = (depth[head[u]] > dlca ? head[u] : lca);
            fup(aligned_id[p] + (p == lca), aligned_id[u]), u = par[p];
        }
        static std::vector<std::pair<int, int>> lrs;
        int sz = 0;
        while (v >= 0 and depth[v] >= dlca) {
            const int p = (depth[head[v]] >= dlca ? head[v] : lca);
            if (int(lrs.size()) == sz) lrs.emplace_back(0, 0);
            lrs.at(sz++) = {p, v}, v = par.at(p);
        }
        while (sz--)
            fdown(aligned_id[lrs.at(sz).first], aligned_id[lrs.at(sz).
second]);
    }

    // query for edges on path [u, v]
    void for_each_edge(int u, int v, const auto &f) const {
        static_assert(std::is_invocable_r_v<void, decltype(f), int, int>);
        assert(tree_id[u] == tree_id[v] and tree_id[u] >= 0);
        while (true) {
            if (aligned_id[u] > aligned_id[v]) std::swap(u, v);
            if (head[u] != head[v]) {
                f(aligned_id[head[v]], aligned_id[v]);
                v = par[head[v]];
            } else {
                if (u != v) f(aligned_id[u] + 1, aligned_id[v]);
                break;
            }
        }
    }

    // lowest_common_ancestor: O(log V)
    int lowest_common_ancestor(int u, int v) const {
        assert(tree_id[u] == tree_id[v] and tree_id[u] >= 0);
        while (true) {
            if (aligned_id[u] > aligned_id[v]) std::swap(u, v);
            if (head[u] == head[v]) return u;
            v = par[head[v]];
        }
    }

    int distance(int u, int v) const {
        assert(tree_id[u] == tree_id[v] and tree_id[u] >= 0);
        return depth[u] + depth[v] - 2 * depth[lowest_common_ancestor(u, v)];
    }

    // Level ancestor, O(log V)
    // if k-th parent is out of range, return -1
    int kth_parent(int v, int k) const {

```

```

    if (k < 0) return -1;
    while (v >= 0) {
        int h = head.at(v), len = depth.at(v) - depth.at(h);
        if (k <= len) return aligned_id_inv.at(aligned_id.at(v) - k);
        k -= len + 1, v = par.at(h);
    }
    return -1;
}
// Jump on tree, O(log V)
int s_to_t_by_k_steps(int s, int t, int k) const {
    if (k < 0) return -1;
    if (k == 0) return s;
    int lca = lowest_common_ancestor(s, t);
    if (k <= depth.at(s) - depth.at(lca)) return kth_parent(s, k);
    return kth_parent(t, depth.at(s) + depth.at(t) - depth.at(lca) * 2
- k);
}
};

```

## 8.26 Kruskal

**Description:** Find the minimum spanning tree of a graph.

**Time:**  $O(E \log E)$

```

#include "../Data Structures/DSU.cpp"
vector<tuple<ll, int, int>> kruskal(int n, vector<tuple<ll, int, int>> &
edges) {
    DSU dsu(n);
    vector<tuple<ll, int, int>> ans;
    sort(all(edges));
    for (auto [a, b, c] : edges) {
        if (dsu.same_set(b, c)) continue;
        ans.emplace_back(a, b, c);
        dsu.union_set(b, c);
    }
    return ans;
}

```

## 8.27 Lowest Common Ancestor

**Description:** Given two nodes of a tree find their lowest common ancestor, or their distance

```

#pragma once
#include "../Contest/template.cpp"
template <typename T>
struct SparseTable {
    vector<T> v;
    int n;
    static const int b = 30;
    vi mask, t;
    int op(int x, int y) { return v[x] < v[y] ? x : y; }
    int msb(int x) { return __builtin_clz(1) - __builtin_clz(x); }
};

```

```

SparseTable() {}
SparseTable(const vector<T> &v_) : v(v_), n(v.size()), mask(n), t(n) {
    for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
        at = (at << 1) & ((1 << b) - 1);
        while (at and op(i, i - msb(at & -at)) == i) at ^= at & -at;
    }
    for (int i = 0; i < n / b; i++)
        t[i] = b * i + b - 1 - msb(mask[b * i + b - 1]);
    for (int j = 1; (1 << j) <= n / b; j++)
        for (int i = 0; i + (1 << j) <= n / b; i++)
            t[n / b * j + i] = op(t[n / b * (j - 1) + i],
t[n / b * (j - 1) + i + (1 << (j -
1))]);
}
int small(int r, int sz = b) { return r - msb(mask[r] & ((1 << sz) -
1)); }
T query(int l, int r) {
    if (r - l + 1 <= b) return small(r, r - l + 1);
    int ans = op(small(l + b - 1), small(r));
    int x = l / b + 1, y = r / b - 1;
    if (x <= y) {
        int j = msb(y - x + 1);
        ans = op(ans, op(t[n / b * j + x], t[n / b * j + y - (1 << j) +
1]));
    }
    return ans;
}
};

struct LCA {
    SparseTable<int> st;
    int n;
    vi v, pos, dep;
    vll wdep;
    LCA(const Graph &g, int root) : n(len(g)), pos(n), wdep(n) {
        dfs(root, 0, -1, g);
        st = SparseTable<int>(vector<int>(all(dep)));
    }
    void dfs(int i, int d, int p, const Graph &g) {
        v.eb(len(dep)) = i, pos[i] = len(dep), dep.eb(d);
        for (auto [w, j] : g[i])
            if (j != p) {
                wdep[j] = wdep[i] + w;
                dfs(j, d + 1, i, g);
                v.eb(len(dep)) = i, dep.eb(d);
            }
    }
    int lca(int a, int b) {
        int l = min(pos[a], pos[b]);
        int r = max(pos[a], pos[b]);
        return v[st.query(l, r)];
    }
    ll dist(int a, int b) { return wdep[a] + wdep[b] - 2ll * wdep[lca(a, b
)]; }
};

```

## 8.28 Lowest Common Ancestor (Binary Lifting)

**Description:** Given a directed tree, finds the LCA between two nodes using binary lifting, and answer a few queries with it.

**Usage:**

- lca: returns the LCA between the two given nodes
- on\_path: finds if  $c$  is in the path from  $a$  to  $b$

**Time:** build  $O(N \cdot \text{MAXLOG}2)$ , all queries  $O(\text{MAXLOG}2)$

```
struct LCA {
    int n;
    const int maxlog;
    vector<vector<int>> up;
    vector<int> depth;
    LCA(const vector<vector<int>> &tree)
        : n(tree.size()),
          maxlog(ceil(log2(n))),
          up(n, vector<int>(maxlog + 1)),
          depth(n, -1) {
        for (int i = 0; i < n; i++) {
            if (depth[i] == -1) {
                depth[i] = 0;
                dfs(i, -1, tree);
            }
        }
    }
    void dfs(int u, int p, const vector<vector<int>> &tree) {
        if (p != -1) {
            depth[u] = depth[p] + 1;
            up[u][0] = p;
            for (int i = 1; i <= maxlog; i++) {
                up[u][i] = up[up[u][i - 1]][i - 1];
            }
            for (int v : tree[u]) {
                if (v == p) continue;
                dfs(v, u, tree);
            }
        }
    }
    int kth_jump(int u, int k) {
        for (int i = maxlog; i >= 0; i--) {
            if ((1 << i) & k) {
                u = up[u][i];
            }
        }
        return u;
    }
    int lca(int u, int v) {
        if (depth[u] < depth[v]) swap(u, v);
        int diff = depth[u] - depth[v];
        u = kth_jump(u, diff);
        if (u == v) return u;
        for (int i = maxlog; i >= 0; i--) {
            if (up[u][i] != up[v][i]) {
                u = up[u][i];
                v = up[v][i];
            }
        }
        return up[u][0];
    }
};
```

```
        v = up[v][i];
    }
    return up[u][0];
}
bool on_path(int u, int v, int s) {
    int uv = lca(u, v), us = lca(u, s), vs = lca(v, s);
    return (uv == s or (us == uv and vs == s) or (vs == uv and us == s));
}
int dist(int u, int v) {
    return depth[u] + depth[v] - 2 * depth[lca(u, v)];
}
};
```

## 8.29 Maximum flow (Dinic)

**Description:** Finds the **maximum flow** in a graph network, given the **source**  $s$  and the **sink**  $t$ . Add edge from  $a$  to  $b$  with capacity  $c$ .

**Time:** In general  $O(E \cdot V^2)$ , if every capacity is 1, and every vertex has in degree equal 1 or out degree equal 1 then  $O(E \cdot \sqrt{V})$ ,

**Warning:** Suffle the edges list for every vertex may take you out of the worst case

```
struct Dinic {
    struct Edge {
        int to, rev;
        ll c, oc;
        ll flow() { return max(oc - c, 0LL); } // if you need flows
    };
    vi lvl, ptr, q;
    vector<vector<Edge>> adj;
    Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
    void addEdge(int a, int b, ll c, ll rcap = 0) {
        adj[a].pb({b, len(adj[b]), c, c});
        adj[b].pb({a, len(adj[a]) - 1, rcap, rcap});
    }
    ll dfs(int v, int t, ll f) {
        if (v == t || !f) return f;
        for (int &i = ptr[v]; i < len(adj[v]); i++) {
            Edge &e = adj[v][i];
            if (lvl[e.to] == lvl[v] + 1)
                if (ll p = dfs(e.to, t, min(f, e.c))) {
                    e.c -= p, adj[e.to][e.rev].c += p;
                    return p;
                }
        }
        return 0;
    }
    ll maxFlow(int s, int t) {
        ll flow = 0;
        q[0] = s;
        rep(L, 0, 31) {
            do { // 'int L=30' maybe faster for random
                ptr = vector<int>(n);
                lvl = vector<int>(n, -1);
                q.clear();
                q.pb(s);
                while (!q.empty()) {
                    int v = q.back(); q.pop_back();
                    for (int &u : adj[v]) {
                        if (lvl[u.to] < lvl[v] + 1) {
                            lvl[u.to] = lvl[v] + 1;
                            q.pb(u.to);
                        }
                    }
                }
                f = 0;
                while (f < len(q)) {
                    int v = q[f];
                    ll p = dfs(v, t, INF);
                    f++;
                    flow += p;
                }
            } while (f < len(q));
        }
        return flow;
    }
};
```

```

        // data
        lvl = ptr = vi(len(q));
        int qi = 0, qe = lvl[s] = 1;
        while (qi < qe && !lvl[t]) {
            int v = q[qi++];
            for (Edge e : adj[v])
                if (!lvl[e.to] && e.c >> (30 - L))
                    q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
        }
        while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
    } while (lvl[t]);
}
return flow;
}
bool leftOfMinCut(int a) { return lvl[a] != 0; }
};

```

### 8.30 Minimum Cost Flow

**Description:** Given a network find the minimum cost to achieve a flow of at most  $f$ . Works with **directed** and **undirected** graphs

**Usage:**

- **add(u, v, c, w):** adds an edge from  $u$  to  $v$  with capacity  $c$  and cost  $w$ .
- **flow(f):** return a pair ( $flow, cost$ ) with the maximum flow until  $f$  with source at  $s$  and sink at  $t$ , with the minimum cost possible.

**Time:**  $O(N \cdot M + f \cdot m \log n)$

```

template <typename T>
struct MinCostFlow {
    struct Edge {
        int to;
        ll c, rc; // capacity, residual capacity
        T w;      // cost
    };
    int n, s, t;
    vector<Edge> edges;
    vi2d g;
    vector<T> dist;
    vi pre;
    MinCostFlow() {}
    MinCostFlow(int n_, int _s, int _t) : n(n_), s(_s), t(_t), g(n) {}
    void addEdge(int u, int v, ll c, T w) {
        g[u].pb(len(edges));
        edges.eb(v, c, 0, w);
        g[v].pb(len(edges));
        edges.eb(u, 0, 0, -w);
    }
    // {flow, cost}
    pair<ll, T> flow(ll flow_limit = LLONG_MAX) {
        ll flow = 0;
        T cost = 0;
        while (flow < flow_limit and dijkstra(s, t)) {
            ll aug = LLONG_MAX;
            for (int i = t; i != s; i = edges[pre[i] ^ 1].to) {

```

```

                aug = min({flow_limit - flow, aug, edges[pre[i]].c});
            }
            for (int i = t; i != s; i = edges[pre[i] ^ 1].to) {
                edges[pre[i]].c -= aug;
                edges[pre[i] ^ 1].c += aug;
                edges[pre[i]].rc += aug;
                edges[pre[i] ^ 1].rc -= aug;
            }
            flow += aug;
            cost += (T)aug * dist[t];
        }
        return {flow, cost};
    }
    // Needs to be called after flow method
    vi2d paths() {
        vi2d p;
        for (;;) {
            int cur = s;
            auto &res = p.eb();
            res.pb(cur);
            while (cur != t) {
                bool found = false;
                for (auto i : g[cur]) {
                    auto &[v, c, cost] = edges[i];
                    if (c > 0) {
                        --c;
                        res.pb(cur = v);
                        found = true;
                        break;
                    }
                }
                if (!found) break;
            }
            if (cur != t) {
                p.ppb();
                break;
            }
        }
        return p;
    }
private:
    bool bellman_ford(int s, int t) {
        dist.assign(n, numeric_limits<T>::max());
        pre.assign(n, -1);
        vc inq(n, false);
        queue<int> q;
        dist[s] = 0;
        q.push(s);
        while (len(q)) {
            int u = q.front();
            q.pop();
            inq[u] = false;
            for (int i : g[u]) {

```

```

        auto [v, c, w, _] = edges[i];
        auto new_dist = dist[u] + w;
        if (c > 0 and dist[v] > new_dist) {
            dist[v] = new_dist;
            pre[v] = i;
            if (not inq[v]) {
                inq[v] = true;
                q.push(v);
            }
        }
    }
    return dist[t] != numeric_limits<T>::max();
}

bool dijkstra(int s, int t) {
    dist.assign(n, numeric_limits<T>::max());
    pre.assign(n, -1);
    dist[s] = 0;
    using PQ = pair<T, int>;
    pqmn<PQ> pq;
    pq.emp(0, s);
    while (len(pq)) {
        auto [cost, u] = pq.top();
        pq.pop();
        if (cost != dist[u]) continue;
        for (int i : g[u]) {
            auto [v, c, _, w] = edges[i];
            auto new_dist = dist[u] + w;
            if (c > 0 and dist[v] > new_dist) {
                dist[v] = new_dist;
                pre[v] = i;
                pq.emp(new_dist, v);
            }
        }
    }
    return dist[t] != numeric_limits<T>::max();
};

```

### 8.31 Minimum Vertex Cover (already divided)

**Description:** Given a bipartite graph  $g$  with  $n$  vertices at left and  $m$  vertices at right, where  $g[i]$  are the possible right side matches of vertex  $i$  from left side, find a minimum vertex cover. The size is the same as the size of the maximum matching, and the complement is a maximum independent set.

```

vector<int> min_vertex_cover(vector<vector<int>> &g, int n, int m) {
    vector<int> match(m, -1), vis;
    auto find = [&](auto &&self, int j) -> bool {
        if (match[j] == -1) return 1;
        vis[j] = 1;
        int di = match[j];
        for (int e : g[di])

```

```

            if (!vis[e] and self(self, e)) {
                match[e] = di;
                return 1;
            }
        return 0;
    };
    for (int i = 0; i < (int)g.size(); i++) {
        vis.assign(match.size(), 0);
        for (int j : g[i]) {
            if (find(find, j)) {
                match[j] = i;
                break;
            }
        }
    }
    int res = (int)match.size() - (int)count(match.begin(), match.end(), -1);
    vector<char> lfound(n, true), seen(m);
    for (int it : match)
        if (it != -1) lfound[it] = false;
    vector<int> q, cover;
    for (int i = 0; i < n; i++)
        if (lfound[i]) q.push_back(i);
    while (!q.empty()) {
        int i = q.back();
        q.pop_back();
        lfound[i] = 1;
        for (int e : g[i])
            if (!seen[e] and match[e] != -1) {
                seen[e] = true;
                q.push_back(match[e]);
            }
    }
    for (int i = 0; i < n; i++)
        if (!lfound[i]) cover.push_back(i);
    for (int i = 0; i < m; i++)
        if (seen[i]) cover.push_back(n + i);
    assert((int)size(cover) == res);
    return cover;
}

```

### 8.32 Prim (MST)

**Description:** Given a graph with  $N$  vertex finds the minimum spanning tree, if there is no such three returns inf, it starts using the edges that connect with each  $s_i \in s$ , if none is provided than it starts with the edges of node 0.

**Time:**  $O(V \log E)$

```

#include "../Contest/template.cpp"
const int MAXN(1'000'000);
int N;
vector<pair<ll, int>> G[MAXN];

```

```

ll prim(vi s = vi(1, 0)) {
    priority_queue<pair<ll, int>, vector<pair<ll, int>>, greater<pair<ll, int>>>
        pq;
    vector<char> ingraph(MAXN);
    int ingraphcnt(0);
    for (auto si : s) {
        ingraphcnt++;
        ingraph[si] = true;
        for (auto &[w, v] : G[si]) pq.emplace(w, v);
    }
    ll mstcost = 0;
    while (ingraphcnt < N and !pq.empty()) {
        ll w;
        int v;
        do {
            tie(w, v) = pq.top();
            pq.pop();
        } while (not pq.empty() and ingraph[v]);
        mstcost += w, ingraph[v] = true, ingraphcnt++;
        for (auto &[w2, v2] : G[v]) {
            pq.emplace(w2, v2);
        }
    }
    return ingraphcnt == N ? mstcost : oo;
}

```

### 8.33 Reachability Tree

```

#include <bits/stdc++.h>
using namespace std;
const int MAXN = 20000 + 100;
int dsu[MAXN];
int n;
const int MAXM = 100000;
int m;
int U[MAXN], V[MAXN];
vector<int> adj[MAXN];
int getRoot(int u) {
    if (u == dsu[u]) return u;
    return dsu[u] = getRoot(dsu[u]);
}
void addEdge(int u, int v) {
    u = getRoot(u);
    v = getRoot(v);
    dsu[n] = n;
    dsu[u] = dsu[v] = n;
    adj[n].push_back(u);
    if (u != v) adj[n].push_back(v);
    ++n;
}
void build() {

```

```

        for (int i = 0; i < n; ++i) dsu[i] = i;
        for (int i = 0; i < m; ++i) addEdge(U[i], V[i]);
    }
    int32_t main() {
        ios_base::sync_with_stdio(0);
        cin.tie(0);
    }

```

### 8.34 Shortest Path With K-edges

**Description:** Given an adjacency matrix of a graph, and a number  $K$  computes the shortest path between all nodes that uses exactly  $K$  edges, so for  $0 \leq i, j \leq N - 1$  ans[i][j] = "the shortest path between  $i$  and  $j$  that uses exactly  $K$  edges, remember to initialize the adjacency matrix with  $\infty$ .

**Time:**  $O(N^3 \cdot \log K)$

```

template <typename T>
vector<vector<T>> prod(vector<vector<T>> &a, vector<vector<T>> &b) {
    const T _oo = numeric_limits<T>::max();
    int n = a.size();
    vector<vector<T>> c(n, vector<T>(n, _oo));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            for (int k = 0; k < n; k++)
                if (a[i][k] != _oo and b[k][j] != _oo)
                    c[i][j] = min(c[i][j], a[i][k] + b[k][j]);

    return c;
}

template <typename T>
vector<vector<T>> shortest_with_k_moves(vector<vector<T>> adj, long long k) {
    if (k == 1) return adj;
    auto ans = adj;
    k--;
    while (k) {
        if (k & 1) ans = prod(ans, adj);
        k >>= 1;
        adj = prod(adj, adj);
    }
    return ans;
}

```

### 8.35 Strongly Connected Components (struct)

**Description:** Find the connected component for each edge (already in a topological order), some additional functions are also provided.

**Time:** Build:  $O(V + E)$

```

struct SCC {
    int n, num_sccs;
    vi2d adj;

```

```

vi scc_id;
SCC(int _n) : n(_n), num_sccs(0), adj(n), scc_id(n, -1) {}
void add_edge(int u, int v) { adj[u].eb(v); }
void find_sccs() {
    int timer = 1;
    vi tin(n), st;
    st.reserve(n);
    function<int(int)> dfs = [&](int u) -> int {
        int low = tin[u] = timer++, siz = len(st);
        st.eb(u);
        for (int v : adj[u])
            if (scc_id[v] < 0) low = min(low, tin[v] ? tin[v] : dfs(v));
        if (tin[u] == low) {
            rep(i, siz, len(st)) scc_id[st[i]] = num_sccs;
            st.resize(siz);
            num_sccs++;
        }
        return low;
    };
    for (int i = 0; i < n; i++)
        if (!tin[i]) dfs(i);
}
vector<set<int>> build_g SCC() {
    vector<set<int>> g SCC;
    for (int i = 0; i < len(adj); ++i)
        for (auto j : adj[i])
            if (scc_id[i] != scc_id[j]) g SCC[scc_id[i]].emplace(scc_id[j]);
    return g SCC;
}
vi2d per_comp() {
    vi2d ret(num_sccs);
    rep(i, 0, n) ret[scc_id[i]].eb(i);
    reverse(all(ret)); // already in topological order ;
    return ret;
}
};

```

### 8.36 Topological Sorting (Kahn)

**Description:** Finds the topological sorting in a **DAG**, if the given graph is not a **DAG** than an empty vector is returned, need to 'initialize' the **INCNT** as you build the graph.

**Time:**  $O(V + E)$

```

const int MAXN(2 '00' 000);
int INCNT[MAXN];
vi2d GOUT(MAXN);
int N;
vi toposort() {
    vi order;
    queue<int> q;
    for (int i = 0; i < N; i++)

```

```

        if (!INCNT[i]) q.emplace(i);
    while (!q.empty()) {
        auto u = q.front();
        q.pop();
        order.emplace_back(u);
        for (auto v : GOUT[u]) {
            INCNT[v]--;
            if (INCNT[v] == 0) q.emplace(v);
        }
    }
    return len(order) == N ? order : vi();
}

```

### 8.37 Topological Sorting (Tarjan)

**Description:** Finds a the topological order for the graph, if there is no such order it means the graph is cyclic, then it returns an empty vector

**Time:**  $O(V + E)$

```

const int maxn(1 '00' 000);
int n, m;
vi g[maxn];
int not_found = 0, found = 1, processed = 2;
int state[maxn];
bool dfs(int u, vi &order) {
    if (state[u] == processed) return true;
    if (state[u] == found) return false;
    state[u] = found;
    for (auto v : g[u]) {
        if (not dfs(v, order)) return false;
    }
    state[u] = processed;
    order.emplace_back(u);
    return true;
}
vi topo_sort() {
    vi order;
    memset(state, 0, sizeof state);
    for (int u = 0; u < n; u++) {
        if (state[u] == not_found and not dfs(u, order)) return {};
    }
    reverse(all(order));
    return order;
}

```

### 8.38 Tree Isomorphism (not rooted)

**Description:** Two trees are considered **isomorphic** if the hash given by *thash()* is the same.

**Time:**  $O(V \cdot \log V)$



```

map<vi, int> mphash;
struct Tree {
    int n;
    vi2d g;
    vi sz, cs;
    Tree(int n_) : n(n_), g(n), sz(n) {}
    void add_edge(int u, int v) {
        g[u].emplace_back(v);
        g[v].emplace_back(u);
    }
    void dfs_centroid(int v, int p) {
        sz[v] = 1;
        bool cent = true;
        for (int u : g[v])
            if (u != p) {
                dfs_centroid(u, v);
                sz[v] += sz[u];
                cent &= not(sz[u] > n / 2);
            }
        if (cent and n - sz[v] <= n / 2) cs.push_back(v);
    }
    int fhash(int v, int p) {
        vi h;
        for (int u : g[v])
            if (u != p) h.push_back(fhash(u, v));
        sort(all(h));
        if (!mphash.count(h)) mphash[h] = mphash.size();
        return mphash[h];
    }
    ll thash() {
        cs.clear();
        dfs_centroid(0, -1);
        if (cs.size() == 1) return fhash(cs[0], -1);
        ll h1 = fhash(cs[0], cs[1]), h2 = fhash(cs[1], cs[0]);
        return (min(h1, h2) << 30ll) + max(h1, h2);
    }
};

```

### 8.39 Tree Isomorphism (rooted)

**Description:** Given a rooted tree find the hash of each subtree, if two roots of two distinct trees have the same hash they are considered isomorphic

**Time:** hash first time in  $O(\log N_v \cdot N_v)$  where  $(N_v)$  is the of the subtree of  $v$

```

map<vi, int> hasher;
int hs = 0;
struct RootedTreeIso {
    int n;
    vi2d adj;
    vi hashes;
    RootedTreeIso(int n_) : n(n_), adj(n), hashes(n, -1) {};
    void add_edge(int u, int v) {
        adj[u].emplace_back(v);

```

```

        adj[v].emplace_back(u);
    }
    int hash(int u, int p = -1) {
        if (hashes[u] != -1) return hashes[u];
        vi children;
        for (auto v : adj[u])
            if (v != p) children.emplace_back(hash(v, u));
        sort(all(children));
        if (!hasher.count(children)) hasher[children] = hs++;
        return hashes[u] = hasher[children];
    }
};

```

### 8.40 Tree diameter (DP)

```

const int MAXN(1'000'000);
int N;
vi G[MAXN];
int diameter, toLeaf[MAXN];
void calcDiameter(int u = 0, int p = -1) {
    int d1, d2;
    d1 = d2 = -1;
    for (auto v : G[u]) {
        if (v != p) {
            calcDiameter(v, u);
            d1 = max(d1, toLeaf[v]);
            tie(d1, d2) = minmax({d1, d2});
        }
    }
    toLeaf[u] = d2 + 1;
    diameter = max(diameter, d1 + d2 + 2);
}

```

### 8.41 Tree edge queries

```

template <typename T = ll, auto E = 0,
          auto F = [](ll a, ll b) { return max(a, b); }>
struct TEQ {
    const int LOG = 20;
    using Graph = vector<vector<pair<ll, int>>>;
    int n;
    vector<int> h;
    vector<vector<int>> par;
    vector<vector<T>> ed;
    TEQ(const Graph& g, int root = 0)
        : n(size(g)),
          h(n, -1),
          par(n, vector<int>(LOG + 1, root)),
          ed(n, vector<T>(LOG + 1, E)) {
        h[root] = 0, dfs(root, g);
    }

```



```

void dfs(int u, const Graph& g) {
    for (auto& [w, v] : g[u]) {
        if (h[v] == -1) {
            h[v] = h[u] + 1, par[v][0] = u, ed[v][0] = w;
            for (int k = 0, p; k < LOG; k++) {
                p = par[v][k];
                par[v][k + 1] = par[p][k];
                ed[v][k + 1] = F(ed[v][k], ed[p][k]);
            }
            dfs(v, g);
        }
    }
}

pair<int, T> up(int u, int dis) {
    T res = E;
    for (int k = 0; k <= LOG; k++) {
        if (dis & (1 << k)) {
            res = F(res, ed[u][k]);
            u = par[u][k];
        }
    }
    return {u, res};
}

pair<int, T> lca(int u, int v) {
    if (h[u] > h[v]) swap(u, v);
    T res = E;
    tie(v, res) = up(v, h[v] - h[u]);
    if (v == u) return {v, res};
    for (int k = LOG; ~k; k--) {
        if (par[u][k] != par[v][k]) {
            res = F(res, ed[v][k]);
            res = F(res, ed[u][k]);
            u = par[u][k], v = par[v][k];
        }
    }
    res = F(res, ed[v][0]);
    res = F(res, ed[u][0]);
    return {par[v][0], res};
}
};

```

## 8.42 Virtual Tree

```

#pragma once
#include "../Contest/template.cpp"
#include "../Lowest common ancestor (sparse table).cpp"
struct VTree {
    int n;
    LCA lca;
    VTree(const Graph& g, int root = 0) : n(len(g)), lca(g, root) {}
    pair<vector<tuple<ll, int, int>>, int> vtree(vector<int> vs) {
        sort(vs.begin(), vs.end(),
            [&](int u, int v) { return lca.pos[u] < lca.pos[v]; });
        for (int i = 0, n = size(vs); i + 1 < n; i++) {

```

```

            vs.pb(lca.lca(vs[i], vs[i + 1]));
        }
        sort(vs.begin(), vs.end(),
            [&](int u, int v) { return lca.pos[u] < lca.pos[v]; });
        vs.erase(unique(all(vs), vs.end()));
        vi st{vs.front()};
        vector<tuple<ll, int, int>> ret;
        for (int i = 1; i < len(vs); i++) {
            int v = vs[i];
            while (len(st) >= 2 && lca.lca(v, st.back()) != st.back()) {
                int a = end(st)[-2];
                int b = st.back();
                ll c = lca.dist(a, b);
                ret.pb(c, a, b);
                st.pop_back();
            }
            st.pb(v);
        }
        while (len(st) >= 2) {
            int a = end(st)[-2];
            int b = st.back();
            ll c = lca.dist(a, b);
            ret.pb(c, a, b);
            st.pop_back();
        }
        return {ret, st.back()};
    }
};

```

## 9 Linear Algebra

### 9.1 Matrix (primitive)

```

#include "../Contest/template.cpp"
template <typename T>
struct Matrix {
    int n, m;
    valarray<valarray<T>> v;
    Matrix(int _n, int _m, int id = 0) : n(_n), m(_m), v(valarray<T>(m), n) {
        if (id) {
            rep(i, 0, n) v[i][i] = 1;
        }
    }
    valarray<T> &operator[](int x) { return v[x]; }
    Matrix transpose() {
        Matrix newv(m, n);
        rep(i, 0, n) rep(j, 0, m) newv[j][i] = (*this)[i][j];
        return newv;
    }
    Matrix operator+(Matrix &b) {
        Matrix ret(*this);

```

```

        return ret.v += b.v;
    }
    Matrix &operator+=(Matrix &b) { return v += b.v; }
    Matrix operator*(Matrix b) {
        Matrix ret(n, b.m);
        rep(i, 0, n) rep(j, 0, m) rep(k, 0, b.m) ret[i][k] +=
            v[i][j] * b.v[j][k];
        return ret;
    }
    Matrix &operator*=(Matrix b) { return *this = *this * b; }
    Matrix power(ll exp) {
        Matrix in = *this;
        Matrix ret(n, n, 1);
        while (exp) {
            if (exp & 1) ret *= in;
            in *= in;
            exp >>= 1;
        }
        return ret;
    }
    /*
    * Alters current matrix.
    * Does gaussian elimination and puts matrix in
    * upper echelon form (possibly reduced).
    * Returns the determinant of the square matrix
    * with side equal to the number of rows of the
    * original matrix.
    */
    T gaussjordanize(int reduced = 0) {
        T det = T(1);
        int line = 0;
        rep(col, 0, m) {
            int pivot = line;
            while (pivot < n && v[pivot][col] == T(0)) pivot++;
            if (pivot >= n) continue;
            swap(v[line], v[pivot]);
            if (line != pivot) det *= T(-1);
            det *= v[line][line];
            v[line] /= T(v[line][col]);
            if (reduced) rep(i, 0, line) {
                v[i] -= T(v[i][col]) * v[line];
            }
            rep(i, line + 1, n) { v[i] -= T(v[i][col]) * v[line]; }
            line++;
        }
        return det * (line == n);
    }
    /*
    * Needs to be called in a square matrix that
    * represents a system of linear equations. Returns {possible solution

```

```

    * number of solutions (2 if infinite solutions)}
    */
    pair<vector<T>, int> solve_system(vector<T> results) {
        Matrix aux(n, m + 1);
        rep(i, 0, n) {
            rep(j, 0, m) aux[i][j] = v[i][j];
            aux[i][m] = results[i];
        }
        T det = aux.gaussjordanize(1);
        int ret = 1 + (det == T(0));
        int n = results.size();
        rrep(i, n - 1, 0 - 1) {
            int pivot = 0;
            while (pivot < n && aux[i][pivot] == T(0)) pivot++;
            if (pivot == n) {
                if (aux[i][m] != T(0)) ret = 0;
            } else
                swap(aux[i], aux[pivot]);
        }
        rrep(i, n - 1, 0 - 1) rep(j, i + 1, n) aux[i][m] -=
            aux[i][j] * aux[j][m];
        rep(i, 0, n) results[i] = aux[i][m];
        rep(i, 0, n) rep(j, 0, m) v[i][j] = aux[i][j];
        return {results, ret};
    }
    /* Does not alter current matrix. Returns {inverse matrix, is curent
    * matrix invertable} */
    pair<Matrix<T>, bool> find_inverse() {
        int n = v.size();
        Matrix<T> aug(n, 2 * n);
        rep(i, 0, n) rep(j, 0, n) aug[i][j] = v[i][j];
        rep(i, 0, n) aug[i][n + i] = 1;
        T det = aug.gaussjordanize(1);
        Matrix<T> ret(n, n);
        rep(i, 0, n) ret[i] = valarray<T>(aug[i][slice(n, n, 1)]);
        return {ret, det != T(0)};
    }
    /* Returns rank of matrix. does not alter it. */
    int get_rank() const {
        if (m == 0) return 0;
        Matrix<T> aux(*this);
        aux.gaussjordanize();
        int resp = 0;
        rep(i, 0, n) resp += (aux[i] != valarray<T>(m)).sum();
        return resp;
    }
};

```

## 10 Math

### 10.1 Arithmetic Progression Sum

Usage:

- $s$  : first term
- $d$  : common difference
- $n$  : number of terms

```
ll arithmeticProgressionSum(ll s, ll d, ll n) {
    return (s + (s + d * (n - 1))) * n / 2ll;
}
```

### 10.2 Binomial

Time:  $O(N \cdot K)$

Memory:  $O(K)$

```
ll binom(ll n, ll k) {
    if (k > n) return 0;
    vll dp(k + 1, 0);
    dp[0] = 1;
    for (ll i = 1; i <= n; i++)
        for (ll j = k; j > 0; j--) dp[j] = dp[j] + dp[j - 1];
    return dp[k];
}
```

### 10.3 Binomial MOD

Description: find  $\binom{n}{k} \pmod{MOD}$

Time:

- precompute: on first call it takes  $O(MAXNBIN)$  to precompute the factorials
- query:  $O(1)$ .

Memory:  $O(MAXNBIN)$

Warning: Remember to set  $MAXNBIN$  properly !

```
const ll MOD = 998244353;
inline ll binom(ll n, ll k) {
    static const int BINMAX = 2'000'000;
    static vll FAC(BINMAX + 1), FINV(BINMAX + 1);
    static bool done = false;
    if (!done) {
        vll INV(BINMAX + 1);
        FAC[0] = FAC[1] = INV[1] = FINV[0] = FINV[1] = 1;
        for (int i = 2; i <= BINMAX; i++) {
            FAC[i] = FAC[i - 1] * i % MOD;
            INV[i] = MOD - MOD / i * INV[MOD % i] % MOD;
            FINV[i] = FINV[i - 1] * INV[i] % MOD;
        }
        done = true;
    }
    if (n < k || n < 0 || k < 0) return 0;
    return FAC[n] * FINV[k] % MOD * FINV[n - k] % MOD;
}
```

### 10.4 Chinese Remainder Theorem

Description: Find the solution  $X$  to the  $N$  modular equations.

$$\begin{aligned} x &\equiv a_1 \pmod{m_1} \\ &\dots \\ x &\equiv a_n \pmod{m_n} \end{aligned} \quad (1)$$

The  $m_i$  don't need to be coprime, if there is no solution then it returns -1.

```
#include "../Contest/template.cpp"
tuple<ll, ll, ll> ext_gcd(ll a, ll b) {
    if (!a) return {b, 0, 1};
    auto [g, x, y] = ext_gcd(b % a, a);
    return {g, y - b / a * x, x};
}
template <typename T = ll>
struct crt {
    T a, m;
    crt() : a(0), m(1) {}
    crt(T a_, T m_) : a(a_), m(m_) {}
    crt operator*(crt C) {
        auto [g, x, y] = ext_gcd(m, C.m);
        if ((a - C.a) % g != 0) a = -1;
        if (a == -1 || C.a == -1) return crt(-1, 0);
        T lcm = m / g * C.m;
        T ans = a + (x * (C.a - a) / g % (C.m / g)) * m;
        return crt((ans % lcm + lcm) % lcm, lcm);
    }
};
template <typename T = ll>
struct Congruence {
    T a, m;
};
template <typename T = ll>
T chinese_remainder_theorem(const vector<Congruence<T>> &equations) {
    crt<T> ans;
    for (auto &[a_, m_] : equations) {
        ans = ans * crt<T>(a_, m_);
    }
    return ans.a;
}
```

### 10.5 Derangement / Matching Problem

Description: Computes the derangement of  $N$ , which is given by the formula:

$$D_N = N! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^N \frac{1}{N!}\right)$$

Time:  $O(N)$

```
#warning Remember to call precompute !
const ll MOD = 1e9 + 7;
const int MAXN(1'000'000);
ll fats[MAXN + 1];
void precompute() {
```

```

    fats[0] = 1;
    for (ll i = 1; i <= MAXN; i++) {
        fats[i] = (fats[i - 1] * i) % MOD;
    }
}

ll fastpow(ll a, ll p, ll m) {
    ll ret = 1;
    while (p) {
        if (p & 1) ret = (ret * a) % MOD;
        p >>= 1;
        a = (a * a) % MOD;
    }
    return ret;
}

ll divmod(ll a, ll b) { return (a * fastpow(b, MOD - 2, MOD)) % MOD; }

ll derangement(const ll n) {
    ll ans = fats[n];
    for (ll i = 1; i <= n; i++) {
        ll k = divmod(fats[n], fats[i]);
        if (i & 1) {
            ans = (ans - k + MOD) % MOD;
        } else {
            ans = (ans + k) % MOD;
        }
    }
    return ans;
}

```

## 10.6 Euler Phi

**Description:** Computes the number of positive integers less than  $N$  that are coprimes with  $N$ , in  $O(\sqrt{N})$ .

```

int phi(int n) {
    if (n == 1) return 1;
    auto fs = factorization(n); // a vector of pair or a map
    auto res = n;
    for (auto [p, k] : fs) {
        res /= p;
        res *= (p - 1);
    }
    return res;
}

```

## 10.7 Euler phi (in range)

**Description:** Computes the number of positive integers less than  $n$  that are coprimes with  $N$ , in the range  $[1, N]$ , in  $O(N \log N)$ .

```

const int MAX = 1e6;
vi range_phi(int n) {
    bitset<MAX> sieve;
    vi phi(n + 1);

```

```

    iota(phi.begin(), phi.end(), 0);
    sieve.set();
    for (int p = 2; p <= n; p += 2) phi[p] /= 2;
    for (int p = 3; p <= n; p += 2) {
        if (sieve[p]) {
            for (int j = p; j <= n; j += p) {
                sieve[j] = false;
                phi[j] /= p;
                phi[j] *= (p - 1);
            }
        }
    }
    return phi;
}

```

## 10.8 Extended Euclidian algorithm

**Description:** Finds the gcd between  $a$  and  $b$  and  $x$  and  $y$  such that  $ax + by = g$

**Time:**  $O(\log \min(a, b))$

**Warning:** If  $a = b = 0$  then there is infity solutions, but 0 is returned. Be careful about overflow.

```

#pragma once
#include "../Contest/template.cpp"
template <typename T>
tuple<T, T, T> extGcd(T a, T b) {
    if (!b) return {a, 1, 0};
    auto [d, x1, y1] = extGcd(b, a % b);
    T x = y1, y = x1 - y1 * (a / b);
    return {d, x, y};
}

```

## 10.9 FFT convolution and exponentiation

```

const ld PI = acos(-1);
/* change the ld to doulbe may increase
 * performance =D */
struct num {
    ld a{0.0}, b{0.0};
    num() {}
    num(ld na) : a{na} {}
    num(ld na, ld nb) : a{na}, b{nb} {}
    const num operator+(const num &c) const { return num(a + c.a, b + c.b); }
    const num operator-(const num &c) const { return num(a - c.a, b - c.b); }
    const num operator*(const num &c) const {
        return num(a * c.a - b * c.b, a * c.b + b * c.a);
    }
    const num operator/(const ll &c) const { return num(a / c, b / c); }
};

void fft(vector<num> &a, bool invert) {

```

```

int n = len(a);
for (int i = 1, j = 0; i < n; i++) {
    int bit = n >> 1;
    for (; j & bit; bit >>= 1) j ^= bit;
    j ^= bit;
    if (i < j) swap(a[i], a[j]);
}
for (int sz = 2; sz <= n; sz <= 1) {
    ld ang = 2 * PI / sz * (invert ? -1 : 1);
    num wsz(cos(ang), sin(ang));
    for (int i = 0; i < n; i += sz) {
        num w(1);
        rep(j, 0, sz / 2) {
            num u = a[i + j], v = a[i + j + sz / 2] * w;
            a[i + j] = u + v;
            a[i + j + sz / 2] = u - v;
            w = w * wsz;
        }
    }
}
if (invert)
    for (num &x : a) x = x / n;
}

vi conv(vi const a, vi const b) {
    vector<num> fa(all(a));
    vector<num> fb(all(b));
    int n = 1;
    while (n < len(a) + len(b)) n <= 1;
    fa.resize(n);
    fb.resize(n);
    fft(fa, false);
    fft(fb, false);
    rep(i, 0, n) fa[i] = fa[i] * fb[i];
    fft(fa, true);
    vi result(n);
    rep(i, 0, n) result[i] = round(fa[i].a);
    while (len(result) and result.back() == 0) result.pop_back();
    /* Uncomment this line if you want a boolean
    * convolution*/
    // for (auto &xi : result) xi = min(xi, 1ll);
    return result;
}

vll poly_exp(vll &ps, int k) {
    vll ret(len(ps));
    auto base = ps;
    ret[0] = 1;
    while (k) {
        if (k & 1) ret = conv(ret, base);
        k >>= 1;
        base = conv(base, base);
    }
    return ret;
}

```

## 10.10 Factorial Factorization

**Description:** Computes the factorization of  $N!$  in  $\varphi(N) * \log N$

**Time:**  $O(\varphi(N) \cdot \log N)$

```

ll E(ll n, ll p) {
    ll k = 0, b = p;
    while (b <= n) {
        k += n / b;
        b *= p;
    }
    return k;
}

map<ll, ll> factorial_factorization(ll n, const vll &primes) {
    map<ll, ll> fs;
    for (const auto &p : primes) {
        if (p > n) break;
        fs[p] = E(n, p);
    }
    return fs;
}

```

## 10.11 Factorization

**Description:** Computes the factorization of  $N$ .

**Time:**  $O(\sqrt{n})$ .

```

map<ll, ll> factorization(ll n) {
    map<ll, ll> ans;
    for (ll i = 2; i * i <= n; i++) {
        ll count = 0;
        for (; n % i == 0; count++, n /= i);
        if (count) ans[i] = count;
    }
    if (n > 1) ans[n]++;
    return ans;
}

```

## 10.12 Factorization (Pollard's Rho)

**Description:** Factorizes a number into its prime factors.

**Time:**  $O(N^{\frac{1}{4}} * \log(N))$ .

```

ll mul(ll a, ll b, ll m) {
    ll ret = a * b - (ll)((ld)1 / m * a * b + 0.5) * m;
    return ret < 0 ? ret + m : ret;
}

ll pow(ll a, ll b, ll m) {
    ll ans = 1;
    for (; b > 0; b /= 2ll, a = mul(a, a, m)) {
        if (b % 2ll == 1) ans = mul(ans, a, m);
    }
    return ans;
}

bool prime(ll n) {

```

```

if (n < 2) return 0;
if (n <= 3) return 1;
if (n % 2 == 0) return 0;
ll r = __builtin_ctzll(n - 1), d = n >> r;
for (int a : {2, 325, 9375, 28178, 450775, 9780504, 795265022}) {
    ll x = pow(a, d, n);
    if (x == 1 or x == n - 1 or a % n == 0) continue;
    for (int j = 0; j < r - 1; j++) {
        x = mul(x, x, n);
        if (x == n - 1) break;
    }
    if (x != n - 1) return 0;
}
return 1;
}

ll rho(ll n) {
    if (n == 1 or prime(n)) return n;
    auto f = [n](ll x) { return mul(x, x, n) + 1; };
    ll x = 0, y = 0, t = 30, prd = 2, x0 = 1, q;
    while (t % 40 != 0 or gcd(prd, n) == 1) {
        if (x == y) x = ++x0, y = f(x);
        q = mul(prd, abs(x - y), n);
        if (q != 0) prd = q;
        x = f(x), y = f(f(y)), t++;
    }
    return gcd(prd, n);
}

vector<ll> fact(ll n) {
    if (n == 1) return {};
    if (prime(n)) return {n};
    ll d = rho(n);
    vector<ll> l = fact(d), r = fact(n / d);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}

```

### 10.13 Fast Pow

**Description:** Computes  $a^b \pmod{m}$

**Time:**  $O(\log B)$ .

```

ll fpow(ll a, ll b, ll m) {
    ll ret = 1;
    while (b) {
        if (b & 1) ret = (ret * a) % m;
        b >>= 1;
        a = (a * a) % m;
    }
    return ret;
}

```

### 10.14 Find $F(\pmod{m})$ period

**Description:** Finds the size of period of  $F(\pmod{m})$ , essentially find  $p_i^{e_i}$  for the prime

factorization in  $m$ , apply a formula, and takes the lcm of it all.

**Time:**  $O(\sqrt{m})$

**Warning:** Be careful with overflows. To improve performance you may use pollard rho for prime factorization instead the "naive way" below.

```

#include "../Contest/template.cpp"
ll find_fib_mod_m_period(ll m) {
    ll ret = 1;
    for (ll p = 2; p * p <= m; p++) {
        if (m % p) continue;
        ll e, pow;
        for (e = 0, pow = 1; m % p == 0; m /= p, pow *= p, e++);
        ll pp = p == 5 ? 20 : p * p - 1;
        ret = lcm(ret, pow / p * pp);
    }
    if (m != 1) ret = lcm(ret, m == 5 ? 20 : m * m - 1);
    return ret;
}

```

### 10.15 Find linear recurrence (Berlekamp-Massey)

**Description:** Given the first  $N$  terms of a linear recurrence finds the smallest recurrence that matches the sequence.

**Time:**  $O(N^2)$

**Warning:** Works faster if the *mod* is const but can be also be a parameter.

Absolute magic !

```

const ll mod = 998244353;
ll modpow(ll b, ll e) {
    ll ans = 1;
    for (; e; b = b * b % mod, e /= 2)
        if (e & 1) ans = ans * b % mod;
    return ans;
}

vl berlekampMassey(vll s) {
    int n = len(s), L = 0, m = 0;
    if (!n) return {};
    vl C(n), B(n), T;
    C[0] = B[0] = 1;
    ll b = 1;
    rep(i, 0, n) {
        ++m;
        ll d = s[i] % mod;
        rep(j, 1, L + 1) d = (d + C[j] * s[i - j]) % mod;
        if (!d) continue;
        T = C;
        ll coef = d * modpow(b, mod - 2) % mod;
        rep(j, m, n) C[j] = (C[j] - coef * B[j - m]) % mod;
        if (2 * L > i) continue;
        L = i + 1 - L;
        B = T;
        b = d;
        m = 0;
    }
}

```

```

}
C.resize(L + 1);
C.erase(C.begin());
for (ll &x : C) x = (mod - x) % mod;
return C;
}

```

## 10.16 Find multiplicative inverse

```

ll inv(ll a, ll m) { return a > 1ll ? m - inv(m % a, a) * m / a : 1ll; }

```

## 10.17 Floor division

```

template <typename T1, typename T2>
constexpr typename std::common_type<T1, T2>::type floor_div(T1 x, T2 y) {
    assert(y != 0);
    if (y < 0) x = -x, y = -y;
    return x < 0 ? (x - y + 1) / y : x / y;
}

```

## 10.18 GCD

```

template <typename T>
T gcd(T a, T b) {
    return b ? gcd(b, a % b) : a;
}

```

## 10.19 Gauss XOR elimination / XOR-SAT

**Description:** Execute gaussian elimination with xor over the system  $Ax = b$  in. The add method must receive a bitset indicating which variables are present in the equation, and the solution of the equation.

**Time:**  $O(\frac{nm^2}{64})$

```

const int MAXXI = 2009;
using Equation = bitset<MAXXI>;
struct GaussXor {
    vector<char> B;
    vector<Equation> A;
    void add(const Equation &ai, bool bi) {
        A.push_back(ai);
        B.push_back(bi);
    }
    pair<bool, Equation> solution() {
        int cnt = 0, n = A.size();
        Equation vis;
        vis.set();
        Equation x;
        for (int j = MAXXI - 1, i; j >= 0; j--) {

```

```

        for (i = cnt; i < n; i++) {
            if (A[i][j]) break;
        }
        if (i == n) continue;
        swap(A[i], A[cnt]), swap(B[i], B[cnt]);
        i = cnt++;
        vis[j] = 0;
        for (int k = 0; k < n; k++) {
            if (i == k || !A[k][j]) continue;
            A[k] ^= A[i];
            B[k] ^= B[i];
        }
    }
    x = vis;
    for (int i = 0; i < n; i++) {
        int acum = 0;
        for (int j = 0; j < MAXXI; j++) {
            if (!A[i][j]) continue;
            if (!vis[j]) {
                vis[j] = 1;
                x[j] = acum ^ B[i];
            }
            acum ^= x[j];
        }
        if (acum != B[i]) return {false, Equation()};
    }
    return {true, x};
};

```

## 10.20 Guess K-th (Berlekamp-Massey)

```

/* Berlekamp-Massey algorithm
 * Given the first n terms of a linear recurrence relation, this algorithm
 * finds the shortest linear recurrence relation that generates the given
 * sequence.
 * Note: mod needs to have inverse
 * Time complexity:  $O(n^2)$ 
 */
template <typename T>
vector<T> berlekamp_massey(const vector<T> &s) {
    vector<T> cur, best;
    int lf, ld;
    for (int i = 0; i < (int)s.size(); i++) {
        T delta = 0;
        for (int j = 0; j < (int)cur.size(); j++)
            delta += s[i - j - 1] * cur[j];
        if (delta == s[i]) continue;
        if (cur.empty()) {
            cur.resize(i + 1);
            lf = i;
            ld = (int)(delta - s[i]).value();
            continue;

```

```

    }
    T coef = -(s[i] - delta) / ld;
    vector<T> c(i - lf - 1);
    c.push_back(coef);
    for (auto &x : best) c.push_back(-x * coef);
    if (c.size() < cur.size()) c.resize(cur.size());
    for (int j = 0; j < (int)cur.size(); j++) c[j] += cur[j];
    if (i - lf + (int)best.size() >= (int)cur.size())
        best = cur, lf = i, ld = (int)(delta - s[i]).value();
    cur = c;
}
return cur;
}
template <typename T>
T get_kth(const vector<T> &rec, const vector<T> &dp, ll k) {
    int n = (int)rec.size();
    assert(rec.size() <= dp.size());
    // use fft to speed up
    auto mul = [&](const vector<T> &a, const vector<T> &b) {
        vector<T> res(2 * n);
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++) res[i + j] += a[i] * b[j];
        for (int i = 2 * n - 1; i >= n; i--)
            for (int j = 1; j <= n; j++) res[i - j] += res[i] * rec[j - 1];
        res.resize(n);
        return res;
    };
    vector<T> a(n), x(n);
    x[0] = 1;
    if (n != 1)
        a[1] = 1;
    else
        a[0] = rec[0];
    while (k) {
        if (k & 1) x = mul(x, a);
        a = mul(a, a);
        k >>= 1;
    }
    T res = 0;
    for (int i = 0; i < n; i++) res += x[i] * dp[i];
    return res;
}
template <typename T>
T guess_kth_term(const vector<T> &s, ll k) {
    if (k < (int)s.size()) return s[k];
    auto coef = berlekamp_massey(s);
    if (coef.empty()) return 0;
    return get_kth(coef, s, k);
}

```

## 10.21 Integer partition

**Description:** Find the total of ways to partition a given number  $N$  in such way that none of the parts is greater than  $K$ .

**Time:**  $O(N \cdot \min(N, K))$

**Memory:**  $O(N)$

**Warning:** Remember to memset everything to  $-1$  before using it

```

const ll MOD = 1000000007;
const int MAXN(100);
ll memo[MAXN + 1];
ll dp(ll n, ll k = oo) {
    if (n == 0) return 1;
    ll &ans = memo[n];
    if (ans != -1) return ans;
    ans = 0;
    for (int i = 1; i <= min(n, k); i++) {
        ans = (ans + dp(n - i, k)) % MOD;
    }
    return ans;
}

```

## 10.22 LCM

```

ll gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }
ll lcm(ll a, ll b) { return a / gcd(a, b) * b; }

```

## 10.23 Linear Recurrence

**Description:** Find the  $n$ -th term of a linear recurrence, given the recurrence  $rec$  and the first  $K$  values of the recurrence, remember that  $first\_k[i]$  is the value of  $f(i)$ , considering 0-indexing.

**Usage:** Suppose you want the  $N$ -th term of Fibonacci the first  $k$  should be 1,1, and the  $rec$  should be 0,1,1,1.

**Time:**  $O(K^3 \log N)$

```

template <typename T>
vector<vector<T>> prod(vector<vector<T>> &a, vector<vector<T>> &b,
                    const ll mod) {
    assert(a.back().size() == b.size());
    int n = a.size();
    int m = a.back().size();
    vector<vector<T>> c(n, vector<T>(m));
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < m; j++) {
            for (int k = 0; k < m; k++) {
                c[i][j] = (c[i][j] + ((a[i][k] * b[k][j]) % mod)) % mod;
            }
        }
    }
    return c;
}
template <typename T>
vector<vector<T>> fpow(vector<vector<T>> &xs, ll p, ll mod) {
    vector<vector<T>> ans(xs.size(), vector<T>(xs.size()));
    for (int i = 0; i < (int)xs.size(); i++) ans[i][i] = 1;
    for (auto b = xs; p; p >>= 1, b = prod(b, b, mod))
        if (p & 1) ans = prod(ans, b, mod);
}

```



```

    return ans;
}
ll linear_req(vector<vector<ll>> rec, vector<ll> first_k, ll n, const ll
mod) {
    int k = first_k.size();
    if (n <= k) return first_k[n - 1];
    ll n2 = n - k;
    rec = fpow(rec, n2, mod);
    ll ret = 0;
    for (int i = 0; i < k; i++)
        ret = (ret + (rec.back()[i] * first_k[i]) % mod) % mod;
    return ret;
}

```

## 10.24 Linear diophantine equation (count)

**Description:**

**Time:**  $O(\log \min(a, b))$

```

#pragma once
#include "../Contest/template.cpp"
#include "../Extended Euclidian algorithm.cpp"
#include "../Linear diophantine equation (solve).cpp"
template <typename T>
T countSolutionsInRange(T a, T b, T c, T minX, T maxX, T minY, T maxY) {
    auto ss = [&](T &x, T &y, T a, T b, T cnt) { x += cnt * b, y -= cnt *
a; };
    assert(a and b);
    auto sol = diophantineEquationSolution(a, b, c);
    if (!sol) return 0;
    auto [x, y] = *sol;
    auto g = get<0>(extGcd(a, b));
    a /= g;
    b /= g;
    int signA = a > 0 ? +1 : -1;
    int signB = b > 0 ? +1 : -1;
    ss(x, y, a, b, (minX - x) / b);
    if (x < minX) ss(x, y, a, b, signB);
    if (x > maxX) return 0;
    int lx1 = x;
    ss(x, y, a, b, (maxX - x) / b);
    if (x > maxX) ss(x, y, a, b, -signB);
    int rx1 = x;
    ss(x, y, a, b, -(minY - y) / a);
    if (y < minY) ss(x, y, a, b, -signA);
    if (y > maxY) return 0;
    int lx2 = x;
    ss(x, y, a, b, -(maxY - y) / a);
    if (y > maxY) ss(x, y, a, b, signA);
    int rx2 = x;
    if (lx2 > rx2) swap(lx2, rx2);
}

```

```

int lx = max(lx1, lx2);
int rx = min(rx1, rx2);
if (lx > rx) return 0;
return (rx - lx) / abs(b) + 1;
}

```

## 10.25 Linear diophantine equation (solve)

**Description:** Finds a solution for  $ax + by = c$ , where  $a, b, c$ , are given and  $x$  and  $y$  unknown.

**Time:**  $O(\log \min(a, b))$

```

#pragma once
#include "../Contest/template.cpp"
#include "../Extended Euclidian algorithm.cpp"
template <typename T>
optional<pair<T, T>> diophantineEquationSolution(T a, T b, T c) {
    if (a == 0 and b == 0) {
        if (c)
            return nullopt;
        else
            return pair<T, T>{(T)0, (T)0};
    }
    auto [g, x0, y0] = extGcd(a < 0 ? a * -1 : a, b < 0 ? b * -1 : b);
    if (c % g) return nullopt;
    x0 *= c / g, y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    pair<T, T> ret;
    ret.first = x0, ret.second = y0;
    return ret;
}

```

## 10.26 List N elements choose K

**Description:** Process every possible combination of  $K$  elements from  $N$  elements, those index marked as 1 in the index vector says which elements are chosen at that moment.

**Time:**  $O(\binom{N}{K} \cdot O(\text{process}))$

```

void process(vi &index) {
    for (int i = 0; i < len(index); i++) {
        if (index[i]) cout << i << " \n"[i == len(index) - 1];
    }
}
void n_choose_k(int n, in k) {
    vi index(n);
    fill(index.end() - k, index.end(), 1);
    do {
        process(index);
    } while (next_permutation(all(index)));
}

```

## 10.27 List primes (Sieve of Eratosthenes)

```
const ll MAXN = 2e5;
vll list_primes(ll n = MAXN) {
    vll ps;
    bitset<MAXN + 1> sieve;
    sieve.set();
    sieve.reset(1);
    for (ll i = 2; i <= n; ++i) {
        if (sieve[i]) ps.push_back(i);
        for (ll j = i * 2; j <= n; j += i) {
            sieve.reset(j);
        }
    }
    return ps;
}
```

## 10.28 Matrix exponentiation

```
const ll MOD = 1'000'000'007;
template <typename T>
vector<vector<T>> prod(vector<vector<T>> &a, vector<vector<T>> &b) {
    int n = len(a);
    vector<vector<T>> c(n, vector<T>(n));
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            for (int k = 0; k < n; k++) {
                c[i][j] = (c[i][j] + ((a[i][k] * b[k][j]) % MOD)) % MOD;
            }
        }
    }
    return c;
}

template <typename T>
vector<vector<T>> fpow(vector<vector<T>> &xs, ll p) {
    vector<vector<T>> ans(len(xs), vector<T>(len(xs)));
    for (int i = 0; i < len(xs); i++) ans[i][i] = 1;
    auto b = xs;
    while (p) {
        if (p & 1) ans = prod(ans, b);
        p >>= 1;
        b = prod(b, b);
    }
    return ans;
}
```

## 10.29 NTT integer convolution and exponentiation

Time:

- Convolution  $O(N \cdot \log N)$ ,
- Exponentiation:  $O(\log K \cdot N \cdot \log N)$

```
#include <bits/stdc++.h>
using namespace std;
#ifdef LOCAL
#include "debug.cpp"
#else
#define dbg(...)
#endif
#define fastio ios_base::sync_with_stdio(0); \
cin.tie(0);
#define all(j) j.begin(), j.end()
#define rall(j) j.rbegin(), j.rend()
#define len(j) (int)j.size()
#define rep(i, a, b) \
for (common_type_t<decltype(a), decltype(b)> i = (a); i < (b); i++)
#define rrep(i, a, b) \
for (common_type_t<decltype(a), decltype(b)> i = (a); i > (b); i--)
#define trav(xi, xs) for (auto &xi : xs)
#define rtrav(xi, xs) for (auto &xi : ranges::views::reverse(xs))
#define loop while (1)
using ll = long long;
#define endl '\n'
#define pb push_back
#define pf push_front
#define ppb pop_back
#define ppf pop_front
#define eb emplace_back
#define ef emplace_front
#define lb lower_bound
#define ub upper_bound
#define fi first
#define se second
#define emp emplace
#define ins insert
#define divc(a, b) ((a) + (b) - 1ll) / (b)
using str = string;
using ull = unsigned long long;
using ld = long double;
using vll = vector<ll>;
using pll = pair<ll, ll>;
using vll2d = vector<vll>;
using vi = vector<int>;
using vi2d = vector<vi>;
using pii = pair<int, int>;
using vpil = vector<pii>;
using vc = vector<char>;
using vs = vector<str>;

#define TT template <typename T>
#define TTU template <typename T, typename U>
TTU using umap = unordered_map<T, U>;
TT using pqmn = priority_queue<T, vector<T>, greater<T>>;
TT using pqmx = priority_queue<T, vector<T>>;

TTU inline bool chmax(T &a, U const &b) { return (a < b ? a = b, 1 : 0); }
TTU inline bool chmin(T &a, U const &b) { return (a > b ? a = b, 1 : 0); }

// read vector
```

```

// TODO: abstract this to any container.
TT std::istream &operator>>(std::istream &is, std::vector<T> &vec) {
    for (auto &element : vec) {
        is >> element;
    }
    return is;
}

// print vector
// TODO: abstract this to any container.
TT ostream &operator<<(ostream &os, vector<T> &xs) {
    rep(i, os.iword(0), xs.size()) os << xs[i] << (i == xs.size() ? "" : "
");
    os.iword(0) = 0;
    return os;
}

// sum a vector, using the default constructor as initial value
// TODO: abstract this to any container.
TT T sumList(vector<T> &xs) { return accumulate(all(xs), T()); }

/*
 * verify if a `predicate` is valid for
 * values in range [l, r]
 *
 * TODO: put a type in predicate ??
 */
// bool all_of_in_range(T l, T r, auto predicate) {
//     return ranges::all_of(views::iota(l, r + 1),
//         [&](T v) { return predicate(v); });
// }

// checks if a value v is the interval [l, r]
TT bool inline between(T v, T l, T r) { return clamp(v, l, r) == v; }

void __init();
void __run();
void __pre_run();

int32_t main() {
#ifdef LOCAL
    fastio;
#endif
    __init();
    int T = 1;
    rep(t, 0, T) {
        dbg(t);
        __pre_run();
        __run();
    }
}

/*
 *
 */
void __init() {}
void __pre_run() {}

```

```

template <int _mod>
struct ntt_mint {
    ll expo(ll b, ll e) {
        ll ret = 1;
        while (e) {
            if (e % 2) ret = ret * b % _mod;
            e /= 2, b = b * b % _mod;
        }
        return ret;
    }
    ll inv(ll b) { return expo(b, _mod - 2); }
    using m = ntt_mint;
    ll v;
    ntt_mint() : v(0) {}
    ntt_mint(ll v_) {
        if (v_ >= _mod or v_ <= -_mod) v_ %= _mod;
        if (v_ < 0) v_ += _mod;
        v = v_;
    }
    m &operator+=(const m &a) {
        v += a.v;
        if (v >= _mod) v -= _mod;
        return *this;
    }
    m &operator-=(const m &a) {
        v -= a.v;
        if (v < 0) v += _mod;
        return *this;
    }
    m &operator*=(const m &a) {
        v = v * ll(a.v) % _mod;
        return *this;
    }
    m &operator/=(const m &a) {
        v = v * inv(a.v) % _mod;
        return *this;
    }
    m operator-() { return m(-v); }
    m &operator^=(ll e) {
        if (e < 0) {
            v = inv(v);
            e = -e;
        }
        v = expo(v, e);
        // possivel otimizacao:
        // cuidado com 0^0
        // v = expo(v, e%(p-1));
        return *this;
    }
    bool operator==(const m &a) { return v == a.v; }
    bool operator!=(const m &a) { return v != a.v; }
    friend istream &operator>>(istream &in, m &a) {
        ll val;
        in >> val;
        a = m(val);
        return in;
    }

```

```

    }
    friend ostream &operator<<(ostream &out, m a) { return out << a.v; }
    friend m operator+(m a, m b) { return a += b; }
    friend m operator-(m a, m b) { return a -= b; }
    friend m operator*(m a, m b) { return a *= b; }
    friend m operator/(m a, m b) { return a /= b; }
    friend m operator^(m a, ll e) { return a ^= e; }
};

const ll MOD1 = 998244353;
const ll MOD2 = 754974721;
const ll MOD3 = 167772161;

template <int _mod>
void ntt(vector<ntt_mint<_mod>> &a, bool rev) {
    int n = len(a);
    auto b = a;
    assert(!(n & (n - 1)));
    ntt_mint<_mod> g = 1;
    while ((g ^ (_mod / 2)) == 1) g += 1;
    if (rev) g = 1 / g;
    for (int step = n / 2; step; step /= 2) {
        ntt_mint<_mod> w = g ^ (_mod / (n / step)), wn = 1;
        for (int i = 0; i < n / 2; i += step) {
            for (int j = 0; j < step; j++) {
                auto u = a[2 * i + j], v = wn * a[2 * i + j + step];
                b[i + j] = u + v;
                b[i + n / 2 + j] = u - v;
            }
            wn = wn * w;
        }
        swap(a, b);
    }
    if (rev) {
        auto n1 = ntt_mint<_mod>(1) / n;
        for (auto &x : a) x *= n1;
    }
}

template <ll _mod>
vector<ntt_mint<_mod>> convolution(const vector<ntt_mint<_mod>> &a,
                                const vector<ntt_mint<_mod>> &b) {
    vector<ntt_mint<_mod>> l(all(a)), r(all(b));
    int N = len(l) + len(r) - 1, n = 1;
    while (n <= N) n *= 2;
    l.resize(n), r.resize(n);
    ntt(l, false), ntt(r, false);
    for (int i = 0; i < n; i++) l[i] *= r[i];
    ntt(l, true);
    l.resize(N);
    // Uncomment for a boolean convolution :)
    /*
    for (auto& li : l) {
        li.v = min(li.v, 1ll);
    }
    */
    while (len(l) and l.back() == 0) l.ppb();
}

```

```

    return l;
}

template <ll _mod>
vector<ntt_mint<_mod>> poly_exp(const vector<ntt_mint<_mod>> &ps, int k) {
    vector<ntt_mint<_mod>> ret(len(ps));
    auto base = ps;
    ret[0] = 1;
    while (k) {
        if (k & 1) ret = convolution<MOD1>(ret, base);
        k >>= 1;
        base = convolution<MOD1>(base, base);
    }
    return ret;
}

using mint = ntt_mint<MOD1>;
using vmint = vector<mint>;
void clear_poly(vmint &p) {
    return;
    while (len(p) and p.back() == 0) p.ppb();
}

vmint poly_sum(vmint as, vmint bs) {
    if (len(as) < len(bs)) swap(as, bs);
    rep(i, 0, len(bs)) as[i] += bs[i];
    return as;
}

void dbg_poly(const vmint &l) {
    rep(i, 0, len(l)) cerr << l[i] << ' ';
    cerr << '\n';
}

vmint merge(vmint l, vmint r, vmint qs) {
    r = convolution<MOD1>(r, poly_exp<MOD1>(qs, len(l)));
    auto ret = poly_sum(l, r);
    return ret;
}

vmint divide_and_conquer(int l, int r, const vmint &ps, const vmint &qs) {
    if (l == r) {
        return vmint(1, ps[l]);
    }
    int m = midpoint(l, r);
    auto valL = divide_and_conquer(l, m, ps, qs),
        valR = divide_and_conquer(m + 1, r, ps, qs);
    valR = convolution<MOD1>(valR, poly_exp<MOD1>(qs, (m - l + 1)));
    valR = poly_sum(valR, valL);
    return valR;
}

vmint polynomial_composition(const vmint &a, const vmint &b) {
    return divide_and_conquer(0, len(a) - 1, a, b);
}

void __run() {
    int n1;
    cin >> n1;
}

```

```

vector<mint> ps(n1 + 1);
cin >> ps;
int n2;
cin >> n2;
vector<mint> qs(n2 + 1);
cin >> qs;
vector<mint> ans = polynomial_composition(ps, qs);
cout << ans << endl;
}

```

## 10.30 NTT integer convolution and exponentiation

Time:

- Convolution  $O(N \cdot \log N)$ ,
- Exponentiation:  $O(\log K \cdot N \cdot \log N)$

```

template <int _mod>
struct mint {
    ll expo(ll b, ll e) {
        ll ret = 1;
        while (e) {
            if (e % 2) ret = ret * b % _mod;
            e /= 2, b = b * b % _mod;
        }
        return ret;
    }
    ll inv(ll b) { return expo(b, _mod - 2); }
    using m = mint;
    ll v;
    mint() : v(0) {}
    mint(ll v_) {
        if (v_ >= _mod or v_ <= -_mod) v_ %= _mod;
        if (v_ < 0) v_ += _mod;
        v = v_;
    }
    m &operator+=(const m &a) {
        v += a.v;
        if (v >= _mod) v -= _mod;
        return *this;
    }
    m &operator-=(const m &a) {
        v -= a.v;
        if (v < 0) v += _mod;
        return *this;
    }
    m &operator*=(const m &a) {
        v = v * ll(a.v) % _mod;
        return *this;
    }
    m &operator/=(const m &a) {
        v = v * inv(a.v) % _mod;
        return *this;
    }
    m operator-() { return m(-v); }
}

```

```

m &operator^=(ll e) {
    if (e < 0) {
        v = inv(v);
        e = -e;
    }
    v = expo(v, e);
    // possível otimizacao:
    // cuidado com 0^0
    // v = expo(v, e%(p-1));
    return *this;
}
bool operator==(const m &a) { return v == a.v; }
bool operator!=(const m &a) { return v != a.v; }
friend istream &operator>>(istream &in, m &a) {
    ll val;
    in >> val;
    a = m(val);
    return in;
}
friend ostream &operator<<(ostream &out, m a) { return out << a.v; }
friend m operator+(m a, m b) { return a += b; }
friend m operator-(m a, m b) { return a -= b; }
friend m operator*(m a, m b) { return a *= b; }
friend m operator/(m a, m b) { return a /= b; }
friend m operator^(m a, ll e) { return a ^= e; }
};

const ll MOD1 = 998244353;
const ll MOD2 = 754974721;
const ll MOD3 = 167772161;
template <int _mod>
void ntt(vector<mint<_mod>> &a, bool rev) {
    int n = len(a);
    auto b = a;
    assert(!(n & (n - 1)));
    mint<_mod> g = 1;
    while ((g ^ (_mod / 2)) == 1) g += 1;
    if (rev) g = 1 / g;
    for (int step = n / 2; step; step /= 2) {
        mint<_mod> w = g ^ (_mod / (n / step)), wn = 1;
        for (int i = 0; i < n / 2; i += step) {
            for (int j = 0; j < step; j++) {
                auto u = a[2 * i + j], v = wn * a[2 * i + j + step];
                b[i + j] = u + v;
                b[i + n / 2 + j] = u - v;
            }
            wn = wn * w;
        }
        swap(a, b);
    }
    if (rev) {
        auto n1 = mint<_mod>(1) / n;
        for (auto &x : a) x *= n1;
    }
}
template <ll _mod>

```

```

vector<mint<_mod>> convolution(const vector<mint<_mod>> &a,
                             const vector<mint<_mod>> &b) {
    vector<mint<_mod>> l(all(a)), r(all(b));
    int N = len(l) + len(r) - 1, n = 1;
    while (n <= N) n *= 2;
    l.resize(n), r.resize(n);
    ntt(l, false), ntt(r, false);
    for (int i = 0; i < n; i++) l[i] *= r[i];
    ntt(l, true);
    l.resize(N);
    // Uncomment for a boolean convolution :
    /*
    for (auto& li : l) {
        li.v = min(li.v, lll);
    }
    */
    return l;
}

template <ll _mod>
vector<mint<_mod>> poly_exp(vector<mint<_mod>> &ps, int k) {
    vector<mint<_mod>> ret(len(ps));
    auto base = ps;
    ret[0] = 1;
    while (k) {
        if (k & 1) ret = convolution(ret, base);
        k >>= 1;
        base = convolution(base, base);
    }
    return ret;
}

```

### 10.31 NTT integer convolution and exponentiation (2 mods) modules)

**Description:** Computes the convolution between the two polynomials and.

**Time:**  $O(N \log N)$

**Warning:** This is pure magic !

```

template <int _mod>
struct mint {
    ll expo(ll b, ll e) {
        ll ret = 1;
        while (e) {
            if (e % 2) ret = ret * b % _mod;
            e /= 2, b = b * b % _mod;
        }
        return ret;
    }
    ll inv(ll b) { return expo(b, _mod - 2); }
    using m = mint;
    ll v;
    mint() : v(0) {}
    mint(ll v_) {

```

```

        if (v_ >= _mod or v_ <= -_mod) v_ %= _mod;
        if (v_ < 0) v_ += _mod;
        v = v_;
    }
    m &operator+=(const m &a) {
        v += a.v;
        if (v >= _mod) v -= _mod;
        return *this;
    }
    m &operator-=(const m &a) {
        v -= a.v;
        if (v < 0) v += _mod;
        return *this;
    }
    m &operator*=(const m &a) {
        v = v * ll(a.v) % _mod;
        return *this;
    }
    m &operator/=(const m &a) {
        v = v * inv(a.v) % _mod;
        return *this;
    }
    m operator-() { return m(-v); }
    m &operator^=(ll e) {
        if (e < 0) {
            v = inv(v);
            e = -e;
        }
        v = expo(v, e);
        // possivel otimizacao:
        // cuidado com 0^0
        // v = expo(v, e%(p-1));
        return *this;
    }
    bool operator==(const m &a) { return v == a.v; }
    bool operator!=(const m &a) { return v != a.v; }
    friend istream &operator>>(istream &in, m &a) {
        ll val;
        in >> val;
        a = m(val);
        return in;
    }
    friend ostream &operator<<(ostream &out, m a) { return out << a.v; }
    friend m operator+(m a, m b) { return a += b; }
    friend m operator-(m a, m b) { return a -= b; }
    friend m operator*(m a, m b) { return a *= b; }
    friend m operator/(m a, m b) { return a /= b; }
    friend m operator^(m a, ll e) { return a ^= e; }
};

const ll MOD1 = 998244353;
const ll MOD2 = 754974721;
const ll MOD3 = 167772161;

template <int _mod>
void ntt(vector<mint<_mod>> &a, bool rev) {
    int n = len(a);
    auto b = a;

```

```

assert(!(n & (n - 1)));
mint<_mod> g = 1;
while ((g ^ (_mod / 2)) == 1) g += 1;
if (rev) g = 1 / g;
for (int step = n / 2; step; step /= 2) {
    mint<_mod> w = g ^ (_mod / (n / step)), wn = 1;
    for (int i = 0; i < n / 2; i += step) {
        for (int j = 0; j < step; j++) {
            auto u = a[2 * i + j], v = wn * a[2 * i + j + step];
            b[i + j] = u + v;
            b[i + n / 2 + j] = u - v;
        }
        wn = wn * w;
    }
    swap(a, b);
}
if (rev) {
    auto n1 = mint<_mod>(1) / n;
    for (auto &x : a) x *= n1;
}
}

tuple<ll, ll, ll> ext_gcd(ll a, ll b) {
    if (!a) return {b, 0, 1};
    auto [g, x, y] = ext_gcd(b % a, a);
    return {g, y - b / a * x, x};
}

template <typename T = ll>
struct crt {
    T a, m;
    crt() : a(0), m(1) {}
    crt(T a_, T m_) : a(a_), m(m_) {}
    crt operator*(crt C) {
        auto [g, x, y] = ext_gcd(m, C.m);
        if ((a - C.a) % g != 0) a = -1;
        if (a == -1 or C.a == -1) return crt(-1, 0);
        T lcm = m / g * C.m;
        T ans = a + (x * (C.a - a) / g % (C.m / g)) * m;
        return crt((ans % lcm + lcm) % lcm, lcm);
    }
};

template <typename T = ll>
struct Congruence {
    T a, m;
};

template <typename T = ll>
T chinese_remainder_theorem(const vector<Congruence<T>> &equations) {
    crt<T> ans;
    for (auto &[a_, m_] : equations) {
        ans = ans * crt<T>(a_, m_);
    }
    return ans.a;
}

#define int long long

```

```

template <ll m1, ll m2>
vll merge_two_mods(const vector<mint<m1>> &a, const vector<mint<m2>> &b) {
    int n = len(a);
    vll ans(n);
    for (int i = 0; i < n; i++) {
        auto cur = crt<ll>();
        auto ai = a[i].v;
        auto bi = b[i].v;
        cur = cur * crt<ll>(ai, m1);
        cur = cur * crt<ll>(bi, m2);
        ans[i] = cur.a;
    }
    return ans;
}

vll convolution_2mods(const vll &a, const vll &b) {
    vector<mint<MOD1>> l(all(a)), r(all(b));
    int N = len(l) + len(r) - 1, n = 1;
    while (n <= N) n *= 2;
    l.resize(n), r.resize(n);
    ntt(l, false), ntt(r, false);
    for (int i = 0; i < n; i++) l[i] *= r[i];
    ntt(l, true);
    l.resize(N);
    vector<mint<MOD2>> l2(all(a)), r2(all(b));
    l2.resize(n), r2.resize(n);
    ntt(l2, false), ntt(r2, false);
    rep(i, 0, n) l2[i] *= r2[i];
    ntt(l2, true);
    l2.resize(N);
    return merge_two_mods(l, l2);
}

vll poly_exp(const vll &xs, ll k) {
    vll ret(len(xs));
    ret[0] = 1;
    auto base = xs;
    while (k) {
        if (k & 1) ret = convolution_2mods(ret, base);
        k >>= 1;
        base = convolution_2mods(base, base);
    }
    return ret;
}

```

## 10.32 Polynomial Taylor Shift

```

using C = complex<double>;
const ll mod = 998244353;
void fft(vector<C> &a) {
    int n = len(a), L = 31 - __builtin_clz(n);
    static vector<complex<long double>> R(2, 1);
    static vector<C> rt(2, 1);
    for (static int k = 2; k < n; k *= 2) {
        R.resize(n);
        rt.resize(n);
    }
}

```



```

    auto x = polar(1.0L, acos(-1.0L) / k);
    for (int i = k; i < 2 * k; i++)
        rt[i] = R[i] = i & 1 ? R[i / 2] * x : R[i / 2];
}
vector<int> rev(n);
for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
for (int i = 0; i < n; i++)
    if (i < rev[i]) swap(a[i], a[rev[i]]);
for (int k = 1; k < n; k *= 2) {
    for (int i = 0; i < n; i += 2 * k)
        for (int j = 0; j < k; j++) {
            auto x = (double *)&rt[j + k], y = (double *)&a[i + j + k];
            C z(x[0] * y[0] - x[1] * y[1], x[0] * y[1] + x[1] * y[0]);
            a[i + j + k] = a[i + j] - z;
            a[i + j] += z;
        }
}
}

vector<double> conv(const vector<double> &a, const vector<double> &b) {
    if (a.empty() || b.empty()) return {};
    vector<double> res(len(a) + len(b) - 1);
    int L = 32 - __builtin_clz(len(res)), n = 1 << L;
    vector<C> in(n), out(n);
    copy(a.begin(), a.end(), begin(in));
    for (int i = 0; i < len(b); i++) in[i].imag(b[i]);
    fft(in);
    for (C &x : in) x *= x;
    for (int i = 0; i < n; i++) {
        out[i] = in[-i & (n - 1)] - conj(in[i]);
    }
    fft(out);
    for (int i = 0; i < len(res); i++) {
        res[i] = imag(out[i]) / (4 * n);
    }
    return res;
}

template <ll M>
vector<ll> convMod(const vector<ll> &a, const vector<ll> &b) {
    if (a.empty() || b.empty()) return {};
    vector<ll> res(len(a) + len(b) + 1);
    int B = 32 - __builtin_clz(len(res)), n = 1 << B, cut = int(sqrt(M));
    vector<C> L(n), R(n), outs(n), outl(n);
    for (int i = 0; i < len(a); i++) {
        L[i] = C((int)a[i] / cut, (int)a[i] % cut);
    }
    for (int i = 0; i < len(b); i++) {
        R[i] = C((int)b[i] / cut, (int)b[i] % cut);
    }
    fft(L), fft(R);
    for (int i = 0; i < n; i++) {
        int j = -i & (n - 1);
        outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
        outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
    }
    fft(outl), fft(outs);
}

```

```

    for (int i = 0; i < len(res); i++) {
        ll av = ll(real(outl[i]) + .5), cv = ll(imag(outs[i]) + .5);
        ll bv = ll(imag(outl[i]) + .5) + ll(real(outs[i]) + .5);
        res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
    }
    return res;
}

ll fexp(ll b, ll e) {
    ll res = 1;
    while (e > 0) {
        if (e & 1) res = res * b % mod;
        b = b * b % mod;
        e >>= 1;
    }
    return res;
}

ll inv(ll n) { return fexp(n, mod - 2); }

vector<ll> shift(vector<ll> &a, ll v) {
    int n = len(a) - 1;
    vector<ll> f(n + 1), g(n + 1);
    vector<ll> i_fact(n + 1);
    f[0] = a[0];
    g[n] = 1;
    i_fact[0] = 1;
    ll fact = 1, potk = 1;
    for (int i = 1; i < n + 1; i++) {
        fact = fact * i % mod;
        f[i] = fact * a[i] % mod;
        potk = (potk * v % mod + mod) % mod;
        g[n - i] = ((potk * inv(fact)) % mod + mod) % mod;
        i_fact[i] = inv(fact);
    }
    auto p = convMod<mod>(f, g);
    vector<ll> res(n + 1);
    for (int i = 0; i < n + 1; i++) {
        res[i] = (p[i + n] * i_fact[i] % mod + mod) % mod;
    }
    return res;
}
}

```

### 10.33 Polyominoes

**Usage:** *buildPolyominoes(x)* creates every polyomino until size *x*, and put it in *polyominoes[x]*, access *polyomino.v* to find the vector of pairs representing the coordinates of each piece, considering that the polyomino was 'rooted' in coordinate (0,0).

**Warning:** note that when accessing *polyominoes[x]* only the first *x* coordinates are valid.

```

#include "../Contest/template.cpp"
const int MAXP = 10;
using pii = pair<int, int>;
// This implementation considers the rotations as
// distinct
// 0, 10, 10+9, 10+9+8...
int pos[11] = {0, 10, 19, 27, 34, 40, 45, 49, 52, 54, 55};
struct Polyominoes {

```





```

        for (int i = 0, carry = 0;
            i < (int)max(a.size(), v.a.size()) || carry; ++i) {
            if (i == (int)res.a.size()) res.a.push_back(0);
            res.a[i] += carry + (i < (int)a.size() ? a[i] : 0);
            carry = res.a[i] >= base;
            if (carry) res.a[i] -= base;
        }
        return res;
    }
    return *this - (-v);
}

bigint operator-(const bigint &v) const {
    if (sign == v.sign) {
        if (abs() >= v.abs()) {
            bigint res = *this;
            for (int i = 0, carry = 0; i < (int)v.a.size() || carry;
++i) {
                res.a[i] -= carry + (i < (int)v.a.size() ? v.a[i] : 0);
                carry = res.a[i] < 0;
                if (carry) res.a[i] += base;
            }
            res.trim();
            return res;
        }
        return -(v - *this);
    }
    return *this + (-v);
}

void operator*=(int v) {
    if (v < 0) sign = -sign, v = -v;
    for (int i = 0, carry = 0; i < (int)a.size() || carry; ++i) {
        if (i == (int)a.size()) a.push_back(0);
        long long cur = a[i] * (long long)v + carry;
        carry = (int)(cur / base);
        a[i] = (int)(cur % base);
        // asm("divl %%ecx" : "=a"(carry),
        //      "=d"(a[i]) : "A"(cur), "c"(base));
    }
    trim();
}

bigint operator*(int v) const {
    bigint res = *this;
    res *= v;
    return res;
}

void operator*=(long long v) {
    if (v < 0) sign = -sign, v = -v;
    if (v > base) {
        *this = *this * (v / base) * base + *this * (v % base);
        return;
    }
    for (int i = 0, carry = 0; i < (int)a.size() || carry; ++i) {
        if (i == (int)a.size()) a.push_back(0);
        long long cur = a[i] * (long long)v + carry;

```

```

        carry = (int)(cur / base);
        a[i] = (int)(cur % base);
        // asm("divl %%ecx" : "=a"(carry),
        //      "=d"(a[i]) : "A"(cur), "c"(base));
    }
    trim();
}

bigint operator*(long long v) const {
    bigint res = *this;
    res *= v;
    return res;
}

friend pair<bigint, bigint> divmod(const bigint &a1, const bigint &b1)
{
    int norm = base / (b1.a.back() + 1);
    bigint a = a1.abs() * norm;
    bigint b = b1.abs() * norm;
    bigint q, r;
    q.a.resize(a.a.size());
    for (int i = a.a.size() - 1; i >= 0; i--) {
        r *= base;
        r += a.a[i];
        int s1 = r.a.size() <= b.a.size() ? 0 : r.a[b.a.size()];
        int s2 = r.a.size() <= b.a.size() - 1 ? 0 : r.a[b.a.size() -
1];
        int d = ((long long)base * s1 + s2) / b.a.back();
        r -= b * d;
        while (r < 0) r += b, --d;
        q.a[i] = d;
    }
    q.sign = a1.sign * b1.sign;
    r.sign = a1.sign;
    q.trim();
    r.trim();
    return make_pair(q, r / norm);
}

bigint operator/(const bigint &v) const { return divmod(*this, v).
first; }

bigint operator%(const bigint &v) const { return divmod(*this, v).
second; }

void operator/=(int v) {
    if (v < 0) sign = -sign, v = -v;
    for (int i = (int)a.size() - 1, rem = 0; i >= 0; --i) {
        long long cur = a[i] + rem * (long long)base;
        a[i] = (int)(cur / v);
        rem = (int)(cur % v);
    }
    trim();
}

bigint operator/(int v) const {
    bigint res = *this;
    res /= v;
    return res;
}

```

```

int operator%(int v) const {
    if (v < 0) v = -v;
    int m = 0;
    for (int i = a.size() - 1; i >= 0; --i)
        m = (a[i] + m * (long long)base) % v;
    return m * sign;
}

void operator+=(const bigint &v) { *this = *this + v; }
void operator-=(const bigint &v) { *this = *this - v; }
void operator*=(const bigint &v) { *this = *this * v; }
void operator/=(const bigint &v) { *this = *this / v; }

bool operator<(const bigint &v) const {
    if (sign != v.sign) return sign < v.sign;
    if (a.size() != v.a.size())
        return a.size() * sign < v.a.size() * v.sign;
    for (int i = a.size() - 1; i >= 0; i--)
        if (a[i] != v.a[i]) return a[i] * sign < v.a[i] * v.sign;
    return false;
}

bool operator>(const bigint &v) const { return v < *this; }
bool operator<=(const bigint &v) const { return !(v < *this); }
bool operator>=(const bigint &v) const { return !(*this < v); }
bool operator==(const bigint &v) const {
    return !(*this < v) && !(v < *this);
}

bool operator!=(const bigint &v) const { return *this < v || v < *this; }
; }

void trim() {
    while (!a.empty() && !a.back()) a.pop_back();
    if (a.empty()) sign = 1;
}

bool isZero() const { return a.empty() || (a.size() == 1 && !a[0]); }

bigint operator-() const {
    bigint res = *this;
    res.sign = -sign;
    return res;
}

bigint abs() const {
    bigint res = *this;
    res.sign *= res.sign;
    return res;
}

long long longValue() const {
    long long res = 0;
    for (int i = a.size() - 1; i >= 0; i--) res = res * base + a[i];
    return res * sign;
}

friend bigint gcd(const bigint &a, const bigint &b) {
    return b.isZero() ? a : gcd(b, a % b);
}

friend bigint lcm(const bigint &a, const bigint &b) {
    return a / gcd(a, b) * b;
}

```

```

void read(const string &s) {
    sign = 1;
    a.clear();
    int pos = 0;
    while (pos < (int)s.size() && (s[pos] == '-' || s[pos] == '+')) {
        if (s[pos] == '-') sign = -sign;
        ++pos;
    }
    for (int i = s.size() - 1; i >= pos; i -= base_digits) {
        int x = 0;
        for (int j = max(pos, i - base_digits + 1); j <= i; j++)
            x = x * 10 + s[j] - '0';
        a.push_back(x);
    }
    trim();
}

friend istream &operator>>(istream &stream, bigint &v) {
    string s;
    stream >> s;
    v.read(s);
    return stream;
}

friend ostream &operator<<(ostream &stream, const bigint &v) {
    if (v.sign == -1) stream << '-';
    stream << (v.a.empty() ? 0 : v.a.back());
    for (int i = (int)v.a.size() - 2; i >= 0; --i)
        stream << setw(base_digits) << setfill('0') << v.a[i];
    return stream;
}

static vector<int> convert_base(const vector<int> &a, int old_digits,
                                int new_digits) {
    vector<long long> p(max(old_digits, new_digits) + 1);
    p[0] = 1;
    for (int i = 1; i < (int)p.size(); i++) p[i] = p[i - 1] * 10;
    vector<int> res;
    long long cur = 0;
    int cur_digits = 0;
    for (int i = 0; i < (int)a.size(); i++) {
        cur += a[i] * p[cur_digits];
        cur_digits += old_digits;
        while (cur_digits >= new_digits) {
            res.push_back((int)(cur % p[new_digits]));
            cur /= p[new_digits];
            cur_digits -= new_digits;
        }
    }
    res.push_back((int)cur);
    while (!res.empty() && !res.back()) res.pop_back();
    return res;
}

typedef vector<long long> vll;
static vll karatsubaMultiply(const vll &a, const vll &b) {
    int n = a.size();
    vll res(n + n);

```

```

    if (n <= 32) {
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++) res[i + j] += a[i] * b[j];
        return res;
    }
    int k = n >> 1;
    vll a1(a.begin(), a.begin() + k);
    vll a2(a.begin() + k, a.end());
    vll b1(b.begin(), b.begin() + k);
    vll b2(b.begin() + k, b.end());
    vll alb1 = karatsubaMultiply(a1, b1);
    vll a2b2 = karatsubaMultiply(a2, b2);
    for (int i = 0; i < k; i++) a2[i] += a1[i];
    for (int i = 0; i < k; i++) b2[i] += b1[i];
    vll r = karatsubaMultiply(a2, b2);
    for (int i = 0; i < (int)alb1.size(); i++) r[i] -= alb1[i];
    for (int i = 0; i < (int)a2b2.size(); i++) r[i] -= a2b2[i];
    for (int i = 0; i < (int)r.size(); i++) res[i + k] += r[i];
    for (int i = 0; i < (int)alb1.size(); i++) res[i] += alb1[i];
    for (int i = 0; i < (int)a2b2.size(); i++) res[i + n] += a2b2[i];
    return res;
}

bigint operator*(const bigint &v) const {
    vector<int> a6 = convert_base(this->a, base_digits, 6);
    vector<int> b6 = convert_base(v.a, base_digits, 6);
    vll a(a6.begin(), a6.end());
    vll b(b6.begin(), b6.end());
    while (a.size() < b.size()) a.push_back(0);
    while (b.size() < a.size()) b.push_back(0);
    while (a.size() & (a.size() - 1)) a.push_back(0), b.push_back(0);
    vll c = karatsubaMultiply(a, b);
    bigint res;
    res.sign = sign * v.sign;
    for (int i = 0, carry = 0; i < (int)c.size(); i++) {
        long long cur = c[i] + carry;
        res.a.push_back((int)(cur % 1000000));
        carry = (int)(cur / 1000000);
    }
    res.a = convert_base(res.a, 6, base_digits);
    res.trim();
    return res;
}
};

```

## 11.2 Integer Mod

```

#include "../Contest/template.cpp"
template <ll m>
struct mod_int {
    ll x;
    mod_int(ll v = 0) {
        x = v % m;
    }
};

```

```

    if (x < 0) v += m;
}
mod_int &operator+=(mod_int const &b) {
    x += b.x;
    if (x >= m) x -= m;
    return *this;
}
mod_int &operator--(mod_int const &b) {
    x -= b.x;
    if (x < 0) x += m;
    return *this;
}
mod_int &operator*=(mod_int const &b) {
    x = (ll)x * b.x % m;
    return *this;
}
friend mod_int mpow(mod_int a, ll e) {
    mod_int res = 1;
    while (e) {
        if (e & 1) res *= a;
        a *= a;
        e >>= 1;
    }
    return res;
}
friend mod_int inverse(mod_int a) { return mpow(a, m - 2); }
mod_int &operator/=(mod_int const &b) { return *this *= inverse(b); }
friend mod_int operator+(mod_int a, mod_int const b) { return a += b; }
mod_int operator++(int) { return this->x = (this->x + 1) % m; }
mod_int operator++() { return this->x = (this->x + 1) % m; }
friend mod_int operator-(mod_int a, mod_int const b) { return a -= b; }
friend mod_int operator-(mod_int const a) { return 0 - a; }
mod_int operator--(int) { return this->x = (this->x - 1 + m) % m; }
mod_int operator--() { return this->x = (this->x - 1 + m) % m; }
friend mod_int operator*(mod_int a, mod_int const b) { return a *= b; }
friend mod_int operator/(mod_int a, mod_int const b) { return a /= b; }
friend ostream &operator<<(ostream &os, mod_int const &a) {
    return os << a.x;
}
friend bool operator==(mod_int const &a, mod_int const &b) {
    return a.x == b.x;
}
friend bool operator!=(mod_int const &a, mod_int const &b) {
    return a.x != b.x;
}
};

```

## 11.3 Integer Mod (complete)

```

#include "../Contest/template.cpp"
template <ll Mod>
struct modint {
    static constexpr ll mod = Mod;
    ll v;
    modint() : v(0) {}
    template <ll Mod2>
    modint(const modint<Mod2> &x) : v(x.value()) {}
    modint(ll x) : v(safe_mod(x)) {}
    ll &value() { return v; }
    const ll &value() const { return v; }
    static ll safe_mod(ll x) {
        return x >= 0 ? x % mod : ((x % mod) + mod) % mod;
    }
    template <typename T>
    explicit operator T() const {
        return (T)v;
    }
    bool operator==(const modint rhs) const noexcept { return v == rhs.v; }
    bool operator!=(const modint rhs) const noexcept { return v != rhs.v; }
    bool operator<(const modint rhs) const noexcept { return v < rhs.v; }
    bool operator<=(const modint rhs) const noexcept { return v <= rhs.v; }
    bool operator>(const modint rhs) const noexcept { return v > rhs.v; }
    bool operator>=(const modint rhs) const noexcept { return v >= rhs.v; }
    modint operator++(int) {
        modint res = *this;
        *this += 1;
        return res;
    }
    modint operator--(int) {
        modint res = *this;
        *this -= 1;
        return res;
    }
    modint &operator++() { return *this += 1; }
    modint &operator--() { return *this -= 1; }
    modint operator+(const modint &rhs) const { return modint(*this); }
    modint operator-(const modint &rhs) const { return modint(*this); }
    friend modint operator+(const modint lhs, const modint rhs) noexcept {
        return modint(lhs) += rhs;
    }
    friend modint operator-(const modint lhs, const modint rhs) noexcept {
        return modint(lhs) -= rhs;
    }
    friend modint operator*(const modint lhs, const modint rhs) noexcept {
        return modint(lhs) *= rhs;
    }
    friend modint operator/(const modint lhs, const modint rhs) noexcept {
        return modint(lhs) /= rhs;
    }
    modint &operator+=(const modint rhs) {
        v += rhs.v;

```

```

        if (v >= mod) v -= mod;
        return *this;
    }
    modint &operator--(const modint rhs) {
        if (v < rhs.v) v += mod;
        v -= rhs.v;
        return *this;
    }
    modint &operator*=(const modint rhs) {
        v = v * rhs.v % mod;
        return *this;
    }
    modint &operator/=(const modint rhs) { return *this *= rhs.inv(); }
    modint pow(ll p) const {
        static_assert(mod < static_cast<ll>(1) << 32,
            "Modulus must be less than 2**32");
        modint res = 1, a = *this;
        while (p) {
            if (p & 1) res *= a;
            a *= a;
            p >>= 1;
        }
        return res;
    }
    modint inv() const { return pow(mod - 2); }
    modint sqrt() const {
        modint b = 1;
        while (b.pow((mod - 1) >> 1) != 1) b += 1;
        ll m = mod - 1, e = 0;
        while (~m & 1) m >>= 1, e++;
        auto x = pow((m - 1) >> 1);
        auto y = *this * x * x;
        x *= *this;
        auto z = b.pow(m);
        while (y != 1) {
            ll j = 0;
            for (modint t = y; t != 1; t *= t, ++j);
            z.pow(1ll << (e - j - 1));
            x *= z;
            z *= z;
            y *= z;
            e = j;
        }
        return x;
    }
    friend ostream &operator<<(ostream &s, const modint &x) {
        s << x.value();
        return s;
    }
    friend istream &operator>>(istream &s, modint &x) {
        ll value;
        s >> value;
        x = {value};
        return s;
    }
};

```

## 11.4 Matrix

```
template <typename T>
struct Matrix {
    vector<vector<T>> d;
    Matrix() : Matrix(0) {}
    Matrix(int n) : Matrix(n, n) {}
    Matrix(int n, int m) : Matrix(vector<vector<T>>(n, vector<T>(m))) {}
    Matrix(const vector<vector<T>> &v) : d(v) {}
    constexpr int n() const { return (int)d.size(); }
    constexpr int m() const { return n() ? (int)d[0].size() : 0; }
    void rotate() { *this = rotated(); }
    Matrix<T> rotated() const {
        Matrix<T> res(m(), n());
        for (int i = 0; i < m(); i++) {
            for (int j = 0; j < n(); j++) {
                res[i][j] = d[n() - j - 1][i];
            }
        }
        return res;
    }
    Matrix<T> pow(int power) const {
        assert(n() == m());
        auto res = Matrix<T>::identity(n());
        auto b = *this;
        while (power) {
            if (power & 1) res *= b;
            b *= b;
            power >>= 1;
        }
        return res;
    }
    Matrix<T> submatrix(int start_i, int start_j, int rows = INT_MAX,
                        int cols = INT_MAX) const {
        rows = min(rows, n() - start_i);
        cols = min(cols, m() - start_j);
        if (rows <= 0 or cols <= 0) return {};
        Matrix<T> res(rows, cols);
        for (int i = 0; i < rows; i++)
            for (int j = 0; j < cols; j++)
                res[i][j] = d[i + start_i][j + start_j];
        return res;
    }
    Matrix<T> translated(int x, int y) const {
        Matrix<T> res(n(), m());
        for (int i = 0; i < n(); i++) {
            for (int j = 0; j < m(); j++) {
                if (i + x < 0 or i + x >= n() or j + y < 0 or j + y >= m())
                    continue;
                res[i + x][j + y] = d[i][j];
            }
        }
        return res;
    }
}
```

```

}
static Matrix<T> identity(int n) {
    Matrix<T> res(n);
    for (int i = 0; i < n; i++) res[i][i] = 1;
    return res;
}
vector<T> &operator[](int i) { return d[i]; }
const vector<T> &operator[](int i) const { return d[i]; }
Matrix<T> &operator+=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x += value;
    }
    return *this;
}
Matrix<T> operator+(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x + value;
    }
    return res;
}
Matrix<T> &operator-=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x -= value;
    }
    return *this;
}
Matrix<T> operator-(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x - value;
    }
    return res;
}
Matrix<T> &operator*=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x *= value;
    }
    return *this;
}
Matrix<T> operator*(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x * value;
    }
    return res;
}
Matrix<T> &operator/=(T value) {
    for (auto &row : d) {
        for (auto &x : row) x /= value;
    }
    return *this;
}
Matrix<T> operator/(T value) const {
    auto res = *this;
    for (auto &row : res) {
        for (auto &x : row) x = x / value;
    }
}
```

```

    }
    return res;
}
Matrix<T> &operator+=(const Matrix<T> &o) {
    assert(n() == o.n() and m() == o.m());
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            d[i][j] += o[i][j];
        }
    }
    return *this;
}
Matrix<T> operator+(const Matrix<T> &o) const {
    assert(n() == o.n() and m() == o.m());
    auto res = *this;
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            res[i][j] = res[i][j] + o[i][j];
        }
    }
    return res;
}
Matrix<T> &operator-=(const Matrix<T> &o) {
    assert(n() == o.n() and m() == o.m());
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            d[i][j] -= o[i][j];
        }
    }
    return *this;
}
Matrix<T> operator-(const Matrix<T> &o) const {
    assert(n() == o.n() and m() == o.m());
    auto res = *this;
    for (int i = 0; i < n(); i++) {
        for (int j = 0; j < m(); j++) {
            res[i][j] = res[i][j] - o[i][j];
        }
    }
    return res;
}
Matrix<T> &operator*=(const Matrix<T> &o) {
    *this = *this * o;
    return *this;
}
Matrix<T> operator*(const Matrix<T> &o) const {
    assert(m() == o.n());
    Matrix<T> res(n(), o.m());
    for (int i = 0; i < res.n(); i++) {
        for (int j = 0; j < res.m(); j++) {
            auto &x = res[i][j];
            for (int k = 0; k < m(); k++) {
                x += (d[i][k] * o[k][j]);
            }
        }
    }
    return res;
}

```

```

}
friend istream &operator>>(istream &is, Matrix<T> &mat) {
    for (auto &row : mat)
        for (auto &x : row) is >> x;
    return is;
}
friend ostream &operator<<(ostream &os, const Matrix<T> &mat) {
    bool frow = 1;
    for (auto &row : mat) {
        if (not frow) os << '\n';
        bool first = 1;
        for (auto &x : row) {
            if (not first) os << ' ';
            os << x;
            first = 0;
        }
        frow = 0;
    }
    return os;
}
auto begin() { return d.begin(); }
auto end() { return d.end(); }
auto rbegin() { return d.rbegin(); }
auto rend() { return d.rend(); }
auto begin() const { return d.begin(); }
auto end() const { return d.end(); }
auto rbegin() const { return d.rbegin(); }
auto rend() const { return d.rend(); }
};

```

## 12 Probability

### 12.1 k-success-in-binomial-distribution

**Description:** Given a binomial distribution with probability  $p$ , find the probability of getting exactly  $k$  successes in  $n$  trials.

**Warning:** Not tested yet. Remember to call `calc()` before making queries.

```

#include "../Contest/template.cpp"
namespace k_success {
    const int MAX = 1e8;
    double logfact[MAX];
    void calc() {
        logfact[0] = 0;
        for (int i = 1; i < MAX; i++) logfact[i] = logfact[i - 1] + log(i);
    }
    double binom(int n, int k, double p) {
        return exp(logfact[n] - logfact[k] - logfact[n - k] + k * log(p) +
            (n - k) * log(1 - p));
    }
}; // namespace k_success

```



## 13 Problems

### 13.1 2081 - Fixed-Lenght Paths II

```
#include <bits/stdc++.h>
using namespace std;
const int MAXN = 2'000'000;
int N, K1, K2;
vector<int> ADJ[MAXN];
int64_t ans = 0;
int sz[MAXN], removed[MAXN];
void calcSize(int u, int p = -1) {
    sz[u] = 1;
    for (int v : ADJ[u]) {
        if (v != p and !removed[v]) {
            calcSize(v, u);
            sz[u] += sz[v];
        }
    }
}
int findCentroid(int u, int mxSz, int p = -1) {
    for (int v : ADJ[u]) {
        if (!removed[v] and v != p and sz[v] * 2 >= mxSz)
            return findCentroid(v, mxSz, u);
    }
    return u;
}
int64_t cnt[MAXN], totCnt[MAXN], initialSum;
int mxD;
void dfs(int u, int p, int d) {
    if (d > K2) return;
    cnt[d]++;
    mxD = max(mxD, d);
    if (K1 - 1 <= d and d <= K2 - 1) initialSum++;
    for (int v : ADJ[u]) {
        if (v != p and !removed[v]) {
            dfs(v, u, d + 1);
        }
    }
}
void solve(int curRoot) {
    calcSize(curRoot);
    int centroid = findCentroid(curRoot, sz[curRoot]);
    removed[centroid] = true;
    int totMxD = 0;
    initialSum = (K1 == 1);
    // cerr << "centroid: " << centroid << '\n';
    for (int v : ADJ[centroid]) {
        if (!removed[v]) {
            // cerr << "v: " << v << '\n';
            mxD = 0;
            int64_t curSum = initialSum;
            dfs(v, centroid, 1);
            totMxD = max(totMxD, mxD);
        }
    }
}
```

```
for (int d = 1; d <= mxD; d++) {
    // cerr << "d : " << d << " curSum: " << curSum << '\n';
    ans += (curSum * cnt[d]);
    int pl = max(0, K1 - d) - 1;
    if (pl >= 0) curSum += totCnt[pl];
    int pr = K2 - d;
    curSum -= totCnt[pr];
}
for (int d = 1; d <= mxD; d++) totCnt[d] += cnt[d];
fill(&cnt[1], &cnt[1] + mxD + 1, 0);
}
// cerr << "centroid: " << centroid
// << " ans: " << ans << '\n';
for (int d = 1; d <= totMxD; d++) totCnt[d] = 0;
for (int v : ADJ[centroid])
    if (!removed[v]) solve(v);
}
int32_t main() {
    ios_base::sync_with_stdio(!cin.tie(0));
    totCnt[0] = 1;
    cin >> N >> K1 >> K2;
    for (int i = 0; i < N - 1; i++) {
        int u, v;
        cin >> u >> v;
        u--, v--;
        ADJ[u].emplace_back(v);
        ADJ[v].emplace_back(u);
    }
    solve(0);
    cout << ans << '\n';
}
// AC, centroid decomposition
```

### 13.2 Fixed lenght pahts I

```
#include <bits/stdc++.h>
using namespace std;
const int MAXN = 2'000'000;
int N, K;
vector<int> ADJ[MAXN];
int64_t ans;
bool removed[MAXN];
int cnt[MAXN];
int sz[MAXN];
void calcSize(int u, int p = -1) {
    sz[u] = 1;
    for (int v : ADJ[u]) {
        if (v != p and !removed[v]) {
            calcSize(v, u);
            sz[u] += sz[v];
        }
    }
}
```



```

    }
}

int getCentroid(int mxSz, int u, int p = -1) {
    for (int v : ADJ[u]) {
        if (v != p and !removed[v] and sz[v] >= mxSz)
            return getCentroid(mxSz, v, u);
    }
    return u;
}

int mxd;
void dfs(int u, int p, bool upd, int d = 1) {
    if (d > K) return;
    mxd = max(mxd, d);
    upd ? cnt[d]++ : ans += cnt[K - d];
    for (int v : ADJ[u]) {
        if (v != p and !removed[v]) dfs(v, u, upd, d + 1);
    }
}

void solve(int u) {
    calcSize(u);
    int c = getCentroid(sz[u] >> 1, u);
    removed[c] = true;
    mxd = 0;
    cnt[0] = 1;
    for (int v : ADJ[c]) {
        if (!removed[v]) {
            dfs(v, c, false);
            dfs(v, c, true);
        }
    }
    for (int i = 0; i <= mxd; i++) cnt[i] = 0;
    for (int v : ADJ[c]) {
        if (!removed[v]) solve(v);
    }
}

int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    cin >> N >> K;
    for (int i = 0; i < N - 1; i++) {
        int u, v;
        cin >> u >> v;
        u--, v--;
        ADJ[u].emplace_back(v);
        ADJ[v].emplace_back(u);
    }
    solve(0);
    cout << ans << '\n';
    return 0;
}

```

### 13.3 Fixed lenght paths II

```

#include <bits/stdc++.h>
using namespace std;
const int MAXN = 2'000'000;
int N, K1, K2;
vector<int> ADJ[MAXN];
int64_t ans = 0;
int sz[MAXN], removed[MAXN];
void calcSize(int u, int p = -1) {
    sz[u] = 1;
    for (int v : ADJ[u]) {
        if (v != p and !removed[v]) {
            calcSize(v, u);
            sz[u] += sz[v];
        }
    }
}

int findCentroid(int u, int mxSz, int p = -1) {
    for (int v : ADJ[u]) {
        if (!removed[v] and v != p and sz[v] * 2 >= mxSz)
            return findCentroid(v, mxSz, u);
    }
    return u;
}

int64_t cnt[MAXN], totCnt[MAXN], initialSum;
int mxD;
void dfs(int u, int p, int d) {
    if (d > K2) return;
    cnt[d]++;
    mxD = max(mxD, d);
    if (K1 - 1 <= d and d <= K2 - 1) initialSum++;
    for (int v : ADJ[u]) {
        if (v != p and !removed[v]) {
            dfs(v, u, d + 1);
        }
    }
}

void solve(int curRoot) {
    calcSize(curRoot);
    int centroid = findCentroid(curRoot, sz[curRoot]);
    removed[centroid] = true;
    int totMxD = 0;
    initialSum = (K1 == 1);
    // cerr << "centroid: " << centroid << '\n';
    for (int v : ADJ[centroid]) {
        if (!removed[v]) {
            // cerr << "v: " << v << '\n';
            mxD = 0;
            int64_t curSum = initialSum;
            dfs(v, centroid, 1);
            totMxD = max(totMxD, mxD);
            for (int d = 1; d <= mxD; d++) {
                // cerr << "d : " << d << " curSum: " << curSum << '\n';

```

```

        ans += (curSum * cnt[d]);
        int pl = max(0, K1 - d) - 1;
        if (pl >= 0) curSum += totCnt[pl];
        int pr = K2 - d;
        curSum -= totCnt[pr];
    }
    for (int d = 1; d <= mxD; d++) totCnt[d] += cnt[d];
    fill(&cnt[1], &cnt[1] + mxD + 1, 0);
}
// cerr << "centroid: " << centroid
//<< " ans: " << ans << '\n';
for (int d = 1; d <= totMxD; d++) totCnt[d] = 0;
for (int v : ADJ[centroid])
    if (!removed[v]) solve(v);
}

int32_t main() {
    ios_base::sync_with_stdio(!cin.tie(0));
    totCnt[0] = 1;
    cin >> N >> K1 >> K2;
    for (int i = 0; i < N - 1; i++) {
        int u, v;
        cin >> u >> v;
        u--, v--;
        ADJ[u].emplace_back(v);
        ADJ[v].emplace_back(u);
    }
    solve(0);
    cout << ans << '\n';
}
// AC, centroid decomposition

```

## 14 Strings

### 14.1 Z-Function

#### 14.1.1 Find smallest period using Z-function

**Description:** Finds the smallest period  $p$  of a sequence  $s$ , such that  $s$  is a repetition of its prefix of length  $p$ , i.e.  $s = t + t + \dots + t$  with  $t = s[0..p)$ . If no such  $p < n$  exists, returns  $n$  (the full length).

Uses the Z-function to efficiently check where the prefix fully matches the remaining suffix.

**Usage:** The function `z_function_find_period(s)` takes a sequence  $s$  (string, vector, etc.) and returns an integer representing the smallest period.

```

string s = "ababab";
int period = z_function_find_period(s);
// period = 2
vector<int> v = {1, 2, 3, 1, 2, 3};
int period_v = z_function_find_period(v);
// period_v = 3

```

**Time:**  $O(n)$   
**Memory:**  $O(n)$

```

#pragma once
#include "../Contest/template.cpp"
#include "../z-function-build.cpp"
template <typename Seq>
int z_function_find_period(Seq &s) {
    auto z = z_function_build(s);
    int n = s.size();
    for (int i = 1; i < n; i++) {
        if ((n % i) == 0 and i + z[i] == n) {
            return i;
        }
    }
    return n;
}

```

#### 14.1.2 Pattern Matching

**Description:** The Pattern Matching algorithm uses the Z-function to efficiently find all occurrences of a pattern  $p$  in a text  $S$ . The Z-function is first computed for the concatenation of the pattern and text ( $p + \$ + S$ ), where  $\$$  is a unique separator character that does not appear in either  $p$  or  $S$ . The Z-array allows us to quickly determine where in the text the pattern occurs by comparing segments of the text with the pattern.

**Usage:** The function `pattern_matching(s, p)` takes two arguments:  $s$ , the text, and  $p$ , the pattern to search for. It returns a vector of integers, each of which represents the starting index of a match of the pattern  $p$  in the text  $S$ .

```

string text = "abacabadabacaba";
string pattern = "abac";
vector<int> result = pattern_matching(text, pattern);
// result = [0, 8]

string text_v = "ababab";
string pattern_v = "ab";
vector<int> result_v = pattern_matching(text_v, pattern_v);
// result_v = [0, 2, 4]

```

**Time:**  $O(n + m)$ , where  $n$  is the length of the text  $S$  and  $m$  is the length of the pattern  $p$ .

**Memory:**  $O(n + m)$

**Warning:** The pattern must be non-empty, and the text must also be non-empty. Ensure that the separator character  $\$$  is not present in the pattern or text.

```

#pragma once
#include "../Contest/template.cpp"
#include "../z-function-build.cpp"
template <typename Seq>
vector<int> z_function_pattern_matching(const Seq& s, const Seq& p) {
    if (s.empty() or p.empty()) return {};
    vector<int> z = z_function_build(p + "$" + s);
    vector<int> occurrences;
    int m = p.size();

```

```

for (int i = m + 1; i < z.size(); ++i) {
    if (z[i] == m) occurrences.push_back(i - m - 1);
}
return occurrences;
}

```

### 14.1.3 Z-function approximate pattern matching

**Description:** This algorithm finds all positions in a string  $s$  where the pattern  $p$  approximately occurs with up to  $k$  consecutive mismatches. It works by leveraging the Z-function to check how many characters match from the beginning (prefix) and end (suffix) of the pattern at each position in  $s$ . The input sequences  $s$  (text) and  $p$  (pattern) can be strings or any container supporting indexing and reversal.

**Time:**  $O(|s| + |p|)$

**Memory:**  $O(|s| + |p|)$

```

#pragma once
#include "../Contest/template.cpp"
#include "../z-function-build.cpp"
template <typename Seq>
vector<int> z_function_approximate_pattern_matching(Seq s, Seq p, int k) {
    if (s.empty() or p.empty()) return {};
    vector<int> z = z_function_build(p + "$" + s);
    reverse(begin(p), end(p));
    reverse(begin(s), end(s));
    vector<int> zr = z_function_build(p + "$" + s);
    vector<int> occurrences;
    int m = p.size();
    for (int i = m + 1; i + p.size() - 1 < z.size(); ++i) {
        int r = i + p.size() - 1 - 1 - p.size();
        int rr = (p.size() + 1) + (s.size() - 1 - r);
        if (z[i] + zr[rr] >= p.size() - k) occurrences.emplace_back(i - m - 1);
    }
    return occurrences;
}

```

### 14.1.4 Z-function building

**Description:** The Z-function is an algorithm used to compute the Z-array of a given string. For a string  $s$ ,  $Z[i]$  represents the length of the longest common prefix between the string  $s$  and the suffix of  $s$  starting from the index  $i$ .

**Usage:** The function `z_function_build(s)` takes a single argument  $s$ , which is a string (or any container-like structure), and returns a vector of integers representing the Z-function of the input.

```

string s = "abacaba";
vector<int> result = z_function_build(s);
// result = [0, 0, 1, 0, 3, 0, 1]

```

```

vector<int> v = {1, 2, 3, 1, 2, 3};
vector<int> result_v = z_function_build(v);
// result_v = [0, 1, 2, 3, 0, 1]

```

**Time:**  $O(n)$

**Memory:**  $O(n)$

**Warning:** By definition  $Z[0] = 0$ , remember to treat it apart.

```

#pragma once
#include "../Contest/template.cpp"
template <typename Seq>
vector<int> z_function_build(const Seq& s) {
    int n = int(s.size());
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r) z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) ++z[i];
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
    return z;
}

```

## 14.2 Count distinct anagrams

```

const ll MOD = 1e9 + 7;
const int maxn = 1e6;
vll fs(maxn + 1);
void precompute() {
    fs[0] = 1;
    for (ll i = 1; i <= maxn; i++) {
        fs[i] = (fs[i - 1] * i) % MOD;
    }
}
ll fpow(ll a, int n, ll mod = LLONG_MAX) {
    if (n == 0) return 1;
    if (n == 1) return a;
    ll x = fpow(a, n / 2, mod) % mod;
    return ((x * x) % mod * (n & 1 ? a : 1ll)) % mod;
}
ll distinctAnagrams(const string &s) {
    precompute();
    vi hist('z' - 'a' + 1, 0);
    for (auto &c : s) hist[c - 'a']++;
    ll ans = fs[len(s)];
    for (auto &q : hist) {
        ans = (ans * fpow(fs[q], MOD - 2, MOD)) % MOD;
    }
    return ans;
}

```

## 14.3 Double hash range query

```
#include "../Contest/template.cpp"
using ll = long long;
using vll = vector<ll>;
using pll = pair<ll, ll>;
const int MAXN(1'000'000);
const ll MOD = 1000027957;
const ll MOD2 = 1000015187;
const ll P = 31;
ll p[MAXN + 1], p2[MAXN + 1];
void precompute() {
    p[0] = p2[0] = 1;
    for (int i = 1; i <= MAXN; i++)
        p[i] = (P * p[i - 1]) % MOD, p2[i] = (P * p2[i - 1]) % MOD2;
}
struct Hash {
    int n;
    vll h, h2, hi, hi2;
    Hash() {}
    Hash(const string &s) : n(s.size()), h(n), h2(n), hi(n), hi2(n) {
        h[0] = h2[0] = s[0];
        for (int i = 1; i < n; i++)
            h[i] = (s[i] + h[i - 1] * P) % MOD,
            h2[i] = (s[i] + h2[i - 1] * P) % MOD2;
        hi[n - 1] = hi2[n - 1] = s[n - 1];
        for (int i = n - 2; i >= 0; i--)
            hi[i] = (s[i] + hi[i + 1] * P) % MOD,
            hi2[i] = (s[i] + hi2[i + 1] * P) % MOD2;
    }
    pll query(int l, int r) {
        ll hash = (h[r] - (l ? h[l - 1] * p[r - l + 1] % MOD : 0));
        ll hash2 = (h2[r] - (l ? h2[l - 1] * p2[r - l + 1] % MOD2 : 0));
        return {(hash < 0 ? hash + MOD : hash),
                (hash2 < 0 ? hash2 + MOD2 : hash2)};
    }
    pll query_inv(int l, int r) {
        ll hash = (hi[l] - (r + 1 < n ? hi[r + 1] * p[r - l + 1] % MOD : 0));
        ll hash2 = (hi2[l] - (r + 1 < n ? hi2[r + 1] * p2[r - l + 1] % MOD2 : 0));
        return {(hash < 0 ? hash + MOD : hash),
                (hash2 < 0 ? hash2 + MOD2 : hash2)};
    }
};
```

## 14.4 Hash range query

```
#include "../Contest/template.cpp"
const ll P = 31;
const ll MOD = 1e9 + 9;
const int MAXN(1e6);
```

```
ll ppow[MAXN + 1];
void pre_calc() {
    ppow[0] = 1;
    for (int i = 1; i <= MAXN; i++) ppow[i] = (ppow[i - 1] * P) % MOD;
}
struct Hash {
    int n;
    vll h, hi;
    Hash(const string &s) : n(s.size()), h(n), hi(n) {
        h[0] = s[0];
        hi[n - 1] = s[n - 1];
        for (int i = 1; i < n; i++) {
            h[i] = (s[i] + h[i - 1] * P) % MOD;
            hi[n - i - 1] = (s[n - i - 1] + hi[n - i - 1] * P) % MOD;
        }
    }
    ll qry(int l, int r) {
        ll hash = (h[r] - (l ? h[l - 1] * ppow[r - l + 1] % MOD : 0));
        return hash < 0 ? hash + MOD : hash;
    }
    ll qry_inv(int l, int r) {
        ll hash = (hi[l] - (r + 1 < n ? hi[r + 1] * ppow[r - l + 1] % MOD : 0));
        return hash < 0 ? hash + MOD : hash;
    }
};
```

## 14.5 Hash unsigned long long $2^{64} - 1$

**Description:** Arithmetic mod  $2^{64} - 1$ . 2x slower than mod  $2^{64}$  and more code, but works on evil test data (e.g. Thue-Morse, where ABBA... and BAAB... of length  $2^{10}$  hash the same mod  $2^{64}$ ).  
"typedef ull H;" instead if you think test data is random.

```
#include "../Contest/template.cpp"
struct H {
    ull x;
    H(ull x = 0) : x(x) {}
    H operator+(H o) { return x + o.x + (x + o.x < x); }
    H operator-(H o) { return *this + ~o.x; }
    H operator*(H o) {
        auto m = ((__uint128_t)x * o.x);
        return H((ull)m) + (ull)(m >> 64);
    }
    ull get() const { return x + !x; }
    bool operator==(H o) const { return get() == o.get(); }
    bool operator<(H o) const { return get() < o.get(); }
};
static const H C = (long long)1e11 + 3; // (order ~ 3e9; random also ok)
struct Hash {
    int n;
    vector<H> ha, pw;
```

```

Hash(string &str) : n(str.size()), ha((int)str.size() + 1), pw(ha) {
    pw[0] = 1;
    for (int i = 0; i < (int)str.size(); i++)
        ha[i + 1] = ha[i] * C + str[i], pw[i + 1] = pw[i] * C;
}
H query(int a, int b) { // hash [a, b]
    b++;
    return ha[b] - ha[a] * pw[b - a];
}
};

vector<H> getHashes(string &str, int length) {
    if ((int)str.size() < length) return {};
    H h = 0, pw = 1;
    for (int i = 0; i < length; i++) h = h * C + str[i], pw = pw * C;
    vector<H> ret = {h};
    for (int i = length; i < (int)str.size(); i++)
        ret.push_back(h = h * C + str[i] - pw * str[i - length]);
    return ret;
}

H hashString(string &s) {
    H h{};
    for (char c : s) h = h * C + c;
    return h;
}

```

## 14.6 K-th digit in digit string

**Description:** Find the  $k$ -th digit in a *digit string*, only works for  $1 \leq k \leq 10^{18}$  !

**Time:** precompute  $O(1)$ , query  $O(1)$

```

using ull = vector<ull>;
ull pow10;
vector<array<ull, 4>> memo;
void precompute(int maxpow = 18) {
    ull qtd = 1;
    ull start = 1;
    ull end = 9;
    ull curlenght = 9;
    ull startstr = 1;
    ull endstr = 9;
    for (ull i = 0, j = 1ll; (int)i < maxpow; i++, j *= 10ll) pow10.eb(j);
    for (ull i = 0; i < maxpow - 1ull; i++) {
        memo.push_back({start, end, startstr, endstr});
        start = end + 1ll;
        end = end + (9ll * pow10[qtd]);
        curlenght = end - start + 1ull;
        qtd++;
        startstr = endstr + 1ull;
        endstr = (endstr + 1ull) + (curlenght)*qtd - 1ull;
    }
}

char kthDigit(ull k) {
    int qtd = 1;

```

```

    for (auto [s, e, ss, es] : memo) {
        if (k >= ss and k <= es) {
            ull pos = k - ss;
            ull index = pos / qtd;
            ull nmr = s + index;
            int i = k - ss - qtd * index;
            return ((nmr / pow10[qtd - i - 1]) % 10) + '0';
        }
        qtd++;
    }
    return 'X';
}

```

## 14.7 KMP

```

/**
 * Author: Johan Sannemo
 * Date: 2016-12-15
 * License: CC0
 * Description: pi[x] computes the length of the longest prefix of s that
 * ends
 * at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to
 * find
 * all occurrences of a string. Time: O(n) Status: Tested on
 * kattis:stringmatching
 */
/*
 * @Title: Prefix function - Knuth-Morris-Pratt
 * @Description: Given a string $$$ builds an array $$$ such that
 * $$$i$$$ is the longest suffix that ends in $$$i$$$ and is also a prefix
 * of $$$0$$$.
 */
#pragma once

vi pi(const string& s) {
    vi p(sz(s));
    rep(i, 1, sz(s)) {
        int g = p[i - 1];
        while (g && s[i] != s[g]) g = p[g - 1];
        p[i] = g + (s[i] == s[g]);
    }
    return p;
}

vi match(const string& s, const string& pat) {
    vi p = pi(pat + '\0' + s), res;
    rep(i, sz(p) - sz(s), sz(p)) if (p[i] == sz(pat))
        res.push_back(i - 2 * sz(pat));
    return res;
}

```

## 14.8 Longest Palindrome Substring (Manacher)

**Description:** Finds the longest palindrome substring, manacher returns a vector where

the  $i$ -th position is how much is possible to grow the string to the left and the right of  $i$  and keep it a palindrome.

**Time:**  $O(N)$

---

```

vi manacher(const string &s) {
    int n = len(s) - 2;
    vi p(n + 2);
    int l = 1, r = 1;
    for (int i = 1; i <= n; i++) {
        p[i] = max(0, min(r - i, p[l + (r - i)]));
        while (s[i - p[i]] == s[i + p[i]]) p[i]++;
        if (i + p[i] > r) l = i - p[i], r = i + p[i];
        p[i]--;
    }
    return p;
}

string longest_palindrome(const string &s) {
    string t("$#");
    for (auto c : s) t.push_back(c), t.push_back('#');
    t.push_back('^');
    vi xs = manacher(t);
    int mpos = max_element(all(xs)) - xs.begin();
    string p;
    for (int k = xs[mpos], i = mpos - k; i <= mpos + k; i++)
        if (t[i] != '#') p.push_back(t[i]);
    return p;
}

```

## 14.9 Longest palindrome

---

```

string longest_palindrome(const string &s) {
    int n = (int)s.size();
    vector<array<int, 2>> dp(n);
    pii odd(0, -1), even(0, -1);
    pii ans;
    for (int i = 0; i < n; i++) {
        int k = 0;
        if (i > odd.second)
            k = 1;
        else
            k = min(dp[odd.first + odd.second - i][0], odd.second - i + 1);
        while (i - k >= 0 and i + k < n and s[i - k] == s[i + k]) k++;
        dp[i][0] = k--;
        if (i + k > odd.second) odd = {i - k, i + k};
        if (2 * dp[i][0] - 1 > ans.second) ans = {i - k, 2 * dp[i][0] - 1};
        k = 0;
        if (i <= even.second)
            k = min(dp[even.first + even.second - i + 1][1], even.second - i + 1);
        while (i - k - 1 >= 0 and i + k < n and s[i - k - 1] == s[i + k])
            k++;
        dp[i][1] = k--;
        if (i + k > even.second) even = {i - k - 1, i + k};
    }
}

```

```

        if (2 * dp[i][1] > ans.second) ans = {i - k - 1, 2 * dp[i][1]};
    }
    return s.substr(ans.first, ans.second);
}

```

## 14.10 Lyndon factorization

---

```

vi lyndon_factorization(string S) {
    auto sa = suffix_array(S);
    vi ans;
    vi mex(len(S) + 1, 0);
    int p = 0;
    rtrav(si, sa) {
        if (si == p) {
            ans.eb(si);
        }
        mex[si] = 1;
        while (mex[p]) p++;
    }
    ans.eb(len(S));
    return ans;
}

```

## 14.11 Rabin-Karp

---

```

size_t rabin_karp(const string &s, const string &p) {
    if (s.size() < p.size()) return 0;
    auto n = s.size(), m = p.size();
    const ll p1 = 31, p2 = 29, q1 = 1e9 + 7, q2 = 1e9 + 9;
    const ll p1_1 = fpow(p1, q1 - 2, q1), p1_2 = fpow(p1, m - 1, q1);
    const ll p2_1 = fpow(p2, q2 - 2, q2), p2_2 = fpow(p2, m - 1, q2);
    pair<ll, ll> hs, hp;
    for (int i = (int)m - 1; ~i; --i) {
        hs.first = (hs.first * p1) % q1;
        hs.first = (hs.first + (s[i] - 'a' + 1)) % q1;
        hs.second = (hs.second * p2) % q2;
        hs.second = (hs.second + (s[i] - 'a' + 1)) % q2;
        hp.first = (hp.first * p1) % q1;
        hp.first = (hp.first + (p[i] - 'a' + 1)) % q1;
        hp.second = (hp.second * p2) % q2;
        hp.second = (hp.second + (p[i] - 'a' + 1)) % q2;
    }
    size_t occ = 0;
    for (size_t i = 0; i < n - m; i++) {
        occ += (hs == hp);
        int fi = s[i] - 'a' + 1;
        int fm = p[i + m] - 'a' + 1;
        hs.first = (hs.first - fi + q1) % q1;
        hs.first = (hs.first * p1_1) % q1;
        hs.first = (hs.first + fm * p1_2) % q1;
    }
}

```

```

        hs.second = (hs.second - fi + q2) % q2;
        hs.second = (hs.second * p2_1) % q2;
        hs.second = (hs.second + fm * p2_2) % q2;
    }
    occ += hs == hp;
    return occ;
}

```

## 14.12 Suffix Automaton

**Description:** A suffix automaton  $A$  for a string  $s$  is a minimal finite automaton that recognizes the suffixes of  $s$ .

```

#pragma once
#include "../Contest/template.cpp"
struct SuffixAutomaton {
    int n;
    vi suffix_link, max_len;
    vi2d transition;
    SuffixAutomaton(const string &s, int alphabet_size='z'-'a'+1, int norm
        = 'a') : n(len(s), suffix_link(n<<1), max_len(n<<1), transition(n
        <<1, vi(alphabet_size, -1)){
        int last = 0;
        trav(c, s) {
            int max_len_cur = max_len[last] + 1;
            while
        }
    }
};

```

## 14.13 Suffix array

```

#include <bits/stdc++.h>
using namespace std;
#ifdef LOCAL
#include "debug.cpp"
#else
#define dbg(...)
#endif
#define endl '\n'
#define fastio \
    ios_base::sync_with_stdio(0); \
    cin.tie(0);
#define int long long
#define all(j) j.begin(), j.end()
#define rall(j) j.rbegin(), j.rend()
#define len(j) (int)j.size()
#define rep(i, a, b) \
    for (common_type_t<decltype(a), decltype(b)> i = (a); i < (b); i++)
#define rrep(i, a, b) \
    for (common_type_t<decltype(a), decltype(b)> i = (a); i > (b); i--)
#define trav(xi, xs) for (auto &xi : xs)
#define rtrav(xi, xs) for (auto &xi : ranges::views::reverse(xs))

```

```

#define pb push_back
#define pf push_front
#define ppb pop_back
#define ppf pop_front
#define eb emplace_back
#define lb lower_bound
#define ub upper_bound
#define fi first
#define se second
#define emp emplace
#define ins insert
#define divc(a, b) ((a) + (b) - 1ll) / (b)
using str = string;
using ll = long long;
using ull = unsigned long long;
using ld = long double;
using vll = vector<ll>;
using pll = pair<ll, ll>;
using vll2d = vector<vll>;
using vi = vector<int>;
using vi2d = vector<vi>;
using pii = pair<int, int>;
using vpii = vector<pii>;
using vc = vector<char>;
using vs = vector<str>;
template <typename T, typename T2>
using umap = unordered_map<T, T2>;
template <typename T>
using pqmn = priority_queue<T, vector<T>, greater<T>>;
template <typename T>
using pqmx = priority_queue<T, vector<T>>;
template <typename T, typename U>
inline bool chmax(T &a, U const &b) {
    return (a < b ? a = b, 1 : 0);
}
template <typename T, typename U>
inline bool chmin(T &a, U const &b) {
    return (a > b ? a = b, 1 : 0);
}
vector<int> sort_cyclic_shifts(string const &s) {
    int n = s.size();
    const int alphabet = 128;
    vector<int> p(n), c(n), cnt(max(alphabet, n), 0);
    for (int i = 0; i < n; i++) cnt[s[i]]++;
    for (int i = 1; i < alphabet; i++) cnt[i] += cnt[i - 1];
    for (int i = 0; i < n; i++) p[--cnt[s[i]]] = i;
    c[p[0]] = 0;
    int classes = 1;
    for (int i = 1; i < n; i++) {
        if (s[p[i]] != s[p[i - 1]]) classes++;
        c[p[i]] = classes - 1;
    }
    vector<int> pn(n), cn(n);
    for (int h = 0; (1 << h) < n; ++h) {
        for (int i = 0; i < n; i++) {

```



```

        pn[i] = p[i] - (1 << h);
        if (pn[i] < 0) pn[i] += n;
    }
    fill(cnt.begin(), cnt.begin() + classes, 0);
    for (int i = 0; i < n; i++) cnt[c[pn[i]]]++;
    for (int i = 1; i < classes; i++) cnt[i] += cnt[i - 1];
    for (int i = n - 1; i >= 0; i--) p[--cnt[c[pn[i]]]] = pn[i];
    cn[p[0]] = 0;
    classes = 1;
    for (int i = 1; i < n; i++) {
        pair<int, int> cur = {c[p[i]], c[(p[i] + (1 << h)) % n]};
        pair<int, int> prev = {c[p[i - 1]], c[(p[i - 1] + (1 << h)) %
n]};
        if (cur != prev) ++classes;
        cn[p[i]] = classes - 1;
    }
    c.swap(cn);
}
return p;
}
vector<int> suffix_array(string s) {
    s += "$";
    vector<int> p = sort_cyclic_shifts(s);
    p.erase(p.begin());
    return p;
}
vector<int> longestCommonPrefix(const string &s, const vector<int> &suf) {
    int n = s.size();
    vector<int> isuf(n), res(n - 1);
    for (int i = 0; i < n; ++i) isuf[suf[i]] = i;
    int k = 0;
    for (; isuf[k] != n - 1; ++k) {
        int cmp_i = suf[isuf[k] + 1];
        int r = k == 0 ? 0 : max(res[isuf[k - 1]] - 1, (int)0);
        while (k + r < n && cmp_i + r < n && s[k + r] == s[cmp_i + r]) ++r;
        res[isuf[k]] = r;
    }
    ++k;
    for (int i = k; i < n; ++i) {
        int cmp_i = suf[isuf[i] + 1];
        int r = i == k ? 0 : max(res[isuf[i - 1]] - 1, (int)0);
        while (i + r < n && cmp_i + r < n && s[i + r] == s[cmp_i + r]) ++r;
        res[isuf[i]] = r;
    }
    return res;
}
ll distinct_substrings(const string &s, const vi &sa) {
    int n = len(s);
    vi lcp = longestCommonPrefix(s, sa);
    ll ans = n - sa[0];
    rep(i, 1, n) { ans += n - sa[i] - lcp[i - 1]; }
    return ans;
}
void run();

```

```

int32_t main() {
#ifdef LOCAL
    fastio;
#endif
    int T = 1;
    /*cin >> T;*/
    rep(t, 0, T) {
        dbg(t);
        run();
    }
}
void run() {
    string S;
    cin >> S;
    auto sa = suffix_array(S);
    cout << distinct_substrings(S, sa) << endl;
}

```

#### 14.14 Suffix array (supreme)

```

template <typename T = ll,
        auto cmp = [](T &src1, T &src2, T &dst) { dst = min(src1, src2);
}>
class SparseTable {
private:
    int sz;
    vi logs;
    vector<vector<T>> st;
public:
    SparseTable() {}
    SparseTable(const vector<T> &v) : sz(len(v)), logs(sz + 1) {
        rep(i, 2, sz + 1) logs[i] = logs[i >> 1] + 1;
        st.resize(logs[sz] + 1, vector<T>(sz));
        rep(i, 0, sz) st[0][i] = v[i];
        for (int k = 1; (1 << k) <= sz; k++) {
            for (int i = 0; i + (1 << k) <= sz; i++) {
                cmp(st[k - 1][i], st[k - 1][i + (1 << (k - 1))], st[k][i])
            }
        }
    }
    T query(int l, int r) {
        r++;
        const int k = logs[r - l];
        T ret;
        cmp(st[k][l], st[k][r - (1 << k)], ret);
        return ret;
    }
};
template <typename T>
using RMQ = SparseTable<T, [](T &a, T &b, T &c) { c = min(a, b); }>;
// éCrditos: ShahjalalShohag

```



```

// O(N)
struct SA {
    string s;
    int n;
    vector<int> sa, lcp, pos;
    RMQ<int> rmq;
    void induced_sort(vector<int> &vec, int val, vector<int> &sa,
        vector<bool> &sl, vector<int> &lms) {
        vector<int> l(val), r(val);
        for (int c : vec) {
            if (c + 1 < val) l[c + 1]++;
            r[c]++;
        }
        partial_sum(l.begin(), l.end(), l.begin());
        partial_sum(r.begin(), r.end(), r.begin());
        fill(sa.begin(), sa.end(), -1);
        for (int i = lms.size() - 1; i >= 0; i--) sa[--r[vec[lms[i]]]] =
lms[i];
        for (int i : sa) {
            if (i >= 1 && sl[i - 1]) sa[l[vec[i - 1]]++] = i - 1;
        }
        fill(r.begin(), r.end(), 0);
        for (int c : vec) r[c]++;
        partial_sum(r.begin(), r.end(), r.begin());
        for (int k = sa.size() - 1, i = sa[k]; k >= 1; --k, i = sa[k]) {
            if (i >= 1 && !sl[i - 1]) sa[--r[vec[i - 1]]] = i - 1;
        }
    }
    vector<int> build_sa(vector<int> &vec, int val) {
        int n = vec.size();
        vector<int> sa(n), lms;
        vector<bool> sl(n);
        sl[n - 1] = false;
        for (int i = n - 2; i >= 0; i--) {
            sl[i] =
                (vec[i] > vec[i + 1] || (vec[i] == vec[i + 1] && sl[i +
1]));
            if (sl[i] && !sl[i + 1]) lms.push_back(i + 1);
        }
        reverse(lms.begin(), lms.end());
        induced_sort(vec, val, sa, sl, lms);
        vector<int> new_lms(lms.size()), lms_vec(lms.size());
        for (int i = 0, k = 0; i < n; i++) {
            if (!sl[sa[i]] && sa[i] >= 1 && sl[sa[i] - 1]) new_lms[k++] =
sa[i];
        }
        int cur = 0;
        sa[n - 1] = cur;
        for (int k = 1; k < (int)new_lms.size(); k++) {
            int i = new_lms[k - 1], j = new_lms[k];
            if (vec[i] != vec[j]) {
                sa[j] = ++cur;
                continue;
            }
            bool flag = false;
            for (int a = i + 1, b = j + 1;; ++a, ++b) {

```

```

                if (vec[a] != vec[b]) {
                    flag = true;
                    break;
                }
                if ((!sl[a] && sl[a - 1]) || (!sl[b] && sl[b - 1])) {
                    flag = !((!sl[a] && sl[a - 1]) && (!sl[b] && sl[b -
1]));
                    break;
                }
            }
            sa[j] = (flag ? ++cur : cur);
        }
        for (int i = 0; i < (int)lms.size(); i++) lms_vec[i] = sa[lms[i]];
        if (cur + 1 < (int)lms.size()) {
            auto lms_sa = build_sa(lms_vec, cur + 1);
            for (int i = 0; i < (int)lms.size(); i++)
                new_lms[i] = lms[lms_sa[i]];
        }
        induced_sort(vec, val, sa, sl, new_lms);
        return sa;
    }
    vector<int> suffix_array() {
        vector<int> vec(n + 1);
        copy(begin(s), end(s), begin(vec));
        vec.back() = '$';
        auto sa = build_sa(vec, 256);
        sa.erase(sa.begin());
        return sa;
    }
    vector<int> build_lcp() {
        int n = (int)s.size(), k = 0;
        vector<int> rank(n), lcp(n);
        for (int i = 0; i < n; i++) rank[sa[i]] = i;
        for (int i = 0; i < n; i++, k -= !!k) {
            if (rank[i] == n - 1) {
                k = 0;
                continue;
            }
            int j = sa[rank[i] + 1];
            while (i + k < n && j + k < n && s[i + k] == s[j + k]) k++;
            lcp[rank[i]] = k;
        }
        return lcp;
    }
    SA() {}
    SA(string _s) : s(_s), n(len(s)), pos(n) {
        sa = suffix_array();
        lcp = build_lcp();
        rmq = RMQ<int>(lcp);
        for (int i = 0; i < n; i++) pos[sa[i]] = i;
    }
    int get_lcp(int i,
        int j) { // lcp na căposio i, indica o lcp
                // das căposies i e i+1 do sa
        if (i == j) return n - i;

```

```

    int l = pos[i], r = pos[j];
    if (l > r) swap(l, r);
    return rmq.query(l, r);
}
// string s = a + '+' + b;
tuple<int, int, int> lcs(int n) { // m é o tamanho da string a
    int m = len(s) - n - 1;
    int best_len = 0;
    int index_s = 0;
    int index_t = 0;
    for (int i = 0; i < n + m; ++i) {
        if ((sa[i] < n && sa[i + 1] >= n + 1) ||
            (sa[i] >= n + 1 && sa[i + 1] < n)) {
            if (lcp[i] > best_len) {
                best_len = lcp[i];
                index_s = min(sa[i], sa[i + 1]);
                index_t = max(sa[i], sa[i + 1]) - n - 1;
            }
        }
    }
    /*int maior = 0, pos = -1;*/
    /*for (int i = 2; i < n; i++) {*/
    /* if ((sa[i] < n) != (sa[i - 1] < n)) {*/
    /*     if (lcp[i - 1] > maior)*/
    /*         maior = lcp[i - 1], pos = sa[i];*/
    /*     }*/
    /* }*/
    /*return {maior, pos};*/
    return {best_len, index_s, index_t};
}
ll distinct_subs() { // n*(n+1)/2 - sum(lcp[i])
    ll resp = (ll)n * ((ll)n + 1) / 2;
    return resp - accumulate(lcp.begin(), lcp.end(), 0LL);
}
};

```

## 14.15 Suffix automaton

```

#include <bits/stdc++.h>
using namespace std;
#ifdef LOCAL
#include "debug.cpp"
#else
#define dbg(...)
#endif
#define endl '\n'
#define fastio \
    ios_base::sync_with_stdio(0); \
    cin.tie(0);
#define int long long
#define all(j) j.begin(), j.end()
#define rall(j) j.rbegin(), j.rend()
#define len(j) (int)j.size()
#define rep(i, a, b) \

```

```

    for (common_type_t<decltype(a), decltype(b)> i = (a); i < (b); i++)
#define rrep(i, a, b) \
    for (common_type_t<decltype(a), decltype(b)> i = (a); i > (b); i--)
#define trav(xi, xs) for (auto &xi : xs)
#define rtrav(xi, xs) for (auto &xi : ranges::views::reverse(xs))
#define pb push_back
#define pf push_front
#define ppb pop_back
#define ppf pop_front
#define eb emplace_back
#define lb lower_bound
#define ub upper_bound
#define fi first
#define se second
#define emp emplace
#define ins insert
#define divc(a, b) ((a) + (b) - 1ll) / (b)
using str = string;
using ll = long long;
using ull = unsigned long long;
using ld = long double;
using vll = vector<ll>;
using pll = pair<ll, ll>;
using vll2d = vector<vll>;
using vi = vector<int>;
using vi2d = vector<vi>;
using pii = pair<int, int>;
using vpil = vector<pii>;
using vc = vector<char>;
using vs = vector<str>;
template <typename T, typename T2>
using umap = unordered_map<T, T2>;
template <typename T>
using pqmn = priority_queue<T, vector<T>, greater<T>>;
template <typename T>
using pqmx = priority_queue<T, vector<T>>;
template <typename T, typename U>
inline bool chmax(T &a, U const &b) {
    return (a < b ? a = b, 1 : 0);
}
template <typename T, typename U>
inline bool chmin(T &a, U const &b) {
    return (a > b ? a = b, 1 : 0);
}
}
struct SuffixAutomaton {
    struct state {
        int len, link, cnt, firstpos;
        // this can be optimized using a vector with
        // the alphabet size
        map<char, int> next;
        vi inv_link;
    };
    vector<state> st;
    int sz = 0;
    int last;

```

```

vc cloned;
SuffixAutomaton(const string &s, int maxlen)
: st(maxlen * 2), cloned(maxlen * 2) {
    st[0].len = 0;
    st[0].link = -1;
    sz++;
    last = 0;
    for (auto &c : s) add_char(c);
    // precompute for count occurrences
    for (int i = 1; i < sz; i++) {
        st[i].cnt = !cloned[i];
    }
    vector<pair<state, int>> aux;
    for (int i = 0; i < sz; i++) {
        aux.push_back({st[i], i});
    }
    sort(all(aux),
        [](const pair<state, int> &a, const pair<state, int> &b) {
            return a.fi.len > b.fi.len;
        });
    for (auto &[stt, id] : aux) {
        if (stt.link != -1) {
            st[stt.link].cnt += st[id].cnt;
        }
    }
    // for find every occurende position
    for (int v = 1; v < sz; v++) {
        st[st[v].link].inv_link.push_back(v);
    }
}

void add_char(char c) {
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    st[cur].firstpos = st[cur].len - 1;
    int p = last;
    // follow the suffix link until find a
    // transition to c
    while (p != -1 and !st[p].next.count(c)) {
        st[p].next[c] = cur;
        p = st[p].link;
    }
    // there was no transition to c so create and
    // leave
    if (p == -1) {
        st[cur].link = 0;
        last = cur;
        return;
    }
    int q = st[p].next[c];
    if (st[p].len + 1 == st[q].len) {
        st[cur].link = q;
    } else {
        int clone = sz++;
        cloned[clone] = true;
        st[clone].len = st[p].len + 1;

```

```

        st[clone].next = st[q].next;
        st[clone].link = st[q].link;
        st[clone].firstpos = st[q].firstpos;
        while (p != -1 and st[p].next[c] == q) {
            st[p].next[c] = clone;
            p = st[p].link;
        }
        st[q].link = st[cur].link = clone;
    }
    last = cur;
}

bool checkOccurrence(const string &t) { // 0(len(t))
    int cur = 0;
    for (auto &c : t) {
        if (!st[cur].next.count(c)) return false;
        cur = st[cur].next[c];
    }
    return true;
}

ll totalSubstrings() { // distinct, 0(len(s))
    ll tot = 0;
    for (int i = 1; i < sz; i++) {
        tot += st[i].len - st[st[i].link].len;
    }
    return tot;
}

// count occurrences of a given string t
int countOccurrences(const string &t) {
    int cur = 0;
    for (auto &c : t) {
        if (!st[cur].next.count(c)) return 0;
        cur = st[cur].next[c];
    }
    return st[cur].cnt;
}

// find the first index where t appears a
// substring 0(len(t))
int firstOccurrence(const string &t) {
    int cur = 0;
    for (auto c : t) {
        if (!st[cur].next.count(c)) return -1;
        cur = st[cur].next[c];
    }
    return st[cur].firstpos - len(t) + 1;
}

vi everyOccurrence(const string &t) {
    int cur = 0;
    for (auto c : t) {
        if (!st[cur].next.count(c)) return {};
        cur = st[cur].next[c];
    }
    vi ans;
    getEveryOccurrence(cur, len(t), ans);
    return ans;
}

void getEveryOccurrence(int v, int P_length, vi &ans) {

```

```

        if (!cloned[v]) ans.pb(st[v].firstpos - P_length + 1);
        for (int u : st[v].inv_link) getEveryOccurence(u, P_length, ans);
    }
};
void run();
int32_t main() {
#ifdef LOCAL
    fastio;
#endif
    int T = 1;
    /*cin >> T;*/
    rep(t, 0, T) {
        dbg(t);
        run();
    }
}
void run() {
    string S;
    cin >> S;
    SuffixAutomaton sa(S, len(S));
    cout << sa.totalSubstrings() << endl;
}

```

## 14.16 Suffix-Tree (Ukkonen's Algorithm)

```

/**
 * Author: Unknown
 * Date: 2017-05-15
 * Source: https://e-maxx.ru/algo/ukkonen
 * Description: Ukkonen's algorithm for online suffix tree construction.
 * Each node contains indices [l, r] into the string, and a list of child
 * nodes. Suffixes are given by traversals of this tree, joining [l, r)
 * substrings. The root is 0 (has l = -1, r = 0), non-existent children
 * are -1.
 * To get a complete tree, append a dummy symbol -- otherwise it may
 * contain
 * an incomplete path (still useful for substring matching, though).
 * Time:  $O(26N)$ 
 * Status: stress-tested a bit
 */
#pragma once
#include "../Contest/template.cpp"
struct SuffixTree {
    enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
    int toi(char c) { return c - 'a'; }
    string a; // v = cur node, q = cur position
    int t[N][ALPHA], l[N], r[N], p[N], s[N], v = 0, q = 0, m = 2;
    void ukkadd(int i, int c) {
        suff:
        if (r[v] <= q) {
            if (t[v][c] == -1) {
                t[v][c] = m;
                l[m] = i;
            }

```

```

                p[m++] = v;
                v = s[v];
                q = r[v];
                goto suff;
            }
            v = t[v][c];
            q = l[v];
        }
        if (q == -1 || c == toi(a[q]))
            q++;
        else {
            l[m + 1] = i;
            p[m + 1] = m;
            l[m] = l[v];
            r[m] = q;
            p[m] = p[v];
            t[m][c] = m + 1;
            t[m][toi(a[q])] = v;
            l[v] = q;
            p[v] = m;
            t[p[m]][toi(a[l[m]])] = m;
            v = s[p[m]];
            q = l[m];
            while (q < r[m]) {
                v = t[v][toi(a[q])];
                q += r[v] - l[v];
            }
            if (q == r[m])
                s[m] = v;
            else
                s[m] = m + 2;
            q = r[v] - (q - r[m]);
            m += 2;
            goto suff;
        }
    }
}
SuffixTree(string a) : a(a) {
    fill(r, r + N, len(a));
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t);
    fill(t[1], t[1] + ALPHA, 0);
    s[0] = 1;
    l[0] = l[1] = -1;
    r[0] = r[1] = p[0] = p[1] = 0;
    rep(i, 0, len(a)) ukkadd(i, toi(a[i]));
}
// example: find longest common substring (uses ALPHA = 28)
pii best;
int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    rep(c, 0, ALPHA) if (t[node][c] != -1) mask |=
        lcs(t[node][c], i1, i2, len);
    if (mask == 3) best = max(best, {len, r[node] - len});
    return mask;
}

```

```

}
static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, len(s), len(s) + 1 + len(t), 0);
    return st.best;
}
};

```

## 14.17 Trie

### Description:

- build with the size of the alphabet ( $\sigma$ ) and the first char ( $norm$ )
- $insert(s)$  insert the string in the trie  $O(|s| * \sigma)$
- $erase(s)$  remove the string from the trie  $O(|s|)$
- $find(s)$  return the last node from the string  $s$ , 0 if not found  $O(|s|)$

---

```

#include "../Contest/template.cpp"
struct Trie {
    vi2d to;
    vi end, pref;
    int sigma;
    char norm;
    Trie(int sigma_ = 'z' - 'a' + 1, char norm_ = 'a')
        : sigma(sigma_), norm(norm_) {
        to = {vector<int>(sigma)};
        end = {0}, pref = {0};
    }
    int next(int node, char key) { return to[node][key - norm]; }
    void insert(const string &s) {
        int x = 0;
        for (auto c : s) {

```

```

            int &nxt = to[x][c - norm];
            if (!nxt) {
                nxt = len(to);
                to.push_back(vi(sigma));
                end.emplace_back(0), pref.emplace_back(0);
            }
            x = nxt, pref[x]++;
        }
        end[x]++, pref[0]++;
    }
    void erase(const string &s) {
        int x = 0;
        for (char c : s) {
            int &nxt = to[x][c - norm];
            x = nxt, pref[x]--;
            if (!pref[x]) nxt = 0;
        }
        end[x]--, pref[0]--;
    }
    int find(const string &s) {
        int x = 0;
        for (auto c : s) {
            x = to[x][c - norm];
            if (!x) return 0;
        }
        return x;
    }
};

```

## 15 numerical-methods