Modelling order picking problem

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1 Conjuntos

- $\mathcal{O}$: Conjunto de pedidos no *backlog*.
- $\mathcal{I}_o$: Subconjunto de itens solicitados pelo pedido $o \in \mathcal{O}$.
- $\mathcal{I}$: Conjunto de itens, onde $\mathcal{I} = \bigcup_{o \in \mathcal{O}} \mathcal{I}_o$.
- $\mathcal{A}_i$: Subconjunto de corredores contendo pelo menos uma unidade do item i.
- $\mathcal{A}$: Conjunto de corredores, onde $\mathcal{A} = \bigcup_{i \in \mathcal{I}} \mathcal{A}_i$.

2 Constantes

- u_{oi}: Número de unidades do item $i \in \mathcal{I}$ solicitado pelo pedido $o \in \mathcal{O}$.
- u_{ai}: Número de unidades do item $i \in \mathcal{I}$ disponíveis no corredor $a \in \mathcal{A}$.
- **LB / UB**: Limite inferior / superior do *tamanho da wave*.

3 First model

3.1 Non-linear

$$\max \quad \frac{\sum_{o \in O} \sum_{i \in I} x_o u_{oi}}{\sum_{a \in A} y_a}$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{o \in O} \sum_{i \in I} x_o u_{oi} \geq LB \\
& \sum_{o \in O} \sum_{i \in I} x_o u_{oi} \leq UB \\
& \sum_{o \in O} x_o u_{oi} \leq \sum_{a \in A} y_a u_{ai}, \quad \forall i \in I
\end{aligned}$$

$$x_o = \begin{cases} 1, & \text{if } o \in O \text{ is in the solution ,} \\ 0, & \text{otherwise} \end{cases}$$

$$y_a = \begin{cases} 1, & \text{if } a \in A \text{ is in the solution ,} \\ 0, & \text{otherwise} \end{cases}$$

3.2 Linearization

If $w = \frac{1}{\sum_{a \in A} y_a}$, then:

$$\max \quad \sum_{o \in O} \sum_{i \in I} x_o w u_{oi}$$

We know that $w \leq 1$. So, using the big M method, we can linearize the function $x_o w$ by introducing a variable z_o that will be equal to $x_o w$ with some new restrictions.

- $z_o \leq w, \forall o \in O.$
- $z_o \leq x_o, \forall o \in O.$
- $z_o \geq w + (x_o - 1), \forall o \in O.$

Now, we have:

$$\max \quad \sum_{o \in O} \sum_{i \in I} z_o u_{oi}$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{o \in O} \sum_{i \in I} x_o u_{oi} \geq LB \\
& \sum_{o \in O} \sum_{i \in I} x_o u_{oi} \leq UB \\
& \sum_{o \in O} x_o u_{oi} \leq \sum_{a \in A} y_a u_{ai}, \quad \forall i \in I \\
& w = \frac{1}{\sum_{a \in A} y_a} \\
& z_o \leq w, \forall o \in O \\
& z_o \leq x_o, \forall o \in O \\
& z_o \geq w + (x_o - 1), \forall o \in O \\
& z_o \geq 0, \forall o \in O \\
\\
& x_o \in B, \forall o \in O \\
& y_a \in B, \forall a \in A
\end{aligned}$$