Evaluation of Local Dependencies of Images Wavelet Decomposition[1]

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Abstract—An approach to identifying local area structure that is used in the spatial interaction models adapted to the image characteristic properties using mutual information criterion is reviewed in this article. Experimental results that demonstrate the value of using the represented method are shown.

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INTRODUCTION

Usually, different algorithms of image processing are based on a mathematical image model. One of the most popular ways to describe an image is using spatial interaction models [3]. This method uses the assumption that every image point depends on its local area as follows: $p(x_{i,j}|\mathbf{X}\backslash x_{i,j}) = p(x_{i,j}|\mathbf{L}(x_{i,j}))$, where $x_{i,j}$ is the current point and $\mathbf{L}(x_{i,j})$ are the points of local area $\mathbf{L}$ by $x_{i,j}$ element. The efficiency of image-processing algorithms based on spatial interaction models depends on local area structure. Normally, an octa-element neighborhood is viewed as the local area structure for each image element [1].

The appearance of wavelet transform led to the description of spatial interaction models in the field of wavelet coefficients [2]. Wavelet transform allows the image to be decomposed at several frequency bands. Each frequency band has a different character of image features. Consequently, for connections between wavelet coefficients at one level of decomposition, description is more efficient using local areas adapted to the features of frequency bands.

Existing approaches to calculating the local area are based on either exhaustive search methods or the solution of optimization problems, which, in this case, does not always allow one to obtain a more profitable structure. In this work, an approach to choosing local area structure based on a mutual information criterion is supported.

COVERAGE OF EXISTING APPROACHES TO THE CHOICE OF LOCAL AREA

Let us suggest that there is an end two-dimensional set of elements $\mathbf{X}$, $\mathbf{X} = \{x_{i,j}, 1 \le i \le P, 1 \le j \le T\}$. It is necessary to define the local area $\mathbf{L}$ structure, size $N \times N$ for the given set of elements. We need to pick out the following area $|x - \hat{x}_a| \le |x - \hat{x}|$, where $\hat{x}_a = \sum_{i,j \in \mathbf{L_a}} \alpha_{i,j} \cdot x_{i,j}$ is an evaluation of the data element value based on adapted local area $\mathbf{L_a}$ elements significances; $\alpha_{i,j}$ are the weight coefficients of local area elements; $\hat{x} = \sum_{i,j \in \mathbf{L}} \alpha_{i,j} \cdot x_{i,j}$ are the evaluations of data element value based on the values of fast local area $\mathbf{L}$ elements.

At present, there is a large amount of approaches to solving this kind of task, e.g., an approach based on AIC criterion [7], an approach that uses the Bayesian solving rule [1], and an approach based on MDL criterion [5]. An adaptive way to evaluate local area structure is also supported in [6]. It is based on a mistake between the source images and generated on a predetermined model with using the assessed local area image minimization. In [4], the method of constructing correlation charts of wavelet coefficients is supported. Obtained charts may be used for local area structure calculation.

ALGORITHM OF LOCAL AREA STRUCTURE SEARCH BASED ON MUTUAL INFORMATION

When search the local area $\mathbf{L}$ structure as a criterion of optimality, the mutual information value $I(\mathbf{X}, \mathbf{Y})$ may be used. The mutual information value $I(\mathbf{X}, \mathbf{Y})$ shows how much information about $\mathbf{X}$ is contained in $\mathbf{Y}$ [8] $I(\mathbf{X}, \mathbf{Y}) = \sum\sum p(\mathbf{X}, \mathbf{Y}) \log \frac{p(\mathbf{X}, \mathbf{Y})}{p(\mathbf{X})p(\mathbf{Y})}$, where $\mathbf{X}$, $\mathbf{Y}$ are two-dimensional sets of data in the defined case of two matrices of wavelet coefficients; $p(\mathbf{X})$, $p(\mathbf{Y})$ are probability accommodation compactnesses of wavelet coefficients $\mathbf{X}$ and $\mathbf{Y}$; and $p(\mathbf{X}, \mathbf{Y})$ is the combined probabil-

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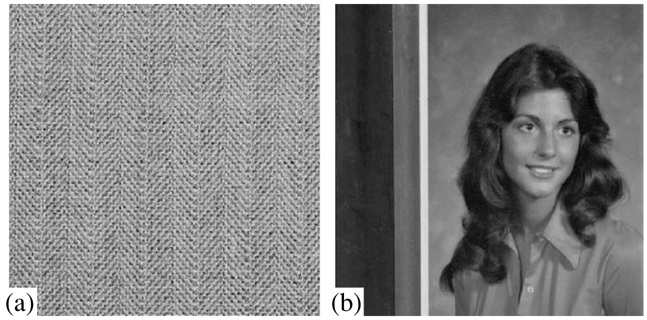

Fig. 1. Test images (a) "texture" and (b) "portrait."

ity accommodation compactness of wavelet coefficients **X** and **Y**.

The supported method is based on the calculation of values of mutual information for each element of local area **L** size $N \times N$ with a center at point (0, 0), where N is an uneven number.

Let us suggest that it is necessary to calculate the value of mutual information between the center element of local area L and the element with coordinates (k, l), $k \in \left[-\frac{N}{2}; \frac{N}{2}\right]$; $l \in \left[-\frac{N}{2}; \frac{N}{2}\right]$, which belongs to local area. For this, it is necessary to perform the source set of wavelet coefficients **X** shifted in such a way as to achieve the element with coordinates (k, l) moving to the center of the local area. For performing this operation, it is necessary to perform the shearing of all of wavelet coefficients set by k elements in the OX direction and by l elements in the OY direction (the sign before k and l will show what direction the set of wavelet coefficients will be displaced to, i.e., to the right or left, up or down).

If this operation is performed for the entire wavelet coefficients set and the wavelet coefficients set **Y** are found, the mutual information value between source **X** and displaced **Y** wavelet coefficients sets may be calculated. The obtained value will be the mutual information value between the local area element with coordinates (k, l) and the center local area element.

By considering all of the possible movements of a certain element to the center of a local area, i.e., by considering all of the possible source sets of wavelet coefficients shearings, the matrix of mutual information values $\mathbf{M}_{\mathrm{I}(\mathbf{X},\mathbf{Y})}$ between the center local area element and any other local area elements will be obtained.

The resulting matrix of mutual information values $\mathbf{M}_{\mathrm{I}(\mathbf{X},\mathbf{Y})}$ may be used to estimate the local area structure. For this, cutting by threshold procedure $\mathbf{L}(k, l) = \begin{cases} 1, & \mathbf{M}_{\mathrm{I}(\mathbf{X},\mathbf{Y})}(k, l) \ge Thr \\ 0, & \mathbf{M}_{\mathrm{I}(\mathbf{X},\mathbf{Y})}(k, l) < Thr \end{cases}$ may be applied, where $Thr \in [0, 1]$ is a threshold rated by tentative way.

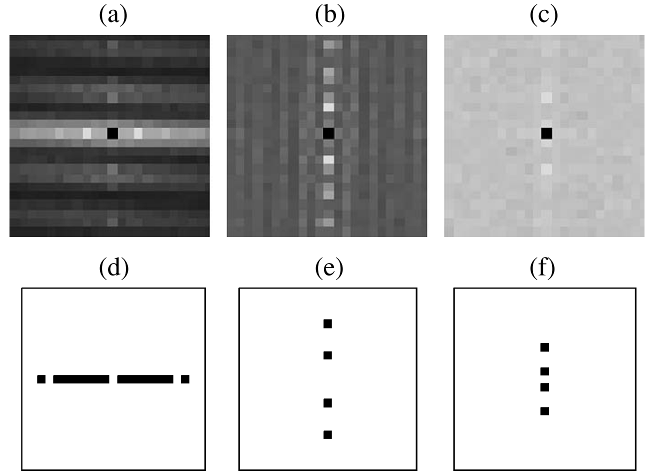

Fig. 2. Calculation results of local area structure for wavelet decomposition of images like "texture;" matrix of mutual information for (a) LH band; (b) HL band; (c) HH band; chosen structure of local area for (d) LH band; (e) HL band; (f) HH band.

EXPERIMENTAL

During the analysis of spontaneous images characteristics, as well as before constructing models and image processing algorithms, it is first necessary to analyze the test images that have some pronounced features. By studying the test images, it may traverse to the spontaneous images calculated parameters processing and analyze.

In order to explore the results and estimation of the supported algorithm's work, the following test images may be used:

(1) test image no. 1, which consists of a field of correlated noise, a coefficient of horizontal correlation r_h = 0.97, and a coefficient of vertical correlation $r_v \approx 0$;

(2) test image no. 2, which consists of a field of correlated noise, a coefficient of horizontal correlation $r_h \approx$ 0, and a coefficient of vertical correlation $r_v = 0.96$;

(3) test image like "texture";

(4) test image like "portrait."

The studied test images are shown in Fig. 1.

The results of the suggested algorithm for test image like "texture" work are shown in Fig. 2.

As far as Fig. 2e it can be seen that the calculated value of mutual information is considerably accurate with estimation of texture's step (pronounced horizontal bars); moreover, from Fig. 2f, which is more difficult to see, it says of a greater horizontal correlation then a vertical one.

The work of the suggested algorithm for test image like "portrait" results are shown in Fig. 3.

In this work, an approach to local area structure for estimating wavelet coefficients is viewed, but the suggested approach allows one to calculate the structure of

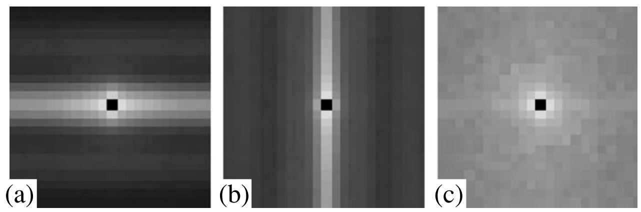

Fig. 3. Calculation results of local area for wavelet decomposition of images like "portrait;" calculated matrix of mutual information for (a) LH band; (b) HL band; (c) HH band.

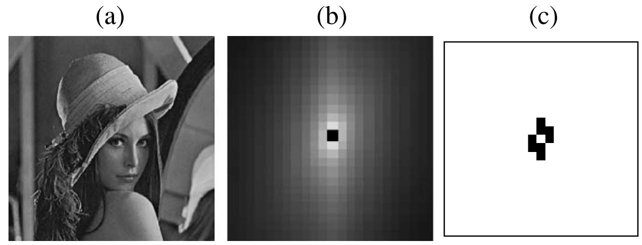

Fig. 4. Source image and calculated structure of local area: (a) source image; (b) graphic representation of value of mutual information between the center point and its neighbors; (c) calculated structure of local area, *Thr* = 0.8.

local area for not only wavelet coefficients, but also images (see below in Fig. 4).

The coefficient of spontaneous images correlation is about 0.8–0.9. In this case, the threshold value's changing *Thr* in well-broad cutoff points allows for different kinds of local area structures to be obtained. The dependence of the number of points in local area *K* on the chosen threshold *Thr* is shown in Table 1.

In order to choose the accuracy of the local area structure based on the estimation of the supported approach, let us consider the value of mutual information between the point and point $\mathbf{L}_a$ local area chosen during performing the supported algorithm and point and the settled local area $\mathbf{L}_f$ size 3×3. The results given in Table 2 are averaged over the group of images.

As far as Table 2, in the adaptive local area, there is more information about the point than in the settled one.

CONCLUSIONS

Using spatial interaction models allows the efficiency of image processing different algorithms to be raised, as each point of the image is rated, not separately, but in the context of its local area. Local area may be either settled or adaptive to image properties.

Table 1. Dependence of the number of points in local area *K* on threshold *Trh*

Trh	0.9	0.7	0.5	0.4	0.35	0.3	0.25
K	2	4	6	10	12	16	20

Table 2. Mutual information between point and its fixed L_f and adaptive L_a local areas

	LH	HL	HH
$\mathbf{L}_a$	0.471	0.466	0.349
$\mathbf{L}_f$	0.435	0.250	0.286

In this work, an approach to local area structure's adaptive to representative image properties determination is supported. The results of research show the possibility of using this approach to search for local area structure and wavelet coefficients for either image. The supported algorithm of searching the local area structure may be used in the estimation of the spatial interaction model's parameters, as well as in image processing (when solving the segmentation, filtering, and the contour preparation generation tasks). Also, in this work, a study of dependences between the value of mutual information and the value of source image correlation has been conducted.

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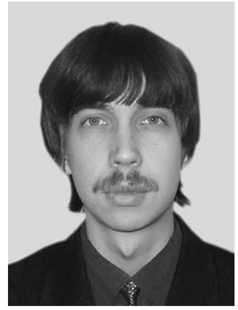

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