A mixed integer linear programming model for short-term open pit mine production scheduling

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Short-term production scheduling in open pit mines usually generates a scheme to enable mining operations to deliver the budgeted tonnes and grades to the mill while following the long-term mine plan. The goal of this paper is to develop and test an optimisation model for short-term open pit mine production scheduling. A multidestination mixed integer linear programming (MILP) model is proposed, which minimises the overall cost of mining operations including mining, processing, haulage, rehandling and rehabilitation costs. The model is solved by branch-and-cut algorithm using TOMLAB/CPLEX optimiser. The model incorporates buffer and blending stockpiles, horizontal directional mining, and decisions on ramps while controlling technical constraints. A monthly production schedule in an iron ore mine is developed over 12 months to illustrate the practicality of the developed model. Three scenarios with different mining horizontal directions are considered. The scenario with minimum number of drop-cuts is selected as the best practical schedule.

Keywords: Short-term production scheduling, Mixed integer linear programming, Open pit, Cost minimisation

Introduction

Production scheduling of open pit mines is a decision making process in which sequence of materials' extraction and their corresponding destinations, e.g. processes, stockpiles and waste dumps, over a time horizon are determined. The block scheduling problem determines the time of extracting blocks within a predetermined pit limit. As the block sequencing problem considers the time of extraction, some physical and technical constraints must be met. Different aspects of mining operations such as mining resources, transportation resources, processing capacity and demand for ore may change during the scheduling time horizon. The aim of short-term production scheduling is usually generating a scheme to enable mining operations to deliver the budgeted tonnes and grade to the mill while following the long-term mine plan (Chanda, 1992). As a decision making problem, mine production scheduling problems have mainly been tackled with operations research techniques.

The objective of this paper is to formulate, implement, and verify a mixed integer linear programming (MILP) model for short-term open pit mine production scheduling at the operational level. The practicality of the MILP formulation is presented by an iron ore mine case study. The MILP formulation considers the objective function as the minimisation of operations' costs while linking the monthly short-term production schedule to the long-term plan. The operations' costs include mining, haulage, processing, rehandling stockpiles and waste rehabilitation costs. Since short-term scheduling is performed at the operational level of mining, some issues, such as taking into account of stockpiles, different routes of haulage for material to final destinations and horizontal directional mining, are included in the formulation. In addition, operational constraints, such as quantity requirements (e.g. mining and processing capacities) and quality requirements (e.g. head grade requirement for processing), are taken into account in the form of constraints in the formulation. Roughly, minimisation of operations' costs leads to maximisation of long-term profit, because the revenue from the extraction of blocks in the short-term period have been expected in the long-term plan. In other words, the formulation models the block sequencing problem in the short-term time horizon with tracking the long-term plan to synchronise the long-term and short-term plans.

Many studies have been conducted on long-term open pit mine production scheduling using mathematical programming. A number of noticeable recent contributions in this area are carried out by Bienstock and Zuckerberg (2010), Bley *et al.* (2010), Askari-Nasab *et al.* (2010a) and Boland *et al.* (2009). For a detailed literature review on applications of mathematical programming in long-term open pit mine production scheduling, see Newman *et al.* (2010) and Osanloo *et al.* (2008).

Despite numerous research in the area of long-term open pit mine production scheduling, limited research

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has been conducted on short-term mine production scheduling using mathematical programming. Linear programming (LP), a widely used mathematical programming technique, has been applied in planning of mining operations for the purpose of ore blending, cut-off grade optimisation, equipment allocation, refining, process control, product mix and environmental control (Gershon, 1982a, b). Linear programming has also been applied in short-term mine production scheduling.

As one early work, Wilke and Reimer (1979) propose an LP model for short-term open pit mine production planning in an iron ore mine. The proposed LP model tries to maximise profit subject to two sets of constraints: blending requirement and mining/processing capacities. Gershon (1982a, b) introduces an MILP model for mine scheduling problem including sequencing and blending. Gershon (1982a, b) claims that the MILP formulation models mining operations with one ore type and dynamic cut-off grade. Generally, the MILP model aims to maximise net present value subject to blending, processing, and slope constraints. Also, Gershon (1982a, b) claims that the MILP model can incorporate other objectives such as minimising costs, maximising the life of mine or maximising production.

Fytas *et al.* (1987) develop an LP model for short-term mine production scheduling. The LP model maximises the revenue subject to the following constraints: minimum/maximum ore tonnage to be mined, minimum/maximum concentrate tonnage to be produced, maximum waste stripping capacity, minimum waste volume to be removed, and minimum/maximum quality and quantity targets of ore to be produced. Youdi *et al.* (1992) develop a two-step approach for short-term open pit mine planning. The first step consists of mid-term planning using goal programming (GP) to obtain the optimal schedule in the midterm period (i.e. quarterly mine plan) and a macro for control of short-term planning (i.e. monthly or weekly plan). The single-period GP model considers penalty values for deviations from coal production target and quality requirement. The second step tackles short-term production scheduling through the application of computer aided design technique and systems simulation. They arrange and adjust the mining and stripping production schedule developed in the first step using an interactive graphic design systems simulation.

Chanda (1992) introduces a combined linear GP and a deterministic mining simulation model designed to get an optimal short-term production schedule in open pit mines with strata-form orebodies. The linear GP formulation maximises metal content in the blender and minimises deviations from ore tonnage and targeted grade fed into the blending process. The proposed GP model is a single-period formulation which can handle ore bodies with at most three elements. Fytas *et al.* (1993) present an LP model for short-term open pit mine scheduling in multiple periods. The proposed model, solved using simplex algorithm, maximises the revenue while considering constraints regarding the head grade, targeted concentrate production and targeted stripping ratio. Kumral and Dowd (2002) develop an integrated Lagrangian parameterisation and multi-objective simulated annealing model (MOSA) to determine the optimal short-term production schedule in the following three stages:

(i) determining pit limit and blend requirement limits using LP and Lerchs–Grossmann method (Lerchs and Grossmann, 1965)
(ii) achieving a suboptimal schedule using Lagrangian parameterisation
(iii) applying MOSA for improving the schedule to reach the optimal or near-optimal schedule.

The LP model of MOSA minimises three objectives as deviations from the quantity of ore production requirements, deviations from ore quality requirements, and variance of quality of ore fed to processes. Also, the LP model considers accessibility requirement as the critical constraint to ensure enough room for mining equipment.

To consider uncertainties, Jewbali (2006) presents a stochastic approach for short-term mine production scheduling with three steps:

(i) a sequential Gaussian cosimulation component to model the future grade control data
(ii) a conditional simulation by successive residuals component to update orebody model with new information obtained from infill drillholes
(iii) a stochastic MILP model to generate the optimum short-term mine production schedule by use of the set of updated orebody models.

The MILP model maximises discounted cash flow while minimising deviations from the production targets.

Gholamnejad (2008) formulates an MILP model to incorporate block accessibility constraints. The proposed MILP model maximises the production of element of interest subject to constraints, such as grade blending, slope, mining and processing capacities. In addition, the MILP model formulates a constraint regarding the accessibility of blocks during extraction. The accessibility constraint ensures that a block can only be extracted, whenever there is enough space for mining equipment to do so. Mintec. Inc. has developed a production scheduling tool called MineSight Schedule Optimiser (MSSO) (Huang *et al.*, 2009). MSSO develops a MILP model while considering multiple mining areas, multiple destinations, multiple processes and blending requirements. The objective function of the developed MILP model in MSSO is usually maximising the net present value or the metal content. But it is flexible to consider other objectives such as minimisation of stripping ratio, haulage distances and shovel/truck hours. In addition, the MILP model takes into account some constraints on geometrical, mining and processing capacity, head grade, mining equipment usage, and stockpile capacity and reclamation. Rahman and Asad (2010) propose an MILP model for short-term blending optimisation for the cement quarry operations. The goal of the proposed MILP is to develop a raw mix stockpile feed through a combination of materials extracted from the mine and materials purchased from the market. To do so, the mathematical model attempts to minimise the operational costs (mining costs and purchasing of raw material costs) while following the blending, slope, mining capacity and quality constraints.

In this paper, an MILP formulation for short-term open pit mine production scheduling problem is developed. The proposed formulation has three significant improvements over the previous research in the context of mathematical programming models for short-term mine production scheduling:

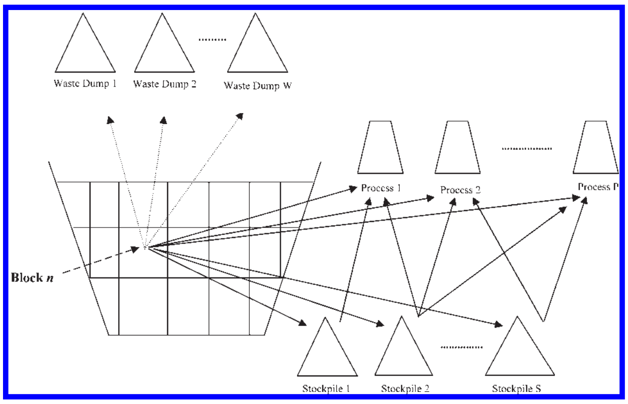
(i) consideration of multiple destinations: the proposed formulation takes into account multiple destinations, stockpiles, processes and waste dumps, to model the real world short-term mining operations better. Apart from Gershon (1982a, b) and Huang *et al.* (2009) that claim they have included multiple destinations in their models, no other study has considered multiple destinations into the short-term mine scheduling
(ii) consideration of routing/ramps: one of the most important issues at the mining operations is haulage cost. In this paper, the proposed mathematical model decides on selecting the routes/ramps, which minimises the haulage costs
(iii) consideration of horizontal directional extraction of blocks: in mining operations, preferably, blocks in each bench are mined in one or two preferred horizontal mining directions. The formulated model is applied in three horizontal mining direction scenarios in an iron ore mine case study.

In this paper, 'short-term' refers to a production schedule time horizon, expanding one to three years with a monthly resolution. It takes into account the defined development sequences, production characteristics, quantities and rates from the life-of-mine plan. By nature, the input data used for mine production scheduling are uncertain, which could be specified by probability distributions. The input data into the short-term production scheduler, such as geological block model, grades, costs, prices, recoveries and practical mining constraints, are based on the best point estimates available at the time of optimisation. Any change in the input data requires a re-run of the model with the new input parameters. This is aligned with the mining industry practice of updating annual, quarterly, monthly and weekly mine plans as new data become available and uncertainty is reduced overtime. In other words, the current model does not capture the cost of uncertainty. It is also assumed that any infill drillhole data, blast-hole data, grade control data and geologist updates are reflected in the input block model.

The rest of this paper is structured as follows: in the section on 'Statement of the problem', the problem definition of scheduling of mine production in the short-term is presented in detail. The section on 'The proposed MILP formulation' reflects the proposed MILP formulation. First, different parameters and decision variables are defined. Then, the mathematical formulation is presented. Explanation of the MILP model follows the mathematical formulation. In the section on 'Case study and discussion of results', the proposed model is applied to scheduling an iron ore mine over 12 months. Finally, conclusions and future research directions are discussed.

Statement of the problem

Figure 1 shows a schematic view of a multidestination open pit mine production scheduling problem. Finding the optimal short-term mining schedule for an open pit mine, which operates with S stockpiles, P processes and W waste dumps is the main purpose of the present paper. The deposit has E elements, which one is labelled as the major product in association with strategic decisions made by mine managers. Stockpiles are modelled to:



1 Schematic view of problem

(i) top up processes at the end of a period when mining rate has been met
(ii) buffer the processes from the changes in extraction of materials from the mine to avoid future stripping hurdles
(iii) blend material to meet head grade constraints
(iv) store material above economic cut-off when there is no process available at prestripping stage.

Each destination can receive material with a specific rock-type or a combination of rock-types. Also, each element has an acceptable grade range at different processes and stockpiles. Generally, stockpiles are separated by rock-type and grade range of ore. In addition, it is assumed that stockpiles are homogeneous and the ore reclaimed from each stockpile has a certain grade equivalent to the average grade of stockpile material. Regarding the haulage of materials, there are numerous routes to send extracted materials from the mine to different destinations. Mined material of each block could be hauled to corresponding destination through some routes. Also, a number of ramps have been designed in each bench for haulage of extracted material of blocks in that bench.

Briefly, the developed short-term open pit mine production scheduler makes a decision about:

(i) sequence of block extraction including amount of extraction of each block in each period through the time horizon (K periods with a span of t) in accordance with the long-term mine plan. Therefore, the optimal sequence of extraction of these blocks is determined in short-term mine production scheduling
(ii) amount of total material that must be extracted from the mine in each period, which indicates the total amount of ore and waste materials that are mined and sent to different destinations in each period
(iii) destinations that the mined materials are sent to. This decision is primarily made based on the rock-type of material, demand of processes, haulage cost of sending material to different possible destinations, grade of elements and type of material (ore or waste)
(iv) amount of ore reclaimed from the stockpiles to the processes in each period.

The proposed MILP formulation

In this section, the proposed MILP formulation for short-term open pit mine production scheduling is presented. First, parameters and decision variables are defined. Then, the mathematical model including the objective function (minimisation of total operations' cost) in association with physical and technical constraints is elaborated.

Parameters

t period of scheduling (t=1, …, K)
e element e (e=1, …, E)
p process p (p=1, …, P)
s stockpile s (s=1, …, S)
w waste dump w (w=1, …, W)
r ramp r
$J(n)$ set of blocks that must be extracted during the short-term time horizon (set in long-term planning)
N number of blocks in $J(n)$
ME^t minimum fraction of each block that could be extracted in period t
O_n tonnage of mineralised zone in block n
R_n rock tonnage of block n
g_n^e grade of element e in block n
$VPB(n)$ set of vertical precedent blocks for block n
$N_{VPB(n)}$ number of blocks in set $VPB(n)$
$R(n)$ set of corresponding ramps to haul extracted materials of block n
$P(n)$ set of processes that can receive ore from block n
$SP(n)$ set of stockpiles that can receive ore from block n
$W(n)$ set of waste dumps that can receive waste material from block n
MC^t unit mining cost in period t
WC_w^t unit waste rehabilitation cost in period t for waste dump w (this cost includes both the unit cost of waste rehabilitation and unit cost of haulage of waste to waste dump w)
PC_p^t unit processing cost in period t for process p (this cost includes both the unit cost of processing by process p and unit cost of haulage of ore to process p)
$RH_s^{t,p}$ unit rehandling cost for stockpile s sending ore to process p in period t (this cost includes the unit cost of hauling ore from stockpile s to process p)
H^t unit haulage cost in period t (this cost includes the unit cost of haulage of material inside pit to the pit exit)
MU^t maximum acceptable tonnage that could be mined based on maximum available mining capacity in period t
ML^t minimum acceptable tonnage that could be mined based on minimum available mining capacity in period t
PU_p^t maximum acceptable ore tonnage that process p can process based on maximum process p capacity in period t
PL_p^t process p minimum ore acceptable tonnage in period t
$gu_p^{t,e}$ upper bound on acceptable grade of element e for process p in period t
$gl_p^{t,e}$ lower bound on acceptable grade of element e for process p in period t
$OGSP_s^e$ grade of element e in ore which is sent from stockpile s to processes
$UGSP_s^e$ upper bound of grade of element e in stockpile s
$LGSP_s^e$ lower bound of grade of element e in stockpile s
CSP_s maximum storage capacity of stockpile s
IMC_s^e initial metal content of element e in stockpile s
dE_n^r distance of block n to the pit exit by the route passing from ramp r
$P[SP(s)]$ set of processes that can receive ore from stockpile s (destination of stockpile s)
$SP[P(p)]$ set of stockpiles that can send ore to process p (source of process p)

Decision variables

u_n^t fraction of block n extracted in period t ($n \in J(n)$)
$uw_n^{t,w}$ fraction of block n extracted in period t and sent to waste dump w ($w \in W(n)$)
$us_n^{t,s}$ fraction of block n extracted in period t and sent to stockpile s ($s \in SP(n)$)
$up_n^{t,p}$ fraction of block n extracted in period t and sent to process p ($p \in P(n)$)
b_n^t binary variable, if block n is extracted in period t it gets value of one otherwise zero
$x_n^{t,r}$ amount of extracted material of block n in period t which is sent to exit through ramp r
$b_n^{t,r}$ binary variable, if ramp r is selected for haulage of extracted material from block n in period t, it gets value of one, otherwise zero
$y_s^{t,p}$ ore tonnage sent from stockpile s to process p in period t ($s \in SP[P(p)]$is the set of stockpiles that send ore to process p)
I_s^t inventory of stockpile s at the end of period t

Mathematical formulation

Subsequent to defining the parameters and decision variables, the mathematical MILP formulation is proposed as

$$\text{Minimise total cost} = \underbrace{\sum_{t=1}^{K}\sum_{n=1}^{N} R_n \times u_n^t \times MC^t}_{\text{total miningcost}} +$$
$$\underbrace{\sum_{t=1}^{K}\sum_{n=1}^{N}\sum_{p\in P(n)} up_n^{t,p} \times R_n \times PC_p^t + \sum_{t=1}^{K}\sum_{p=1}^{P}\sum_{s\in SP[P(p)]} y_s^{t,p} \times PC_p^t}_{\text{total processingcost}} +$$
$$\underbrace{\sum_{t=1}^{K}\sum_{n=1}^{N}\sum_{w\in W(n)} uw_n^{t,w} \times R_n \times WC_w^t}_{\text{total waste rehabilitationcost}} + \underbrace{\sum_{t=1}^{K}\sum_{s=1}^{S}\sum_{p\in P[SP(s)]} y_s^{t,p} \times RH_s^{t,p}}_{\text{total rehandlingcost}} +$$
$$\underbrace{\sum_{t=1}^{K}\sum_{n=1}^{N}\sum_{r\in R(n)} x_n^{t,r} \times dE_n^r \times H^t}_{\text{total haulage cost}} \quad (1)$$

Subject to

$$\sum_{t=1}^{K} u_n^t = 1, \ \forall n = 1,\cdots,N \quad (2)$$

$$\sum_{p\in P(n)} up_n^{t,p} + \sum_{s\in SP(n)} us_n^{t,s} + \sum_{w\in W(n)} uw_n^{t,w} = u_n^t,$$
$$\forall n=1,\ldots,N, \forall t=1,\ldots,K \tag{3}$$

$$R_n \times \sum_{t=1}^{K}\left(\sum_{p\in P(n)} up_n^{t,p} + \sum_{s\in SP(n)} us_n^{t,s}\right) = O_n,\ \forall n=1,\cdots,N \tag{4}$$

$$u_n^t \le b_n^t,\ \forall n=1,\cdots,N,\ \forall t=1,\cdots,K \tag{5}$$

$$ME^t \times b_n^t \le u_n^t,\ \forall n=1,\cdots,N,\ \forall t=1,\cdots,K \tag{6}$$

$$ML^t \le \sum_{n=1}^{N} R_n \times u_n^t \le MU^t,\ \forall t=1,\cdots,K \tag{7}$$

$$PL_p^t \le \sum_{\forall n: p\in P(n)} R_n \times up_n^{t,p} + \sum_{s\in SP[P(p)]} y_s^{t,p} \le PU_p^t,$$
$$\forall t=1,\ldots,K,\ \forall p=1,\ldots,P \tag{8}$$

$$N_{VPB(n)} \times b_n^t \le \sum_{\tau=1}^{t}\sum_{i\in VPB(n)} u_i^{\tau},\ \forall n=1,\cdots,N,\ \forall t=1,\cdots,K \tag{9}$$

$$gl_p^{t,e} \le \frac{\sum_{s\in SP[P(p)]} y_s^{t,p}\times OGSP_s^e + \sum_{\forall n: p\in P(n)} up_n^{t,p}\times R_n \times g_n^e}{\sum_{s\in SP[P(p)]} y_s^{t,p} + \sum_{\forall n: p\in P(n)} up_n^{t,p}\times R_n} \le gu_p^{t,e},$$
$$\forall t=1,\ldots,K,\ \forall p=1,\ldots,P,\ \forall e=1,\ldots,E \tag{10}$$

$$\sum_{\forall n: s\in SP(n)} R_n \times us_n^{t,s} - \sum_{p\in P[SP(s)]} y_s^{t,p} + I_s^{t-1} = I_s^t,$$
$$\forall t=1,\ldots,K, \forall s=1,\ldots,S \tag{11}$$

$$\sum_{p\in P[SP(s)]} y_s^{t,p} \le I_s^{t-1},\ \forall t=1,\cdots,K,\ \forall s=1,\cdots,S \tag{12}$$

$$LGSP_s^e \le \frac{\sum_{\forall n: s\in SP(n)} us_n^{t,s}\times R_n \times g_n^e}{\sum_{\forall n: s\in SP(n)} us_n^{t,s}\times R_n} \le UGSP_s^e,\ \forall t=1,\ldots,K,$$
$$\forall s=1,\ldots,S,\ \forall e=1,\ldots,E \tag{13}$$

$$\sum_{p\in P[SP(s)]} y_s^{t,p} \le \frac{N_s^{e,t-1} - LGSP_s^e \times I_s^{t-1}}{OGSP_s^e - LGSP_s^e},\ \forall t=1,\ldots,K,$$
$$\forall s=1,\ldots,S, \forall e=1,\ldots,E \tag{14}$$

$$\sum_{p\in P[SP(s)]} y_s^{t,p} \le \frac{-N_s^{e,t-1} + UGSP_s^e \times I_s^{t-1}}{-OGSP_s^e + UGSP_s^e},\ \forall t=1,\ldots,K,$$
$$\forall s=1,\ldots,S,\ \forall e=1,\ldots,E \tag{15}$$

$$\sum_{r\in R(n)} x_n^{t,r} = R_n \times u_n^t,\ \forall n=1,\cdots,N,\ \forall t=1,\cdots,K \tag{16}$$

$$x_n^{t,r} \le M \times b_n^{t,r},\ \forall n=1,\ldots,N,\ \forall t=1,\ldots,K,$$
$$\forall r=1,\ldots,R(n)\quad (M \text{ is a large number}) \tag{17}$$

$$\sum_{r\in R(n)} b_n^{t,r} = b_n^t,\ \forall n=1,\cdots,N,\ \forall t=1,\cdots,K \tag{18}$$

$$0 \le u_n^t \le 1,\ \forall n=1,\cdots,N, \forall t=1,\cdots,K \tag{19}$$

$$0 \le uw_n^{t,w} \le 1,\ \forall n=1,\ \cdots,\ N,\ \forall t=1,\ \cdots,\ K,\ \forall w\in W(n) \tag{20}$$

$$0 \le up_n^{t,p} \le 1,\ \forall n=1,\ \cdots,\ N,\ \forall t=1,\ \cdots,\ K,\ \forall p\in P(n) \tag{21}$$

$$0 \le us_n^{t,s} \le 1,\ \forall n=1,\ \cdots,\ N,\ \forall t=1,\ \cdots,\ K,\ \forall s\in SP(n) \tag{22}$$

$$0 \le x_n^{t,r},\ \forall n=1,\ \cdots,\ N,\ \forall t=1,\ \cdots,\ K,\ \forall r\in R(n) \tag{23}$$

$$0 \le y_s^{t,p},\ \forall t=1,\ \cdots,\ K,\ \forall s=1,\ \cdots,\ S,\ \forall p\in P[SP(s)] \tag{24}$$

$$0 \le I_s^t \le CSP_s,\ \forall t=1,\ \cdots,\ K,\ \forall s=1,\ \cdots,\ S \tag{25}$$

$$b_n^t \text{ and } b_n^{t,r} = 0/1,\ \forall n=1,\ \ldots,\ N,$$
$$\forall t=1,\ \ldots,\ K,\ \forall r\in R(n) \tag{26}$$

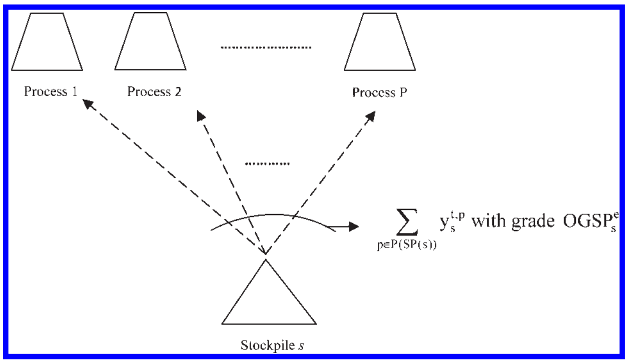
Equation (1) presents the objective function. The objective function includes the following cost terms:

(i) total mining cost that includes drilling, blasting and loading of material throughout K periods
(ii) total processing cost that is the overall cost of processing the ore throughout K periods
(iii) total waste rehabilitation cost that is the summation of costs for rehabilitating of waste materials in the waste dumps during K periods
(iv) total rehandling cost that is total haulage cost of ore that is sent to processes from stockpiles in K periods
(v) total haulage cost that is the overall haulage cost of sending materials extracted inside the mine to the exit of the open pit.

The objective function is minimised under some constraints that are reflected in equations (2)–(26). Equation (2) shows the constraints regarding the complete extraction of all blocks, all blocks must be mined completely during K periods. As mentioned, from the long-term plan, a set of blocks that are going to be extracted in a period, are determined. Equation (2) enforces obeying the long-term plan in regard to staying in the pit space in the given year of the life-of-mine. An extracted portion of each block is sent to different destinations, this is presented by equation (3). To ensure the entire extraction of mineralised zone, equation (4) is imposed. According to equation (4), the mineralised zone of each block is going to be extracted completely through K periods. In other words, after K periods, all the mineral portion of each block will be extracted completely. Equations (2) and (4) lead to complete mining of waste portion of each block. Equation (2) ensures complete mining of each block, while equation (4) enforces complete extraction of mineralised portion of each block.

Equation (5) defines the relationship between binary variable b_n^t and the continuous variable u_n^t. Based on equation (5), only if a portion of block n is mined, $u_n^t > 0$, the corresponding binary variable b_n^t is enforced to get value one. Equation (6) guarantees minimum extraction of blocks in each period. According to equation (6), if a portion of a block is mined, this portion cannot be less than a certain fraction. Equation (6) is imposed to avoid extraction of very small fraction of blocks in each period.

Equations (7) and (8) indicate limitations on capacity of mining equipment and processes respectively. These constraints prevent the mine production in the short-term plan to deviate from the long-term target production. In fact, these constraints enforce the short-term mine schedule to completely obey the long-term plan in a given year of life-of-mine. Equation (7) presents that the total rock tonnage mined in each period must be in a certain acceptable range, according to the available mining equipment capacity. Equation (8) forces that the total tonnage of ore sent to each process in each period must be in an acceptable range based on the process's capacity. Equation (9) guarantees the vertical precedence in extraction of blocks (slope constraint). According to equation (9), extraction of each block can start only when all blocks, directly placed above that block, have already been mined or simultaneously are mined exhaustively. For further elaboration, suppose a typical block x with nine blocks placed above it ($N_{\mathrm{VPB(x)}}=9$). Equation (9) indicates that in each period, mining of block x can be performed only when all of its nine precedent blocks have been extracted entirely through the previous periods and the current period. The right-hand side of equation (9) shows the cumulative portions of all nine vertical precedent blocks by period t and the current period as well. The value of right-hand side would be less than or equal to nine. A value less than nine means that all nine vertical precedent blocks have not been mined completely. Therefore, the extraction of block x cannot begin. Dividing the value of right-hand side by nine, a value less than one would be obtained which enforces $b_{\mathrm{n}}^{\mathrm{t}}$ to be zero. However, if the value of right-hand side equals to nine, division by nine would equal to one which indicates that mining of block x could initiate from the current period t. Equation (10) imposes the head grade constraint of each process in each period. Each process can work with ore with certain grade for different elements. Thus, average grade of elements in the ore sent to each process should be in a certain acceptable grade range specified for the process and different elements. The numerator of equation (10) represents the total amount of metal content of element e available in the ore sent to process p from stockpiles and mine in period t. The denominator stands for ore tonnage sent to process p from stockpiles and mine in period t. Equation (11) shows the balancing equation for inventory of stockpiles thorough K periods. Inventory of stockpiles at the end of period t is the summation of total received material in period t and the current inventory minus total ore tonnage reclaimed in period t. $\forall n{:}s\in SP(n)$ represents the set of blocks that can send ore to stockpile s. Equation (12) stresses that total ore tonnage sent out from each stockpile to the processes in each period should be less than the inventory of that stockpile at the beginning of that period. The main assumption hidden in equation (12) is that stockpiles can send materials to processes only at the beginning of each period. Equation (13) constraints the grade value of element e in the ore sent from the mine to stockpiles to be in an acceptable range corresponding to each stockpile and elements. The numerator and denominator of equation (13) represent the total metal content tonnage of element e carried from the mine to stockpile p in period t and the total tonnage of ore sent from the mine to stockpile p in period t respectively.



2 Procedure of sending ore from stockpile *s* to processes

Equations (14) and (15) guarantee that after reclaiming ore from one stockpile, the average grade of elements in a stockpile remains in its acceptable grade range. Figure 2 indicates the procedure of reclaiming ore from stockpile s in period t. Suppose $\mathrm{N}_{\mathrm{s}}^{\mathrm{e,t-1}}$ stands for the metal content of element e in stockpile s at the beginning of period t (or at the end of period $t-1$). Since ore is sent out with average grade $OGSP_s^e$, the remaining metal content for element e at the end of period t is calculated by following equation

$$N_{\mathrm{s}}^{\mathrm{e,t-1}}-OGSP_{\mathrm{s}}^{\mathrm{e}}\times\sum_{\mathrm{p}\in\mathrm{P[SP(s)]}}y_{\mathrm{s}}^{\mathrm{t,p}},\forall e=1,\cdots,E \tag{27}$$

Also, the remaining ore tonnage available in stockpile s is calculated by equation (28)

$$I_{\mathrm{s}}^{\mathrm{t-1}}-\sum_{\mathrm{p}\in\mathrm{P[SP(s)]}}y_{\mathrm{s}}^{\mathrm{t,p}} \tag{28}$$

As the average grade of element e in stockpile s should be between its predefined lower and upper bound, then

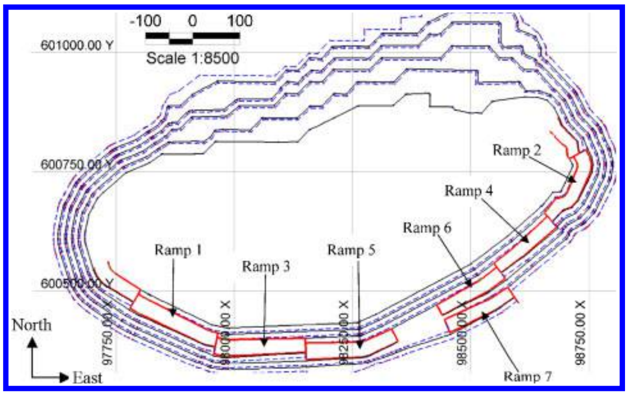
$$LGSP_{\mathrm{s}}^{\mathrm{e}}\le\frac{N_{\mathrm{s}}^{\mathrm{e,t-1}}-OGSP_{\mathrm{s}}^{\mathrm{e}}\times\sum_{\mathrm{p}\in\mathrm{P[SP(s)]}}y_{\mathrm{s}}^{\mathrm{t,p}}}{I_{\mathrm{s}}^{\mathrm{t-1}}-\sum_{\mathrm{p}\in\mathrm{P[SP(s)]}}y_{\mathrm{s}}^{\mathrm{t,p}}}\le UGSP_s^e,\ \forall e=1,\ldots,E \tag{29}$$

After re-organisation of equation (29), equations (14) and (15) are obtained. It should be mentioned that the metal content of element e in stockpile s in period $t-1$ is calculated by equation (30)

$$N_{\mathrm{s}}^{\mathrm{e,t-1}}=\sum_{\tau=1}^{t-1}\sum_{\forall\mathrm{n:s\in SP(n)}}us_{\mathrm{n}}^{\tau,\mathrm{s}}\times R_{\mathrm{n}}\times g_{\mathrm{n}}^{\mathrm{e}}+IMC_{\mathrm{s}}^{\mathrm{e}}-\sum_{\tau=1}^{t-1}\sum_{\mathrm{p}\in\mathrm{P[SP(s)]}}y_{\mathrm{s}}^{\tau,\mathrm{p}}\times OGSP_{\mathrm{s}}^{\mathrm{e}},$$
$$\forall e=1,\ldots,E,\ \forall t=1,\ldots,K,\ \forall s=1,\ldots,S \tag{30}$$

In equation (30), the metal content of element e in period $t-1$ for stockpile s is calculated by the summation of all received metal tonnages in the previous periods and the initial metal content of element e, minus the total metal tonnages that the stockpile has sent to the processes by period $t-1$.

Equations (16)–(18) indicate that total rock tonnage mined from each block in period t is hauled to



3 **Plan view of benches scheduled over a year showing location of ramps**

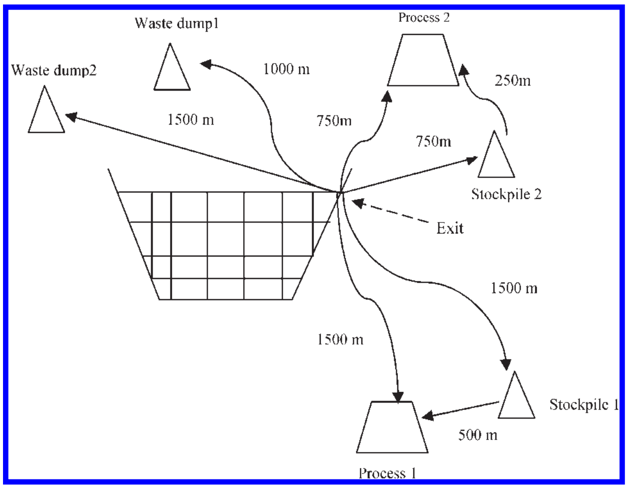
destinations by just one of the possible ramps that the block has access. Here, the decision making is on selection of one possible ramp for haulage of extracted material of each block to destinations. Equation (16) stress that total hauled material of block n in period t to different ramps equals to the total extracted material from that block in that period. Equations (17) and (18) indicate the extracted tonnage can be sent only to one of the possible ramps. Equations (19)–(26) represent the sign constraints regarding decision variables. Among these constraints, equation (25) shows the limitation on storage capacity of stockpiles.

Case study and discussion of results

To show the practicality of the proposed MILP model, it is implemented in TOMLAB/CPLEX environment (Holmstrom, 2009). The model is thoroughly verified with small synthetic datasets as well as real data. In this section, an illustrative example of short-term planning of Gol-E-Gohar iron ore complex in south of Iran is presented to verify the proposed MILP model. The main element of interest in the deposit is iron. The process employs magnetic separators; therefore, the main criterion in selecting ore to be sent to the concentrator is the magnetic weight recovery (per cent MWT) of iron ore. The contaminants are phosphor and sulphur that are considered as secondary elements to be controlled. The open pit mine has 20 benches which only blocks from benches 14, 15, 16 and 17 with 3089 blocks are used for the purpose of short-term planning over 12 months. Blocks are clustered into 150 mining-cuts using fuzzy C-means method. For more information on clustering algorithms for block aggregation, see Tabesh and Askari-Nasab (2011), and Askari-Nasab *et al.* (2010b,

Table 1 General information of problem

	Bench no.			
	17	16	15	14
Number of blocks	614	726	820	929
Number of Cuts	30	35	40	45
Ramps no.	1, 2	3, 4	5, 6	7
Number of periods/months	12			
Total number of blocks	3089			
Total number of cuts	150			
Block size/m^3	$25\times25\times15$			
Total rock tonnage/$\times10^6$ t	94			
Total mineral tonnage/$\times10^6$ t	47			



4 **Map of case study**

c). Block aggregation is required for two reasons: to reduce the number of variables, especially binary variables in the MILP formulation, and to generate a practical mining schedule that follows a selective mining unit. In this study, the MILP model is implemented at the mining-cut resolution.

For the pit, only one exit has been designed. Extracted materials are hauled to this unique exit through different routes/ramps and then sent to final destinations. Also, two ramp accesses have been designed for each bench, on average. Figure 3 shows the plan view of four benches and location of ramp entrances and pit exit. As can be seen, seven ramps are accessible in the four benches. Table 1 represents the general information of the problem. Blocks or mining-cuts can be sent to all destinations.

For implementation of the proposed mathematical model, six destinations including two processes (P1 and P2), two stockpiles (SP1 and SP2) and two waste dumps (W1and W2) are considered.

Tables 2 and 3 indicate the acceptable grade range for different elements and the processing capacity at each process respectively. The acceptable grade ranges and capacities are same over 12 months of scheduling.

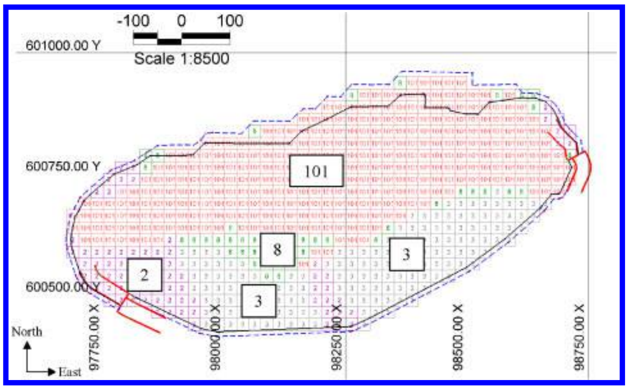
Figure 4 illustrates a schematic view of the distances from the pit exit to the final possible destinations. Stockpile 1 only feeds process 1 and stockpile 2 only feeds process 2. The initial inventory and maximum capacity of stockpiles are zero and two million tonnes, respectively. There is enough space for constructing waste

Table 2 Processes' main features

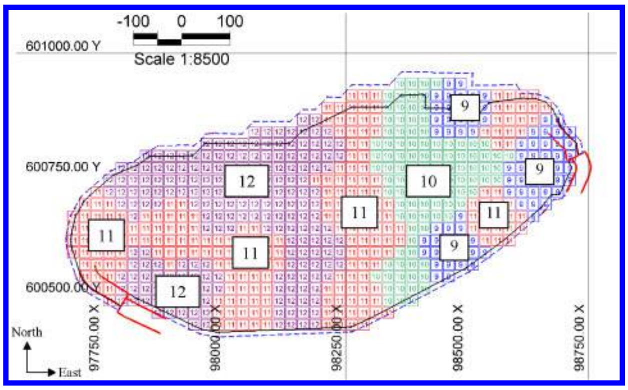
	Lower grade/%			Upper grade/%			Capacity/$\times10^6$ t	
Process	MWT	*S*	*P*	MWT	*S*	*P*	Min.	Max.
Process 1	70	0	0	85	4	3·5	1	2
Process 2	70	0	0	85	4	3·5	1	1·75

Table 3 Stockpiles' main features

	Lower grade/%			Upper grade/%			Output grade/%		
Stockpile	MWT	*S*	*P*	MWT	*S*	*P*	MWT	*S*	*P*
Stockpile 1	55	0	0	65	8	7	60	4	3·5
Stockpile 2	65	0	0	78	8	7	71	4	3·5



5 Plan view of bench 17 showing waste and mineralised zone



6 Plan view of bench 17 showing optimal schedule in basic scenario

dumps, thus there is no limit on their capacity. Also, all different rock types can be sent to the waste dumps.

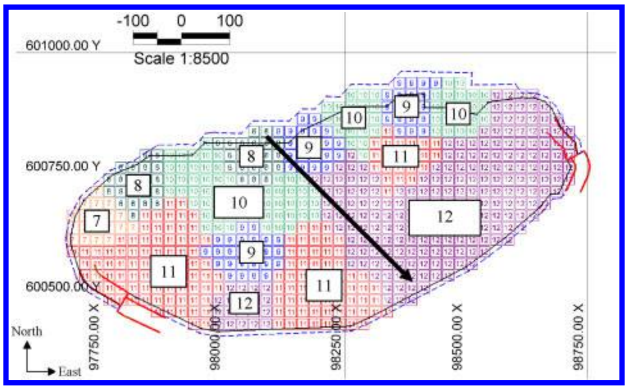
Table 4 shows different unit costs. In Table 4, *PC*, *RH*, *WR*, *MC* and *H* stand for unit processing cost, unit rehandling cost, unit waste rehabilitation cost, unit mining cost and unit haulage cost respectively. The unit cost of haulage of a tonne ore per meter is 0·001$/(t m). For example, in Table 4, unit processing cost of process 2, 5·75$/t, is the summation of 5 which is unit processing cost of process 2 and the value of 0·75 (multiplication of 750 m as the distance from exit location to process 2 in 0·001$/(t m) which is the haulage cost of a tonne of ore to this process). Also, rehandling cost for stockpile 1 is calculated by multiplying unit haulage cost (0·001$/(t m)) to the distance between process 1 and stockpile 1 (500 m).

The proposed MILP model is applied to the case study in three scenarios as follows:

(i) scenario 1: implementing the proposed MILP without imposing any horizontal directional mining
(ii) scenario 2: implementing the proposed MILP with imposing northwest to southeast (NW–SE) horizontal directional extraction. In this scenario, extraction of materials in each bench is performed from the northwest corner of bench and advances to southeast corner
(iii) scenario 3: applying the proposed MILP with imposing southeast to northwest (SE–NW) horizontal extraction. This scenario enforces the extraction of materials in each bench from the southeast corner and advancing to north-west corner.

Table 4 Unit costs

PC/ ($/t)		*RH*/ ($/t)		*WR*/ ($/t)		*MC*/ ($/t)	*H*/ [$/(t m)]
P1	P2	SP1 to p1	SP2 to p2	W1	W2	1	0·001
5·5	5·75	0·5	0·25	1·75	2		



7 Plan view of bench 17 showing optimal schedule in NW–SE scenario

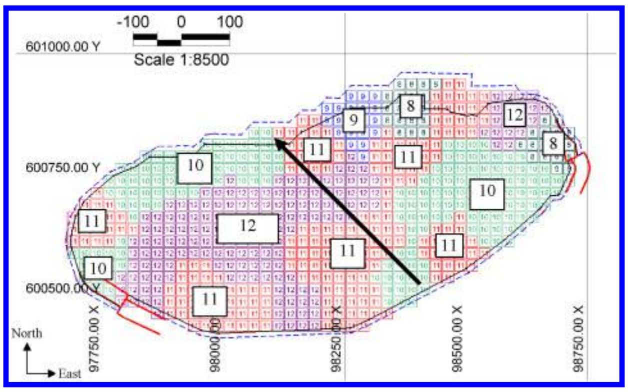
To apply the horizontal precedence constraints in extraction of blocks, a set of equations similar to equation (9) (vertical precedence constraints) is imposed based on the desired horizontal precedence among mining-cuts. To apply the horizontal direction precedence constraint, a list of precedent mining-cuts in the preferred horizontal direction is formed for each mining-cut *n*. It should be mentioned that horizontal precedent mining-cuts of a typical mining-cut *n* are selected from the set of mining-cuts located in the same bench as mining-cut *n*. Equation (31) shows the horizontal precedence constraint in extraction of blocks

$$N_{\mathrm{HPB(n)}} \times b_{\mathrm{n}}^{\mathrm{t}} \leq \sum_{\tau=1}^{\mathrm{t}} \sum_{\mathrm{i} \in \mathrm{HPB(n)}} u_{\mathrm{i}}^{\tau},$$

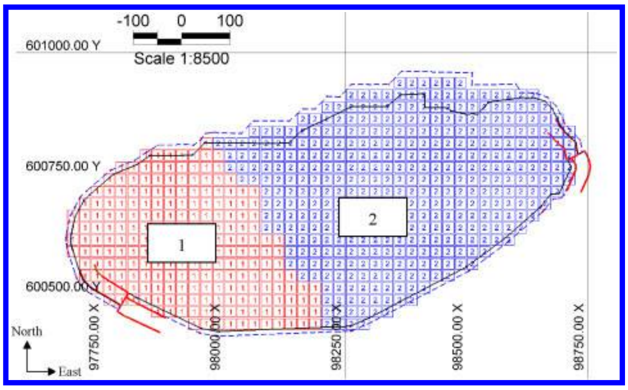
$$\forall n=1, \ldots, N, \forall t=1, \ldots, K \tag{31}$$

As can be seen, the above equation is similar to equation (9). Only set of precedent blocks *HPB*(*n*), is different. $N_{\mathrm{HPB(n)}}$ represents the number of precedent blocks in the corresponding direction for block *n*.

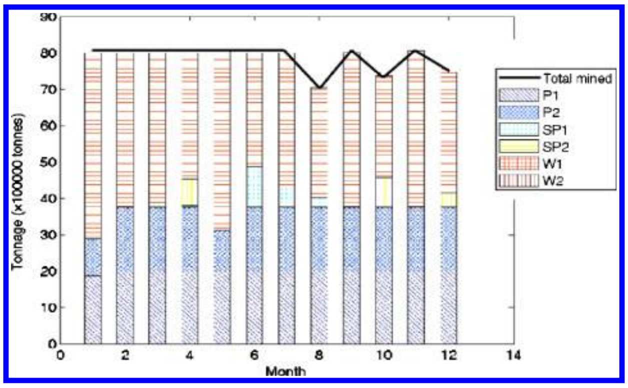
Figure 5 sketches the rock code of blocks in bench 17. The mineralised zone is indicated by rock code 101. All other rock codes show waste region. Also, Figs. 6–8 present plan view plots of level 17 for three scenarios



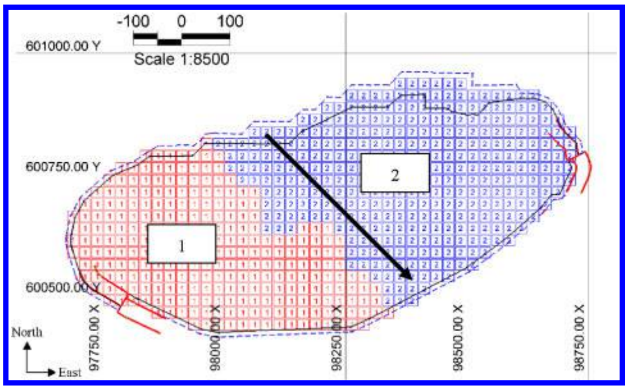
8 Plan view of bench 17 showing optimal schedule in SE–NW scenario



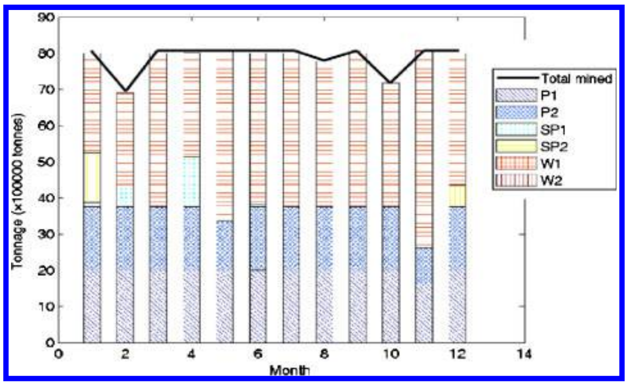
9 Plan view of bench 17 showing selected ramps in basic scenario



12 Optimal production schedule in basic scenario



10 Plan view of bench 17 showing selected ramps in NW–SE scenario



13 Optimal production schedule in NW–SE scenario

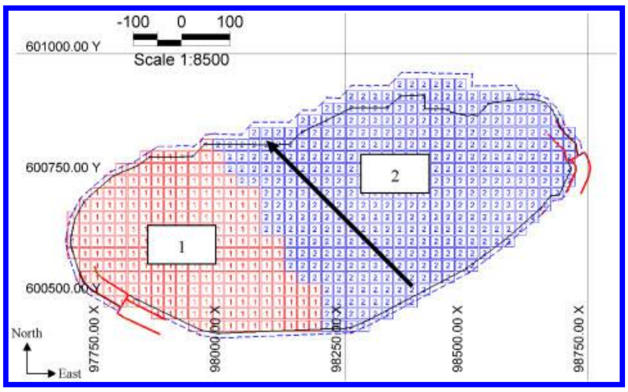
based on their optimal schedules. The numbers inside each region represent the month that maximum portion is extracted from that region. For instance, 12 placed in a block refers that most of rock tonnage of that block is extracted in month 12.

Figures 9–11 show the plan view plots indicating the selected ramps for sending extracted materials in bench 17 to the pit exit. According to the figures, the ramp nearer to a block is normally selected to haul extracted material of that block. This was expected because the objective function tries to minimise haulage cost which depends on the distance (or routes' length).

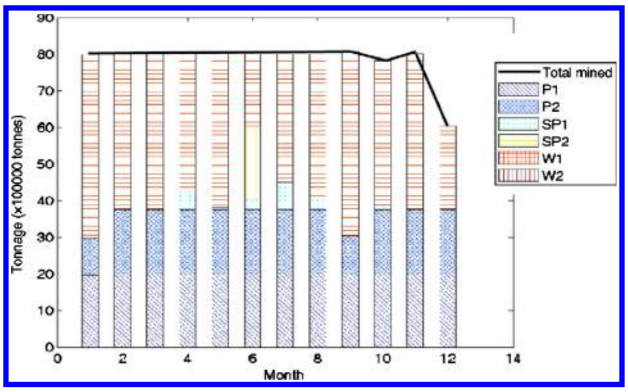
Figures 12–14 show the results of implementation of MILP in the three scenarios in terms of production charts. Based on these figures, fluctuations of mine production for the third scenario is the least. For the first 11 months, mine production in the third scenario (SE–NW) is more stable than scenarios 1 and 2. In addition, in the earlier months, only scenario 3 feeds processes to their maximum capacity which leads to more production than other scenarios. According to time value of money, more production in the earlier months makes more revenue and as a result profit.

Figures 15–17 reflect the average grade of MWT in the ore sent to two processes from mine and stockpiles within 12 months in the three scenarios. As it can be seen, the average grade is around 80% for both processes while fluctuates gently in all scenarios. Therefore, it can be inferred that in all scenarios, processes can highly trust in the mine production to supply the ore with a reliable grade of MWT.

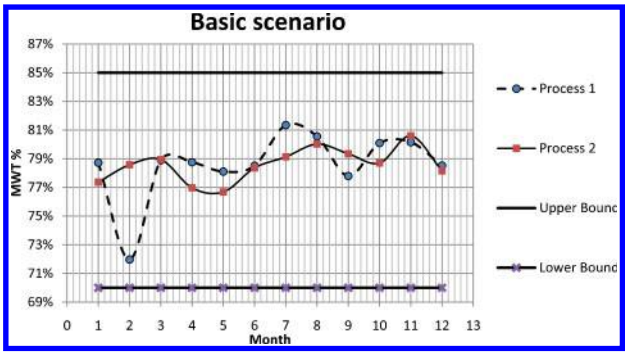
Figures 18–20 show the status of inventory of stockpiles within 12 months for three scenarios. The initial



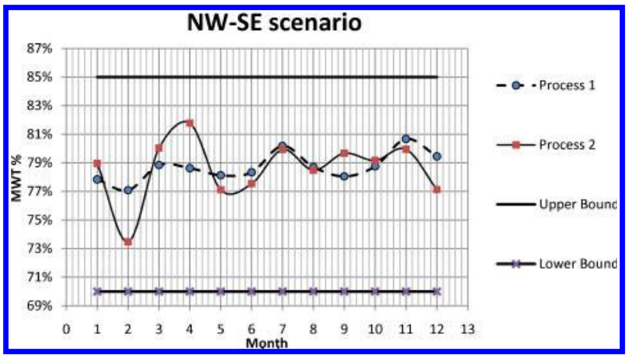
11 Plan view of bench 17 showing selected ramps in SE–NW scenario



14 Optimal production schedule in SE–NW scenario



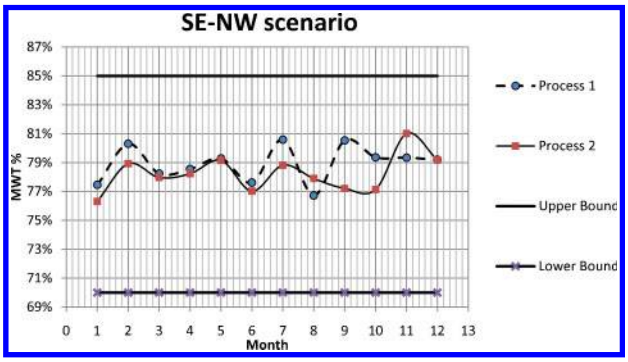
15 Monthly average grade of MWT (%) delivered to processes 1 and 2 in basic scenario



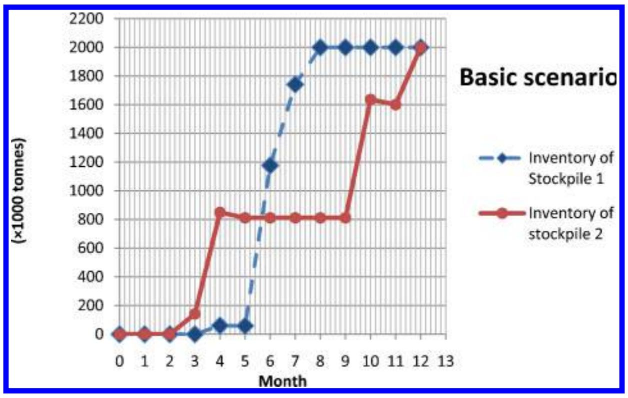
16 Monthly average grade of MWT (%) delivered to processes 1 and 2 in NW–SE scenario

inventory of stockpiles is zero. Then, as long as mine produces ore and sends it to the stockpiles, the value of inventory for each of stockpiles increases. In the periods that the inventory decreases from the previous month, illustrates that ore is reclaimed from the stockpile to balance the production and meet the processes' head grades.

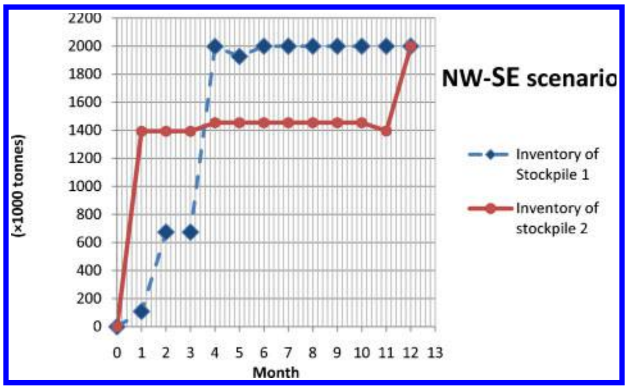
Figure 21 presents a comparison of the total cost in the three schedules resulting from the three scenarios. According to this figure, the first and second scenarios incur the minimum and maximum total cost. Obviously, the first scenario should have the least total cost because the other scenarios accompany an extra type of constraints – preferred horizontal directional mining – which makes the feasible solution space smaller.



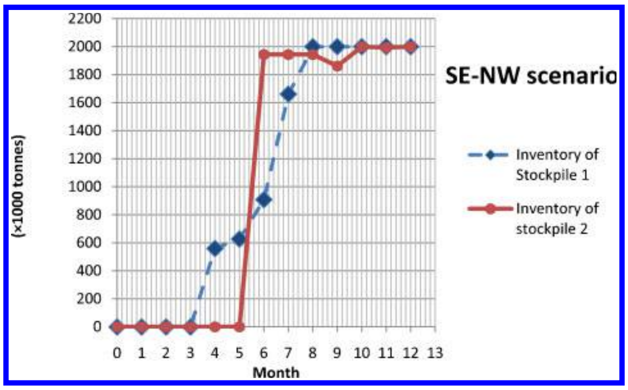
17 Monthly average grade of MWT (%) delivered to processes 1 and 2 in SE–NW scenario



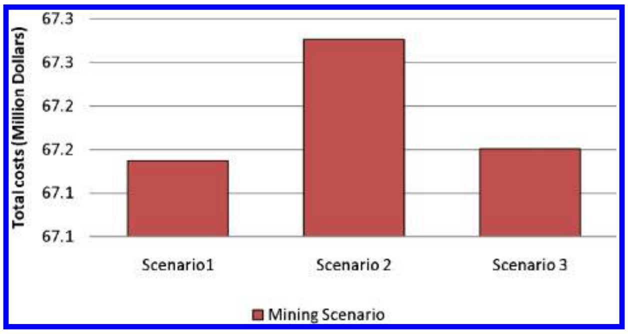
18 Monthly inventory of stockpiles in basic scenario



19 Monthly inventory of stockpiles in NW–SE scenario



20 Monthly inventory of stockpiles in SE–NW scenario



21 Comparison of optimal total cost of three scenarios

After comparison of the three scenarios, the third scenario is selected as the best case because:

(i) the third scenario leads to the minimum fluctuations in mining equipment usage in the earlier months
(ii) the third scenario feeds processes at their maximum capacity in the earlier months that result in more revenue and profit in association with time value of money concept
(iii) although the total cost of the third scenario is more than the first scenario, the third scenario is more practical from mining point of view than the first one. Comparison of Figs. 6–8 illustrates that the third scenario requires fewer number of drop-cuts comparing to the other scenarios.

Conclusions and future work

Mine production scheduling is a challenging problem in the mining industry. In this paper, a MILP model for the short-term scheduling of open pit mines has been formulated and solved. The proposed model considers different destinations, processes, stockpiles, and waste dumps. The objective function is to minimise total operations' cost including mining and haulage, processing, rehandling and waste rehabilitation costs. The model incorporates stockpiles, horizontal directional mining and decisions on ramps, while controlling technical and operational constraints, such as head grade, precedence and mining and processing capacity constraints.

The main contribution of the present paper is an MILP formulation for short-term open pit mine production scheduling optimisation. The significant improvements over the previous research in the context of mathematical programming models for short-term production scheduling are: consideration of multiple destinations including stockpiles, consideration of routing and ramps and consideration of horizontal directional mining. The proposed model is solved by TOMLAB/CPLEX for a set of blocks in four benches of an iron ore mine in three different directional mining scenarios. The schedules resulting from the three scenarios are compared with each other. The comparison showed that the third scenario (SE–NW) is the best because of minimum overall cost and minimum number of drop-cuts that are required to mine each bench. Also, in the third scenario, more share of available capacity of mining equipment is used in earlier months to extract materials than the other two scenarios.

In this paper, 'short-term' refers to a production schedule time horizon, expanding one to three years with a monthly resolution. The input data into the short-term production scheduler, such as geological block model, grades, costs, prices, recoveries, and practical mining constraints are based on the best point estimates available at the time of optimisation. Any change in the input data requires a re-run of the model with the new input parameters. This is aligned with the mining industry practice of updating yearly, quarterly, monthly, and weekly mine plans as new data becomes available and uncertainty is reduced overtime. In other words, the limitation of this model is that it does not capture the cost of uncertainty.

For the future research directions, simulation study of the developed schedules would be of high value. Discrete event simulation could capture the uncertainties in equipment availability, utilisation, break-down times, and unscheduled maintenance. Also, the variability in cycle times due to season and different road and ramp conditions could be taken into account. Considering these uncertainties using simulation and examining the schedules can show the strength and weakness points of MILP models more in detail. Also, another future line of research is extension of formulation for pits with multiple exit locations.

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