# OPEN PIT MINE PLANNING WITH STOCKPILING

by

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A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of PhD (Operations Research with Engineering).

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# ABSTRACT

This dissertation consists of three papers; the first is published in *European Journal of Operational Research*, the second is nearing submission to *Optimization and Engineering*, and the third is nearing submission to *International Journal of Mining, Reclamation and Environment*. These papers apply operations research techniques to open pit mine production scheduling with stockpiling (OPMPS+S). The first paper, "Linear Models for Stockpiling in Open-pit Mine Production Scheduling Problems," reviews existing models to solve OPMPS+S and shows that a nonlinear-integer model provides an exact solution but is intractable even for medium-size data sets. Then, we present an approximation to that nonlinear-integer model, solve the nonlinear-integer and proposed models for multiple data sets and show that the latter model provides solutions very close to those provided by the nonlinear-integer model. By pairing this novel formulation with recently developed linear programming algorithms and heuristics, we demonstrate a dramatic decrease in solution time. The second paper, "Practical Performance of an Open Pit Mine Scheduling Model Considering Blending and Stockpiling," introduces an extension to the model presented in the first paper to blend material with multiple grades and a contaminant in the stockpile. Existing state-of-the-art algorithms which exploit the problem structure produce integer solutions with associated net present value at least 18.7% higher than that provided by commercial of-the-shelf software. The third paper, "Open Pit Mine Planning with Degradation Due to Stockpiling," presents three new models that consider degradation due to stockpiling, compare them with models without degradation, and show that degradation has a major impact on the value a stockpile provides.

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## ACKNOWLEDGMENTS

I would like to thank everyone who helped me along this journey, specially my committee members. Thanks.

I wish to thank my dear wife, Sara Bourbour, for her helps and inspiration that she gave me. I would like to thank my children, Mohammad Mahdi and Fatemeh Zahra.

# CHAPTER 1
# INTRODUCTION

Operations research is a field that uses various analytical techniques such as mathematical modeling to make more informed decisions, resulting in optimal or near-optimal solutions to complex problems. Applying optimization techniques to mining operations is very important, because (i) the mining industry involves large operations for which efficiency is very crucial, and (ii) with continued extraction, deposits are becoming rarer and less easily accessible, making informed decisions increasingly important. We apply operations research techniques to open pit mine production scheduling with stockpiling (OPMPS+S), and provide a new linear-integer model to solve this problem. This model aims to improve the profitably of open pit mine operations in long-term by determining which notional three-dimensional blocks of ore and waste to extract, and when, and where to send them. Restrictions on the model include operational, resource-availability, precedence and blending constraints.

In this thesis, we show that the proposed integer and mixed-integer programming models are capable of increasing net present value of mining operations with stockpiling by millions of dollars. Traditionally, optimization models have focused on surface mining, and either do not consider stockpiling or present nonlinear-integer models which are difficult to solve. The purpose of this research is: (i) to develop a realistic linear-integer program to solve open pit mine scheduling problem with stockpiling, (ii) extend the proposed model in (i) to blend material with multiple grades and a contaminant in the stockpile, and (iii) consider degradation due to stockpiling.

This dissertation contributes to mine planning in an operations research context. We add to the operations research literature by introducing linear-integer models that are capable of solving very close approximations of OPMPS+S for large data sets in a few seconds to a few minutes. This dissertation is organized as follows: in Chapter 2, we develop a new linear-integer programming model to solve OPMPS+S, which is tractable even for large data sets. We solve this model for multiple data sets and show that this approximation model provides a solution very close to that of a nonlinear-integer model, traditionally thought to provide the most accurate solutions to mine planning models that include stockpiling. Chapter 3 introduces extensions to the model presented in Chapter 2; these extensions solve OPMPS+S for an operational mine with multiple metals and a contaminant. In Chapter 4, we propose three new linear-integer programming models to solve the OPMPS+S by considering degradation due to stockpiling. We also examine the effect of degradation on the value of stockpiling.

CHAPTER 2

# LINEAR MODELS FOR STOCKPILING IN OPEN-PIT MINE PRODUCTION SCHEDULING PROBLEMS

A paper published in *European Journal of Operational Research*[1]

Eduardo Moreno[2], Mojtaba Rezakhah[3], Alexandra Newman[4], Felipe Ferreira[5]

The open pit mine production scheduling (OPMPS) problem seeks to determine when, if ever, to extract each notional, three-dimensional block of ore and/or waste in a deposit and what to do with each, e.g., send it to a particular processing plant or to the waste dump. This scheduling model maximizes net present value subject to spatial precedence constraints , and resource capacities. Certain mines use stockpiles for blending different grades of extracted material, storing excess until processing capacity is available, or keeping low-grade ore for possible future processing. Common models assume that material in these stockpiles, or "buckets," is theoretically immediately mixed and becomes homogeneous.

We consider stockpiles as part of our open pit mine scheduling strategy, propose multiple models to solve the OPMPS problem, and compare the solution quality and tractability of these linear-integer and nonlinear-integer models. Numerical experiments show that our proposed models are tractable, and correspond to instances which can be solved in a few seconds up to a few minutes in contrast to previous nonlinear models that fail to solve.

## 2.1 Introduction

Open pit mine production scheduling (OPMPS) is a decision problem involving which blocks, within the final pit limits, should be mined in each year, and where the blocks should

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[1]Reprinted with permission of *European Journal of Operational Research*, January 13, 2017.
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be sent, e.g., mill, waste dump or stockpile, to maximize the net present value (NPV) subject to the constraints that: (i) mining and processing consume limited resources and affect the production profile in each period; and (ii) spatial precedence must be obeyed among the blocks (Figure 3.3).

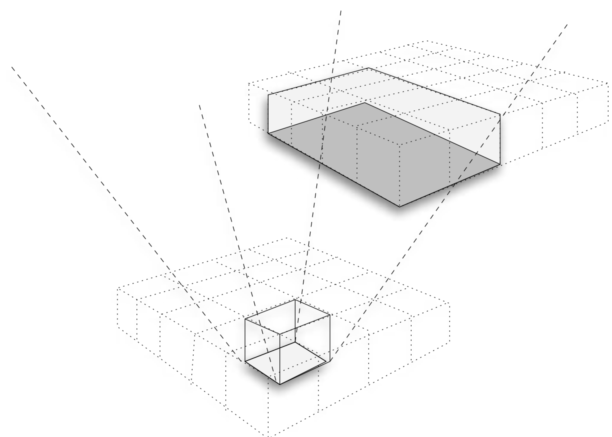


Figure 2.1: Block precedence relationships. In order to extract the lower blocks, all upper blocks inside a predefined slope should be previously extracted. Figure courtesy of Nelson Morales.

In open pit mine scheduling, the question arises as to how mathematically to model the stockpile and determine a strategy, and how to assess the value associated with using a stockpile. While some researchers do not consider a stockpile as part of OPMPS, others suggest using a stockpile without providing the mathematical framework. In this research, we focus on proposing tractable models which provide practical solutions.

Initially, researchers proposed linear programs to solve OPMPS without considering a stockpile. Johnson [1] describes the first such model to maximize net present value (NPV) of an open pit mine while determining whether each block should be sent to the mill or the waste dump, subject to precedence and operational resource constraints. Because his model contains only continuous-valued variables, his precedence constraints enforce that in order to extract a certain amount of block $b'$, at least that same amount of predecessor block $b$ must be extracted. The author uses Dantzig-Wolfe decomposition to solve several instances. Given hardware and software limitations at the time, he illustrates with some small examples.

An important challenge in solving OPMPS is that model instances can contain many blocks and time periods, and each block-time period combination has an associated binary decision variable in order to capture the more realistic constraint that all of a predecessor block must be extracted before any of a successor block is extracted. One way to decrease the number of decision variables in these linear-integer programs is to aggregate some blocks with similar characteristics. Askari-Nasab et al. [2] discuss different aggregation techniques that can be used to fit the geology of the deposit and the time fidelity of the model. They also develop an open-pit production method which depicts the stochastic dynamic expansion of an open pit using discrete incremental pushbacks in different directions.

Ramazan [3] uses the concept of "fundamental trees" to aggregate blocks for an open pit production scheduling problem. Boland et al. [4] suggest that variables or constraints which are determined to be "similar" according to some criteria can be grouped together into new variables or constraints, called aggregates. The new OPMPS problem is then solved, causing some decisions to lose their fidelity in the aggregated model. By disaggregating, i.e., reverting to the original variables, a solution for the initial problem, which is usually not optimal and possibly infeasible, is obtained. Jélvez et al. [5] present a number of heuristics to tackle the open-pit block scheduling problem. Their approach is mainly based on block aggregation. The authors first solve the aggregated problem and then obtain a feasible solution for the original instance.

Bienstock and Zuckerberg [6] provide a new algorithm for solving the linear programming relaxation of the precedence constrained production scheduling problem by reformulating it such that many constraints are modeled as a single one. They also consider multiple processing options. Their maximum weight closure problem can be solved as a minimum cut problem with a small number of side constraints, making it amenable to Lagrangian-based approaches. Chicoisne et al. [7] propose a new algorithm to solve linear programming relaxations of large instances of the same problem, and a set of heuristics to solve the corresponding integer program.

[8] present a mixed-integer model to schedule long- and short-term underground production which minimizes deviations from preplanned production quantities while adhering to operational constraints. The authors develop an optimization-based decomposition heuristic that solves large instances quickly. O'Sullivan and Newman [9] schedule extraction and backfill at an underground Lead-Zinc mine that uses three different underground methods; their heuristic enables them to solve real-world instances.

Shishvan and Sattarvand [10] present a metaheuristic approximation based on Ant Colony Optimization for open-pit mine production planning which considers any type of objective function and nonlinear constraints. [11] propose a risk-based method which incorporates geological uncertainty to optimize mining operations comprised of multiple pits, stockpiles, blending requirements, processing paths, operating alternatives and transportation systems. Their method perturbs an initial solution iteratively to improve the objective function. Lamghari and Dimitrakopoulos [12] and, similarly, de Freitas Silva et al. [13] propose different heuristics such as tabu search and variable neighborhood descent to solve models that consider metal uncertainty and multiple destinations for the extracted material; low-grade material sent to the stockpile is mixed homogeneously, and the corresponding average grade is successively approximated.

Although linear and mixed integer programming models are recognized as having significant potential for optimizing production scheduling in both open pit and underground mines, most of these approaches focus on the extraction sequence and do not consider the material flow post-extraction. In particular, the use of stockpiling to manage processing plant capacity, and the interplay of material flows from the mine to a stockpile, the mine to a processing plant, and a stockpile to a plant, have not been treated as an integrated part of mine extraction sequence optimization. While industrial uses of mine planning software with stockpiling exist, these have limited benefit due to the nature of their modeling and solution techniques.

### 2.1.1 Existing Industrial Software

While some mining software such as MineSight [14] and MineMax [15] have tried to consider the stockpile as part of open pit mine scheduling, such software does not guarantee global optimal solutions. Whittle, one of the leading pieces of software in mine planning, has a stockpiling module and considers mixing material with different grades in the stockpile:

> *As material is moved to the stockpile, the tonnage and metal information is accumulated, so that at any point in time, the average grade is known. Stock withdrawals are considered to be at the average grade. Stockpiles are only used if they return a positive cash flow* [16].

Whittle does not use optimization techniques to model the stockpile, so there is no guarantee of obtaining an optimal solution with respect to the number of stockpiles and/or the grade contained in each stockpile. Academic researchers have been developing models to address these shortcomings.

### 2.1.2 Linear-Integer Models Considering a Stockpile

Smith [17] uses mixed integer programming to solve a short-term production scheduling problem with blending, considering stockpiles both at the mine and at the mill. He notes that correctly capturing the contents of the stockpile requires nonlinear constructs, and enhances tractability of the original model by introducing piecewise linear constructs to approximate separable terms (after reformulation) representing the product of the average grade in the stockpile and the quantity retrieved from the stockpile in a given time period. After aggregation and variable elimination, he applies the model results to a phosphorus mine in Idaho. This research represents an early attempt to correctly model the grade of a stockpile, but requires approximations whose accuracies are not quantified, to ensure tractability.

Caccetta and Hill [18] propose an exact approach to solve a monolithic OPMPS problem by defining variables representing whether a block is mined by time period $t$. The model includes constraints on: precedence, operational resources, and processing grade requirements. They also discuss the possibility of considering a stockpile in their model but without an associated mathematical formulation. The authors propose a branch-and-cut strategy combined with a heuristic. Asad [19] describes a simple optimization model designed to assess the tradeoffs between cutoff grades and stockpile levels for a two-mineral deposit. His static model omits production scheduling decisions. Ramazan and Dimitrakopoulos [20] explain that the OPMPS problem typically contains uncertainty in the geological and economic input data. They use a stochastic framework to incorporate stockpiling since the amount of material to be stockpiled is determined by the block grades in the orebody model. In these models, the authors ignore mixing of material in the stockpile. Koushavand et al. [21] quantify oregrade uncertainty by including a term for its cost in the objective function; their model captures typical constraints on extraction and processing limits, and on block precedence, as well as on blending, and on over- and under-production. Stockpile levels are bounded above and below, and are tracked in aggregate by time period; the authors demonstrate their model using a case study in which they assume that the stockpile has its grade set a priori and that it is used to mitigate uncertainty, i.e., overproduction can be carried over until the next time period. [22] use a mixed-integer program (MIP) that maximizes recovered copper and accounts for constraints on shovel, extraction, stockpiling, and processing capacities, as well as blending. Here, the stockpiling constraints result in an optimistic bound on the model, in that each block is retrieved from the stockpile having preserved its characteristics upon entry to the stockpile. The authors' life-of-mine model, solved using a sliding time window heuristic to incorporate a 60-month horizon, yields information regarding stripping ratios and qualities and quantities of ore mined.

### 2.1.3 Nonlinear-Integer Models Considering a Stockpile

Nevertheless, some researchers do consider material mixing in the stockpile. When placing an ore block on a stockpile, the block characteristics (e.g., grade and tonnage) are known. However, as blocks are mixed in the stockpile, the characteristics of the material removed from the stockpile must be treated as variables. Since the amount of ore removed from the stockpile is not known a priori, the model has some non-convex, nonlinear constraints. Efforts to solve this problem result in local optimal solutions or consist of linearizing the model, which might introduce unrealistic assumptions.

Tabesh et al. [23] acknowledge that stockpiling should theoretically be modeled nonlinearly to optimize a comprehensive open-pit mine plan, and linearizes the formulation by using a “sufficient number” of stockpiles, each with a tight range of grades. No numerical results are given, however. (We will return to this model later.)

Although there have been efforts to consider stockpiling as part of OPMPS, some of these models result in locally optimal solutions and/or are intractable for big data sets. Attempting to decrease the size of the problem instances results in aggregation, which causes a loss of information regarding each type of material [24].

Bley et al. [25] propose two different models considering *one* stockpile with the following assumptions:

1. Material in the stockpile mixes, resulting in a grade equal to the average grade of all the material inside the stockpile.

2. Material is extracted from the stockpile at the beginning of each period, so the grade of the resulting material is the average of that of the material at the end of the previous period.

In Section 2.3.2, we present ($\mathcal{P}^b$), which tracks the ore and mineral in the stockpile in each period, considering material mixing by adding a non-convex quadratic constraint for each period. In Section 2.3.3, we discuss ($\mathcal{P}^w$), in which the fraction of each block in the stockpile in each period is tracked, and additional non-convex constraints force the fraction of each block in the stockpile that is sent to be processed in a given time period to be the same. Bley et al. [25] prove that ($\mathcal{P}^b$) and ($\mathcal{P}^w$) are equivalent, but the latter model provides a stronger formulation of the problem, resulting in a better upper bound.

Bley et al. [25] focus on exact algorithmic approaches. They study a relaxation of ($\mathcal{P}^w$) by removing the non-linear constraints, and instead enforcing these restrictions using a scheme, integrated within a branch-and-bound framework, that (i) branches on the variable representing the value of the proportion of metal (versus ore) removed from the stockpile in each time period, and (ii) forces the violation of all non-linear constraints to be arbitrarily close to 0. Additionally, the authors propose a primal heuristic to obtain feasible solutions of the exact problem from a relaxed solution, and cuts and inequalities to strengthen the relaxation. Finally, they apply these techniques on two small instances, showing the impact of each solution procedure they propose.

Our research, by contrast, focuses on proposing new models, rather than on developing new algorithms, and compares how their assumptions affect solution quality and tractability. These linear-integer models include blending requirements without unrealistic assumptions, and yield good approximations using state-of-the-art methodologies on large-scale instances.

We organize the remainder of this paper as follows. In Section 2.2, we explain an existing model that does not incorporate stockpiling; in Section 2.3, we present existing nonlinear models that incorporate stockpiling. In Section 2.4, we propose linear models with stockpiling. In Section 2.8, we graphically represent the difference between our proposed models, and in Section 2.9, we compare the results. We conclude with Section 2.10.

## 2.2 Lower Bound Model

In this section, we present the formulation of a model that provides a lower bound on the objective function value of the OPMPS problem in which the option of stockpiling does not exist; such a model can be found in Caccetta and Hill [18], Boland et al. [4], and as a special case of Bienstock and Zuckerberg [6]. The first subsection introduces notation, and the following subsections provide the math. We use the term “material” to include ore, i.e., rock that contains sufficient minerals including metals that can be economically extracted, and to include waste.

### 2.2.1 Notation

Indices and Sets:

$b \in \mathcal{B}$ : blocks; $1, ..., B$
$\hat{b} \in \hat{\mathcal{B}}_b$ : blocks that must be mined directly before block $b$
$r \in \mathcal{R}$ : resources $\{1 = \text{mine}, 2 = \text{mill}\}$
$t \in \mathcal{T}$ : time periods; $1, ..., T$

Parameters:

$\delta_t$ : discount factor for time period $t$ (fraction)
$C^m$ : mining cost per ton of material (dollars /ton)
$C^p$ : processing cost per ton of material (dollars /ton)
$P$ : profit generated per ton of metal (dollars /ton)
$W_b$ : tonnage of block $b$ (ton)
$M_b$ : metal obtained by completely processing block $b$ (ton)

Decision Variables:

$y_{bt}^m$ : fraction of block $b$ mined in time period $t$
$y_{bt}^p$ : fraction of block $b$ mined in time period $t$ and sent (directly) to the mill
$y_{bt}^w$ : fraction of block $b$ mined in time period $t$ and sent to waste
$x_{bt}$ : 1 if block $b$ has finished being mined by time $t$; 0 otherwise

### 2.2.2 Model without Stockpiling ($\mathcal{P}^{ns}$)

The following model omits stockpiling:

$$(\mathcal{P}^{ns}) : \max \sum_{t \in \mathcal{T}} \delta_t \left[ P \left( \sum_{b \in \mathcal{B}} M_b y_{bt}^p \right) - C^p \left( \sum_{b \in \mathcal{B}} W_b y_{bt}^p \right) - C^m \left( \sum_{b \in \mathcal{B}} W_b y_{bt}^m \right) \right] \tag{2.1}$$

$$y_{bt}^p + y_{bt}^w = y_{bt}^m \qquad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \tag{2.2}$$

$$\sum_{t \in \mathcal{T}} y_{bt}^m \leq 1 \qquad \forall b \in \mathcal{B} \tag{2.3}$$

$$x_{bt} \leq \sum_{t' \leq t} y_{bt'}^m \qquad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \tag{2.4}$$

$$\sum_{t' \leq t} y_{bt'}^m \leq x_{\hat{b}t} \qquad \forall b \in \mathcal{B}, \hat{b} \in \hat{\mathcal{B}}_b, t \in \mathcal{T} \tag{2.5}$$

$$(x, y) \in \Omega \qquad \text{(other constraints)} \tag{2.6}$$

The objective function is the sum of the revenues of blocks sent directly to the mill, minus the sum of the extraction and processing costs. All terms are multiplied by an appropriate discount rate according to the time period, $t$.

The first constraint forces the material sent to the mill or waste to equal the quantity of extracted material. Constraint (2.3) ensures that extracted fractions of each block summed across all time periods must be less than or equal to one. Constraint (2.4) forces the sum of the fractional variables to 1 by time $t$ if the block has been mined by that time. Constraint (2.5) enforces mining precedence constraints by ensuring that for each block, all predecessors are completely mined before any amount of the successor block is mined. Constraint (2.6) might represent geometrical and operational restrictions (e.g., block-level or bench-phase scheduling, mining and processing bounds, blending constraints, and/or the maximum number of phases opened), and could involve bound and integrality constraints on $x$ and $y$.

### 2.3 Nonlinear Models that Consider Stockpiling

In this section, we provide nonlinear formulations that consider a stockpile. Because we propose models with just one stockpile, we define "buckets" that represent different parts of a stockpile, where each bucket incorporates material within a specific grade range. The grade of material when removing it from the stockpile is the minimum grade of the associated bucket. First, we define additional notation:

### 2.3.1 Notation

Indices and Sets:

$k \in \mathcal{K}$ : buckets; $1, ..., K$

Parameters:

$C^h$ : rehandling cost per ton of material (dollars /ton)
$L$ : average grade in the stockpile (grams/ton)
$L_k$ : average grade in bucket $k$ (grams/ton)

Decision Variables:

$y^s_{bt}$ : fraction of block $b$ mined in time period $t$ and sent to the stockpile
$\bar{y}^s_{bkt}$ : fraction of block $b$ mined in time period $t$ and sent to bucket $k$ of the stockpile
$z^p_{bt}$ : fraction of block $b$ sent from the stockpile to the mill in time period $t$
$z^s_{bt}$ : fraction of block $b$ remaining in the stockpile at the end of time period $t$
$f_t$ : relative proportion of blocks from the stockpile processed in time period $t$
$i^p_t, m^p_t$ : tonnage of ore and metal sent from the stockpile to the mill in time period $t$, respectively
$i^s_t, m^s_t$ : tonnage of ore and metal remaining in the stockpile at the end of time period $t$, respectively
$\bar{\imath}^p_{kt}$ : tonnage of ore sent from bucket $k$ in the stockpile to the mill in time period $t$
$\bar{\imath}^s_{kt}$ : tonnage of ore remaining in bucket $k$ in the stockpile at the end of time period $t$

### 2.3.2 Basic model that considers stockpiling ($\mathcal{P}^b$)

The OPMPS+S with a single stockpile is similar to the formulation in §2.2.2 except that the objective function and constraint (2.2) are modified and five more constraints are added. Close variants of the following two formulations are given in Bley et al. (2012). The objective function becomes:

$$(\mathcal{P}^b) : \max \sum_{t \in \mathcal{T}} \delta_t \left[ P\left(\sum_{b \in \mathcal{B}} M_b y^p_{bt} + m^p_t\right) - C^p\left(\sum_{b \in \mathcal{B}} W_b y^p_{bt} + i^p_t\right) \right.$$
$$\left. -C^m\left(\sum_{b \in \mathcal{B}} W_b y^m_{bt}\right) - C^h i^p_t \right] \quad (2.7)$$

The profit of the blocks sent from the stockpile to the mill is added to $(\mathcal{P}^{ns})$'s objective function, and processing and rehandling costs are subtracted. Constraint (2.2) becomes:

$$y^p_{bt} + y^w_{bt} + y^s_{bt} = y^m_{bt} \qquad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \quad (2.8)$$

The original constraints from $(\mathcal{P}^{ns})$ are:

$$\sum_{t \in \mathcal{T}} y^m_{bt} \leq 1 \qquad \forall b \in \mathcal{B} \quad \text{(2.3 revisited)}$$
$$x_{bt} \leq \sum_{t' \leq t} y^m_{bt'} \qquad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \quad \text{(2.4 revisited)}$$
$$\sum_{t' \leq t} y^m_{bt'} \leq x_{\hat{b}t} \qquad \forall b \in \mathcal{B}, \hat{b} \in \hat{\mathcal{B}}_b, t \in \mathcal{T} \quad \text{(2.5 revisited)}$$
$$(x, y) \in \Omega \qquad \text{(other constraints)} \quad \text{(2.6 revisited)}$$

The five new constraints are:

$$i^p_t \leq i^s_{t-1} \qquad \forall t \in \mathcal{T} \quad (2.9)$$
$$m^p_t \leq m^s_{t-1} \qquad \forall t \in \mathcal{T} \quad (2.10)$$
$$i^s_t = \begin{cases} \sum_{b \in \mathcal{B}} W_b y^s_{bt} & t = 1 \\ i^s_{t-1} - i^p_t + \sum_{b \in \mathcal{B}} W_b y^s_{bt} & t \in \mathcal{T} : t \geq 2 \end{cases} \quad (2.11)$$
$$m^s_t = \begin{cases} \sum_{b \in \mathcal{B}} M_b y^s_{bt} & t = 1 \\ m^s_{t-1} - m^p_t + \sum_{b \in \mathcal{B}} M_b y^s_{bt} & t \in \mathcal{T} : t \geq 2 \end{cases} \quad (2.12)$$
$$\frac{m^p_t}{i^p_t} \leq \frac{m^s_{t-1}}{i^s_{t-1}} \qquad \forall t \in \mathcal{T} \quad (2.13)$$

Constraints (2.9) and (2.10) ensure that what we send from the stockpile to the mill in time period $t$ is less than or equal to the material and the metal, respectively, in the stockpile in time period $t-1$. Constraints (2.11) and (2.12) enforce inventory balance for an initial time period and a general time period $t$, ensuring that the amount of material and metal, respectively, in the stockpile during time period $t$ is equal to that of the last period plus anything that was added and minus anything sent to the mill from the stockpile. Constraint (2.13) forces the ratio of the metal contained in the material (i.e., grade) sent to the processing plant to be less than or equal to that ratio in the stockpile at the end of the previous time period.

### 2.3.3 Warehouse model ($\mathcal{P}^w$)

The formulation with one stockpile and homogeneous mixing requires different variable definitions but follows the same logic as the formulation of the Basic Model ($\mathcal{P}^b$). The new variables (defined in Section 2.3.1.) express stockpile amounts in terms of fractions of the block instead of in terms of tons, which is necessary to track the grade of each block going to the stockpile. The new objective function is:

$$(\mathcal{P}^w) : \max \sum_{t \in \mathcal{T}} \delta_t \left[ P \left( \sum_{b \in \mathcal{B}} M_b (y^p_{bt} + z^p_{bt}) \right) - C^p \left( \sum_{b \in \mathcal{B}} W_b (y^p_{bt} + z^p_{bt}) \right) \right.$$
$$\left. - C^m \left( \sum_{b \in \mathcal{B}} W_b y^m_{bt} \right) - C^h \left( \sum_{b \in \mathcal{B}} W_b z^p_{bt} \right) \right] \quad (2.14)$$

subject to:

$$y^p_{bt} + y^w_{bt} + y^s_{bt} = y^m_{bt} \qquad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \qquad \text{(2.8 revisited)}$$

$$\sum_{t \in \mathcal{T}} y^m_{bt} \leq 1 \qquad \forall b \in \mathcal{B} \qquad \text{(2.3 revisited)}$$

$$x_{bt} \leq \sum_{t' \leq t} y^m_{bt'} \qquad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \qquad \text{(2.4 revisited)}$$

$$\sum_{t' \leq t} y^m_{bt'} \leq x_{\hat{b}t} \qquad \forall b \in \mathcal{B}, \hat{b} \in \hat{\mathcal{B}}_b, t \in \mathcal{T} \qquad \text{(2.5 revisited)}$$

$$z^s_{bt} = \begin{cases} y^s_{bt} & t = 1 \\ z^s_{b,t-1} + y^s_{bt} - z^p_{bt} & t \in \mathcal{T} : t \geq 2 \end{cases} \qquad \forall b \in \mathcal{B}, t \in \mathcal{T} \qquad (2.15)$$

$$\frac{z_{bt}^p}{z_{bt}^p + z_{bt}^s} = f_t \qquad \forall b \in \mathcal{B}, t \in \mathcal{T} \tag{2.16}$$

$$(x, y) \in \Omega \qquad \text{(other constraints)} \tag{2.6 revisited}$$

As before, constraint (2.8) forces the material sent directly to the mill, to the stockpile, or to waste to be equal to the quantity of extracted material. The following constraints duplicate those in ($\mathcal{P}^{ns}$): Constraint (2.3) ensures that extracted fractions of each block summed across all time periods must be less than or equal to one. Constraint (2.4) forces the sum of the fractional variables to 1 by time $t$ if the block has been mined by that time. Constraint (2.5) enforces mining precedence constraints by ensuring that for each block, all predecessors are completely mined before any amount of the successor block is mined. Constraints specific to ($\mathcal{P}^w$) include: Constraint (2.15) indicates that the fraction of block $b$ remaining in the stockpile at period $t$ will be the remaining fraction from the previous period, plus the fraction of $b$ extracted at period $t$ and sent to the stockpile, minus the fraction of $b$ sent from the stockpile to mill at period $t$. Constraint (2.16), which introduces a new variable, $f_t$, requires that the relative proportion of each block in the stockpile that is processed is the same for all blocks in each time period. As in ($\mathcal{P}^{ns}$), constraint (2.6) might represent geometrical and operational restrictions (e.g., block-level or bench-phase scheduling, mining and processing bounds, blending constraints, and/or the maximum number of phases opened), and could involve bound and integrality constraints on $x$ and $y$. Bley et al. (2012) prove that ($\mathcal{P}^b$) and ($\mathcal{P}^w$) are equivalent.

### 2.4 Approximate Linear Models

In Sections 2.4.1-2.5, we formalize results from models in the literature, i.e., Akaike and Dagdelen [26], Hoerger et al. [27] and Tabesh et al. [23], respectively. In the latter case, the authors present a model that is similar to $K$-bucket (see Section 2.5), in which the authors categorize the possible grades in the buckets; there are, however, three differences: (i) they define a lower and an upper bound for the *average* grade sent to each bucket in each period, and (ii) they assess an "output grade" from each bucket, which does not necessarily correspond to the upper or to the lower bound, and (iii) there is no linking constraint between the buckets. The model we present in Section 2.6 is new. We conclude Section 2.4 with a summary of our models and an example of where they appear seminally in the literature.

### 2.4.1 Upper bound model ($\mathcal{P}^{ub}$)

A model that provides an upper bound on ($\mathcal{P}^{w}$) can be obtained by removing the nonlinear constraint (2.16) such that each block can be sent (maintaining its original characteristics) from the stockpile to the processing plant. The solution of this upper bound model, ($\mathcal{P}^{ub}$), is generally infeasible for the basic and warehouse models. A different upper bound can be obtained by removing non-linear constraints (2.13) from ($\mathcal{P}^{b}$); however, Bley et al. [25] show that the corresponding upper bound is never better than that provided by ($\mathcal{P}^{ub}$).

### 2.4.2 L-bound model ($\mathcal{P}^{lb}$)

We can assume that the stockpile has a pre-defined output grade, denoted by $L$. In order to obtain a feasible solution for ($\mathcal{P}^{b}$) or ($\mathcal{P}^{w}$), only blocks with grade greater than or equal to $L$ may be sent to the stockpile. Hence, we replace constraint (2.13) in ($\mathcal{P}^{b}$) by the following constraints:

$$m_t^p = L \cdot i_t^p \qquad \forall t \in \mathcal{T} \tag{2.17}$$

$$y_{bt}^s = 0 \qquad \forall b \in \mathcal{B} \text{ such that } \frac{M_b}{W_b} < L \qquad \forall t \in \mathcal{T} \tag{2.18}$$

**Lemma 1.** Let $v^b$ be the optimal objective function value of the nonlinear model ($\mathcal{P}^{b}$), and $v^{lb}$ be that of model ($\mathcal{P}^{lb}$); then $v^{lb} \leq v^b$.

*Proof.* We will show that the optimal solution of ($\mathcal{P}^{lb}$) is feasible for ($\mathcal{P}^{b}$) to demonstrate that an optimal solution for the latter problem must be at least as good as that for the former. The objective functions of the two models are the same, and all constraints are the same with the exception of (2.13) (see Table 2.1). So, we only need to prove that a solution satisfying (2.17) and (2.18) satisfies (2.13):

Constraints (2.11) and (2.12), which are contained in ($\mathcal{P}^{lb}$), can be rewritten as:

$$i_t^s = \sum_{t' \leq t} \sum_{b \in \mathcal{B}} W_b y_{bt'}^s - \sum_{t' < t} i_{t'}^p \qquad t \in \mathcal{T} \tag{2.19}$$

$$m_t^s = \sum_{t' \leq t} \sum_{b \in \mathcal{B}} M_b y_{bt'}^s - \sum_{t' < t} m_{t'}^p \qquad t \in \mathcal{T} \tag{2.20}$$

by cumulating on $t$. (Note that the case in which $t = 1$ is addressed by the loose inequality for the summation on time in the first term and the corresponding strict inequality in the second for both expressions: $i_t^s$ and $m_t^s$.)

By the contrapositive, constraint (2.18) implies that if $y_{bt}^s > 0$ then $M_b \geq L \cdot W_b$. If we multiply equation (2.19) by $L$ and compare it to equation (2.20), we see from (2.17) that the second term on the right hand side of both equations is the same, while the first term is larger in (2.20), because the condition stated in (2.18) does not hold. This implies that the left hand side of (2.20) is larger than that of (2.19) (with the left hand side multiplied by $L$). This yields that $m_t^s \geq L \cdot i_t^s \Rightarrow \frac{m_t^s}{i_t^s} \geq L \quad \forall t \in \mathcal{T}$. Since $L = \frac{m_t^p}{i_t^p}$ from (2.17), this proves that constraint (2.13) holds. $\square$

## 2.5 K-bucket model ($\mathcal{P}^{kb}$)

The $L$-bound model can be too conservative, because blocks sent from the stockpile with a grade greater than $L$ are undervalued, and blocks with a grade lower than $L$ cannot be sent to the stockpile to make up for this undervaluation. A better lower bound can be obtained assuming that we have several *buckets* of different grades. That is, we define $K$ buckets, each of them with an associated minimum required grade $\hat{L}_k$, such that $\hat{L}_k \leq \hat{L}_{k+1}$ for all $k = 1 \ldots K-1$. Hence, a block that is sent to the stockpile can go to any bucket if it has the minimum required grade. We replace variables $y_{bt}^s$ by variables $\bar{y}_{bkt}^s$, representing the fraction of each block sent to the $k$th-bucket of the stockpile. Then, constraint (2.8) is replaced by:

$$y_{bt}^p + \sum_{k \in \mathcal{K}} \bar{y}_{bkt}^s + y_{bt}^w = y_{bt}^m \qquad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \tag{2.21}$$

Also, we track the material in each bucket by using variables $\bar{\imath}^s_{kt}$ and $\bar{\imath}^p_{kt}$ for each bucket $k$, replacing constraints (2.9) and (2.11) by

$$\bar{\imath}^p_{kt} \leq \bar{\imath}^s_{k,t-1} \qquad \forall t \in \mathcal{T} : t \geq 2, \forall k \in \mathcal{K} \tag{2.22}$$

$$\bar{\imath}^s_{kt} = \begin{cases} \sum_{b\in\mathcal{B}} W_b \bar{y}^s_{bkt} & t = 1 \\ \bar{\imath}^s_{k,t-1} - \bar{\imath}^p_{kt} + \sum_{b\in\mathcal{B}} W_b \bar{y}^s_{bkt} & \forall t \in \mathcal{T} : t \geq 2 \end{cases}, \forall k \in \mathcal{K} \tag{2.23}$$

and, as in the $L$-bound model ($\mathcal{P}^{lb}$), we replace constraint (2.13) by:

$$m^p_t = \sum_{k\in\mathcal{K}} \hat{L}_k \cdot \bar{\imath}^p_{kt} \qquad \forall t \in \mathcal{T} \tag{2.24}$$

$$\bar{y}^s_{bkt} = 0 \qquad \forall b \in \mathcal{B} \text{ such that } \frac{M_b}{W_b} < \hat{L}_k, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{2.25}$$

where $\hat{L}_k$ is the pre-defined output grade of bucket $k$. Note that with constraints (2.21)-(2.25) (see Table 2.1), the model is not a lower bound nor an upper bound of the nonlinear models ($\mathcal{P}^b$) and ($\mathcal{P}^w$). In fact, for $K = 1$, we recover the $L$-bound model with $L = \hat{L}_1$ because we have just one bucket in the stockpile; for $K = B$ and $\hat{L}_k = \frac{M_k}{W_k}$ for all $k = 1 \ldots K$, we recover the upper bound model because we have as many buckets as we have blocks. In order to obtain a lower bound on the objective function of the nonlinear model ($\mathcal{P}^b$), we must add these constraints:

$$\bar{\imath}^p_{kt} = \bar{\imath}^p_{k't} \qquad \forall k, k' \in \mathcal{K}, \forall t \in \mathcal{T} \tag{2.26}$$

$$\sum_{b\in\mathcal{B}} W_b \bar{y}^s_{bkt} = \sum_{b\in\mathcal{B}} W_b \bar{y}^s_{bk't} \qquad \forall k, k' \in \mathcal{K}, \forall t \in \mathcal{T} \tag{2.27}$$

**Lemma 2.** Let $v^b$ be the optimal objective function value of the nonlinear model ($\mathcal{P}^b$), and $v^{kb}$ be that of model ($\mathcal{P}^{kb}$), then $v^{kb} \leq v^b$.

*Proof.* We show that from the optimal solution of ($\mathcal{P}^{kb}$) we can construct a feasible solution for ($\mathcal{P}^b$). Define $i^p_t = \sum_{k=1}^K \bar{\imath}^p_{kt}$, $i^s_t = \sum_{k=1}^K \bar{\imath}^s_{kt}$ and $y^s_{bt} = \sum_{k=1}^K \bar{y}^s_{bkt}$. Constraint (2.2) is satisfied owing to the revision expressed in (2.21); constraints (2.9) and (2.11) are satisfied based on our variable definitions in which we cumulate over $k$. From constraints (2.24), we have $\forall t \in \mathcal{T}$:

$$\begin{aligned}
m_t^p &= \sum_{k=1}^{K} \hat{L}_k \cdot \bar{\imath}_{kt}^p \\
&= \left( \sum_{k=1}^{K} \hat{L}_k \right) \cdot \bar{\imath}_{1t}^p \\
&= \left( \frac{1}{K} \sum_{k=1}^{K} \hat{L}_k \right) \cdot (K \cdot \bar{\imath}_{1t}^p) \\
&= \left( \frac{1}{K} \sum_{k=1}^{K} \hat{L}_k \right) \cdot i_t^p
\end{aligned}$$

where the last implication follows from the definition of $i_t^p$ at the beginning of the proof and equation (2.26). Hence, as in the proof of Lemma 1, this equality implies that

$$\begin{aligned}
m_t^s &= \sum_{b \in \mathcal{B}} \sum_{t' \leq t} M_b y_{bt'}^s - \sum_{t' < t} m_{t'}^p \\
&= \sum_{b \in \mathcal{B}} \sum_{k=1}^{K} \sum_{t' \leq t} M_b \bar{y}_{bkt'}^s - \left( \frac{1}{K} \sum_{k=1}^{K} \hat{L}_k \right) \sum_{t' < t} i_{t'}^p \\
&\geq \sum_{b \in \mathcal{B}} \sum_{k=1}^{K} \sum_{t' \leq t} \hat{L}_k W_b \bar{y}_{bkt'}^s - \left( \frac{1}{K} \sum_{k=1}^{K} \hat{L}_k \right) \sum_{t' < t} i_{t'}^p \quad \text{(by} \\
&= \sum_{k=1}^{K} \hat{L}_k \sum_{b \in \mathcal{B}} \sum_{t' \leq t} W_b \bar{y}_{bkt'}^s - \left( \frac{1}{K} \sum_{k=1}^{K} \hat{L}_k \right) \sum_{t' < t} i_{t'}^p \\
&= \left( \sum_{k=1}^{K} \hat{L}_k \right) \cdot \left( \sum_{b \in \mathcal{B}} \sum_{t' \leq t} W_b \bar{y}_{b1t'}^s \right) - \left( \frac{1}{K} \sum_{k=1}^{K} \hat{L}_k \right) \sum_{t' < t} i_{t'}^p \\
&= \left( \frac{1}{K} \sum_{k=1}^{K} \hat{L}_k \right) \cdot \left( K \cdot \sum_{b \in \mathcal{B}} \sum_{t' \leq t} W_b \bar{y}_{b1t'}^s \right) - \left( \frac{1}{K} \sum_{k=1}^{K} \hat{L}_k \right) \sum_{t' < t} i_{t'}^p \\
&= \left( \frac{1}{K} \sum_{k=1}^{K} \hat{L}_k \right) \cdot \sum_{b \in \mathcal{B}} \sum_{t' \leq t} W_b y_{bt'}^s - \left( \frac{1}{K} \sum_{k=1}^{K} \hat{L}_k \right) \sum_{t' < t} i_{t'}^p \\
&= \left( \frac{1}{K} \sum_{k=1}^{K} \hat{L}_k \right) \cdot \left( \sum_{b \in \mathcal{B}} \sum_{t' \leq t} W_b y_{bt'}^s - \sum_{t' < t} i_{t'}^p \right) \\
&= \left( \frac{1}{K} \sum_{k=1}^{K} \hat{L}_k \right) i_t^s
\end{aligned}$$

proving the result. $\square$

### 2.6 L-average bound model ($\mathcal{P}^{la}$)

A novel way to approximate the nonlinear models similar to the $L$-bound is to fix the grade of the material leaving the stockpile to a fixed value $L$, but instead of requiring each block to have a grade greater than or equal to $L$, this model requires all the blocks that are going into the stockpile to have a grade greater than or equal to $L$ "on average." Formally, we replace constraint (2.13) with the following constraints:

$$m_t^p = L \cdot i_t^p \qquad \forall t \in \mathcal{T} \tag{2.28}$$

$$L \cdot \sum_{b \in \mathcal{B}} \sum_{t' < t} W_b y_{bt'}^s \leq \sum_{b \in \mathcal{B}} \sum_{t' < t} M_b y_{bt'}^s \qquad \forall t \in \mathcal{T}. \tag{2.29}$$

**Lemma 3.** Let $v^b$ be the optimal objective function value of nonlinear model ($\mathcal{P}^b$), and $v^{la}$ be that value of model ($\mathcal{P}^{la}$), then $v^{la} \leq v^b$.

*Proof.* Let us reconsider the definitions of the variables $i_t^s$ and $m_t^s$ as given in constraints (2.19) and (2.20). We can multiply constraint (2.19) by $L$ and use the following version of constraint (2.28): $\sum_{t'<t} m_{t'}^p = L \cdot \sum_{t'<t} i_{t'}^p$ to cancel the last term in (2.20) and the last term in (2.19) multiplied by $L$. Constraint (2.29) implies that the first term on the right hand side of (2.20) is greater than that same term in (2.19) multiplied by $L$. We can then compare left-hand sides of these same two equations to conclude that $m_t^s \geq L \cdot i_t^s$ for all $t \in \mathcal{T}$, proving that constraint (2.13) holds (because $L = \frac{m_t^p}{i_t^p}$ by constraint (2.28)). $\square$

Moreover, we can prove that this model provides a better approximation to the nonlinear model ($\mathcal{P}^w$).

**Lemma 4.** Let $v^{lb}$, $v^{kb}$ and $v^{la}$ be the optimal objective function values of models ($\mathcal{P}^{lb}$), ($\mathcal{P}^{kb}$) and ($\mathcal{P}^{la}$), respectively. For the best possible value of parameters $L^{lb}$, $\hat{L}_k^{kb}$ and $L^{la}$, respectively, the following inequalities hold:

$$v^{lb} \leq v^{kb} \leq v^{la}$$

*Proof.* The first inequality holds because for any value of $L^{lb}$ for $(\mathcal{P}^{lb})$, we can define two buckets, one with grade $\hat{L}_1 = L^{lb}$ and a second with grade $\hat{L}_2 \geq L^{lb}$. The optimal solution of model $(\mathcal{P}^{lb})$ can be reassigned to this 2-bucket model with a profit at least equal to $v^{lb}$.

For the second inequality, given an optimal solution of $(\mathcal{P}^{kb})$ with variables $\bar{y}^s_{bkt}$ representing the material in each bucket, if we define $y^s_{bt} = \sum_{k=1}^{K} \bar{y}^s_{bkt}$ and $\bar{L} = \frac{1}{K}\sum_{k=1}^{K} \hat{L}_k$, then $y^s_{bt}$ satisfies constraint (2.29) for grade $L = \bar{L}$. In fact, by (2.25), we have that if $\bar{y}^s_{bkt} > 0$ then $M_b \geq \hat{L}_k \cdot W_b$. Hence,

$$
\begin{aligned}
\sum_{b\in\mathcal{B}} M_b y^s_{bt} = \sum_{b\in\mathcal{B}}\sum_{k=1}^{K} M_b \bar{y}^s_{bkt} &\geq \sum_{b\in\mathcal{B}}\sum_{k=1}^{K} \hat{L}_k W_b \bar{y}^s_{bkt} \\
&= \left(\sum_{k=1}^{K} \hat{L}_k\right)\cdot\left(\sum_{b\in\mathcal{B}} W_b \bar{y}^s_{b1t}\right) \\
&= \left(\frac{1}{K}\sum_{k=1}^{K} \hat{L}_k\right)\cdot\left(K\cdot\sum_{b\in\mathcal{B}} W_b \bar{y}^s_{b1t}\right) \\
&= \bar{L}\cdot\sum_{b\in\mathcal{B}} W_b y^s_{bt}
\end{aligned}
$$

In other words, we can construct an equivalent feasible solution for model $(\mathcal{P}^{la})$ using $L = \bar{L}$ with the same objective, proving the result. □

## 2.7 Summary of All Models

This section provides a summary of all models, listing the variables, the objective function and constraints associated with each model (see Table 2.1).

## 2.8 Graphical Representations

We can assume that in a mine, some material is sent to the stockpile in the first time period and is processed at the mill in the second period. Since profit per ton is a linear function of the grade, we assume that the "grade" $g$ of the material is defined by units of profit per ton. We can represent the total tonnage sent to the stockpile with grade greater than or equal to $g$ using a function $G(g)$. An illustrative example of this function appears

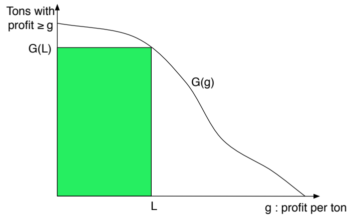
Table 2.1: Summary of principal open-pit mine scheduling models that include inventory; we provide: (i) our naming convention for each model, (ii) the section of this paper in which we introduce it, (iii) the associated variables, objective function and constraints, and (iv) an example of related, seminal work that uses such a model or a close approximation thereof.

| Model | Section | Variables | Objective | Constraints | Related seminal reference |
|---|---|---|---|---|---|
| $(\mathcal{P}^{ns})$ | 2.2.2 | $x_{bt}, y^m_{bt}, y^p_{bt}, y^w_{bt}$ | (2.1) | (2.2)-(2.6) | Johnson [1] |
| $(\mathcal{P}^{b})$ | 2.3.2 | $x_{bt}, y^p_{bt}, y^w_{bt}, y^s_{bt}, y^m_{bt}$ $i^s_t, i^p_t, m^s_t, m^p_t$ | (2.7) | (2.3)-(2.6), (2.8)-(2.13) | Bley et al. [25] |
| $(\mathcal{P}^{w})$ | 2.3.3 | $x_{bt}, y^m_{bt}, y^p_{bt}, y^w_{bt}$ $y^s_{bt}, z^p_{bt}, z^s_{bt}, f_t$ | (2.14) | (2.3)-(2.6), (2.8), (2.15), (2.16) | Bley et al. [25] |
| $(\mathcal{P}^{ub})$ | 2.4.1 | $x_{bt}, y^m_{bt}, y^p_{bt}, y^w_{bt}$ $y^s_{bt}, z^p_{bt}, z^s_{bt}$ | (2.14) | (2.3)-(2.6), (2.8), (2.15) | Akaike and Dagdelen [26] |
| $(\mathcal{P}^{lb})$ | 2.4.2 | $x_{bt}, y^m_{bt}, y^p_{bt}, y^w_{bt}, y^s_{bt}$ $i^p_t, m^p_t, i^s_t, m^s_t$ | (2.7) | (2.3)-(2.6), (2.8)-(2.12), (2.17), (2.18) | Hoerger et al. [27] |
| $(\mathcal{P}^{kb})$ | 2.5 | $x_{bt}, y^m_{bt}, y^p_{bt}, y^w_{bt}$ $\bar{y}^s_{bkt}, \bar{\imath}^p_{kt}, m^p_t, \bar{\imath}^s_{kt}, m^s_t$ | (2.7) | (2.3)-(2.6), (2.10), (2.12), (2.21)-(2.27) | Hoerger et al. [27] Tabesh et al. [23] |
| $(\mathcal{P}^{la})$ | 2.6 | $x_{bt}, y^m_{bt}, y^p_{bt}, y^w_{bt}, y^s_{bt}$ $i^p_t, m^p_t, i^s_t, m^s_t$ | (2.7) | (2.3)-(2.6), (2.8)-(2.12), (2.28), (2.29) | New model |

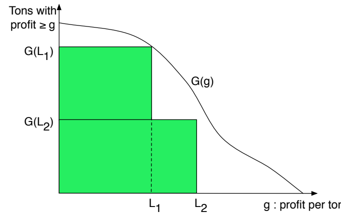
in Figure 2.2. Note that if all of this material is sent to mill, the total profit recovered from the stockpile is equivalent to the area below $G(g)$.

In the case of the $L$-bound model, the total profit obtained from the stockpile is $L \cdot G(L)$, equivalent to the area of a rectangle below the curve (see Figure 2.2(a)). Material with grade less than $L$ cannot be sent to the stockpile in this model, and the material with grade at least $L$ is extracted with a grade equal to $L$.

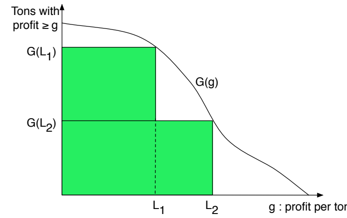
Figure 2.2(b) represents, for the $K$-bucket model, the specific case of two buckets with grade $L_1$ and $L_2$, where $G(L_2)$ tons are extracted at profit $L_2$, and $G(L_1) - G(L_2)$ tons are extracted at grade $L_1$; we obtain a higher profit in this case than for the $L$-bound model, because the grade attributed to the material in the stockpile is more precisely matched with the true grade, and more material overall is allowed to be stockpiled. Note that constraint (2.26) requires that $G(L_1) - G(L_2) = G(L_2)$. Figure 2.2(c) shows a selection for grade $L_2$ that improves the value of the material processed from the stockpile. Hence, the best selection of grades $L_k$ for this example must satisfy the condition that $G(L_k) - G(L_{k+1}) = G(L_K)$ for $k = 1, \ldots, K-1$. Figure 2.2(d) shows an example with five buckets of equal tonnage. When



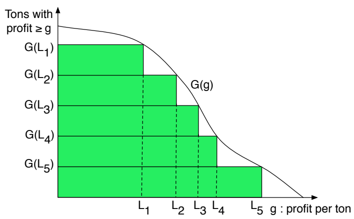

(a) $L$-bound model



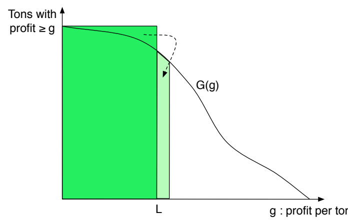

(b) 2-bucket model (case a)



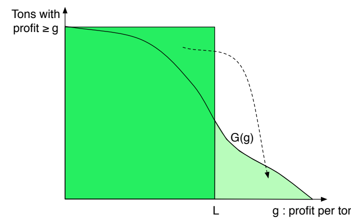

(c) 2-bucket model (case b)




(d) 5-bucket model




(e) $L$-avg model (case a)




(f) $L$-avg model (case b)

Figure 2.2: Graphical examples of $L$-bound, $K$-bucket and $L$-average models. The green area shows the obtained profit by processing the material in the stockpile. The light green color shows the profit above $G(L)$ which compensates for the profit that was not obtained below $G(L)$.

$K$ increases, a higher fraction of the total profit can be recovered from the stockpile.

Figure 2.2(e) shows an example of the $L$-average model in which we can extract more than $G(L)$ tons with grade $L$. This is possible because the green area above the curve is compensated for by the profit of material with a grade greater than $L$ (light green region). The figure shows that we can increase the value of $L$, allowing us to obtain a higher profit by extracting the same material. In fact, there exists a value $L^*$ such that we gain the maximal profit, equivalent to the area below $G(L)$ (see Figure 2.2(f)).

### 2.9 Computational Experiments

In this section, we examine the solution quality associated with different linear-integer and nonlinear-integer models. In Section 2.9.1, we compare the proposed models to the nonlinear model. This requires a reduction in problem size, accomplished by fixing the block extraction time in all models. In Section 2.9.2, we compare two linear-integer models using a customized solver called OMP [28] without fixing the block extraction time. Unless otherwise stated, we perform all computation on a Dell R620 with eight Xeon E5-2670 2.0 Ghz cores and 128 GB RAM.

### 2.9.1 Comparing different linear models to the nonlinear model

Our first computational experiment compares the quality of our linear-integer models against that of the nonlinear models presented by Bley et al. [25]. We coded the models in AMPL [29] and solved models ($\mathcal{P}^{ns}$), ($\mathcal{P}^{ub}$), ($\mathcal{P}^{la}$), ($\mathcal{P}^{kb}$), and ($\mathcal{P}^{lb}$) using CPLEX [30]. Nonlinear models ($\mathcal{P}^{b}$) and ($\mathcal{P}^{w}$) were solved using SCIP 3.1.0 [31] with CPLEX 12.6 as the linear solver.

We use six instances: *newman1, marvin, sm2, zuck-small, zuck-medium*, and *zuck-large*, available on the Minelib website [32]. Table 2.2 presents the unique characteristics of these instances, which include two capacity constraints for each time period: one for the total mined material, and a second for the total processed material; the latter restriction makes inventory relevant. We add one stockpile using a rehandling cost equal to 10% of the original mining cost. Standard solvers are not able to produce optimal solutions for such large instances, even without stockpiles. Hence, in order to compare solver performance more precisely, i.e., through an optimal objective function value, we simplify our instances by fixing the extraction time period (i.e., the $x$-variables) in all models to that of the best-known solution for each instance presented on the Minelib website. Note that resulting problems have only continuous variables. We later relax this assumption.

Table 2.2: Problem Instance Characteristics

| Instance | # blocks | # blocks in ultimate pit | Discount Factor (%) | Mining Cap (MT/year) | Mill Cap (MT/year) |
|---|---|---|---|---|---|
| newman1 | 1,060 | 1,059 | 8 | 2 | 1.1 |
| marvin | 53,271 | 8,516 | 10 | 60 | 20.0 |
| zuck-small | 9,400 | 9,399 | 10 | 60 | 20.0 |
| sm2 | 99,014 | 18,388 | 10 | 12 | 1.4, 1.8, 2.1, 2.2,...,2.2 |
| zuck-medium | 29,277 | 27,387 | 10 | 18 | 8.0 |
| zuck-large | 96,821 | 96,821 | 10 | 3 | 1.2 |

Bley et al. [33] explain that the nonlinear model ($\mathcal{P}^w$) performs considerably better than ($\mathcal{P}^b$), but requires a substantial amount of memory, e.g., we are only able to obtain solutions to ($\mathcal{P}^w$) for instances *newman1*, *zuck-small*, *sm2* and *marvin* due to memory requirements; the latter instance requires more than 100 GB of RAM to obtain a solution within a 0.13% optimality gap after two weeks of run time.

On the contrary, we found the optimal solution using each of the linear models ($\mathcal{P}^{lb}$), ($\mathcal{P}^{kb}$) and ($\mathcal{P}^{la}$) in a few seconds of run time for all instances. For models ($\mathcal{P}^{la}$) and ($\mathcal{P}^{lb}$), we tested several values of $L$ with a view to improving the objective function for which our numerical experiments indicate unimodality in $L$, enabling us to perform a simple line search for its optimal value corresponding to each model and instance. In the case of ($\mathcal{P}^{kb}$), for ease of comparison, we consider four buckets for all instances and explore several bucket $k$ grade values $L_k$ under the assumption that by increasing the number of buckets in the stockpile, the objective function value of ($\mathcal{P}^{kb}$) approaches the upper bound of the problem given by ($\mathcal{P}^{ub}$).

Table 2.3 displays the resulting NPV, normalized to that of the ($\mathcal{P}^{ns}$) model. The objective function values corresponding to all models for the *newman1* instance are the same because we have extra mill capacity and therefore there is no incentive to stockpile. The differences in objective function values between the most extreme models, models ($\mathcal{P}^{ns}$) and ($\mathcal{P}^{ub}$), for the *marvin* and *zuck-small* instances (i.e., those that benefit most from stockpiling) are 2.07% and 1.95%, respectively. For the *newman1*, *marvin*, *sm2* and *zuck-small* instances, that difference between ($\mathcal{P}^w$) and ($\mathcal{P}^{la}$), the theoretical nonlinear "mixing model" and our closest approximation to it, is less than 0.17%, and the difference between ($\mathcal{P}^w$) and ($\mathcal{P}^{kb}$) is less than 0.7%. In other words, the ($\mathcal{P}^{la}$) model provides a very close approximation to the objective function given by ($\mathcal{P}^w$).

A comparison of the objective function values from the models whose instances can all be solved in our numerical experiments indicate a difference between ($\mathcal{P}^{ub}$) and ($\mathcal{P}^{kb}$) of less than 0.97%, and a difference between ($\mathcal{P}^{ub}$) and ($\mathcal{P}^{la}$) of less than 0.42%. In other words, these linear models approximate the nonlinear model very well, provide solutions with corresponding objective function values that are close to the theoretical optimum, and are much more tractable.

Changing the maximum mill production capacity illustrates that there is some trade-off between stockpiling and this parameter. In order to better demonstrate the difference between our proposed models, we decrease the milling capacity relative to the mining capacity, which increases the value of the stockpile, especially with a fixed extraction sequence, and a corresponding relative increase in the amount of material left on the stockpile relative to what is extracted. Table 2.3 illustrates the trade-off between stockpiling and mill capacity for all instances. Decreasing mill capacity results in as much as 20% value added at half of the original for the *newman1* instance. Relative to each other, the models perform similarly; the difference in objective function value between ($\mathcal{P}^{w}$) and ($\mathcal{P}^{la}$) is less than 0.03% for the three instances that we were able to solve.

Table 2.3: NPV normalized to ($\mathcal{P}^{ns}$) model

| Instance | Original mill capacity | | | | | 50% of original mill capacity | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | ($\mathcal{P}^{ub}$) | ($\mathcal{P}^{w}$) | ($\mathcal{P}^{la}$) | ($\mathcal{P}^{kb}$) | ($\mathcal{P}^{lb}$) | ($\mathcal{P}^{ub}$) | ($\mathcal{P}^{w}$) | ($\mathcal{P}^{la}$) | ($\mathcal{P}^{kb}$) | ($\mathcal{P}^{lb}$) |
| newman1 | 1 | 1 | 1 | 1 | 1 | 1.1957 | 1.1858 | 1.1857 | 1.1145 | 1.0959 |
| marvin | 1.0212 | 1.0170 | 1.0169 | 1.0114 | 1.0081 | 1.2823 | 1.2246 | 1.2226 | 1.1647 | 1.1420 |
| zuck-small | 1.0199 | 1.0157 | 1.0157 | 1.0104 | 1.0073 | 1.2735 | 1.2165 | 1.2132 | 1.1548 | 1.1342 |
| sm2 | 1.0025 | 1.0019 | 1.0016 | 1.0011 | 1.0005 | 1.2589 | - | 1.1406 | 1.0749 | 1.0693 |
| zuck-medium | 1.0159 | - | 1.0126 | 1.0086 | 1.0062 | 1.3237 | - | 1.2429 | 1.1608 | 1.1451 |
| zuck-large | 1.0061 | - | 1.0047 | 1.0031 | 1.0029 | 1.1883 | - | 1.1262 | 1.1013 | 1.0935 |

### 2.9.2 Comparing linear-integer models considering the extraction sequence

Our set of computational experiments in §2.9.1 shows that ($\mathcal{P}^{la}$) provides a very close objective function value to that of ($\mathcal{P}^{w}$) for a model with a predefined extraction sequence. In this subsection, we first compare the quality of the *LP relaxation* provided by ($\mathcal{P}^{ns}$), ($\mathcal{P}^{ub}$), and ($\mathcal{P}^{la}$) without fixing the extraction time. Note that state-of-the art solvers based on the Bienstock-Zuckerberg algorithm only solve the corresponding linear program of our linear-integer production scheduling problems in an exact way, and then apply other techniques that use this relaxed solution to generate a near-optimal integer solution, e.g., the academic solver OMP. The nonlinear model cannot be solved with the current state-of-the-art algorithms, e.g., SCIP and BARON, even with the size of machine we use. Therefore, we focus on the difference between the LP-relaxation of our $L$-average and the upper bound models, because they can be computed in an exact way.

Table 2.4 displays the resulting NPV, normalized to that of the no-stockpile model. The time required by OMP to solve these problems varies from a few seconds (*newman1*) to 19 minutes (*zuck_large*). Because of the limitations of the OMP solver, and for the sake of consistency between models, for these instances, we do not consider rehandling costs. The upper bound model and the solution provided by the $L$-average model differ in objective function value by less than 0.9%, on average, with a difference for the *zuck-small* instance of 1.4%. These ranges show that the $L$-average model provides upper bounds that are a good approximation to those provided by the nonlinear model for the more general case in which we include extraction decisions.

Table 2.4: NPV normalized to $(\mathcal{P}^{ns})$ for the original mill capacity case without fixing the extraction time, and solved with OMP

| Instance | $(\mathcal{P}^{ub})$ | $(\mathcal{P}^{la})$ |
|---|---|---|
| newman1 | 1 | 1 |
| marvin | 1.0500 | 1.0365 |
| zuck-small | 1.0516 | 1.0377 |
| sm2 | 1.0087 | 1.0047 |
| zuck-medium | 1.0489 | 1.0373 |
| zuck-large | 1.0108 | 1.0091 |

Finally, we show that standard rounding techniques, e.g., TopoSort [7], still provide good integrality gaps for the mixed-integer version of the $(\mathcal{P}^{la})$ model. Table 2.5 shows the objective value of the LP relaxation compared with that from the integer solution obtained by running TopoSort considering a stockpile, i.e., using model $(\mathcal{P}^{la})$ and without considering a stockpile,i.e., using model $(\mathcal{P}^{ns})$. Standard rounding heuristics yield near-optimal solutions for both models, demonstrating that the additional variables and constraints required to model a stockpile do not loosen the LP relaxation, a crucial property for solving large-scale instances of the problem. In fact, this is not true for the ($\mathcal{P}^{ub}$) model.

Table 2.5: Objective function values of the LP relaxation (LP), the integer program obtained via TopoSort (IP), and the corresponding integrality gaps (Int. Gap) obtained with OMP for our best stockpiling model, ($\mathcal{P}^{la}$), and the corresponding model with no stockpiling, ($\mathcal{P}^{ns}$)

| Instance | LP ($\mathcal{P}^{la}$) | IP ($\mathcal{P}^{la}$) | Int. Gap | LP ($\mathcal{P}^{ns}$) | IP ($\mathcal{P}^{ns}$) | Int. Gap |
|---|---|---|---|---|---|---|
| newman1 | 24.486 | 24.486 | 0.000% | 24.486 | 24.486 | 0.000% |
| marvin | 944.767 | 935.024 | 1.031% | 911.480 | 889.729 | 2.386% |
| zuck-small | 939.728 | 927.599 | 1.291% | 905.544 | 879.886 | 2.833% |
| sm2 | 1,660.120 | 1,658.475 | 0.099% | 1,652.393 | 1,650.818 | 0.095% |
| zuck-medium | 776.037 | 718.464 | 7.419% | 748.150 | 710.379 | 5.049% |
| zuck-large | 58.468 | 57.891 | 0.986% | 57.938 | 57.391 | 0.945% |

All the models we present can be easily adapted to the case of scheduling clusters of blocks (e.g., predefined bench-phases, bins or panels) by redefining extraction binary variables $x_{bt}$. Specifically, we can replace these variables by $x_{ct}$ for clusters $c \in C$ of blocks, and modify (2.4) and (2.5) such that these constraints apply to each block in its corresponding cluster. In this way, the number of binary variables can be reduced considerably, making it possible to obtain optimal integer solutions using a Bienstock-Zuckerberg algorithm embedded in a branch-and-bound scheme, e.g., within the spatial branching proposed by Bley et al. [25], enabling us to solve these instances exactly; however, doing so is outside of the scope of this paper.

## 2.10 Conclusion

Considering stockpiling as part of open pit mine planning presents numerous challenges: (i) the most precise model in the literature at the time of this writing is nonlinear and integer, yielding a non-convexity and therefore no guarantee of a global optimum; (ii) nonlinear-integer models are often intractable, especially for realistically sized instances; (iii) even if we obtain a solution for these models, the way in which some assumptions are handled, in particular, that of homogeneous mixing of the material in a single stockpile in each time period, is unrealistic. This paper proposes several variants of linear-integer models that expedite solutions. Computational experiments show that the linear-integer model, $(\mathcal{P}^{la})$, the best for the realistic instances we test, is tractable and possesses an objective function value very close to that of $(\mathcal{P}^{w})$ and $(\mathcal{P}^{b})$, the nonlinear models.

Blending ore with contaminants can be modeled with $(\mathcal{P}^{la})$; in this case, the economical impact of stockpiles could be considerably higher than in our cases. The proposed $L$-average model can easily be extended to consider more than one grade, and to account for degradation in the stockpile. In practice, since model instances solve sufficiently quickly, it is possible to iteratively determine an optimal value of $L$ using a binary search. However, an interesting avenue for future research would integrate the branching scheme proposed by Bley et al. [25] to this end, which would prove to be of particular importance for the trivial extension of the $L$-average model to the case in which a different value of $L$ exists for each time period.

# CHAPTER 3

# PRACTICAL PERFORMANCE OF AN OPEN PIT MINE SCHEDULING MODEL CONSIDERING BLENDING AND STOCKPILING

A paper nearing submission to *Optimization and Engineering*

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Open pit mine production scheduling (OPMPS) is a decision problem which seeks to maximize net present value (NPV) by determining the extraction time of each block of ore and/or waste in a deposit and the destination to which this block must be sent, e.g., a processing plant or waste dump. Spatial precedence constraints are critical, as are resource capacities. Stockpiles can be used to maintain low-grade ore for future processing, to store extracted material until processing capacity is available, or to blend material based on single or multiple block characteristics (i.e., metal grade and/or contaminant).

We modify an existing integer-linear program to maximize NPV and provide a schedule and stockpiling strategy for an operational open pit mine, in which the stockpile is used to blend materials based on multiple block characteristics. We observe that the LP relaxation of our objective function is unimodal for different grade combinations (metals and contaminants) in the stockpile, which allows us to search systematically for an optimal grade combination to fix when solving the integer program; we then use an enhanced rounding heuristic to produce integer solutions with a 19% higher NPV than those produced by state-of the-art mine scheduling software. Furthermore, our proposed model provides schedules for large instances in a few seconds up to a few minutes with significantly different stockpiling and material flow strategies.

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## 3.1 Introduction

Mining is the extraction of economically valuable minerals or materials from the earth. The open pit mine production scheduling (OPMPS) problem consists of (i) identifying a mineralized zone through exploration, which involves drilling and mapping, (ii) developing an ore body model to numerically represent the mineral deposit by dividing the field into three-dimensional rectangular blocks, (iii) assigning attributes such as grades that are estimated by sampling drill cores, and (iv) using these attributes to estimate the economic value of each block, i.e., the difference between the expected revenue from selling the ore and the cost of mining and processing. Given this data, we seek to maximize the net present value (NPV) of the mining operation by determining the extraction time of each block of ore and/or waste in a deposit and the destination to which this block must be sent, e.g., a processing plant or waste dump. Solving the OPMPS problem with annual time fidelity is important in that it determines the rate and quality of production involving large cash flows, which can reach hundreds of millions of dollars.

In order to facilitate solutions for the OPMPS problem, mine planners often first solve the ultimate pit limit problem, which determines the boundary that maximizes undiscounted ore value, and balances the stripping ratio, i.e., the ratio of waste to ore, with the cumulative value of blocks within the pit boundaries. The solution of various instances of the ultimate pit limit problem with different ore prices results in a series of nested pits, pushbacks, or phases. Lerchs and Grossmann [34] provide a tractable approach for solving the ultimate pit limit problem. However, this method only specifies the economic envelope of profitable blocks given pit-slope requirements, ignoring the time aspect of the production scheduling problem and the associated operational resource constraints such as blending limits, bounds on production and processing, inventory constraints, and/or constraints regarding cutoff grade, i.e., the grade that differentiates ore and waste in a deposit.

The OPMPS problem is commonly formulated as an integer program with binary variables representing if and when each block is extracted. Some researchers present nonlinear-integer models to solve open pit mine production scheduling with stockpiling (OPMPS+S) problems. In their models, the authors assume that the material mixes homogeneously in the stockpile and that the grade of material leaving the stockpile is equal to the average grade of all the material within the stockpile, but these models are difficult to solve. Here, we use a linear-integer model to approximate the (OPMPS+S) problem, and instead of computing the average grade in the stockpile, we force the stockpile to have an average grade above a specific limit. We compare the output of this model in terms of a schedule and stockpiling strategy against that provided by state-of-the-art software.

We develop a production schedule for an operational open pit mine with two metal types and a contaminant. Our data show that a considerable number of blocks contain both high metal grade and a high level of contaminant. The processing plant requires us to keep the contaminant level of material below a specific limit. Therefore, blending material in the stockpile based on the metal's grade and contaminant level may result in a higher NPV. In other words, blocks with high metal content and a high contaminant level should be mixed with other material to satisfy the plant requirement; without a good blending strategy, we might lose the value of the aforementioned blocks by sending them to waste. Existing models either do not consider stockpiling as part of OPMPS problem, or just control one grade in the stockpile.

### 3.1.1 Models without a Stockpile

A mine schedule can be optimized with respect to maximizing NVP, maximizing metal content, minimizing mining and processing costs, or minimizing the variance of the grade at the mill. Using optimization techniques for mine production scheduling dates back to 1969 when Johnson proposed the first linear model to maximize NPV of an open pit mine. A challenge to solve an OPMPS problem is that its scale can be very large, since there may be more than one million blocks, many time periods, and several constraints associated with each block-time period combination. Smith [35] develops an aggregation-based method to decrease the number of binary variables in the formulation. In more recent work, Ramazan and Dimitrakopoulos [36] propose handling the large number of integer variables also by means of aggregation. Maximal groups of blocks are created by solving a linear program so that any group has a positive value in total and the blocks forming the boundary of the aggregate obey slope-based constraints. Osanloo et al. [37] review different models and algorithms for long-term open pit mine production planning. Topal and Ramazan [38] propose a network model to efficiently optimize the production schedule of a large operation.

Silva et al. [39] propose a heuristic for stochastic mine production scheduling and apply their approach to a relatively large gold deposit with multiple processing options and a stockpile. Their method finds an initial feasible solution by sequentially solving the stochastic open-pit mine production scheduling problem period-by-period, and then a network-flow algorithm searches for improvements. In their network, the nodes represent blocks, and the goal is to find higher value and lower risk schedules by advancing or postponing the processing of each block. [40] present an integer program for the optimal stope design problem which satisfies geomechanical requirements, and avoids unstable and irregular geometries to find feasible stopes.

### 3.2 Integer-Linear and Nonlinear Models Considering a Stockpile

Jupp et al. [41] explain that there are four different reasons for stockpiling before material processing: buffering, blending, storing, and grade separation. Stockpiles can be used as buffers so that extraction and processing can operate independently, which often provides the economic justification for the expense of stockpiling. Moreover, stockpiles can be used for blending material with different grades to reduce grade variation, which in turn, increases efficiency at the mill. Furthermore, stockpiles can be used to store lower grade material for processing later. Finally, stockpiles can separate different grades of material.

In order for the mill to operate efficiently, the mill feed grade should deviate from a predefined grade as little as possible. Some deposits have more than one element of economic interest, and may contain impurities that have to be minimized in the run-of-mine ore. Stockpiles can be used as part of a blending strategy to ensure constant mill feed grade.

For some mining operations, ore is produced from several pits to provide feed for the mill. Robinson [42] discusses the grade variation reduction by blending material in the stockpile.

Linear and mixed integer programming models have been used significantly to optimize open pit production scheduling by focusing on the extraction sequence. Akaike and Dagdelen [26] propose a model for long-term mine planning considering a stockpile. The authors use a graph theory-based method which considers an infinite number of stockpiles, meaning that every block has its associated stockpile, i.e., there is no blending in the stockpile and the material grade in a stockpile is the same as that of the associated block. Hoerger et al. [27] describe a mixed integer-linear programming model to optimize mine scheduling for Newmont Mining Corporation's Nevada operations. The authors consider multiple stockpiles, each of which has a specific grade range. When the material is removed from the stockpile, its grade is considered to be the minimum of the associated grade range.

Asad [43] presents a cutoff grade optimization algorithm for open pit mining operations with stockpiling in a deposit with two minerals. Ramazan and Dimitrakopoulos [20] consider uncertainty in the geological and economic input data and use a stochastic framework to solve the OPMPS+S problem. The authors do not consider mixing material in the stockpile, meaning that when the material leaves the stockpile, it has the same characteristics as when it enters.

However, a more realistic assumption is that material in the stockpile is mixed homogeneously. In other words, the characteristics (e.g., grade) of the material change by entering the stockpile and should be treated as variables. Bley et al. [25] propose two different non-linear integer models for stockpiling and assume that the grade of the material removed from the stockpile is the weighted average of the material inside the stockpile. Due to the non-linearity of this model, it cannot be used for solving real-sized instances of the problem. Recently, Moreno et al. [44] propose different linear integer models to consider stockpiling in open pit mine scheduling, compare the objective function values of these models, and suggest that their ($\mathcal{P}^{la}$) model provides more accurate solutions in which material with a single grade is mixed in the stockpile. However, a deposit usually contains more than one metal and sometimes a contaminant. The use of stockpiling to blend extracted material in these cases has not been treated in previous linear and mixed integer programming models.

### 3.2.1 Existing Industrial Software

Various industrial software packages facilitate open pit mine planning with stockpiling. Ray [45] explains that the evolution of such software started towards the late 1970s with a focus on operational gold mines in which avoiding any wasteful mining was crucial. He adds that many of the current packages were initiated by existing mining companies or undertaken by universities as research projects. Kaiser et al. [46] also explain that mine planning and design software packages have existed for decades and that their application has greatly improved the mine design quality, as well as the overall economics throughout the mining process.

In the last few decades, an increasing number of international mining companies have used planning software in which historical information is employed to build large databases and models of existing mines. The first direct benefit of such software is simply the usage and validation of largely unutilized information. The 3D modeling capabilities of mine planning software are important in assessing the environmental impact of new deposits. Available mining software covers a large range of capabilities, such as visualization, modeling, database management, reserve calculation, mine design, and scheduling. Mine planning software can contain a number of different algorithms.

While some mining software such as [14] and MineMax [15] provide open pit mine scheduling with stockpiling, this software blends material in the stockpile by considering just one grade. Also, there is no guarantee of obtaining an optimal number of stockpiles and optimal material grade for each stockpile. Moreover, if the material contains more than one metal and/or contaminant, the schedule provided by the software is not guaranteed to be even near-optimal. To obtain the grade of material leaving the stockpile for the mill, Vulcan [47] records what has entered it. MineMarket [48] creates inventory models and catalogs each batch of material as it is delivered to the stockpile and as it leaves it during reclamation, i.e., the process of restoring land that has been mined. Whittle [16] considers mixing material in the stockpile, and the grade of the material sent from the stockpile to the mill can be obtained by accumulating the tonnage and metal of material in it. Although the aforementioned mining software packages consider stockpiling, either the software does not use optimization techniques or it does not blend material in the stockpile with more than one metal grade or contaminant level.

In this research, we modify the model ($\mathcal{P}^{la}$) proposed by Moreno et al. [44] to blend material in the stockpile considering different metal and contaminant grades. Then, we solve this model using a data set from an operational mine, compare its results with those produced by state-of-the-art software, and analyze the difference in metal grade and contaminant level in the stockpile. We show that the capability of blending material with more than one grade heavily influences the determination of the optimal schedule, especially when the material contains a contaminant. Also, we explore the difference in NPV by comparing the schedules between ($\hat{\mathcal{P}}^{la}$) and state-of-the-art-software. Moreover, we show that the LP relaxation of the objective function value is unimodal with respect to blending criteria in the stockpile, which allows us to search systematically for an optimal grade combination for the LP. Finally, we use an enhanced heuristic to create an integer solution from a corresponding LP solution.

We have organized the remainder of this paper as follows. In Section 3.3, we describe the data from an operational mine; in Section 3.4, we explain the modifications we make to ($\mathcal{P}^{la}$) for our data set. In Section 3.5, we compare the result of our model with state-of-the-art software. Section 3.8 concludes that our model provides a better blending strategy which results in a higher NPV compared to that obtained with state-of-the-art software.

### 3.3 Data

Our initial data set is taken from an operational open pit mine in Southeast Asia and consists of 30,100 blocks, four phases, 56 benches, and 16 time periods. A phase corresponds to a sub-region of the pit, and can be obtained by employing the Lerchs-Grossmann (1965) algorithm. A bench is a ledge that forms a single level of operation to extract both mineral and waste materials. Figure 3.1 shows the pit, which includes four phases differentiated by shading.

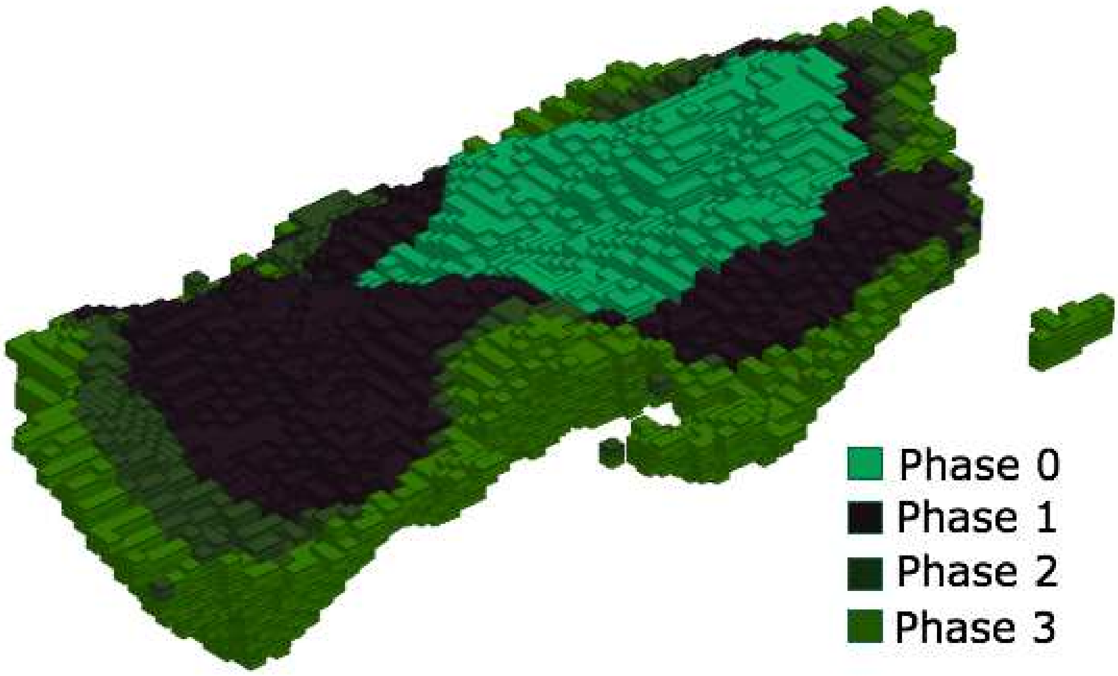



Figure 3.1: The pit contains four phases, differentiated by shading. The center light gray area is Phase 0, the black region is Phase 1, the dark gray region is Phase 2, and the gray perimeter region is Phase 3.

Each block may contain two metals, gold and copper, whose selling prices are assumed as \$20 per gram and \$2.5 per pound, respectively. Copper and gold recovery rates are 60% and 80%, respectively. The mining and processing costs are \$2.5 and \$7.5 per ton, respectively. For the sake of simplicity, we represent the copper and gold contained in each block as a copper-equivalent by jointly considering their selling prices, mining and processing costs, and their recovery rates at the mill.

Blocks also contain arsenic as a contaminant, which should be limited at the mill to 150 particles per million (ppm). In other words, the average grade of arsenic processed in each period should not exceed 150 ppm. Figure 3.2 shows the grade distribution of each block.

In this data set, 24% of the blocks have a copper-equivalent grade greater than zero and an arsenic level that exceeds the limit at the mill. To maximize NPV, we need to process the blocks containing high copper-equivalent as soon as possible, but Figure 3.2 shows that there is a considerable number of blocks whose material includes both high-copper-equivalent grade and a high arsenic level. Here, material blending is very important because, without a good blending strategy, we might send some blocks with high-copper-equivalent grade to the waste dump.

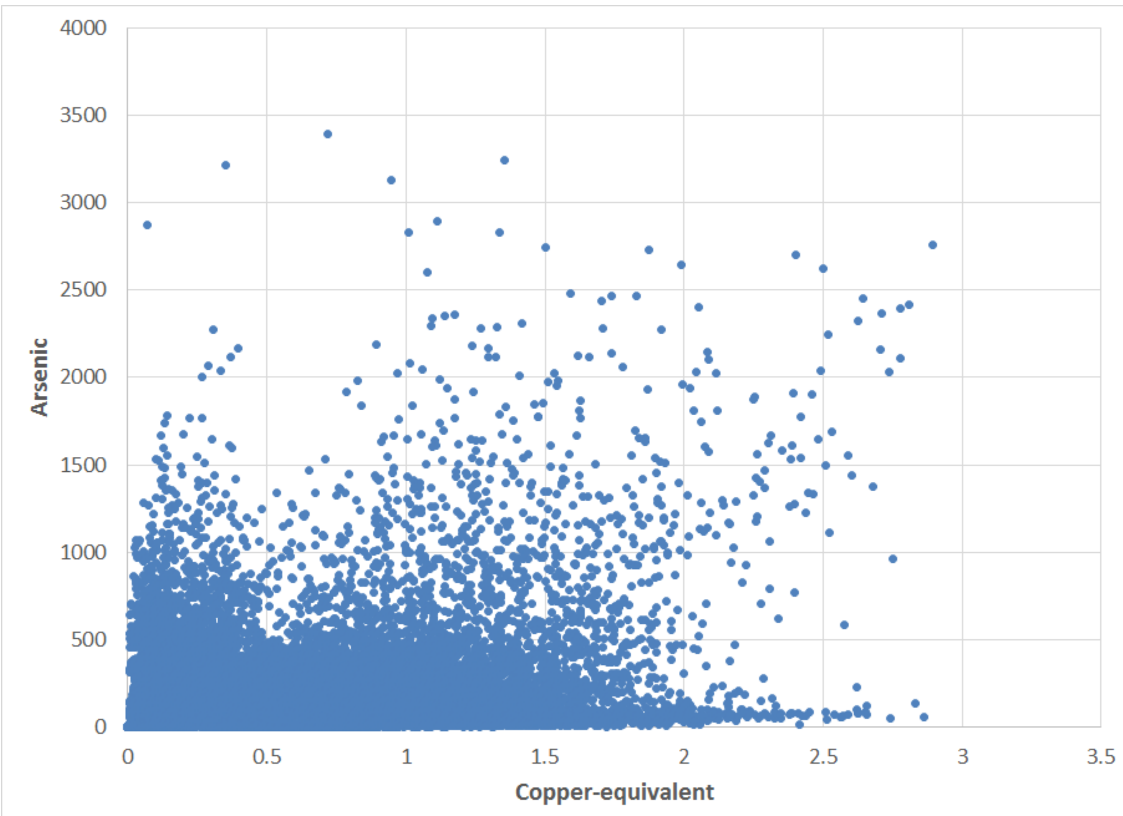



Figure 3.2: Distribution of copper-equivalent and arsenic contained in each block in the data set. A considerable number of blocks have both high-copper-equivalent grade and a high arsenic level.

There are 56 benches in the mine and the height of each bench is 12 meters. The aforementioned blocks are contained in four predefined phases, computed using the Whittle commercial mining software, resulting in 220 phase-benches to schedule. To extract any block in a given phase-bench, the predecessor phase-benches must be mined completely (see

Figure 3.3). As a phase-bench is mined, equal proportions must be taken from each block. We emphasize that our approach solves OPMPS+S for this data set at the block level, and predefined phase-benches force precedence constraints between blocks in different bench-phases. Mining capacity is 116.8 million tons per year and processing capacity is 67.89 million tons per year. The cash flow discount factor is 10% per year. For each extracted block, there are three destinations: waste dump, mill, and stockpile.

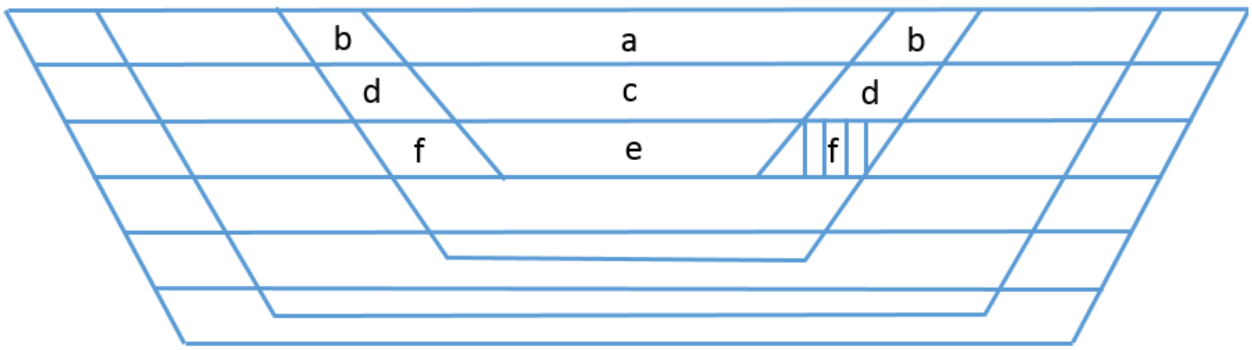



Figure 3.3: Blocks are differentiated by vertical lines, and are aggregated in each phase-bench. Before mining any block in phase-bench *f*, both phase-benches *d* and *e* must have been mined.

## 3.4 An Optimization Model considering stockpiles and blending constraints

Bley et al. [25] propose a nonlinear-integer model to solve the OPMPS+S problem. Although their model provides the most accurate solution, it is not tractable. Moreno et al. [44] propose a linear-integer model, ($\mathcal{P}^{la}$), which is tractable and provides an objective function value that is very close to the nonlinear-integer model proposed by Bley et al. [25] for the numerical experiments presented. We modify ($\mathcal{P}^{la}$) in such a way that it (i) enforces the precedences between phase-benches, (ii) homogeneously mixes material in the stockpile by controlling both metal content and contaminant level, (iii) satisfies the contaminant level at the mill by blending material from the mine and the stockpile, and (iv) considers mining and processing resource constraints explicitly. The first subsection introduces notation, and the following subsections provide the mathematical formulation.

### 3.4.1 Notation

Indices and Sets:

$b \in \mathcal{B}$ : blocks; $1, ..., B$
$n \in \mathcal{N}$ : phase-benches; $1, ..., N$
$b \in \tilde{\mathcal{B}}_n$ : all blocks in a phase-bench $n$ that must be mined together
$\hat{n} \in \hat{\mathcal{N}}_n$ : all phase-benches that must be mined directly before phase-bench $n$
$r \in \mathcal{R}$ : resources $\{1 = \text{mine}, 2 = \text{mill}\}$
$t \in \mathcal{T}$ : time periods; $1, ..., T$

Parameters:

$\delta_t$ : discount factor for time period $t$ (fraction)
$C^m$ : mining cost per ton of material (\$/ton)
$C^p$ : processing cost per ton of material (\$/ton)
$P$ : profit generated per ton of metal (\$/ton)
$W_b$ : tonnage of block $b$ (ton)
$M_b$ : metal obtained by completely processing block $b$ (ton)
$C^h$ : rehandling cost per ton of material (\$/ton)
$L_b$ : grade of metal in block $b$
$\bar{L}$ : average grade of metal in the stockpile
$G_b$ : grade of contaminant in block $b$ (ppm)
$\bar{G}$ : average grade of contaminant in the stockpile (ppm)
$\hat{G}$ : contaminant limit at the mill (ppm)
$R_{rt}$ : maximum amount of resource $r$ available in time $t$ (tons/yr)

Decision Variables:

$y^m_{bt}$ : fraction of block $b$ mined in time period $t$
$y^p_{bt}$ : fraction of block $b$ mined in time period $t$ and sent to the mill
$y^s_{bt}$ : fraction of block $b$ mined in time period $t$ and sent to the stockpile
$y^w_{bt}$ : fraction of block $b$ mined in time period $t$ and sent to waste
$x_{nt}$ : 1 if all blocks in $\tilde{\mathcal{B}}_n$ have finished being mined by time $t$; 0 otherwise
$i^p_t$ : tonnage of ore sent from the stockpile to the mill in time period $t$
$i^s_t$ : tonnage of ore remaining in the stockpile at the end of time period $t$

### 3.4.2 L-average bound model ($\hat{\mathcal{P}}^{la}$)

Here, we use a modified version of ($\mathcal{P}^{la}$) to explore the effect of blending material while controlling different grades in the stockpile. This model requires the blocks that enter the stockpile to have an average metal grade of at least $\bar{L}$ and an average contaminant grade of at most $\bar{G}$. The model is as follows:

$$(\hat{\mathcal{P}}^{la}) : \max \sum_{t \in \mathcal{T}} \delta_t \Bigg[ P \Bigg( \sum_{b \in \mathcal{B}} M_b y^p_{bt} + \bar{L} i^p_t \Bigg) - C^p \Bigg( \sum_{b \in \mathcal{B}} W_b y^p_{bt} + i^p_t \Bigg) - C^m \Bigg( \sum_{b \in \mathcal{B}} W_b y^m_{bt} \Bigg) - C^h i^p_t \Bigg] \quad (3.1)$$

$$\sum_{t \in \mathcal{T}} y^m_{bt} \leq 1 \qquad \forall b \in \mathcal{B} \qquad (3.2)$$

$$y^p_{bt} + y^w_{bt} + y^s_{bt} = y^m_{bt} \qquad \forall b \in \mathcal{B}, t \in \mathcal{T} \qquad (3.3)$$

$$x_{nt} \leq \sum_{t' \leq t} y^m_{bt'} \qquad \forall b \in \tilde{\mathcal{B}}_n, n \in \mathcal{N}, t \in \mathcal{T} \qquad (3.4)$$

$$\sum_{t' \leq t} y^m_{bt'} \leq x_{n't} \qquad \forall n \in \mathcal{N}, b \in \tilde{\mathcal{B}}_n, n' \in \hat{\mathcal{N}}_n, t \in \mathcal{T} \qquad (3.5)$$

$$\sum_{b \in \mathcal{B}} W_b y^m_{bt} \leq R_{rt} \qquad \forall t \in \mathcal{T}, r \in \mathcal{R} : r = 1 \qquad (3.6)$$

$$\sum_{b \in \mathcal{B}} W_b y^p_{bt} + i^s_t \leq R_{rt} \qquad \forall t \in \mathcal{T}, r \in \mathcal{R} : r = 2 \qquad (3.7)$$

$$\sum_{b \in \mathcal{B}} G_b W_b y^p_{bt} + \bar{G} i^s_t \leq \hat{G}(\sum_{b \in \mathcal{B}} W_b y^p_{bt} + i^s_t) \qquad \forall t \in \mathcal{T} \qquad (3.8)$$

$$i^p_t \leq i^s_{t-1} \qquad \forall t \in \mathcal{T} : t \geq 2 \qquad (3.9)$$

$$i^s_t = \begin{cases} \sum_{b \in \mathcal{B}} W_b y^s_{bt} & t = 1 \\ i^s_{t-1} - i^p_t + \sum_{b \in \mathcal{B}} W_b y^s_{bt} & t \in \mathcal{T} : t \geq 2 \end{cases} \qquad (3.10)$$

$$\sum_{b \in \mathcal{B}} \sum_{t' \leq t} L_b W_b y^s_{bt'} \geq \bar{L} \sum_{b \in \mathcal{B}} \sum_{t' \leq t} W_b y^s_{bt'} \qquad \forall t \in \mathcal{T} \qquad (3.11)$$

$$\sum_{b \in \mathcal{B}} \sum_{t' \leq t} G_b W_b y^s_{bt'} \leq \bar{G} \sum_{b \in \mathcal{B}} \sum_{t' \leq t} W_b y^s_{bt'} \qquad \forall t \in \mathcal{T}. \qquad (3.12)$$

$$0 \leq y^m_{bt}, y^p_{bt}, y^w_{bt}, y^s_{bt} \leq 1; \; x_{nt} \in \{0, 1\} \qquad \forall b \in \mathcal{B}, n \in \mathcal{N}, t \in \mathcal{T} \qquad (3.13)$$

The objective function is the sum of the revenues of blocks sent to the mill from the mine or the stockpile, minus the sum of the processing, mining and rehandling costs. To represent NPV, all terms are multiplied by a discount rate associated with each time period $t$.

Constraint (3.2) ensures that each block is not extracted more than once. Constraint (3.3) establishes that the amount of material sent to different destinations is equal to the amount of extracted material. Constraint (3.4) forces all of the blocks in each phase-bench to be mined by time period $t$ if the binary variable associated to that phase-bench is set to one. Constraint (3.5) enforces mining precedence constraints by ensuring that all phase-bench predecessors are completely mined before the successor phase-bench.

Constraints (3.6) and (3.7) represent extraction and processing restrictions, respectively. Constraint (3.8) requires that the average arsenic level (ppm) of material at the mill in each time period should not exceed $\hat{G}$. Constraint (3.9) ensures that the amount of material sent from the stockpile to the mill in time period $t$ is at most the amount of material in the stockpile in time period $t-1$. Constraint (3.10) enforces inventory balance for an initial time period and a general time period $t$, ensuring that the amount of material in the stockpile during time period $t$ is equal to that of the previous period plus or minus any material added or subtracted from the stockpile and sent to the mill, respectively. Constraint (3.11) ensures that blocks in the stockpile in any time period have an average metal grade of at least $\bar{L}$, and Constraint (3.12) guarantees that blocks in the stockpile in any time period have an average contaminant level of at most $\bar{G}$.

## 3.5 Resulting schedule and its comparison with a commercial software package

We compare the schedule produced by solving ($\hat{\mathcal{P}}^{la}$) to that of provided by state-of-the-art software. Here, we call the schedule provided by ($\hat{\mathcal{P}}^{la}$) *the Experiment*, and that of obtained from the software *the Control*. Although the Control uses three stockpiles, for the sake of simplicity, the Experiment considers just one. If the Experiment allowed for three stockpiles, we may achieve even better solutions.

## 3.6 Solving $(\hat{\mathcal{P}}^{la})$

The $(\hat{\mathcal{P}}^{la})$ model sends material to the stockpile according to a predefined average grade of copper-equivalent, $\bar{L}$, and an arsenic level, $\bar{G}$. Changing both $\bar{L}$ and $\bar{G}$ alters the schedule and NPV for the OPMPS+S problem.

To select these parameters, we consider eight different values for $\bar{L}$ and thirteen different values for $\bar{G}$, shown in Table 3.1. We use OMP [28] to solve the LP relaxation of $(\hat{\mathcal{P}}^{la})$ to find the highest NPV among all combinations. We emphasize that fixing $\bar{L}$ and $\bar{G}$ means both a linear model and one whose structure we can exploit such that we can use OMP as the solution algorithm, which, when combined with the modified version of the TopoSort algorithm [7], is able to obtain a near-optimal solution to $(\hat{\mathcal{P}}^{la})$, even for large instances. We remark that a problem of this size cannot be solved using general integer programming solvers, nor can a corresponding nonlinear-integer model be solved in a reasonable amount of time[25].

Our numerical experiments show that the objective function of the relaxed problem is unimodal in $\bar{L}$ and $\bar{G}$. Table 3.1 displays the resulting NPV. Figure 3.4 shows the surface plot of the LP relaxation of NPV for different combinations of copper-equivalent grade and arsenic levels in the stockpile. Table 3.1 and Figure 3.4 show that the highest LP relaxation value of the NPV is associated with a copper-equivalent of 1 and an arsenic level of 900 ppm, meaning that we should set the optimal average copper-equivalent grade greater than or equal to 1 and average arsenic level less than or equal to 900 ppm for the material sent from the stockpile to the mill.

To obtain an integer-feasible solution for this problem, we use a modified version of the TopoSort heuristic, which is a two-step algorithm that creates an integer solution from the LP relaxation values of each unit (i.e., block or bench-phase). Based on these values, it computes a topological order, breaking ties according to the expected (weighted) LP extraction times. Then, given a set of capacity constraints, i.e., mining and processing, per period and a set of resources used by each unit, TopoSort assigns units with the lowest topological order to be

Table 3.1: LP relaxation of the objective function (NPV (M$)) for $(\hat{\mathcal{P}}^{la})$ for different combinations of copper-equivalent grade and arsenic level in the stockpile.

| As grade (ppm) | copper-equivalent grade (%) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0.80 | 0.85 | 0.9 | 0.95 | 1 | 1.1 | 1.2 | 1.3 |
| 500 | 9,344 | 9,334 | 9,320 | 9,303 | 9,284 | 9,246 | 9,215 | 9,187 |
| 600 | 9,382 | 9,377 | 9,370 | 9,359 | 9,349 | 9,318 | 9,284 | 9,250 |
| 700 | 9,397 | 9,398 | 9,398 | 9,394 | 9,388 | 9,365 | 9,343 | 9,312 |
| 800 | 9,384 | 9,399 | 9,405 | 9,406 | 9,406 | 9,395 | 9,374 | 9,353 |
| 900 | 9,347 | 9,375 | 9,393 | 9,403 | 9,407 | 9,407 | 9,396 | 9,377 |
| 1000 | 9,308 | 9,338 | 9,366 | 9,384 | 9,397 | 9,403 | 9,403 | 9,392 |
| 1100 | 9,279 | 9,304 | 9,331 | 9,357 | 9,377 | 9,396 | 9,396 | 9,392 |
| 1200 | 9,257 | 9,278 | 9,301 | 9,325 | 9,349 | 9,380 | 9,390 | 9,386 |
| 1400 | 9,224 | 9,241 | 9,259 | 9,277 | 9,297 | 9,337 | 9,365 | 9,374 |
| 1600 | 9,200 | 9,215 | 9,230 | 9,245 | 9,260 | 9,293 | 9,328 | 9,350 |
| 1800 | 9,182 | 9,195 | 9,208 | 9,221 | 9,235 | 9,262 | 9,291 | 9,320 |
| 2000 | 9,169 | 9,180 | 9,019 | 9,203 | 9,215 | 9,238 | 9,263 | 9,289 |
| 2200 | 9,158 | 9,168 | 9,178 | 9,188 | 9,199 | 9,220 | 9,241 | 9,264 |

extracted in the first time period as resource capacity allows, and then units to successive time periods, based on their topological ordering. Recall that material in the stockpile is not tracked by unit. Therefore, for each time period, we can fix the amount of material in the stockpile ($i_t^s$) and the amount of material sent from the stockpile to the mill ($i_t^p$) based simply on the value from the LP solution; the latter fixed value reduces available plant capacity.

The TopoSort heuristic provides a finishing extraction time for each bench-phase (an assignment of values to the $x_{nt}$ binary variables). However, the values of the other variables from the LP solution can be incompatible with the $x_{nt}$ variables, leading to an infeasible overall solution. For example, the value of other variables may violate the blending constraints or the total material available in the stockpile during each period. In order to obtain a feasible solution, we re-solve $(\hat{\mathcal{P}}^{la})$ by fixing the values of variables $x_{nt}$ according to the solution obtained from TopoSort. Note that fixing the $x$ variables considerably reduces the size of the problem and the resulting model only contains continuous variables, allowing us to solve it using a standard linear programming solver in a few seconds. This heuristic gives us an integer solution with a 2% integrality gap based on an objective in which the contribution of each unit is weighted as the average of the profit obtained from the different fractions of

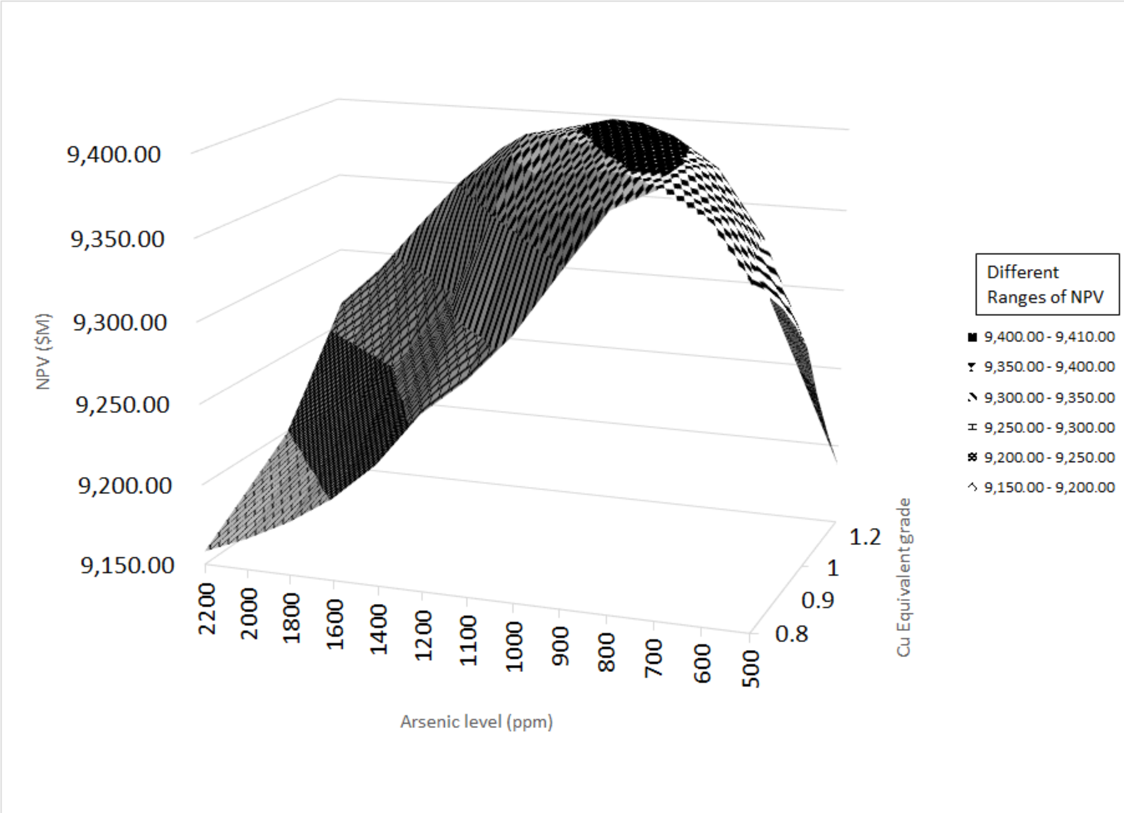


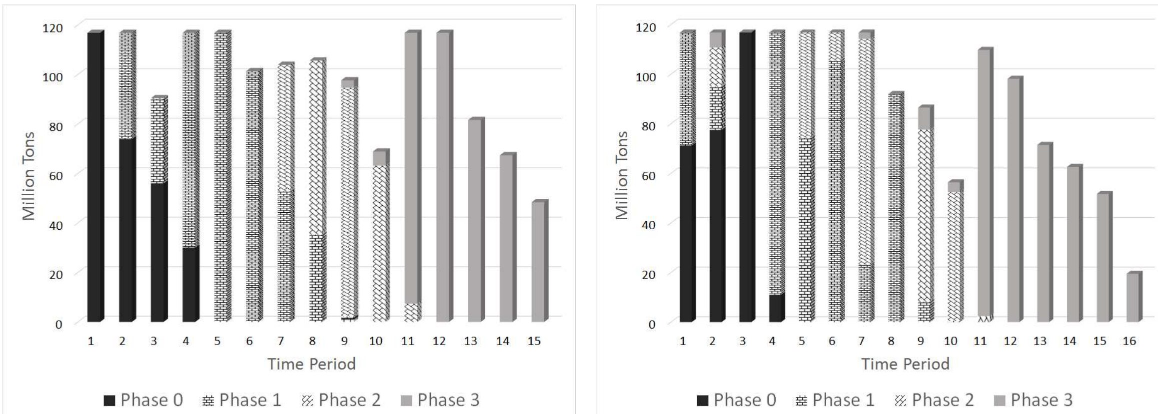

Figure 3.4: LP relaxation of the objective function (NPV (M\$)) for different combinations of copper-equivalent grade and arsenic level in the stockpile. We observe that the LP relaxation value is unimodal.

that unit (assigned to various destinations) according to the LP solution; the profit of the fractions of blocks sent to the stockpile is computed assuming an $\bar{L}$ metal grade.

### 3.6.1 Comparison with Commercial Software

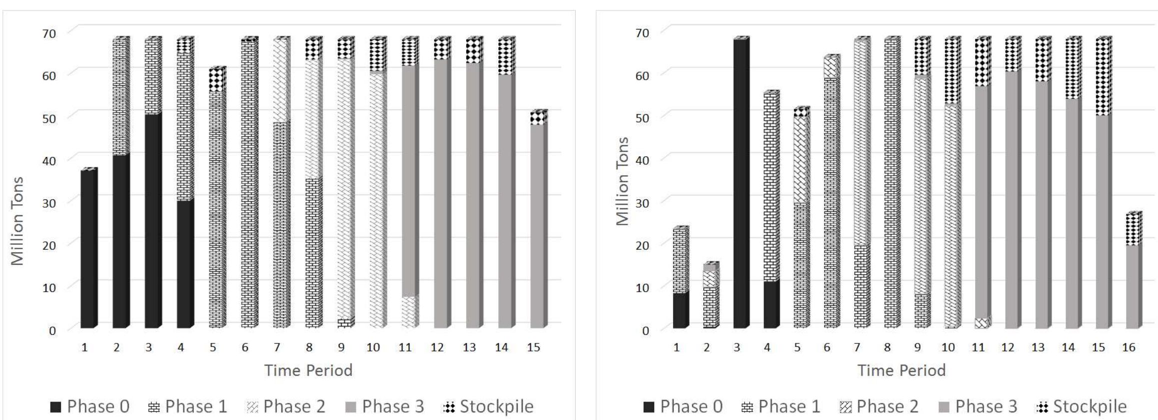
We compare the schedule obtained by solving $(\hat{\mathcal{P}}^{la})$ with the results from state-of-the-art strategic and operational mine planning software, which "optimizes" schedules considering stockpiles and blending at a block level, re-binning these blocks if the problem size becomes so large that the instances are intractable. The exact model that the software uses internally to schedule the blocks is unknown, but the algorithm used to solve the model is CPLEX [30].

Our analysis compares the tonnage of extracted and processed material, the average grades of the material sent to the different destinations, and the resulting NPV from both schedules.




(a) Extracted tonnage by Experiment (b) Extracted tonnage by Control

Figure 3.5: Extracted tonnage comparison between Experiment and Control cases over the life of the mine.

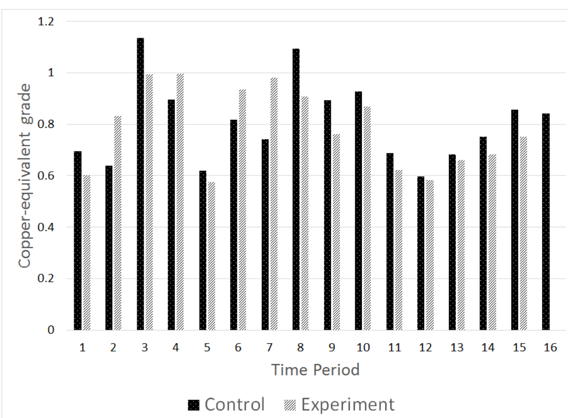



(a) Processed tonnage by Experiment (b) Processed tonnage by Control

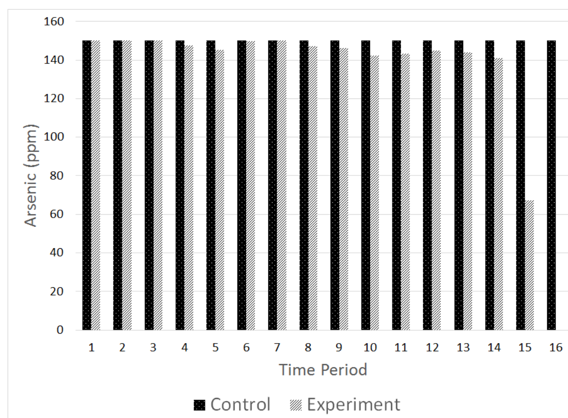
Figure 3.6: Processed tonnage comparison between Experiment and Control cases over the life of the mine.

Figure 3.5 compares the extraction schedule of the Experiment and Control cases in each time period. Different patterns show different phases. This figure shows that the Experiment's extraction occurs mostly in a specific phase and after finishing that phase, the extraction of the next phase begins. In practice, this is useful so that the equipment does not have to be moved multiple times between phases. However, the Control extracts material from different phases within one time period, which could be expensive and/or impractical. We emphasize that having less equipment movement in the Experiment is a coincidence, meaning that this feature is not controlled in ($\hat{\mathcal{P}}^{la}$), except for the precedence defined between different phase-benches. Although the amount of extracted material is about the same for both the Experiment and Control cases, Figure 3.5 shows that the resulting schedule for the Control is one year longer than that obtained by the Experiment.

Figure 3.6 compares the processing schedule of the Experiment and Control cases in each time period. Due to the discount factor applied each year, it is preferable to obtain the highest possible profit sooner rather than later, which is effected by the efficient use of processing capacity. The figure shows that the Experiment has a better utilization of the mill, processing 5% more material than the Control.



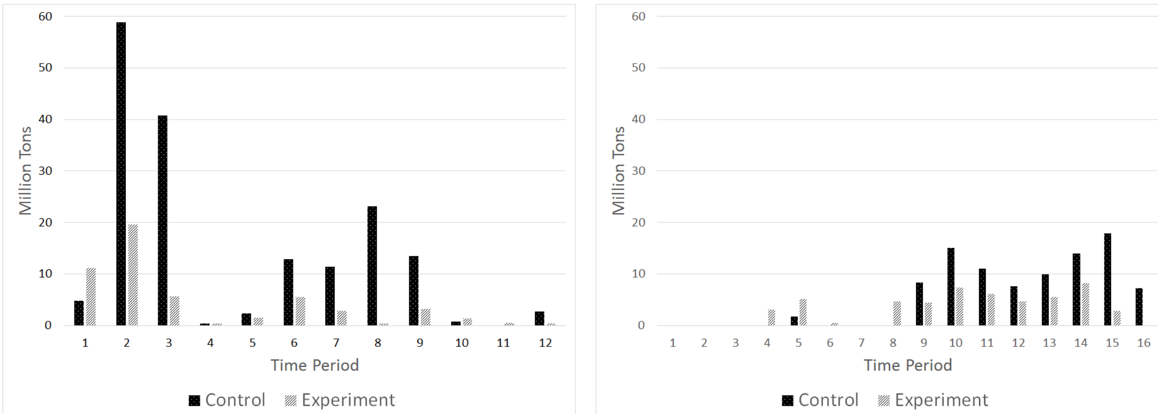

(a) Average copper-equivalent grade




(b) Average arsenic level

Figure 3.7: Comparison of average grades at the mill between the Experiment and Control cases.

Figure 3.7 compares the average copper-equivalent and average arsenic per ton processed at the mill between the Experiment and Control cases in each time period. It can be seen that the high level of arsenic in the mine and its limit at the plant is a binding constraint for the problem, requiring a good blending strategy to satisfy this requirement. Both schedules process ore with similar copper-equivalent grades.

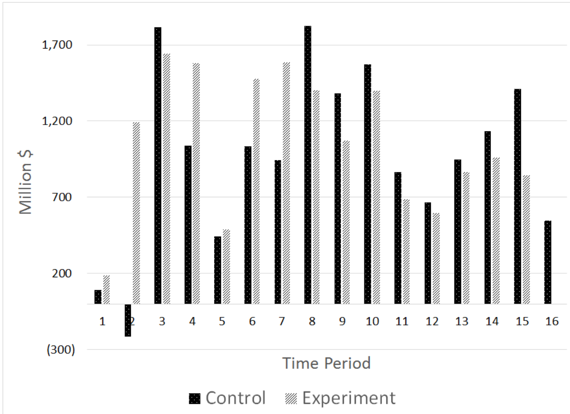



(a) Tonnage sent from the mine to the stockpile (b) Tonnage sent from the stockpile to the mill

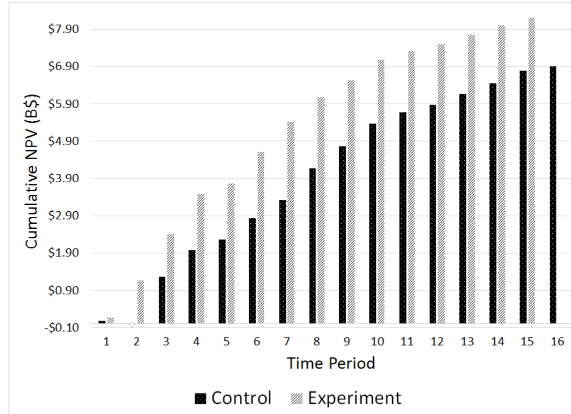
Figure 3.8: Comparison of material movement at the stockpile

Regarding the use of the stockpile, Figure 3.8 shows that the Control case sends more material from the mine to the stockpile than the Experiment case. In other words, although the Control extracts more material than the Experiment, it sends more material to the stockpile. Figure 3.8(b) compares the material flow from the stockpile to the mill and shows that the Experiment sends the material in stockpile to the mill sooner than the Control. In other words, the Experiment blends the material in the stockpile with the material sent directly from the mine to the mill in such a way that the arsenic limit at the mill is not violated. On the other hand, the Control sends material in the stockpile to the mill primarily after time period 9. By the end of the mine life, the Experiment uses all of the material in the stockpile, but the Control still contains some high arsenic material. Figures 3.8(a) and 3.8(b) show that the Experiment possesses a better stockpile management strategy, because it sends less material to the stockpile than the Control, and it uses the material in the stockpile in such a way that mill requirements are met.

Finally, Figure 3.9 compares the cash flow between the Experiment and Control cases. It shows that in the first two time periods, the Experiment provides considerably higher cash



(a) Undiscounted cash flow



(b) Cumulative discounted cash flow

Figure 3.9: Comparison of cash flows between the Experiment and Control cases.

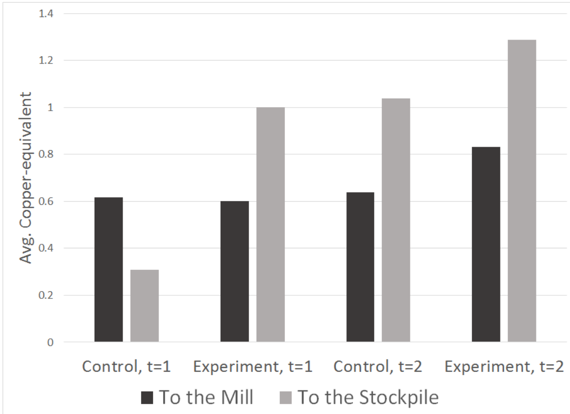
flow than the Control, because the Experiment achieves a higher NPV early in the early life. The difference obtained in these two periods is maintained throughout the remainder of the time horizon (Figure 3.9(b)).

In summary, the schedule produced by the Experiment extracts about the same amount of material as the Control, but processes 5% more ore than the Control, with different copper-equivalent grade at the mill in different years, obtaining an NPV that is 18.7% higher. Also, the Experiment presents a better use of the stockpile and the schedule satisfies the predefined order of the phases in a superior manner. Because the main difference in the NPV is obtained at the beginning of the mine life, we provide a deeper analysis of the first two periods.

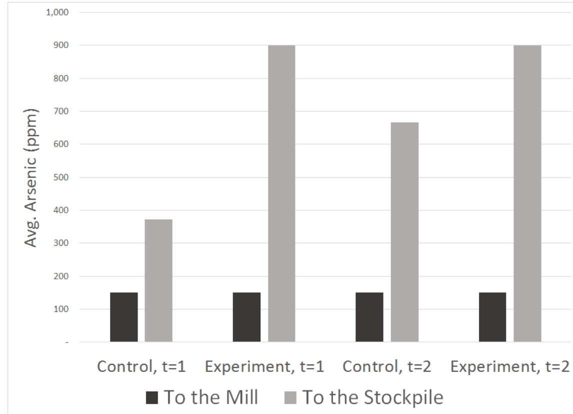
## 3.7 Analysis of the first two periods

In order to understand how the schedule obtained by the Experiment results in sending almost three time more material to the mill in the first two periods than the Control (see Figure 3.6), we analyze the characteristics of the extracted material, such as tonnage and grades of metal and contaminant in the first two time periods.

Figure 3.10(a) shows that the average copper-equivalent grade of material sent to the plant is greater for the Experiment. In other words, the Experiment processes more ore and more valuable material than the Control, resulting in a considerably higher cash flow




(a) Average copper-equivalent grade

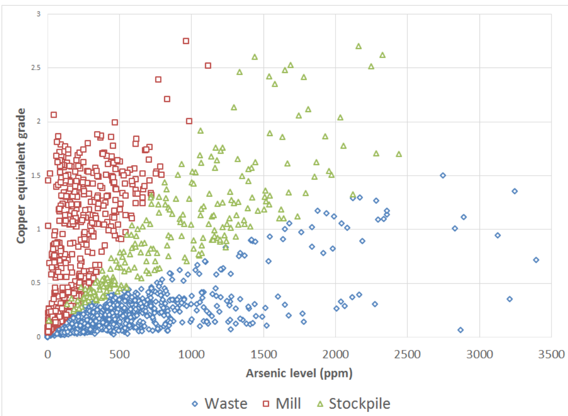



(b) Average arsenic level

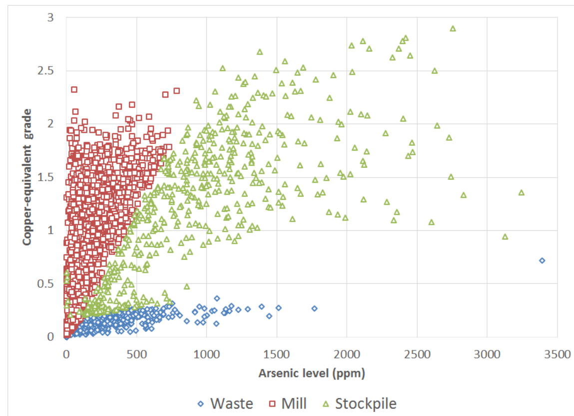
Figure 3.10: Comparison of copper-equivalent and average arsenic level in extracted ore between the Experiment and Control cases for the first two time periods.

in the first two time periods. Figures 3.10(a) and 3.10(b) show that the Experiment sends material with higher copper-equivalent grade and higher arsenic level to the stockpile than the Control, meaning that the Experiment can use the valuable material with high arsenic levels by blending it with material in the stockpile. In other words, the Experiment's blending strategy sends the material with high copper-equivalent (but high arsenic levels) to the stockpile instead of sending it to waste or to the mill.

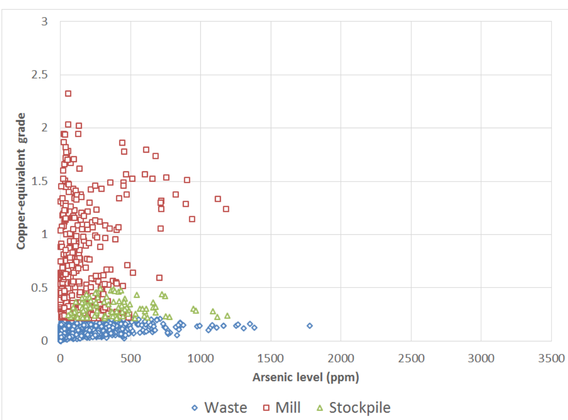
Figure 3.11 shows important differences between the schedules of the Experiment and Control cases; each dot represents a block, located using its corresponding grades, and shows the destination of the block in either schedule. The Control sends to waste all blocks with a copper-equivalent grade less than the economic cutoff grade of 0.21%. In other words, if processing a block does not yield a positive profit, then it is sent to waste. On the contrary, the Experiment sends to the mill blocks with a copper-equivalent grade below this economic cutoff grade if they contain a low level of arsenic. This allows these blocks to be blended with material containing a high arsenic and copper-equivalent grade, resulting in the Experiment attaining the maximum arsenic level at the mill to a greater extent than the Control. Second, material with a high ratio of arsenic to copper is sent to the waste by the Experiment. On



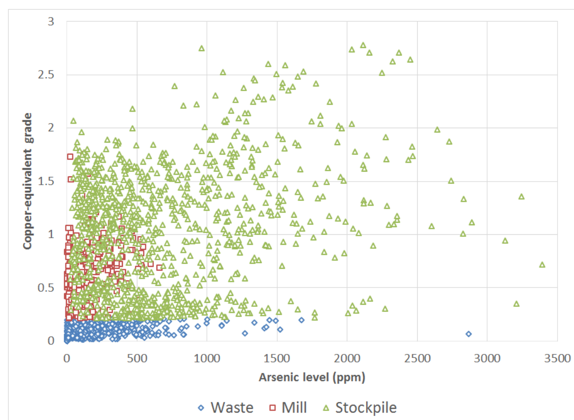

(a) Experiment- first time period




(b) Experiment- second time period




(c) Control- first time period




(d) Control- second time period

Figure 3.11: Destination of extracted material in the Experiment and Control cases in the first two time periods.

the contrary, the Control sends all material above the economic cutoff grade to either the stockpile or to the mill, obtaining higher levels of arsenic at the stockpile, and explaining the remaining material at the stockpile at the end of the mine life. Finally, we note that the Experiment provides a clear delineator regarding the destination of each block based on the ratio of arsenic to copper, whereas the Control does not do this, resulting in blocks with similar characteristics sent to different destinations. This may be due to the fact that the software requires aggregating blocks in the same bench-phase in order to reduce the size of its problem.

### 3.8 Conclusion

We use a modified version of ($\mathcal{P}^{la}$) presented by Moreno et al. [44] to provide long-term planning for an operational open pit mine and compare its schedule with that from state-of-the-art software. We observe through our numerical experiments that the objective function LP relaxation of ($\hat{\mathcal{P}}^{la}$) is unimodal regarding blending criteria in the stockpile, which allows us to find the optimal grade combination for the LP. Then, we use TopoSort to create an IP solution from the highest possible LP relaxation. By comparing the schedule from the ($\hat{\mathcal{P}}^{la}$) with that of state-of-the-art software, we detect that ($\hat{\mathcal{P}}^{la}$) provides a schedule with higher NPV. We show that ($\hat{\mathcal{P}}^{la}$) sends more material to the processing plant, especially early in the mine life. Also, ($\hat{\mathcal{P}}^{la}$) produces a blending strategy in the stockpile that controls more than one grade. Here, we recognize that the economic impact of a suitable blending strategy not only results in sending less material to the stockpile, but also using processing plant capacity more efficiently.

CHAPTER 4

# OPEN PIT MINE PLANNING WITH DEGRADATION DUE TO STOCKPILING

A paper nearing submission to *Optimization and Engineering*

Mojtaba Rezakhah[9], Alexandra Newman[10]

The open pit mine production planning with stockpiling (OPMPS+S) problem decides when to extract each block of ore and/or waste in a deposit. In addition, this problem determines whether to send each block to a particular processing plant, a stockpile, or a waste dump. The objective function maximizes net present value (NPV), subject to constraints such as precedence, and capacities for mining and processing. Since the material within the stockpile is exposed to the environment, some time-dependent changes occur in the material's properties, which results in decreased value.

In this research, we create three new linear-integer stockpiling models which consider degradation within the stockpile(s). We compare results from these models on a data set from an operational mine, and suggest the most accurate one. Finally, we show that the material degradation within a stockpile has a considerable impact on the value that a stockpile provides.

## 4.1 Introduction

Operations research uses mathematical modeling to make better decisions, and find optimal or near-optimal solutions to complex problems. The open pit mine production scheduling with stockpiling (OPMPS) problem determines the extraction sequence of blocks within the final pit limits, and their destinations, e.g., waste dump or processing plant. To develop a mathematical model to solve the OPMPS problem, three questions must be addressed: (i) what are the variables of the problem? (ii) what is the objective function that needs to be

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achieved to determine the optimum solution? (iii) what constraints must be imposed on the variables to satisfy the limitations of the operation? Here, variables define the extraction time and the destination for each block. The objective function maximizes the NPV subject to a set of constraints including mining and processing limits, precedence, and cutoff grade.

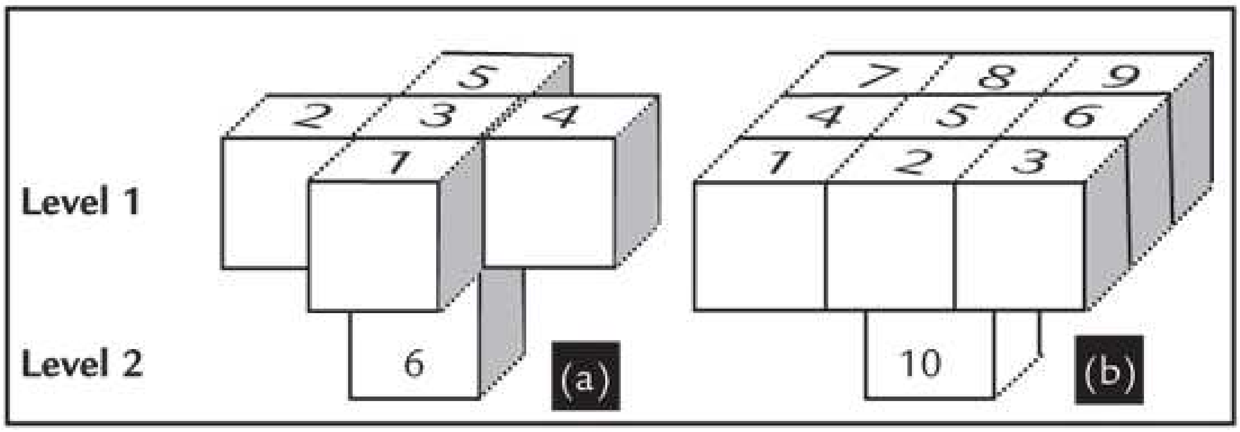


Figure 4.1: Blocks' precedence relationship. a: before extracting block 6, the five blocks above should be mined, b: before extracting block 10, the nine blocks above should be mined.

Figure 4.1 shows the precedence between blocks. The principal structural constraint associated with open pit mining, known as the "slope angle," is that any block $b$ may not be extracted before an upward facing (generally) circular cone that sits above $b$ is extracted. There are, moreover, a number of destinations, e.g, waste or mill, for each block once it has been extracted. There are often too many blocks, on the order of hundreds of thousands, within the final pit limits to determine the optimum annual production schedule. To reduce the size of the problem, it is common to partition the material within the final pit limits into smaller volumes called "pushbacks" using one of the existing methods including those described in Seymour [49], Ramazan and Dagdelen [50] and Whittle [51].

The principal approaches applied to solve the open pit mine production scheduling (OPMPS) problem consist of heuristics and exact methods. A famous heuristic approach is the Lerchs and Grossmann [34] algorithm known as the "nested pits" method, which assumes that any block of ore is defined by a single value, and that the objective value $v$ of that block in any period $t$ is the present value in period 1 of $v$, obtained in period $t$, considering some discount rate. In this method, the problem is converted to one in which there are no time periods, the capacity constraints are ignored, and the only decision is whether or not each block is to be extracted. By decrementing the sale price of final product from its true value, this algorithm creates a series of nested ultimate pits. This rough approach can be fairly ineffective when there are (i) precedence constraints, (ii) multiple capacity constraints, and/or (iii) multiple processing options. Moreover, it is often the case that there is a huge increase in size between some set of blocks and the next set in the sequence, and the algorithm can give no guidance as to how to decrease the difference between the sets into manageable chunks.

Another heuristic algorithm is a "floating cone" method, which is computationally simpler than the Lerchs-Grossmann method. Dowd and Onur [52] describe a dynamic programming approach to the OPMPS problem which, they note, is unlikely to solve large problem instances. Moreover, Tolwinski and Underwood [53] combine ideas from dynamic programming and stochastic-search heuristics to produce feasible solutions to the OPMPS problem. They show that the solution time of this approach grows exponentially with the number of blocks: "For even a small model where 10,000 blocks need to be considered, the number of states is enormous, and to generate all of them is impractical."

Ramazan and Dimitrakopoulos [20] state that mixed-integer and linear programming models have significant potential for optimizing production scheduling in open pit mines in which the objective is to maximize total discounted profit.

### 4.1.1 Integer-Linear and Nonlinear Models Considering a Stockpile

Optimization techniques use a mathematical model to represent the optimal solution for a mining operation given the constraints placed on the model. The mine schedule can be optimized with respect to maximizing the NVP, maximizing metal content, minimizing mining and processing costs, or minimizing the variance of the grade.

Hoerger et al. [27] describe a mixed integer-linear programming model to optimize mine scheduling for Newmont Mining Corporation's Nevada operations. The authors consider multiple stockpiles, each of which has a specific grade range. When the material is removed from the stockpile, its grade is considered to be the minimum of the associated grade range. Asad [43] presents a cutoff grade optimization algorithm for open pit mine production with stockpiling with stockpiling (OPMPS+S) in a deposit with two minerals. Ramazan and Dimitrakopoulos [20] propose a stochastic model to solve the OPMPS+S problem, and do not consider material mixing in the stockpile.

However, some researchers do consider material mixing. In other words, material with different grades mixes within the stockpile, and the original characteristics of the material changes. Bley et al. [25] propose two different models. The authors assume that blocks going to the stockpile are mixed homogeneously, so the characteristics of the material removed from the stockpile must be treated as variables. Since the amount of ore removed from the stockpile is not known a priori, their model for solving OPMPS+S has some non-convex, nonlinear constraints. Efforts to solve this problem result in local optimal solutions and/or are intractable for big data sets. Attempting to decrease the size of the instances results in aggregation, which causes a loss of information regarding each type of material. Moreno et al. [44] review different stockpiling models and propose a new linear-integer model, ($\mathcal{P}^{la}$), which is tractable. The authors show that this model closely approximates OPMPS+S with respect to the nonlinear-integer model presented by Bley et al. [25].

### 4.1.2 Material Degradation Due to Stockpiling

Degradation or corrosion means deterioration of material when exposed to an environment, resulting in the loss of material. The main consequences of degradation include changes in physical and mechanical properties, weight loss or gain, and material loss. Lovich and Bainbridge [54] state that the most obvious forms of degradation in mining operations occur in stockpiles and waste dumps. The authors emphasize that degradation in toxic tailings may have a major effect on the environment. Different factors affect the degradation rate in the stockpile, including oxygen, temperature, and humidity. Robinson [55] discusses that predicting stockpile complications before critical issues occur is very important, and compares different methods to characterize the time-dependent changes in material properties within the stockpile. Pantelis et al. [56] review the degradation of sulphidic material in a waste dump and present a mathematical model to predict the effect of gas, water, and heat transport on material degradation. Ghose and Kundu [57] investigate soil degradation in a large opencast coal mine, claim that major deterioration occurs when the soil is stockpiled for a few years, and suggest that stockpiling should generally be avoided.

In this research, we propose that degradation results in a reduction in the amount of metal that can be recovered at mill. Here, we consider an exponential reduction in the recoverable material within the stockpile. For example, a 5% annual degradation rate signifies that keeping the material in the stockpile for one year results in 5% lower recoverable metal at the mill. Although we only consider degradation as a reduction in the amount of recoverable metal, our formulation is also capable of considering increased processing cost as a result of degradation. Here, we modify three existing models to solve the OPMPS+S problem for an operational mine. Also, we create three new models that consider the material degradation due to stockpiling, and suggest the most accurate one. Moreover, we compare these models and explore the value of stockpiling and how it alters with a change in the mill processing capacity. Furthermore, we show how the grade distribution in a mine can affect the strategy of sending material to different stockpiles. Finally, we demonstrate that material degradation within the stockpile has a considerable effect on the value that a stockpile provides.

We have organized the remainder of this paper as follows. In Section 4.2, we describe the modifications made to the models proposed by Moreno et al. [44], and also create three new models to consider the material degradation within the stockpile. In Section **??**, we compare the result of these six models. We conclude in Section **??** that there is a trade-off in sending the material to the stockpile when considering degradation, and an increased degradation rate decreases the value of stockpiling.

### 4.2 The OPMPS+S Problem with and without Degradation

In order to explore the effects of stockpiling in open pit mining, we attempt to solve six different OPMPS+S models: $(\mathcal{P}^w)$ has as many stockpiles as there are blocks and therefore allows each block to be stored separately; $(\mathcal{P}^{la})$ is a linear-integer model that considers a single stockpile with homogeneous mixing. The grade of the material when removing it from the stockpile is the average grade. $(\mathcal{P}^{ms})$ considers several stockpiles and assigns the minimum allowable grade to each stockpile which has a range of acceptable grades (e.g., 0.1 to 0.3) so that only bins within that range can be stored there. When taking material out of a stockpile, we assume that the grade corresponds to that of the lower bound on the range (e.g., 0.1 in the example above) so that the objective function value provides a pessimistic estimate. $(\mathcal{P}^w_d)$, $(\mathcal{P}^{la}_d)$, and $(\mathcal{P}^{ms}_d)$ are reformulations of $(\mathcal{P}^w)$, $(\mathcal{P}^{la})$, and $(\mathcal{P}^{ms})$ respectively, but consider degradation.

Solving $(\mathcal{P}^w)$ provides an upper bound on the NPV of the mine, yet storing each block separately is unrealistic. $(\mathcal{P}^{la})$ is a tractable model that provides the most reasonable estimate of NPV, and its objective function value is less than that of $(\mathcal{P}^w)$. Also, this model can provide a better solution based on the mill's needs because we can impose processing requirements. $(\mathcal{P}^{ms})$ provides another reasonable alternative to $(\mathcal{P}^w)$; its objective function value is less than that of $(\mathcal{P}^w)$ since it maximizes a pessimistic estimate. $(\mathcal{P}^w_d)$, $(\mathcal{P}^{la}_d)$, and $(\mathcal{P}^{ms}_d)$ provide a more accurate NPV than $(\mathcal{P}^w)$, $(\mathcal{P}^{la})$, and $(\mathcal{P}^{ms})$, respectively, for cases in which degradation exists.

We consider an operational open pit mine in Africa with a multi-phase pit design, in which a phase corresponds to a sub-region of the pit. A bench is a ledge that forms a single level of operation from which to extract material. A phase-bench consists of all of the material in a specific phase that resides within a predefined vertical distance. We call any phase-bench a block. We first introduce notation for the models, and then we provide the mathematical formulations.

### 4.2.1 Notation

Indices and Sets:

| | |
|---|---|
| $b \in \mathcal{B}$ | blocks $b$ |
| $p \in \mathcal{P}$ | phases $p$, e.g., {2,3,4,5} |
| $n \in \mathcal{N}_b$ | bins in block $b$ |
| $n \in \hat{\mathcal{N}_b}$ | non-waste bins in block $b$ |
| $d \in \mathcal{D}$ | bin destinations {1 = mill, 2 = stockpile, 3 = waste} |
| $d \in \hat{\mathcal{D}}_{bn}$ | set of allowable destinations for bin $n$ in block $b$ |
| $\hat{b} \in \hat{\mathcal{B}}_b$ | blocks that must be mined directly before block $b$ |
| $\check{b} \in \check{\mathcal{B}}_p$ | blocks in phase $p$ |
| $r \in \mathcal{R}$ | resources {1 = mine, 2 = mill} |
| $t \in \mathcal{T}$ | time periods |
| $s \in \mathcal{S}$ | stockpiles |

Parameters:

| | |
|---|---|
| $C^m_{nb}$ | mining cost for bin $n$ in block $b$ ($/ton) |
| $C^p_{nb}$ | revenue generated by bin $n$ in block $b$ ($/ton) |
| $M_{nb}$ | material requiring extraction in bin $n$ of block $b$ (tons) |
| $C^h$ | rehandling cost of removing material from the stockpile ($/ton) |
| $\underline{R}_{rt}, \bar{R}_{rt}$ | minimum, maximum amount of resource $r$ available in time $t$ (tons/yr) |
| $\delta_t$ | discount factor for time period $t$ (fraction) |
| $W$ | maximum number of blocks that can be extracted per phase per year |
| $V_{st}$ | the revenue per ton of metal sent from the stockpile $s$ to the processing plant in time period $t$ ($/ton) [($\mathcal{P}^{ms}$) and ($\mathcal{P}^{ms}_d$) only] |
| $L$ | average revenue provided by the stockpile ($/ton) [($\mathcal{P}^{la}$) and ($\mathcal{P}^{la}_d$) only] |
| $\gamma_{tt'}$ | amount by which material degrades in value having been in the stockpile from time period $t$ to time period $t'$ (fraction) |

Decision Variables:

$y_{nbdt}$ fraction of bin $n$ in block $b$ mined in time $t$ and sent to location $d$ [$(\mathcal{P}^w)$, $(\mathcal{P}^{la})$, $(\mathcal{P}^{ms})$ and $(\mathcal{P}^w_d)$ only]

$\hat{y}_{nbt}$ fraction of bin $n$ from block $b$ in the stockpile in time $t$

$\tilde{y}_{nbt}$ fraction of bin $n$ from block $b$ sent to the processing plant from the stockpile in time $t$

$y_{nbtdl}$ fraction of bin $n$ in block $b$ mined in time $t$, sent to location $d$ and processed in time period $l$ [$(\mathcal{P}^{ms}_d)$ and $(\mathcal{P}^{la}_d)$ only]

$x_{bt}$ 1 if block $b$ has finished being mined by time $t$; 0 otherwise

$i_t$ total amount of ore remaining in the stockpile at the beginning of time period $t$ (ton) [$(\mathcal{P}^{la})$ and $(\mathcal{P}^{la}_d)$ only]

$\hat{i}_t$ total amount of ore sent from the stockpile to processing plant in time period $t$ (ton) [$(\mathcal{P}^{la})$ and $(\mathcal{P}^{la}_d)$ only]

${i'}_{st}$ total amount of ore remaining in the stockpile $s$ at the beginning of time period $t$ (tons) [$(\mathcal{P}^{ms})$ and $(\mathcal{P}^{ms}_d)$ only]

$\hat{i'}_{st}$ total amount of ore sent from the stockpile $s$ to the processing plant in time period $t$ (tons) [$(\mathcal{P}^{ms})$ and $(\mathcal{P}^{ms}_d)$ only]

### 4.2.2 Individual Block Storage: $(\mathcal{P}^w)$

The OPMPS+S formulation with an infinite number of stockpiles is displayed below. This model provides an upper bound on NPV.

$$\max \sum_{b\in\mathcal{B}} \sum_{n\in\mathcal{N}_b} \sum_{t\in\mathcal{T}} \delta_t [C^p_{nb} M_{nb}(y_{nb1t} + \tilde{y}_{nbt}) - \sum_{d\in\mathcal{D}} C^m_{nb} M_{nb} y_{nbdt} - C^h M_{nb} \tilde{y}_{nbt}] \tag{4.1}$$

$$\text{s.t.} \sum_{d\in\mathcal{D}} \sum_{t\in\mathcal{T}} y_{nbdt} \le 1 \qquad \forall b \in \mathcal{B}, n \in \mathcal{N}_b \tag{4.2}$$

$$\sum_{d\in\mathcal{D}} y_{1bdt} = \sum_{d\in\mathcal{D}} y_{nbdt} \qquad \forall b \in \mathcal{B}, n \in \mathcal{N}_b : n \ge 2, t \in \mathcal{T} \tag{4.3}$$

$$x_{bt} \le \sum_{d\in\mathcal{D}} \sum_{t'\le t} y_{nbdt'} \qquad \forall b \in \mathcal{B}, n \in \mathcal{N}_b, t \in \mathcal{T} \tag{4.4}$$

$$x_{bt} \le x_{b,t+1} \qquad \forall b \in \mathcal{B}, t \in \mathcal{T} : t \ne |\mathcal{T}| \tag{4.5}$$

$$\sum_{d\in\mathcal{D}} \sum_{t'\le t} y_{1bdt'} \le x_{\hat{b}t} \qquad \forall b \in \mathcal{B}, \hat{b} \in \hat{\mathcal{B}}_b, t \in \mathcal{T} \tag{4.6}$$

$$\underline{R}_{rt} \leq \sum_{b \in \mathcal{B}} \sum_{n \in \mathcal{N}_b} \sum_{d \in \mathcal{D}} M_{nb} y_{nbdt} \leq \bar{R}_{rt} \qquad \forall t \in \mathcal{T}, r \in \mathcal{R} : r = 1 \tag{4.7}$$

$$\underline{R}_{rt} \leq \sum_{b \in \mathcal{B}} \sum_{n \in \hat{\mathcal{N}}_b} M_{nb}(y_{nb1t} + \tilde{y}_{nbt}) \leq \bar{R}_{rt} \qquad \forall t \in \mathcal{T}, r \in \mathcal{R} : r = 2 \tag{4.8}$$

$$\hat{y}_{nb,t+1} = \hat{y}_{nbt} - \tilde{y}_{nbt} + y_{nb2t} \qquad \forall b \in \mathcal{B}, n \in \mathcal{N}_b, t \in \mathcal{T} : t \neq |\mathcal{T}| \tag{4.9}$$

$$\hat{y}_{nb0} = \tilde{y}_{nb0} = 0 \qquad \forall b \in \mathcal{B}, n \in |\mathcal{N}_b| \tag{4.10}$$

$$\sum_{b \in \check{\mathcal{B}}_p} \sum_{d \in \mathcal{D}} y_{1bdt} \leq W \qquad \forall p \in \mathcal{P}, t \in \mathcal{T} \tag{4.11}$$

$$0 \leq y_{nbdt}, \hat{y}_{nbt}, \tilde{y}_{nbt} \leq 1; \;\; x_{bt} \in \{0,1\} \quad \forall b \in \mathcal{B}, n \in \mathcal{N}_b, d \in \mathcal{D}, t \in \mathcal{T} \tag{4.12}$$

The objective function is the sum of the revenues of bins sent directly to the mill, added to the sum of revenues of blocks sent from the stockpile to the mill, subtracted from the sum of the stockpiling and extraction costs. All terms are multiplied by an appropriate discount rate according to the year, $t$.

Constraint (4.2) ensures that each block is mined only once. The extracted fractions of each bin summed across all destinations and time periods must equal one if the block is extracted completely and less than one if not. Constraint (4.3) precludes selective mining within a block. In other words, different bins in each block are extracted from the ground in equal proportions. Constraint (4.4) turns on the binary variable for fully mined blocks, and Constraint (4.5) ensures that this binary variable retains a value of one to enforce the "by" definition of the variable. Constraint (4.6) enforces mining precedence constraints by ensuring that for each block, all predecessors are completely mined before any amount of the successor block is mined. Constraint (4.7) enforces bounds on mining capacity and Constraint (4.8) enforces bounds on mill production capacity. Constraint (4.9) is an inventory balance constraint ensuring that the amount of material in the stockpile during the current time period is equal to that of the last period plus anything that was added, less anything sent to the mill from the stockpile. Constraint (4.10) sets the initial conditions on inventory. Constraint (4.11) ensures that at most $w$ blocks can be extracted per phase per year. Combined with the precedence Constraints, (4.11) ensures that mining cannot go any deeper than a given number of vertically stacked blocks per year; in our case, this number is three.

### 4.2.3 One Stockpile: ($\mathcal{P}^{la}$)

The formulation with one stockpile and homogeneous mixing follows the same logic as the formulation of ($\mathcal{P}^{w}$), (4.2)-(4.7) and (4.11)-(4.12), with the addition of a stockpile, and requires some new variables (defined in Section 4.2.1) which express stockpile amounts in terms of tons instead of in terms of fractions of the block. This is necessary because different blocks and bins have different tonnages but, in this formulation, they are mixed together. The new objective function is:

$$\max \sum_{b\in\mathcal{B}} \sum_{n\in\mathcal{N}_b} \sum_{t\in\mathcal{T}} (\delta_t C^p_{nb} M_{nb} y_{nb1t} - \sum_{d\in\mathcal{D}} \delta_t C^m_{nb} M_{nb} y_{nbdt}) + \sum_{t\in\mathcal{T}} \delta_t (L - C^h)\hat{i}_t \tag{4.13}$$

Constraint (4.8) is substituted with this constraint:

$$\underline{R}_{rt} \leq \sum_{b\in\mathcal{B}} \sum_{n\in\hat{\mathcal{N}}_b} M_{nb} y_{nb1t} + \hat{i}'_t \leq \bar{R}_{rt} \qquad \forall t \in \mathcal{T}, r \in \mathcal{R} : r = 2 \tag{4.14}$$

Significant changes in constraints include the inventory constraints displayed below:

$$i_{t+1} = i_t + \sum_{b\in\mathcal{B}} \sum_{n\in\mathcal{N}_b} y_{nb2t} - \hat{i}_t \qquad \forall t \in \mathcal{T} : t < |\mathcal{T}| \tag{4.15}$$

$$i_0 = \hat{i}_0 = i_T = 0 \tag{4.16}$$

$$\hat{i}_t \leq i_t \qquad \forall t \in \mathcal{T} : t > 0 \tag{4.17}$$

Also, we add a new mixing constraint:

$$\sum_{b\in\mathcal{B}} \sum_{n\in\mathcal{N}_b} \sum_{t'<t} C^p_{nb} M_{nb} y_{nbt'd} \geq L \sum_{b\in\mathcal{B}} \sum_{n\in\mathcal{N}_b} \sum_{t'<t} M_{nb} y_{nbt'd} \qquad \forall t, t' \in \mathcal{T}, d = 3. \tag{4.18}$$

Constraint (4.18) forces blocks going to the stockpile to have an average revenue of at least $L$.

### 4.2.4 Multiple Stockpiles with Lower Bound Grade: ($\mathcal{P}^{ms}$)

This model uses a similar framework as the formulation in ($\mathcal{P}^w$), (4.2)-(4.7) and (4.11)-(4.12), with the addition of multiple stockpiles, and each stockpile has a predetermined grade. The new objective function is:

$$\max \sum_{b\in\mathcal{B}} \sum_{n\in\mathcal{N}_b} \sum_{t\in\mathcal{T}} (\delta_t C^p_{nb} M_{nb} y_{nb1t} - \sum_{d\in\mathcal{D}} \delta_t C^m_{nb} M_{nb} y_{nbdt}) + \sum_{t\in\mathcal{T}} \sum_{s\in\mathcal{S}} \delta_t (V_{st} - C^h) \hat{i}'_{st} \tag{4.19}$$

Constraint (4.8) is substituted with this constraint:

$$\underline{R}_{rt} \leq \sum_{b\in\mathcal{B}} \sum_{n\in\hat{\mathcal{N}}_b} M_{nb} y_{nb1t} + \sum_{s\in\mathcal{S}} \hat{i}'_{ts} \leq \bar{R}_{rt} \qquad \forall t \in \mathcal{T}, r \in \mathcal{R} : r = 2 \tag{4.20}$$

Significant changes in constraints include inventory, displayed below:

$$i'_{s,t+1} = i'_{st} + \sum_{b\in\mathcal{B}} \sum_{n\in\mathcal{N}_b} y_{nb2t} - \hat{i}'_{st} \qquad \forall t \in \mathcal{T} : t < |\mathcal{T}|, s \in \mathcal{S} \tag{4.21}$$

$$i'_{s0} = \hat{i}'_{s0} = i'_{sT} = V_{s0} = 0 \qquad \forall s \in \mathcal{S} \tag{4.22}$$

$$\hat{i}'_{st} \leq i'_{st} \qquad \forall t \in \mathcal{T} : t > 0, s \in \mathcal{S} \tag{4.23}$$

In general, we can have any number of stockpiles in this model, but since this particular mine has six types of grades or bins (not including waste), the minimum number of stockpiles considered in this paper is six. A couple of the bins have large variations in grade so they are further split in order to make nine, and then fifteen, stockpiles. We did not determine the cut-off grade for this problem, but rather just used the predetermined cut-off grade by the data provider.

### 4.2.5 Individual Block Storage Considering Degradation: ($\mathcal{P}^w_d$)

This model is a reformulation of ($\mathcal{P}^w$) in that it considers degradation in stockpiles. So, the definition of set of destinations is different. The new destination set is:

$d \in \mathcal{D}$ bin location-time period destinations $\{1, ..., |\mathcal{T}| = \text{mill at time period } d\}$
$|\mathcal{T}| + 1 = \text{waste}$

The formulation for this model is as follows:

$$\max \sum_{b\in\mathcal{B}} \sum_{n\in\mathcal{N}_b} \sum_{t\in\mathcal{T}} \sum_{d\in\mathcal{D}} [\delta_d C^p_{nb} M_{nb}(\gamma_{td} y_{nbdt}) - \delta_t C^m_{nb} M_{nb} y_{nbdt}]$$
$$- \sum_{b\in\mathcal{B}} \sum_{n\in\mathcal{N}_b} \sum_{t\in\mathcal{T}} \sum_{d\in\mathcal{D}} \delta_d C^h M_{nb} y_{nbdt}$$

$$\text{s.t.} \sum_{d\in\mathcal{D}} \sum_{t\in\mathcal{T}} y_{nbdt} \le 1 \qquad \forall b \in \mathcal{B}, n \in \mathcal{N}_b \tag{4.24}$$

$$\sum_{d\in\mathcal{D}} y_{1bdt} = \sum_{d\in\mathcal{D}} y_{nbdt} \qquad \forall b \in \mathcal{B}, n \in \mathcal{N}_b : n \ge 2, t \in \mathcal{T} \tag{4.25}$$

$$x_{bt} \le \sum_{d\in\mathcal{D}} \sum_{t'\le t} y_{nbdt'} \qquad \forall b \in \mathcal{B}, n \in \mathcal{N}_b, t \in \mathcal{T} \tag{4.26}$$

$$x_{bt} \le x_{b,t+1} \qquad \forall b \in \mathcal{B}, t \in \mathcal{T} : t \ne |\mathcal{T}| \tag{4.27}$$

$$\sum_{d\in\mathcal{D}} \sum_{t'\le t} y_{1bdt'} \le x_{\hat{b}t} \qquad \forall b \in \mathcal{B}, \hat{b} \in \hat{\mathcal{B}}_b, t \in \mathcal{T} \tag{4.28}$$

$$\underline{R}_{rt} \le \sum_{b\in\mathcal{B}} \sum_{n\in\mathcal{N}_b} \sum_{d\in\mathcal{D}} M_{nb} y_{nbdt} \le \bar{R}_{rt} \qquad \forall t \in \mathcal{T}, r \in \mathcal{R} : r = 1 \tag{4.29}$$

$$\underline{R}_{rd} \le \sum_{b\in\mathcal{B}} \sum_{n\in\acute{\mathcal{N}}_b} \sum_{t\le d} M_{nb} y_{nbdt} \le \bar{R}_{rd} \qquad \forall d \in \mathcal{D} : d \ne |\mathcal{D}|, r \in \mathcal{R} : r = 2 \tag{4.30}$$

$$\sum_{b\in\check{\mathcal{B}}_p} \sum_{d\in\mathcal{D}} y_{1bdt} \le W \qquad \forall p \in \mathcal{P}, t \in \mathcal{T} \tag{4.31}$$

$$0 \le y_{nbdt} \le 1;\ x_{bt} \in \{0, 1\} \quad \forall b \in \mathcal{B}, n \in \mathcal{N}_b, d \in \mathcal{D}, t \in \mathcal{T} \tag{4.32}$$

This model is a reformulation of $(\mathcal{P}^w)$, and besides the extraction time, tracks the processing time of each block. The degradation rate depends on the difference between processing and extraction time. In this model, each destination, with an exception of last one, represents the processing time period. In other words, for each block, the destination should be greater than or equal to the extraction time. The last destination represents waste dump.

### 4.2.6 One Stockpile with Average Grade Considering Degradation: $(\mathcal{P}_d^{la})$

This model is a reformulation of $(\mathcal{P}^{la})$, which considers degradation in the stockpile, and requires using a new variable. In this model we have three destinations, which is defined in Section 4.2.1. The model is as follows:

$$\max \sum_{b\in\mathcal{B}}\sum_{n\in\mathcal{N}_b}\sum_{t\in\mathcal{T}}\sum_{d\in\mathcal{D}}\sum_{l\in\mathcal{T}:l\geq t}\delta_t M_{nb}(C^p_{nb}y_{nbt1l}-C^m_{nb}y_{nbtdl})$$
$$+\sum_{b\in\mathcal{B}}\sum_{n\in\mathcal{N}_b}\sum_{t\in\mathcal{T}}\sum_{l\in\mathcal{T}:l>t}\delta_l M_{nb}(\gamma_{tl}L-C^h)y_{nbt2l}$$

$$\text{s.t.}\sum_{d\in\mathcal{D}}\sum_{t\in\mathcal{T}}\sum_{l\in\mathcal{T}:l\geq t} y_{nbtdl}\leq 1 \qquad \forall b\in\mathcal{B}, n\in\mathcal{N}_b \tag{4.33}$$

$$\sum_{d\in\mathcal{D}} y_{1btdl}=\sum_{d\in\mathcal{D}} y_{nbtdl} \quad \forall b\in\mathcal{B}, n\in\mathcal{N}_b, l\in\mathcal{T}: n\geq 2, t\in\mathcal{T}, l\geq t \tag{4.34}$$

$$x_{bt}\leq\sum_{d\in\mathcal{D}}\sum_{t'\leq t}\sum_{l\in\mathcal{T}:l\geq t} y_{nbt'dl} \qquad \forall b\in\mathcal{B}, n\in\mathcal{N}_b, t\in\mathcal{T} \tag{4.35}$$

$$x_{bt}\leq x_{b,t+1} \qquad \forall b\in\mathcal{B}, t\in\mathcal{T}: t\neq|\mathcal{T}| \tag{4.36}$$

$$\sum_{d\in\mathcal{D}}\sum_{t'\leq t}\sum_{l\in\mathcal{T}:l\geq t} y_{1bt'dl}\leq x_{\hat{b}t} \qquad \forall b\in\mathcal{B}, \hat{b}\in\hat{\mathcal{B}}_b, t\in\mathcal{T} \tag{4.37}$$

$$\underline{R}_{rt}\leq\sum_{b\in\mathcal{B}}\sum_{n\in\mathcal{N}_b}\sum_{d\in\mathcal{D}}\sum_{l\in\mathcal{T}:l\geq t} M_{nb}y_{nbtdl}\leq\bar{R}_{rt} \qquad \forall t\in\mathcal{T}, r\in\mathcal{R}: r=1 \tag{4.38}$$

$$\underline{R}_{rl}\leq\sum_{b\in\mathcal{B}}\sum_{n\in\mathcal{N}_b}\sum_{t\in\mathcal{T}:l\geq t}\sum_{d\in\mathcal{D},d\neq 3} M_{nb}y_{nbtdl}\leq\bar{R}_{rl} \quad \forall l\in\mathcal{T}, r\in\mathcal{R}: r=2 \tag{4.39}$$

$$\sum_{b\in\bar{\mathcal{B}}_p}\sum_{d\in\mathcal{D}}\sum_{l\in\mathcal{T}:l\geq t} y_{1btdl}\leq W \qquad \forall p\in\mathcal{P}, t\in\mathcal{T} \tag{4.40}$$

$$\sum_{b\in\mathcal{B}}\sum_{n\in\mathcal{N}_b}\sum_{t'\leq t}\sum_{l\in\mathcal{T}}\gamma_{t'l}C^p_{nb}M_{nb}x_{nbt'dl}\geq L\sum_{b\in\mathcal{B}}\sum_{n\in\mathcal{N}_b}\sum_{t'\leq t}\sum_{l\in\mathcal{T}}\gamma_{t'l}M_{nb}y_{nbt'dl} \qquad \forall t\in\mathcal{T}, d=2 \tag{4.41}$$

$$0\leq y_{nbtdl}\leq 1;\; x_{bt}\in\{0,1\} \quad \forall b\in\mathcal{B}, n\in\mathcal{N}_b, d\in\mathcal{D}, t\in\mathcal{T}, l\in\mathcal{T} \tag{4.42}$$

There are three destinations in this model, including mill, stockpile and waste. Similar to $(\mathcal{P}_d^w)$, we track the processing time of each block, and consider degradation of the material going to the stockpile. Constraint (4.41) forces blocks going to the stockpile to have an average revenue of at least $L$. The value of the material within the stockpile decreases depending the difference between extraction and processing time.

### 4.2.7 Multiple Stockpiles with Lower Bound Grade Considering Degradation ($\mathcal{P}_d^{ms}$)

This model is a reformulation of ($\mathcal{P}^{ms}$), which considers degradation in the stockpile, and requires using a new variable. We need to define a new destination set for this model:

$$d \in \mathcal{D} \text{ bin destinations } \{1 = \text{mill}, 2 = \text{waste}, 3, \cdots, 3 + |\mathcal{S}| - 1 = \text{stockpiles}\}$$

The formulation for this model is as follows:

$$\begin{aligned}
\max & \sum_{b \in \mathcal{B}} \sum_{n \in \mathcal{N}_b} \sum_{t \in \mathcal{T}} \sum_{d \in \hat{\mathcal{D}}_{bn}} \Big( \sum_{l \in \mathcal{T}: l = t} \delta_t C^p_{nb} M_{nb} y_{nbtdl} - \sum_{l \in \mathcal{T}: l \geq t} \delta_t C^m_{nb} M_{nb} y_{nbtdl} \Big) \\
& + \sum_{b \in \mathcal{B}} \sum_{n \in \mathcal{N}_b} \sum_{t \in \mathcal{T}} \sum_{d \in \hat{\mathcal{D}}_{bn}: d > 2} \sum_{l \in \mathcal{T}: l > t} \delta_l M_{nb} (\gamma_{tl} V_{st} - C^h) y_{nbtdl}
\end{aligned}$$

$$\text{s.t.} \sum_{d \in \hat{\mathcal{D}}_{bn}} \sum_{t \in \mathcal{T}} \sum_{l \in \mathcal{T}: l \geq t} y_{nbtdl} \leq 1 \qquad \forall b \in \mathcal{B}, n \in \mathcal{N}_b \tag{4.43}$$

$$\sum_{d \in \hat{\mathcal{D}}_{bn}} y_{1btdl} = \sum_{d \in \hat{\mathcal{D}}_{bn}} y_{nbtdl} \qquad \forall b \in \mathcal{B}, n \in \mathcal{N}_b, l \in \mathcal{T} : n \geq 2, t \in \mathcal{T}, l \geq t \tag{4.44}$$

$$x_{bt} \leq \sum_{d \in \hat{\mathcal{D}}_{bn}} \sum_{t' \leq t} \sum_{l \in \mathcal{T}: l \geq t} y_{nbt'dl} \qquad \forall b \in \mathcal{B}, n \in \mathcal{N}_b, t \in \mathcal{T} \tag{4.45}$$

$$x_{bt} \leq x_{b,t+1} \qquad \forall b \in \mathcal{B}, t \in \mathcal{T} : t \neq |\mathcal{T}| \tag{4.46}$$

$$\sum_{d \in \hat{\mathcal{D}}_{bn}} \sum_{t' \leq t} \sum_{l \in \mathcal{T}: l \geq t} y_{1bt'dl} \leq x_{\hat{b}t} \qquad \forall b \in \mathcal{B}, \hat{b} \in \hat{\mathcal{B}}_b, t \in \mathcal{T} \tag{4.47}$$

$$\underline{R}_{rt} \leq \sum_{b \in \mathcal{B}} \sum_{n \in \mathcal{N}_b} \sum_{d \in \hat{\mathcal{D}}_{bn}} \sum_{l \in \mathcal{T}: l \geq t} M_{nb} y_{nbtdl} \leq \bar{R}_{rt} \qquad \forall t \in \mathcal{T}, r \in \mathcal{R} : r = 1 \tag{4.48}$$

$$\underline{R}_{rl} \leq \sum_{b \in \mathcal{B}} \sum_{n \in \mathcal{N}_b} \sum_{t \in \mathcal{T}: l \geq t} \sum_{d \in \hat{\mathcal{D}}_{bn}, d \neq 2} M_{nb} y_{nbtdl} \leq \bar{R}_{rl} \quad \forall l \in \mathcal{T}, r \in \mathcal{R} : r = 2 \tag{4.49}$$

$$\sum_{b \in \check{\mathcal{B}}_p} \sum_{d \in \hat{\mathcal{D}}_{bn}} \sum_{l \in \mathcal{T}: l \geq t} y_{1btdl} \leq W \qquad \forall p \in \mathcal{P}, t \in \mathcal{T} \tag{4.50}$$

$$0 \leq y_{nbtdl} \leq 1; \; x_{bt} \in \{0, 1\} \quad \forall b \in \mathcal{B}, n \in \mathcal{N}_b, d \in \hat{\mathcal{D}}_{bn}, t \in \mathcal{T}, l \in \mathcal{T} \tag{4.51}$$

In this model, the first destination represents the mill, the second one indicates the waste dump, and the remainder of the destinations represent multiple stockpiles. Since degradation depends on the time that the material stays within the stockpile, we track the processing time of each block.

## 4.3 Computations

All aforementioned models are coded in AMPL, and are solved using CPLEX (2016). The machine characteristics follow: 1TB hard drive, Intel Xeon processor, and 24GB of RAM, operating under the Linux environment. The optimality gap is set to 0.5%. We first introduce the data set.

### 4.3.1 Data

Our data set consists of 336 blocks. There are seven types of bins, some subset of which are contained in each block, including high-, medium-, and low-grade bins for two material types based on processing properties, and a waste bin. The blocks range in weight from 20,000 to 5,500,000 tons, and there are 1,312 bin-block combinations ranging from 250 to 1,100,000 tons. As a block is mined, equal proportions must be taken from each bin because the grades are not separate in their natural state. The blocks are contained in four phases, numbered 2-5, since the first phase is presented as fully extracted in the data. Mining capacity is 50 million tons per year and the mill capacity is 8 million tons per year. The discount factor is 9% per year.

### 4.3.2 Results

The resulting NPVs are displayed in the following tables and figures. The objective function values of all instances are the same for Phase 2, because we have extra mill capacity and therefore there is no stockpiling, even though it is allowed. Since this particular mine has six types of bins differentiated by grade and geological properties, the minimum number of stockpiles for ($\mathcal{P}^{ms}$) is six, but the result of ($\mathcal{P}^{ms}$) with six stockpiles returns the same solutions as a model without stockpiles, because there is a wide grade range for each material type, implying that high-grade material in the stockpile comes out having a much lower grade; hence, stockpiles are not used. For more explanation of a no stockpiling model, see ($\mathcal{P}^{ns}$) [44]. Rezakhah et al. [58] note that ($\mathcal{P}^{la}$) is unimodal with respect to the average grade in the stockpile, so it is possible to find the optimized average grade; we observe that the corresponding objective function values, for two and three phases, differ from that of ($\mathcal{P}^{w}$) by 0.05% and 0.2%, respectively. To solve ($\mathcal{P}^{ms}$), we consider nine and fifteen stockpiles to provide a finer level of detail with respect to grade. The objective function values of ($\mathcal{P}^{w}$) and ($\mathcal{P}^{ms}$) using nine stockpiles for two and three phases differ by 0.3% and 1%, respectively, and by using fifteen stockpiles in ($\mathcal{P}^{ms}$), the associated results differ by 0.1% and 0.3%, respectively. These results show that by increasing the number of stockpiles, the objective function values approach the upper bound of the problem given by ($\mathcal{P}^{w}$). We observe the same trend for the corresponding models with degradation. Table 4.1 shows the resulting NPVs.

Table 4.1: NPV for Phases 2, 3, 4 (M$). Optimality gap: 0.5%, annual degradation rate: 10%. Note that ($\mathcal{P}^{ms}$) has 15 stockpiles.

| Mill Cap (Mt) | ($\mathcal{P}^{ns}$) | ($\mathcal{P}^{w}$) | ($\mathcal{P}^{la}$) | ($\mathcal{P}^{ms}$) | ($\mathcal{P}^{w}_{d}$) | ($\mathcal{P}^{la}_{d}$) | ($\mathcal{P}^{ms}_{d}$) |
|---|---|---|---|---|---|---|---|
| 10 | 1,653.92 | 1,687.87 | 1,685.25 | 1,684.68 | 1,673.52 | 1,672.35 | 1,672.94 |
| 9 | 1,619.56 | 1,668.10 | 1,665.23 | 1,662.98 | 1,642.49 | 1,640.42 | 1,641.04 |
| 8 | 1,575.51 | 1,639.40 | 1,635.35 | 1,633.75 | 1,601.84 | 1,600.87 | 1,600.03 |
| 7 | 1,520.73 | 1,600.11 | 1,595.32 | 1,592.33 | 1,542.99 | 1,543.54 | 1,543.96 |
| 6 | 1,441.10 | 1,540.64 | 1,534.12 | 1,531.21 | 1,472.00 | 1,470.41 | 1,469.03 |
| 5 | 1,336.04 | 1,449.97 | 1,442.21 | 1,437.02 | 1,362.31 | 1,361.23 | 1,362.54 |
| 4 | 1,211.65 | 1,313.64 | 1,305.95 | 1,299.76 | 1,234.35 | 1,232.64 | 1,231.67 |

Changing the maximum mill production capacity illustrates that there is some trade-off between stockpiling and this parameter. As the milling capacity decreases relative to mining capacity, it is no longer possible to process the extracted material in the same time period, and therefore stockpiling increases NPV.

The value of stockpiling increases as deeper phases of the mine are considered. Figure 4.2 illustrates the trade-off between stockpiling and mill capacity for Phase 2, 3 and 4. The value of stockpiling in ($\mathcal{P}^w$) is 3.9% at the current milling capacity of 8 million tons per years, but this value increases to 7.8% as the milling capacity decreases to half of its original value.

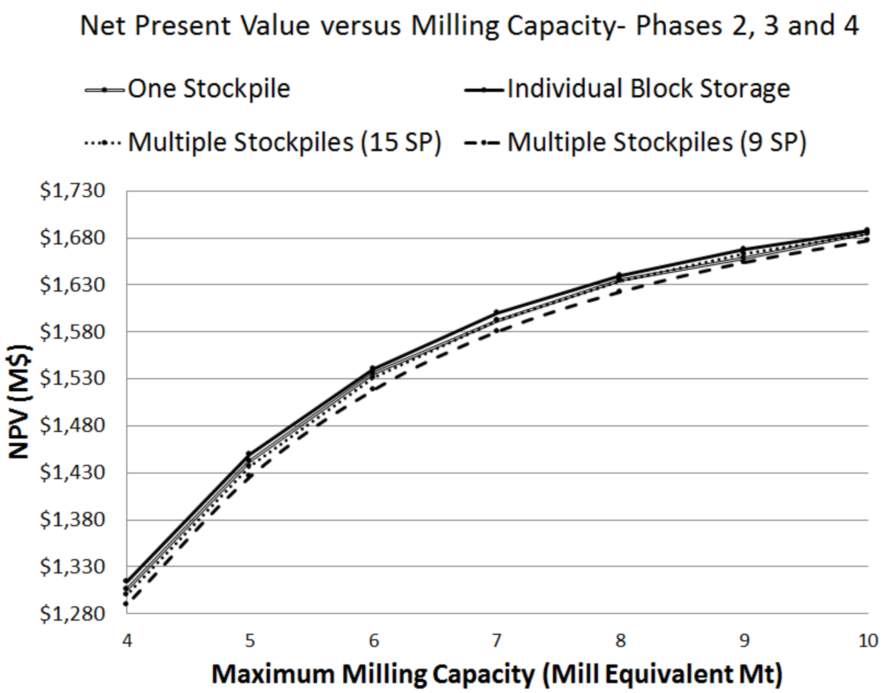


Figure 4.2: NPV vs Mill Capacity for Phases 2, 3, and 4 for the individual block storage model, ($\mathcal{P}^w$), one stockpile, ($\mathcal{P}^{la}$), and multiple stockpiles ($\mathcal{P}^{ms}$) without considering degradation

Figure 4.2 shows the objective function values for three models which do not consider degradation within the stockpile. The objective function values of ($\mathcal{P}^{la}$) and ($\mathcal{P}^{ms}$) for the current mill capacity differ by 0.6% and 0.4% from the upper bound problem, ($\mathcal{P}^w$), respectively, and, because they model a finite number of stockpiles, provide an operationally realistic solution to OPMPS+S. With three phases, the difference between ($\mathcal{P}^{la}$) and ($\mathcal{P}^{ms}$) is 0.57% at the current milling capacity of 8 million tons per year, but this difference increases to 1% as the milling capacity decreases to 4 million tons per year. This result means that considering the minimum grade in fifteen stockpiles provides a lower NPV than considering one stockpile with an optimized average grade for this data set. By increasing the number of stockpiles in ($\mathcal{P}^{la}$), we expect to achieve an even higher NPV.

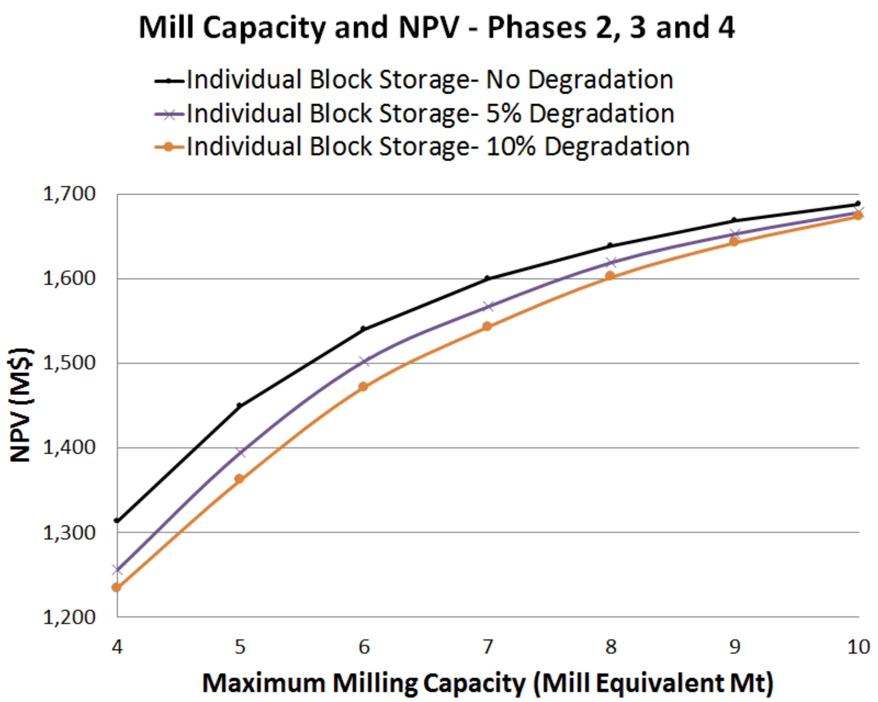


Figure 4.3: NPV vs Mill Capacity for Phases 2, 3 and 4 for the individual block storage model, ($\mathcal{P}^{w}$), with and without degradation.

Figure 4.3 compares objective function values for different degradation rates in the stockpile(s) for ($\mathcal{P}^{w}_{d}$). This figure indicates that an increased degradation rate results in decreased NPV. If we think of the value of a stockpile in each model as the difference between the NPV provided by that model and that of the no stockpiling model, for the original mill capacity, the stockpiling value in ($\mathcal{P}^{w}_{d}$) by considering 5% and 10% annual degradation decreases by 32% and 59%, respectively (Figure 4.3). Also, we observe that those numbers for ($\mathcal{P}^{la}_{d}$) and ($\mathcal{P}^{ms}_{d}$) are 35% and 62%, and 15% and 49%, respectively. These results indicate that the value of stockpiling decreases considerably due to material degradation in the stockpile(s).

Figure 4.3 shows that the difference between ($\mathcal{P}^{w}$) and ($\mathcal{P}^{w}_{d}$) with 10% annual degradation for the material in stockpiles is 2.29% at the current milling capacity of 8 million tons per

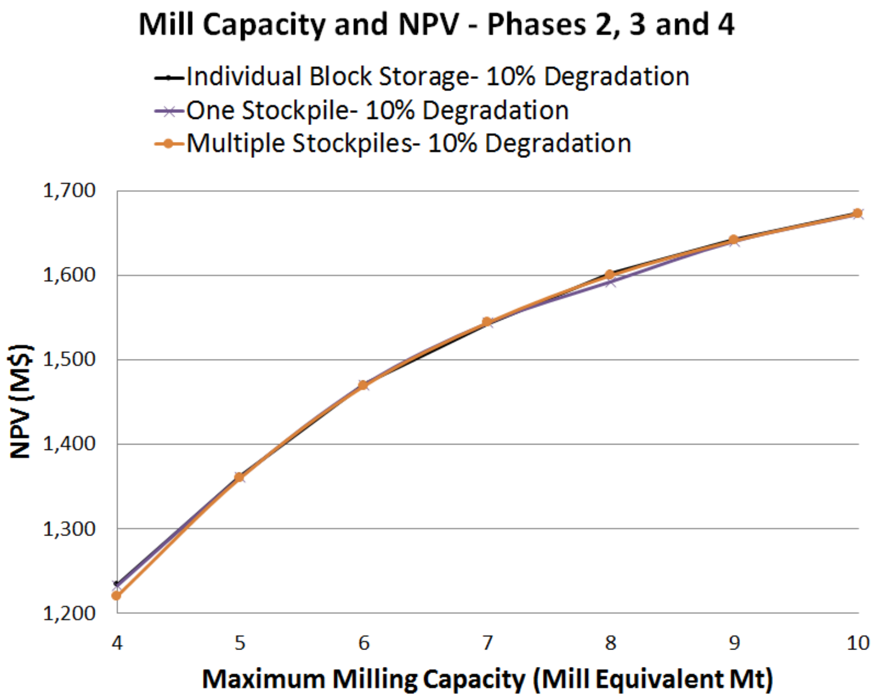


Figure 4.4: NPV vs Mill Capacity for Phases 2, 3 and 4 for individual block stockpiles, $(\mathcal{P}^{w})$, one stockpile, $(\mathcal{P}^{la})$, and multiple stockpiles $(\mathcal{P}^{ms})$ with 10% degradation.

year, but this difference increases to 4.06% as the milling capacity decreases to half of its original value. Also, we observe that the difference between $(\mathcal{P}^{la})$ and $(\mathcal{P}^{la}_d)$ with 10% annual degradation for the material in stockpiles, is 2.15% at the current milling capacity of 8 million tons per year, but this difference increases to 6.20% as the milling capacity decreases to half of its original value. Finally, we detect that with 10% annual degradation for the material in 15 stockpiles, the difference between objective function value of $(\mathcal{P}^{ms})$ and $(\mathcal{P}^{ms}_d)$ at 8 million tons per year of milling capacity is 2.06%, but this difference increases to 5.24% as the milling capacity decreases to 4 million tonnes per year. Figure 4.4 compares the objective function values of $(\mathcal{P}^{w}_d)$ with $(\mathcal{P}^{la}_d)$ and $(\mathcal{P}^{ms}_d)$, and shows that for 8 million tons of mill capacity, the difference is 0.06% and 0.11%, respectively. Also, for 4 million tons of mill capacity, those numbers are 0.38% and 1.14%, respectively. These small differences indicate that $(\mathcal{P}^{w}_d)$ is a very good estimate of $(\mathcal{P}^{la}_d)$ and $(\mathcal{P}^{ms}_d)$ in terms of objective function value.

## 4.4 Conclusion

As illustrated in previous sections, the value of stockpiling increases as the milling capacity decreases relative to the mining capacity, and as deeper phases of the mine are considered. The upper bound on the value of stockpiling, is as high as 7.9% for the open pit mine considered here. While the largest difference with the current milling capacity is only 3.9%, this is expected to increase when the next phase is considered.

In the open pit mine being considered, the largest difference between and the NPV of ($\mathcal{P}^w$) and ($\mathcal{P}^{la}$) is 0.59%, but this difference decreases to 0.25% for the current mill capacity. Also, the difference between ($\mathcal{P}^w$) and ($\mathcal{P}^{ms}$) with 9 stockpiles is 1.9%, but with the current milling capacity, it is only 1.2%. ($\mathcal{P}^{ms}$) with 15 stockpiles returns an NPV closer to that of ($\mathcal{P}^w$) with the largest difference being 1.1% and a difference of only 0.5% for the current milling capacity. These results indicate that without considering degradation, ($\mathcal{P}^w$) provides a reasonable estimate of the NPV for the operation considered here, although this may not be true for other operations.

The final goal of this research is to find the effect of different degradation rates on sending material from the mine to the stockpile and also on NPV. Comparing objective function values of ($\mathcal{P}^w$) and ($\mathcal{P}^w_d$) for 5% and 10% annual degradation indicates that the value of stockpiling decreases by 32% and 59%, respectively. So, it is important to consider degradation due to stockpiling when solving the OPMPS+S problem. Finally, we observe that by considering degradation, ($\mathcal{P}^{la}_d$) and ($\mathcal{P}^{ms}_d$) provide a solution very close to that of ($\mathcal{P}^w_d$) for the operation. Finally, we conclude that ($\mathcal{P}^{la}_d$), which is relatively easy to solve, can be used to provides a reasonable estimate of the NPV, although this may not be true for other operations.

# CHAPTER 5
# CONCLUSION

This dissertation shows that operations research techniques yield solutions that provide better objective values and solve faster than existing and intuitive methods in mining operations. Our results highlight the benefit of using operations research techniques to solve open pit mine production scheduling with stockpiling and indicate that larger, more complicated applications can be examined with these techniques. Traditionally, optimization models that have focused on surface mining either do not consider stockpiling or present nonlinear-integer models which are difficult to solve. We present a new linear-integer program to consider stockpiling as part of open pit mine planning. We show the flexibility of the formulation and its value to mining companies.

## Research Contributions

We present a formulation for solving a good approximation to the OPMPS+S that has fast solution times when using recently developed algorithms. We demonstrate the flexibility of the formulation and its value to mining companies. The contributions of this research follow:

- We develop a new and reasonably exact linear-integer program to solve the OPMPS+S, which is tractable even for large data sets.
- We extend the proposed model in (i) to blend material with multiple grades and a contaminant in the stockpile. We show that the provided NPV by our model for an operational mine is considerably higher than that provided by state-of-the-art software.
- We develop multiple linear-integer models that consider degradation due to stockpiling, and explain the impact of material degradation on stockpiling value.

In the first paper, we develop a linear-integer programming model to solve the OPMPS+S. This model is tractable and is capable of solving OPMPS+S for large data sets in a few minutes. Existing nonlinear-integer models are intractable. We show that our model provides a solutions that are close approximations of those for an existing nonlinear-integer model. In the second paper, we extend the model presented in the first to solve the OPMPS+S for an operational mine in the south east of Asia in which the material includes multiple metals and a contaminant. We solve this model and show that the NPV provided by this model is at least 18.7% higher than that of provided by state-of-the-art software. Here, we have one stockpile and observe that LP relaxation is unimodal regarding blending criteria in the stockpile. The unimodal property for multiple stockpiles needs to be examined. In the third paper, we develop three linear-integer programming models to solve OPMPS+S considering degradation due to stockpiling, compare them, and examine the effect of degradation on the value of stockpiling. We conclude that material degradation should be considered as part of OPMPS+S.

**Suggested Further Research**

Improvements to our model could include considering stochastic characterization of block's metal grades and selling prices. Our methodology only applies a deterministic grade and metal prices, which may result in solutions that are not optimal when considering the fluctuations of block's grade and metal prices. Creating an accurate stochastic model for OPMPS+S represents a significant advance in risk mitigation. A two-stage stochastic model would include: (i) what material to extract, and (ii) to what destination the material is sent based on the realization of the grade and metal price.

Another interesting aspect of open pit mine planning with stockpiling is to consider the capital cost of a processing plant, since there is a trade-off between processing capacity and the value that a stockpile provides.

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