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Universidade Federal do Rio Grande do Sul
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A Comprehensive Approach to Strategic Open Pit Mine Planning with Stockpile Consideration

Mohammad Tabesh and Hooman Askari-Nasab
Mining Optimization Laboratory, University of Alberta, Edmonton, AB, Canada

Rodrigo Peroni
Departamento de Engenharia de Minas, Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil

ABSTRACT: Strategic mine planning is a complicated process that is usually broken down into smaller problems in order to get more practical solutions in shorter time. In this paper, we present a multi-step approach that starts form a pushback design procedure that is able to create pushbacks with controlled ore and waste tonnage. This is a hybrid algorithm that incorporates mathematical programming with heuristics to tackle with large-scale problems. It benefits from the special structures of the mathematical formulation to calculate relaxation bounds and uses heuristics to provide near-optimal solutions. Next, we propose a hierarchical clustering algorithm that creates mining units with minable shapes and homogeneity in rock type and grade within the boundaries of generated pushbacks. The third step of our solution procedure is to form a mathematical model that provides a long-term mining schedule based on the generated mining units as well as the created pushbacks. Our proposed formulation considers various mining and processing constraints and is able to include stockpiling in long-term plans to improve the blending. Finally, we use the idea of piecewise linearization to modify the model to be able to solve it with mixed integer linear programming solvers.

INTRODUCTION

Open-pit mining is the most common and the oldest method of mining valuable material from the ground. It has attracted many researchers to study various aspects of the operation such as production planning, truck-shovel allocations, risk analysis and grade blending. Various heuristic, meta-heuristic and mathematical programming techniques have been implemented on these areas to improve the operation. Their goals are to maximize profit, minimize costs and to optimize the utilization of the resources or the outcome of the mining operation. The mine planning problem has also been studied in different time frames. Long- to short-term plans are usually determined based on different levels of details. Long-term plans usually deal with larger units of production and decide when to extract material and where to send them. Short-term plans, on the other hand, deal with smaller units and make more detailed decisions on the production levels, blending, truck-shovel allocations etc. In this paper, we present a multi-step hybrid approach to deal with the long-term multi-destination open-pit production planning problem by creating controlled push-backs and aggregated mining units and using mathematical programming to solve the problem.

We incorporate the blending constraints and stockpiling in the long-term mine planning decisions to improve the operation and help the mine planners decide if they want to use stockpiles in the operation.

Mathematical programming is not new to mine planning researchers. Johnson (1969) introduced mathematical programming and in particular linear programming to the mine planning research area. He proposed a linear programming model for the long-term multi-destination open-pit production planning problem along with a decomposition approach to solve the problem. However, this initial model was using continuous variables to control precedence constraints which would result in partial extraction of blocks and infeasible solutions (Gershon 1983). Although, introducing binary variables to control the block extraction precedence can solve this problem, it will create another obstacle on the way: curse of dimensionality. In other words, introducing binary variables will make the problem NP-Hard and impossible to solve for real size block models. Therefore, the focus of mine planners in the past few decades has been on breaking the problem into smaller problems, reducing the size of the problem or finding near-optimal solutions to the problem. Interested readers are referred to Osanloo et. al (2008) and Newman et. al (2010) for a complete review on the applications of operations research and mathematical programming on the mine planning problem. On the other hand, most of the proposed models incorporate mining, processing and precedence constraints and do not include grade blending and stockpiling constraints.

PUSHBACK DESIGN

Pushback design is an important step in LTOPP in which the phases of production, pushbacks, are determined. The intersection of pushbacks and mining benches are called bench-phases. These are the most common units of long-term planning in open-pit mines. From manual methods such as fixed lead to more advanced heuristics such as Milawa (Geovia 2012) use the bench-phases to optimize the long-term open-pit production plan. Therefore, how the pushbacks are defined can significantly affect the output. In this paper, we are using a hybrid heuristic-binary programming approach from Mieth (2012) to create the pushbacks. The pushback design procedure is explained in details in Mieth (2012) and Tabesh et al. (2014). The generated bench-phases are then used as units of mining and as boundaries for clustering.

CLUSTERING

Clustering is the process of grouping similar objects together in a way that maximizes the similarity between the objects of the same cluster and the dissimilarity between the objects of different clusters. However, the clustering algorithm we used here not only accounts for the similarities but also respects the size and shape constraints. The clustering algorithm mentioned is a variation of hierarchical agglomerative clustering and is thoroughly explained in Tabesh and Askari-Nasab (2011) and Tabesh and Askari-Nasab (2013). We use the clustering algorithm to create processing units within the boundaries of bench-phases. Therefore, the bench-phases are divided into smaller units with similar rock type and grade which are the basis for making processing and stockpiling decisions.

MATHEMATICAL FORMULATION

As mentioned earlier, various LTOPP mathematical models have been proposed in the literature. However, none of them incorporate stockpiling in long-term planning. One major reason is that calculating the reclamation grade of the stockpiles introduces non-linearity into the model. Bley et al. (2014) model the LTOPP with stockpiling by adding the non-linear constraints and proposing a problem-specific solution method. In this paper, we tried to avoid the non-linear constraints by benefiting from the piecewise linearization technique. We introduce multiple stockpiles with different acceptable grades to be able to assign fixed reclamation grades to each stockpile. These input grade ranges as well as reclamation grades are determined based on histograms of grades to be representative of data.

ORIGINAL MODEL

We first present the original LTOPP mathematical model without the stockpile. The model is a multi-destination LTOPP which uses two different sets of units for making mining and processing decisions. Two sets of variables are defined for bench-phases: $y_m^t \in [0,1]$ is the portion of bench-phase extracted in each period and $b_m^t \in \{0,1\}$ is the binary variable to control the precedence. Since the number of bench-phases is less than number of blocks and clusters, controlling the precedence with bench-phases results in less binary variables and less resource consumption for solving the model. However, making material destination decisions requires more accurate units with distinction between ore and waste. This is achieved by dividing every bench-phase into smaller units using clustering algorithm.

Sets

S^m For each bench-phase m, there is a set of bench-phases (S^m) that have to be extracted prior to extracting bench-phase m to respect slope and precedence constraints

U^m Each bench-phase m is divided into a set of clusters. U^m is the set of clusters that are contained in bench-phase m

Indices

$d \in \{1,\ldots,D\}$ Index for material destinations
$m \in \{1,\ldots,M\}$ Index for bench-phases
$p \in \{1,\ldots,P\}$ Index for clusters
$c \in \{1,\ldots,C\}$ Index for processing plants
$e \in \{1,\ldots,E\}$ Index for elements
$t \in \{1,\ldots,T\}$ Index for scheduling periods

Parameters

D Number of material destinations (including processing plants and waste dumps)
M Total number of bench-phases
P Total number of clusters
E Number of elements in the block model
T Number of scheduling periods
$\overline{MC}^t$ Upper bound on the mining capacity in period t
$\underline{MC}^t$ Lower bound on the mining capacity in period t
$\overline{PC}_c^t$ Maximum tonnage allowed to be sent to plant c in period t
$\underline{PC}_c^t$ Minmum tonnage allowed to be sent to plant c in period t
$\overline{G}_c^{t,e}$ Upper limit on the allowable average grade of element e at processing plant c in period t
$\underline{G}_c^{t,e}$ Lower limit on the allowable average grade of element e at processing plant c in period t

s_m Number of predecessors of bench-phase m (members of S^m)
o_m Total ore tonnage in bench-phase m
w_m Total waste tonnage in bench-phase m
o_p Total waste tonnage in cluster p
w_p Total waste tonnage in cluster p
c^t_m Unit discounted cost of mining material from bench-phase m in period t
$r^t_{p,c}$ Unit discounted revenu of sending material from processing unit p to processing destination p in period t minus the processing costs
g^e_p Average grade of element e in cluster p

Decision Variables

$y^t_m \in [0,1]$ Continous decision variable representing the portion of bench-phase m extracted in period t
$x^t_{p,c} \in [0,1]$ Continous decision variable representing the portion of ore tonnage in cluster p extracted in period t and sent to processing plant c
$b^t_m \in \{0,1\}$ Binary decision variable indicating if all the predecessors of bench-phase m are completely extracted by or in period t

Objective Function

$$\max \sum_{t=1}^{T}\left(\sum_{p=1}^{P}\sum_{c=1}^{C}(r^t_{p,c} \times o_p \times x^t_{p,c}) - \sum_{m=1}^{M}(c^t_m \times (o_m + w_m) \times y^t_m)\right) \tag{1}$$

Constraints

$$\underline{MC}^t \leq \sum_{m=1}^{M}((o_m + w_m) \times y^t_m) \leq \overline{MC}^t \qquad \forall t \in \{1, ..., T\} \tag{2}$$

$$\underline{PC}^t_c \leq \sum_{p=1}^{P}\left(o_p \times x^t_{p,c}\right) \leq \overline{PC}^t_c \qquad \forall t \in \{1, ..., T\}, \forall c \in \{1, ..., C\} \tag{3}$$

$$\sum_{p \in U^m}\sum_{d=1}^{D}\left(o_p \times x^t_{p,d}\right) \leq (o_m + w_m) \times y^t_m \qquad \forall t \in \{1, ..., T\}, \forall m \in \{1, ..., M\} \tag{4}$$

$$\underline{G}^{t,e}_c \leq \frac{\sum_{p=1}^{P}\left(o_p \times g^e_p \times x^t_{p,c}\right)}{\sum_{p=1}^{P}\left(o_p \times x^t_{p,c}\right)} \leq \overline{G}^{t,e}_c \qquad \forall t \in \{1, ..., T\}, \forall c \in \{1, ..., C\}, \forall e \in \{1, ..., E\} \tag{5}$$

$$\sum_{t=1}^{T} y^t_m = 1 \qquad \forall m \in \{1, ..., M\} \tag{6}$$

$$\sum_{i=1}^{t} y^i_m \leq b^t_m \qquad \forall m \in \{1, ..., M\}, \forall t \in \{1, ..., T\} \tag{7}$$

$$s_m \times b^t_m \leq \sum_{i \in S^m}\sum_{j=1}^{t} y^j_i \qquad \forall m \in \{1, ..., M\}, \forall t \in \{1, ..., T\} \tag{8}$$

$$b^t_m \leq b^{t+1}_m \qquad \forall m \in \{1, ..., M\}, \forall t \in \{1, ..., T-1\} \tag{9}$$

The objective function (Equation (1)) is summation of discounted revenue made from sending material to the processing plants minus the total cost of mining material from the ground. Equations (2) and (3) are responsible for controlling the minimum and maximum extraction and processing capacity in each period. Equation (4) controls the relation between the tonnage mined from each bench-phase and the tonnage processed from the clusters within that bench-phase. Note that the difference between the tonnage extracted and the tonnage processed is the waste extracted and sent to the waste dump. However, if we have a waste dump with an extra haulage cost the dump can be defined as a destination with negative revenue. Equation (5) controls the average head grade of material sent to processing plants in each period. However, to avoid non-linearity the equations are rearranged before putting into matrix format. Equation (6) ensures that all the material within the ultimate pit is extracted during mine life. Equations (7) to (9) are the precedence control constraints with the binary variables.

NON-LINEAR MODEL

We can modify the original LTOPP model to account for stockpiling by adding stockpiles as material destinations and introducing $f_c^t \geq 0$ variables. These variables are the tonnages reclaimed from the stockpile and sent to processing plants in each period. The stockpile is added as a destination with the index of c'. In addition, we have to introduce another set of variables $G^{t,e}$ as the reclamation grade of element *e* in period *t*. A new set of parameters is also required; $r_c^{t,e}$ is the unit discounted revenue of processing one unit of element *e* from stockpile in processing destination *c* in period *t* minus the processing and rehandling costs. Accordingly, we can rewrite the LTOPP model be replacing Equations (1), (3) and (5) with Equations (10), (11) and (12) respectively. Note that the objective function is not linear anymore as $G^{t,e}$ is a variable calculated for each element in each period. Moreover, we have to add a constraint for calculating $G^{t,e}$ as in Equation (13) which has a nonlinear term. Equation (14) ensures that the summation of tonnages reclaimed from stockpile from the first period to the current period does not exceed the summation of tonnages sent to the stockpile by the current period.

$$\max \sum_{t=1}^{T}\left(\sum_{p=1}^{P}\sum_{c=1}^{C}(r_{p,c}^{t} \times o_p \times x_{p,c}^{t}) - \sum_{m=1}^{M}(c_m^t \times (o_m + w_m) \times y_m^t) + \sum_{e=1}^{E}\sum_{c=1}^{C}(f_c^t \times G^{t,e} \times r_c^{t,e})\right) \quad \textbf{(10)}$$

$$\underline{PC}_c^t \leq \sum_{p=1}^{P}\left(o_p \times x_{p,c}^t\right) + f_c^t \leq \overline{PC}_c^t \qquad \forall t \in \{1,\dots,T\}, \forall c \in \{1,\dots,C\} \quad \textbf{(11)}$$

$$\underline{G}_c^{t,e} \leq \frac{\sum_{p=1}^{P}\left(o_p \times g_p^e \times x_{p,c}^t\right) + f_c^t \times G^{t,e}}{\sum_{p=1}^{P}\left(o_p \times x_{p,c}^t\right) + f_c^t} \leq \overline{G}_c^{t,e} \qquad \forall t \in \{1,\dots,T\}, \forall c \in \{1,\dots,C\}, \forall e \in \{1,\dots,E\} \quad \textbf{(12)}$$

$$G^{t,e} = \frac{\sum_{p=1}^{P}\left(o_p \times g_p^e \times x_{p,c'}^t\right) - \sum_{t'=1}^{t-1}\sum_{c=1}^{C} f_c^{t'} \times G^{t',e}}{\sum_{p=1}^{P}\left(o_p \times x_{p,c'}^t\right) + \sum_{t'=1}^{t-1}\sum_{c=1}^{C} f_c^{t'}} \qquad \forall t \in \{1,\dots,T\}, \forall e \in \{1,\dots,E\} \quad \textbf{(13)}$$

$$\sum_{t'=1}^{t}\sum_{c=1}^{C} f_c^{t'} \leq \sum_{t'=1}^{t-1}\sum_{p=1}^{P}\left(o_p \times x_{p,c'}^{t'}\right) \qquad \forall t \in \{2,\ldots,T\} \tag{14}$$

LINEARIZED MODEL

In order to have a linear LTOPP model with stockpiling, we assume that there are multiple stockpiles with tight ranges for the acceptable element grades. Therefore, we can assign an average reclamation grade and the corresponding reclamation revenue to each stockpile. The more stockpiles defined the smaller error is introduced into the model. However, more stockpiles sacrifices the complete blending assumption present in most stockpiling scenarios. Therefore, making reasonable assumptions regarding the number of stockpiles to define and the acceptable element grade ranges is crucial to obtaining reasonable results.

In order to create the linear LTOPP model with stockpiling we define *S* stockpiles. G_s^e is the average reclamation grade of element *e* from stockpile *s* and $r_{s,c}^t$ is the unit discounted revenue of reclaiming material from stockpile *s* with the average grade and processing them in plant *c* in period *t* minus the processing and rehandling costs. $\underline{G}_s^e$ and $\overline{G}_s^e$ are the lower and upper bounds on the acceptable element grade *e* for stockpile *s*. $f_{s,c}^t \geq 0$ is the set of variables representing the tonnage of material reclaimed from stockpile *s* in period *t* and sent to processing destination *c*. Now we can rewrite the model by replacing the objective function with Equation (15) and Equations (11) to (14) with Equations (16) to (19) respectively.

$$\max \sum_{t=1}^{T}\left(\sum_{p=1}^{P}\sum_{c=1}^{C}(r_{p,c}^t \times o_p \times x_{p,c}^t) - \sum_{m=1}^{M}(c_m^t \times (o_m + w_m) \times y_m^t) + \sum_{s=1}^{S}\sum_{c=1}^{C}\left(f_{s,c}^t \times r_{s,c}^t\right)\right) \tag{15}$$

$$\underline{PC}_c^t \leq \sum_{p=1}^{P}\left(o_p \times x_{p,c}^t\right) + \sum_{s=1}^{S} f_{s,c}^t \leq \overline{PC}_c^t \qquad \forall t \in \{1,\ldots,T\}, \forall c \in \{1,\ldots,C\} \tag{16}$$

$$\underline{G}_c^{t,e} \leq \frac{\sum_{p=1}^{P}\left(o_p \times g_p^e \times x_{p,c}^t\right) + \sum_{s=1}^{S} f_{s,c}^t \times G_s^e}{\sum_{p=1}^{P}\left(o_p \times x_{p,c}^t\right) + \sum_{s=1}^{S} f_{s,c}^t} \leq \overline{G}_c^{t,e} \qquad \begin{array}{l}\forall t \in \{1,\ldots,T\},\\ \forall c \in \{1,\ldots,C\},\\ \forall e \in \{1,\ldots,E\}\end{array} \tag{17}$$

$$\underline{G}_s^e \leq \frac{\sum_{p=1}^{P}\left(o_p \times g_p^e \times x_{p,s}^t\right)}{\sum_{p=1}^{P}\left(o_p \times x_{p,s}^t\right)} \leq \overline{G}_s^e \qquad \begin{array}{l}\forall t \in \{1,\ldots,T\},\\ \forall s \in \{1,\ldots,S\},\\ \forall e \in \{1,\ldots,E\}\end{array} \tag{18}$$

$$\sum_{t'=1}^{t}\sum_{c=1}^{C} f_{s,c}^{t'} \leq \sum_{t'=1}^{t-1}\sum_{p=1}^{P}\left(o_p \times x_{p,s}^{t'}\right) \qquad \forall t \in \{2,\ldots,T\}, \forall s \in \{1,\ldots,S\} \tag{19}$$

CONCLUSION AND FUTURE WORK

In this paper, we presented a multi-step approach to long-term open-pit production planning by using different resolutions for making mining and processing decisions. We determine the pushbacks based on a hybrid binary programming-heuristic method and use the intersections of pushbacks and mining benches as mining units. Afterwards, we divide the bench-phases into smaller units with similar rock type and grade using an agglomerative hierarchical clustering algorithm.

These units are then used as processing units. Then, we presented a mathematical model to solve the LTOPP problem with the aggregated units. Finally, we added stockpiling to the model with non-linear and linear objective functions and constraints. In the next step, the linear model has to be tested on multiple case studies to verify the simplification assumptions used for linearization.

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