

### STAT 4800 Homework 3

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**Question 1: Derive the link function for the Poisson distribution.**

The image shows a handwritten derivation of the link function for the Poisson distribution on lined paper. The steps are as follows:

$$f(x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \quad \lambda > 0, \quad x = 0, 1, 2, \dots$$
$$\ln(f(x|\lambda)) = x \ln(\lambda) - \lambda - \ln(x!)$$
$$f(x|\lambda) = \exp(x \ln(\lambda) - \lambda - \ln(x!))$$
$$= \frac{1}{x!} \exp(x \ln(\lambda) - \lambda)$$

Below the equation, arrows point from the terms to their corresponding functions in the canonical form:

- $\frac{1}{x!}$  points to  $h(x)$
- $x$  points to  $T(x)$
- $\ln(\lambda)$  points to  $b(\lambda)$
- $-\lambda$  points to  $a(\lambda)$

$$= h(x) \exp(T(x)b(\lambda) - a(\lambda)) \quad \leftarrow \text{canonical form of exponential dist}$$
$$= h(x) e^{T(x)b(\lambda) - a(\lambda)}$$

**Question 2: Use Poisson regression to investigate shot rate in the game of hockey in the 2016-2017 season of the NHL.**

We decided to build our Poisson model based on the period and number of bins. We decided to break up our bins into one-minute intervals because we felt it gave us the most accurate Poisson output. After running the distribution, we created a predictor function that would predict the total number of shots for all teams over all bins. We then divided this number by the number of bins and 30, representing the number of NHL teams, to produce a per minute average. We ended up with a number of about 0.6 per bin, which means that an NHL team averages a little over one-half shot per minute in a game, which seems realistic.

One potential factor missing from this model could be whether the team was home or away. Generally, home teams perform better than away teams, so it seems possible that home teams could have a higher shot rate than away teams. Another potential factor would be any injuries or extenuating circumstances going on in the game. These would be difficult to model, but that could affect the shot rate of the teams.

[ see hw3.ipynb for code ]