Calculus, Volume 1, 2nd Edition - Tom M. Apostal

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Chapter 1

Introduction

1.1 Some Basic Concepts of the Theory of Sets

1.1.1 Excercises

1. Use the roster notation to designate the following sets of real numbers.

   Proposition 1. $A = \{x|x^2 - 1 = 0\}$ can be designated as $\{-1, 1\}$ in roster notation.

   Proof.

   $$\begin{aligned} A &= \{x|x^2 - 1 = 0\} \\ &= \{x|(x-1)(x+1) = 0\} \\ &\therefore \{-1, 1\} \end{aligned} \tag{1.1}$$

   QED

   Proposition 2. $B = \{x|(x-1)^2 = 0\}$ can be designated as $\{1\}$ in roster notation.

   Proof.

   $$\begin{aligned} B &= \{x|(x-1)^2 = 0\} \\ &= \{x|x - 1 = \sqrt{0}\} \\ &= \{x|x = 1\} \\ &\therefore \{1\} \end{aligned} \tag{1.2}$$

QED

Proposition 3. $C = \{x|x + 8 = 9\}$ can be designated as $\{1\}$ in roster notation.

Proof.

$$\begin{aligned} C &= \{x|x + 8 = 9\} \\ &= \{x|x = 9 - 8\} \\ &= \{x|x = 1\} \\ &\therefore \{1\} \end{aligned} \tag{1.3}$$

QED

Proposition 4. $D = \{x|x^3 - 2x^2 + x = 2\}$ can be designated as $\{2\}$ in roster notation.

Proof.

$$\begin{aligned} D &= \{x|x^3 - 2x^2 + x = 2\} \\ &= \{x|x^3 - 2x^2 + x - 2 = 0\} \\ &= \{x|x^2(x - 2) + (x - 2) = 0\} \\ &= \{x|(x^2 + 1)(x - 2) = 0\} \\ &\therefore \{2\} \end{aligned} \tag{1.4}$$

QED

Proposition 5. $E = \{x|(x + 8)^2 = 9^2\}$ can be designated as $\{-17, 1\}$ in roster notation.

Proof.

$$\begin{aligned} E &= \{x|(x + 8)^2 = 9^2\} \\ &= \{x|x + 8 = \pm 9\} \\ &= \{x|x = \pm 9 - 8\} \\ &\therefore \{-17, 1\} \end{aligned} \tag{1.5}$$

QED

Proposition 6. $F = \{x|(x^2 + 16x)^2 = 17^2\}$ can be designated as $\{-17, 1, -8 - \sqrt{47}, -8 + \sqrt{47}\}$ in roster notation.

Proof.

$$\begin{aligned} F &= \{x|(x^2 + 16x)^2 = 17^2\} \\ &= \{x|x^2 + 16x = \pm 17\} \\ &= \{x|x^2 + 16x \pm 17 = 0\} \\ &= \{x|x^2 + 16x \pm 17 = 0\} \end{aligned} \tag{1.6}$$

Using the quadratic formula:

Definition 1.1.1. Quadratic Equation, analytical method for calculating the roots of a quadratic polynomial.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } ax^2 + bx + c = 0 \tag{1.7}$$

Solving when the last term is $+17$:

$$\begin{aligned} x &= \frac{-16 \pm \sqrt{16^2 - 4(1)17}}{2(1)} \\ &= -8 \pm \frac{\sqrt{188}}{2} \\ &= -8 \pm \frac{\sqrt{188}}{\sqrt{2^2}} \\ &= -8 \pm \sqrt{188/4} \\ &= -8 \pm \sqrt{47} \\ &\therefore \{-8 - \sqrt{47}, -8 + \sqrt{47}\} \end{aligned} \tag{1.8}$$

Solving when the last term is -17:

$$\begin{aligned} 0 &= x^2 + 16x - 17 \\ &= (x + 17)(x - 1) \\ &\therefore \{-17, 1\} \end{aligned} \tag{1.9}$$

QED

2. For the sets in Exercise 1, note that $B \subseteq A$. List all the inclusion relations $\subseteq$ that hold among the sets A, B, C, D, E, F.

   (a) $B \subseteq A$

   (b) $B \subseteq C$

   (c) $B \subseteq E$

   (d) $B \subseteq F$

   (e) $C \subseteq A$

   (f) $C \subseteq B$

   (g) $C \subseteq E$

   (h) $C \subseteq F$

   (i) $E \subseteq F$