No Way around It. Introduction.

This book is devoted to the study of permutations. While the overwhelming majority of readers already know what they are, we are going to define them for the sake of completeness. Note that this is by no means the only definition possible.

DEFINITION 0.1 *A linear ordering of the elements of the set* $[n] = \{1, 2, 3, \cdots, n\}$ *is called a* permutation*, or, if we want to stress the fact that it consists of* n *entries, an* n*-permutation.*

In other words, a permutation lists all elements of $[n]$ so that each element is listed exactly once.

Example 0.2
If $n = 3$, then the n-permutations are 123, 132, 213, 231, 312, 321. $\square$

There is nothing magic about the set $[n]$; other sets having n elements would be just as good for our purposes, but working with $[n]$ will simplify our discussion. In Chapter 2, we will extend the definition of permutations to multisets, and in Chapter 3, we will consider permutations from a different perspective. The set of all n-permutations will be denoted by S_n, and the reason for that will become clear in Chapter 3.

For now, we will denote an n-permutation by $p = p_1 p_2 \cdots p_n$, with p_i being the ith entry in the linear order given by p.

The following simple statement is probably the best-known fact about permutations.

PROPOSITION 0.3
The number of n*-permutations is* $n!$.

PROOF When building an n-permutation $p = p_1 p_2 \cdots p_n$, we can choose n entries to play the role of p_1, then $n - 1$ entries for the role of p_2, and so on. $\blacksquare$

We promise the rest of the book will be less straightforward.

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