Preface to the First Edition

A few years ago, I was given the opportunity to teach a graduate combinatorics class on a special topic of my choice. I wanted the class to focus on the combinatorics of permutations. However, I instantly realized that while there were several excellent books that discussed some aspects of the subject, there was no single book that would have contained all, or even most, areas that I wanted to cover. Many areas were not covered in any book, which was easy to understand as the subject is developing at a breathtaking pace, producing new results faster than textbooks are published. Classic results, while certainly explained in various textbooks of very high quality, seemed to be scattered in numerous sources. This was again no surprise; indeed, permutations are omnipresent in modern combinatorics, and there are quite a few ways to look at them. We can consider permutations as linear orders; we can consider them as elements of the symmetric group; we can model them by matrices; or by graphs. We can enumerate them according to countless interesting statistics; we can decompose them in many ways, and we can bijectively associate them to other structures. One common feature of these activities is that they all involve factual knowledge, new ideas, and serious fun. Another common feature is that they all evolve around permutations, and quite often, the remote-looking areas are connected by surprising results. Briefly, they do belong to one book, and I am very glad that now you are reading such a book.

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As I have mentioned, there are several excellent books that discuss various aspects of permutations. Therefore, in this book, I cover these aspects less deeply than the areas that have previously not been contained in any book. Chapter 1 is about descents and runs of permutations. While Eulerian numbers have been given plenty of attention during the last 200 years, most of the research was devoted to analytic concepts. Nothing shows this better than the fact that I was unable to find published proofs of two fundamental results

of the area using purely combinatorial methods. Therefore, in this chapter, I concentrated on purely combinatorial tools dealing with these issues. By and large, the same is true for Chapter 2, whose subject is inversions in permutations, and in permutations of multisets. Chapter 3 is devoted to permutations as products of cycles, which is probably the most-studied of all areas covered in this book. Therefore, while there were many classic results we had to include there for the sake of completeness, nevertheless we still managed to squeeze in less well-known topics, such as applications of Darroch's theorem, or transpositions and trees.

The area of pattern avoidance is a young one, and has not been given significant space in textbooks before. Therefore, we devoted two full chapters to it. Chapter 4 walks the reader through the quest for the solution of the Stanley-Wilf conjecture, ending with the recent spectacular proof of Marcus and Tardos for this 23-year-old problem. Chapter 5 discusses aspects of pattern avoidance other than upper bounds or exact formulae. Chapter 6 looks at random permutations and Standard Young Tableaux, starting with two classic and difficult proofs of Greene, Nijenhaus, and Wilf. Standard techniques for handling permutation statistics are presented. A relatively new concept, that of min-wise independent families of permutations, is discussed in the Exercises. Chapter 7, Algebraic Combinatorics of Permutations, is the one in which we had to be most selective. Each of the three sections of that chapter covers an area that is sufficiently rich to be the subject of an entire book. Our goal with that chapter is simply to raise interest in these topics and prepare the reader for the more detailed literature that is available in those areas. Chapter 8 is about combinatorial sorting algorithms, many of which are quite recent. This is the first time many of these algorithms (or at least, most aspects of them) are discussed in a textbook, so we treated them in depth.

Besides the Exercises, each chapter ends with a selection of Problems Plus. These are typically more difficult than the exercises, and are meant to raise interest in some questions for further research, and to serve as reference material of what is known. Some of the Problems Plus are not classified as such because of their level of difficulty, but because they are less tightly connected to the topic at hand. A solution manual for the even-numbered Exercises is available for instructors teaching a class using this book, and can be obtained from the publisher.