# ΔΙΑΓΩΝΙΣΜΟΣ ΕΠΙΛΟΓΗΣ 2

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1. For an integer $n \geq 1$ , denote by

$$T_n(x) = \sum_{k=1}^{n} (-1)^{k-1} \frac{x^{2k-1}}{(2k-1)!},$$

the $(2n-1)$-th Taylor polynomial of the sine function at 0, and let

$$I_n = \int_0^{\infty} \frac{T_n(x) - \sin(x)}{x^{2n+1}} dx$$

   a) Prove that $I_n = \frac{1}{2n(2n-1)} I_{n-1}, n \geq 1$

   b) Calculate $I_n$.

2. Let $A \in M_n(\mathbb{R})$ be non-zero fixed matrix. Define the function

$$f_A : M_n(\mathbb{R}) \to M_n(\mathbb{R})$$

$$f_A(X) = AX - XA$$

   a) Show that $f_A = 0$ if and only if $A = \lambda I_n$.

   b) Show that $f_A(f_B) = f_B(f_A)$ if and only if $AB = BA$.

   c) If $A$ is a matrix with $n$ distinct real eigenvalues, find the dimension of $\ker(f_A)$.