ΔΙΑΓΩΝΙΣΜΟΣ ΕΠΙΛΟΓΗΣ 2018

31 Αυγούστου 2018

1. $$\lim_{n\to\infty}\left(\frac{\left(1+\frac{1}{n}\right)^n}{e}\right)^n$$

2. Given a nondecreasing differentiable function $f:\mathbb{R}_+\to\mathbb{R}_+$, prove that
$$\int_0^\infty e^{-t-f(t)}\sqrt{1+\left(f'(t)\right)^2}dt\geq\sqrt{\alpha^2+(\alpha-e^{-f(0)})^2},$$
where $\alpha=\int_0^\infty e^{-t-f(t)}dt$.

3. Suppose that A,B and $A+B$ are unitary. Prove that $(AB^*)^3=I$.

4. Suppose $A\in\mathcal{M}_n(\mathbb{C})$. Prove that the sequence $(a_k)_{k\geq 0}$ is nondecreasing, where $a_k=\operatorname{rank}(A^{k+1})-\operatorname{rank}(A^k)$.

5. Let $f:[0,1]\to\mathbb{R}$ be a continuous function on $[0,1]$. Suppose that
$$\int_0^n f(x/n)e^x dx=0,$$
Prove that $f(x)=0$ for every $x\in[0,1]$.