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Problem #5

Game Theory

Due: May 3 by 8:30 AM Pacific

Bargaining Models with the Threat of War

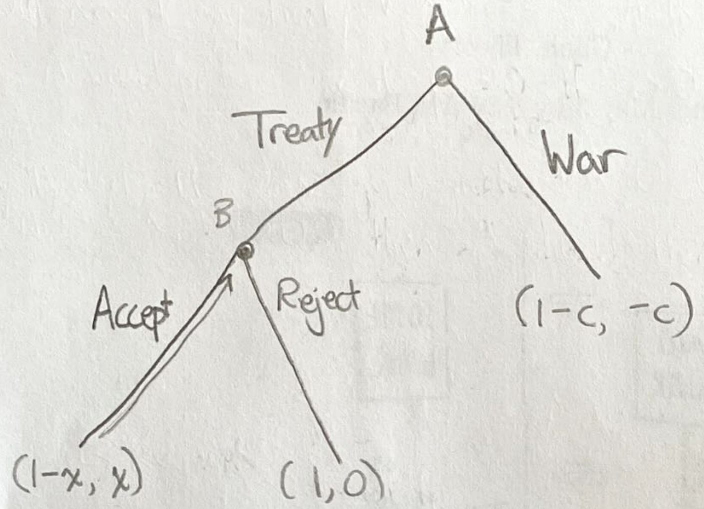
This question is based on Powell (1999), *In the Shadows of Power*, as discussed in McCarthy & Meirowitz (2007). Two countries, A and B, are in conflict over a particular region. We will write the payoffs in the order: (A's payoff, B's payoff). Currently, A controls the region but country B is staking a claim. The overall value of the region is equal to 1 in each period. The game works as follows: A can either offer a treaty with B or attack B and start a war.

If A offers a treaty, the treaty specifies a value x, which is the fraction of the region's value that would go to country B, where $0 \leq x \leq 1$. B can either accept or reject the offer. If B accepts the offer, the payoffs are $(1-x, x)$. If B rejects the offer, the game ends and the payoffs are $(1, 0)$.

If A goes to war, it beats B with probability p. Both countries pay a cost of c after a war, where $0 \leq c \leq 1$. The winner gets the region, so the winner's total payoff is $1-c$ and the loser's payoff is $-c$.

Part a

Assume for now that A definitely would win the war, so $p = 1$. Also suppose x is fixed, say, it's determined by a third party that negotiates the treaty, so neither A nor B can choose it. Draw the extensive form of the game.



Part b

Characterize all the possible SPNE. Be sure to indicate a complete strategy profile for each SPNE.

The SPNE could change based on the relationship between c and x.

c	x	SPNE
0	0	1) (Treaty$_{x=0}$, Accept) 2) (Treaty$_{x=0}$, Reject) $x=0$, accept $x>0$ 3) (War, Accept if Treaty) 4) (War, Reject if Treaty) $x=0$, accept $x>0$
$0<c<x$ $\Rightarrow 1-c>1-x$	$(x>c>0)$	1) (War, Accept if Treaty) any $x \geq 0$
$0<x<c$ $\Rightarrow 1-x>1-c$	$(c>x>0)$	1) (Treaty, Accept) any $x \geq 0$
$0<c=x$ $\Rightarrow 1-x=1-c$		1) (Treaty, Accept) any $x \geq 0$ 2) (War, Accept if Treaty) any $x \geq 0$
$0=c<x$		1) (War, Accept if Treaty) any $x \geq 0$
$0=x<c$		1) (Treaty, Accept) any $x \geq 0$ 2) (Treaty, Reject) $x=0$, accept $x>0$

Part c

Now suppose A can choose x. What offer x should A choose in order to maximize its equilibrium payoff?

B will accept any treaty $\forall\, x > 0$. B might also accept $x = 0$. If A is guaranteed a wartime victory, then A would prefer war if $c = 0$. If $c > 0$, A would maximize its payoff by offering x equal to 0 (provided $x < c$).

Assuming $c > 0$, then A should offer $x = 0$.

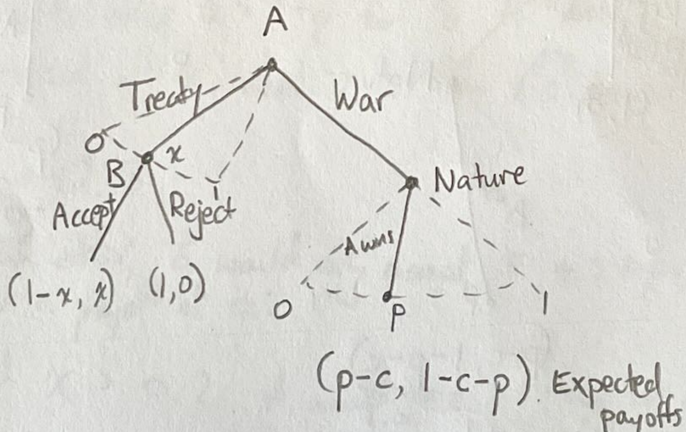
↳ Regardless of B's decision, A receives 1 and B receves 0.

Part d

Now suppose $0 < p < 1$, so A might lose a war if it starts one. If A offers a treaty, it can still choose the value of x. Again, characterize all the SPNE. Note that now, you must take expected values in the situation where A goes to war. (Hint: The answer is simpler than it might seem. However, you must still be sure to specify B's response for *each possible offer*, not just the one A would choose in equilibrium.)

If A goes to war, their expected payoff is $p(1-c) + (1-p)(-c) = p - \cancel{pc} - c + \cancel{pc}$
$= p - c.$

If $p - c > 1 - x$, then A prefers war. Otherwise, A prefers the treaty. B is now faced with a payoff of x if accepting A's treaty offer, and $p(-c) + (1-p)(1-c) = \cancel{-cp} + 1 - c - p + \cancel{pc} = 1 - c - p$ if they go to war.



$x = 1 - p + c$

0.6 0.3

$x = 0.7$

If offered a treaty, B will still accept any $x > 0$. (Might accept $x = 0$).
A would prefer to offer a treaty as long as $1 - x > p - c$
$\Rightarrow$ A would offer $x < 1 - p + c$.

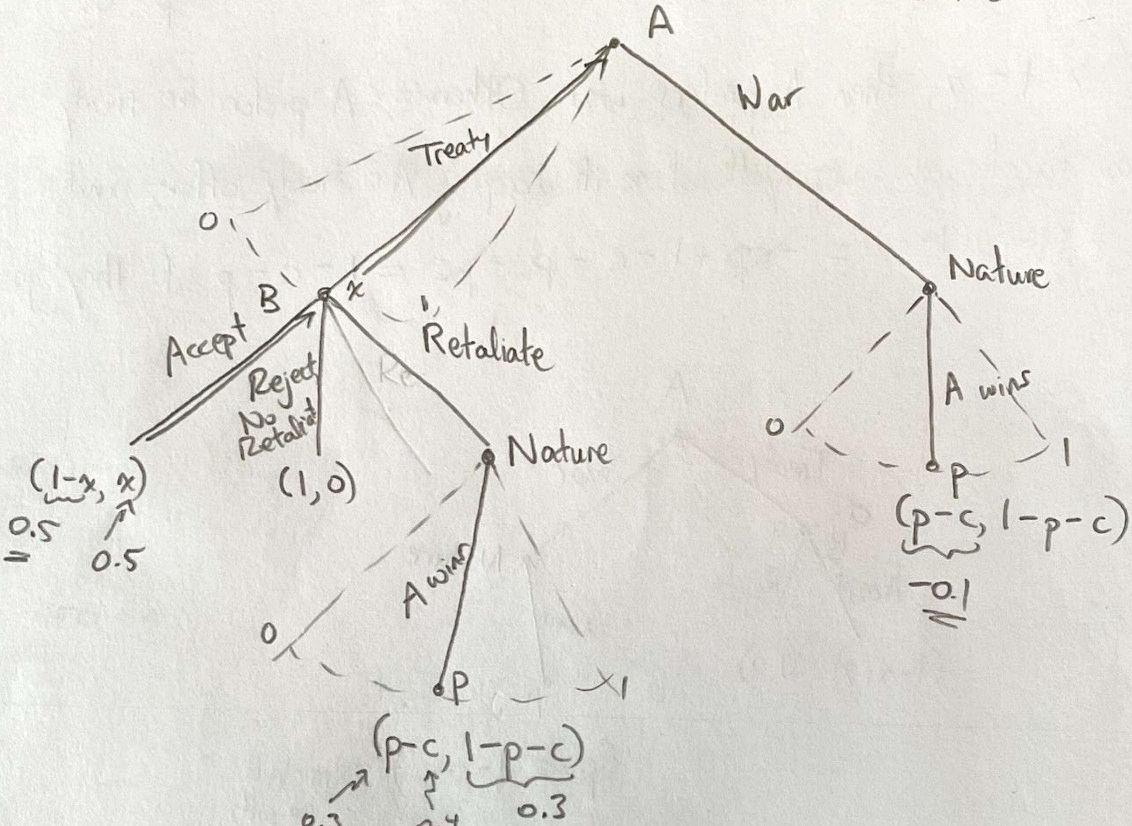
Situation	A's offer	SPNE
$p < c$	$x = 0$. (offer)	1) (Treaty, $x = 0$; Accept any $x \geq 0$) 2) (Treaty, $x = 0$; Reject $x = 0$) ← (accept $x > 0$)
$p > c$	$x = 0$ (offer)	1) (Treaty, $x = 0$; Accept any $x \geq 0$) 2) (Treaty, $x = 0$; Reject $x = 0$) ← (accept $x > 0$)
$p = c$	$x = 0$ (offer)	1) (Treaty, $x = 0$; Accept any $x \geq 0$) 2) (Treaty, $x = 0$; Reject $x = 0$) ← (accept $x > 0$)

no change

Part c

Now let's make things more realistic and allow B to retaliate. After A makes an offer, if B accepts it then the game ends with payoffs as above. But if B rejects it, then B can choose to attack A. Just like if A attacks B, when B attacks A there is a probability p that A wins the war. If B rejects A's offer but does not attack A, the game ends with payoffs $(1, 0)$ as in part (a).

Find *any* SPNE in which A appeases B, i.e. A chooses to make a strictly positive offer $x > 0$ and B accepts the offer. You can choose particular values of x, p, and c and prove that your choices yields an SPNE, or you can work with variables. Either way, you must write the full strategy profile that constitutes the SPNE and you must prove it is indeed an SPNE. You may also assume that when B is indifferent, it chooses to accept an offer (this gets rid of the need to consider mixed strategies). :)



In cases where A definitely wants to avoid war (e.g., $p < c$), they would offer $x > 0$. But for B to accept this offer, it must also be greater than $1-p-c$.

So, the necessary condition is $0 < 1-p-c \leq x$. (and $p < c$).

For example, given $p = 0.3$, $c = 0.4$, A's payoffs are:

- $p-c = -0.1$ in war
- $1-x$ for an offer x.

B would accept any offer $x \geq 1-0.3-0.4 = 0.3$. If A offers $x=0.5$, B would accept. (Offer $x=0.5$; Accept $x \geq 0.3$ / Retaliate if $x < 0.3$)

Part f: Optional, not graded

For the same situation as above, find any SPNE in which A attacks B from the beginning. (Hint this is a knife-edge case where one parameter has a specific value.)

First, war must be preferable for A, i.e. ~~$p > c$.~~ $p - c > 1 - x$.
~~Second, A must offer x such that~~

This would also describe a scenario where the minimum x that B would accept places A in a worse-off situation ~~that~~ than war.

Let $c = 0$ (war is free!). Now, for a given p, B's payoffs are:

- x for an offer x
- $1-p$ in war.

Thus A would need to offer $x > 1-p$ for B to accept. But in such an offer, A would receive $1-x$, which is less than A's payoffs in war ($=p$).

So if $c = 0$, $p = 0.8$:

- If A made an offer, B would only accept if $x > 1-p = 0.2$. Otherwise, B retaliates for a payoff of 0.2.
- If A offered $x > 0.2$, A would receive $1-x < 0.8$, vs. a payoff of 0.8 in war.

(If A offered $x < 0.2$, B would retaliate and war would result anyway.)

SPNE:

(War, Accept if $x > 0.2$/Retaliate if $x < 0.2$)