

## Problem #1

Game Theory, Spring 2022

Due: Tuesday, April 5 by 8:30 AM Pacific

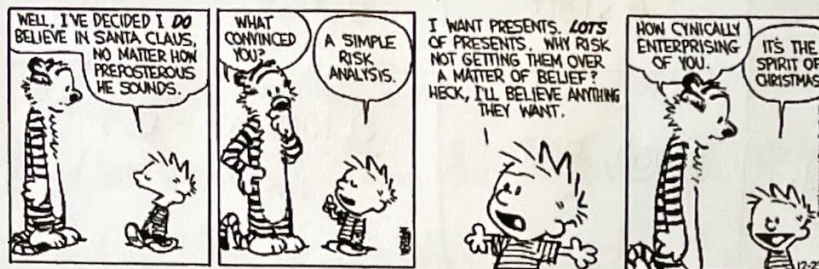


Figure 1: Calvin concludes that believing in Santa is a strictly dominant strategy.

*General guidance for homework assignments:*

- You may work in groups, but each person must turn in their own solution.
- You may fill in your answers in the blank space provided, or write out your answers separately.
- In order to get full credit, you must show all of your work or explain your reasoning; a correct answer with no justification will not get many (if any) points.
- You should upload your answers to Populi or email them directly to the professor, by the deadline—taking a photo or scanning handwritten work is fine.
- If you trust inter-office mail, you can also send it to the professor at Mail Stop W9W, but send an email noting that you shipped it off prior to the day it's due.

### 1 The Terrorist Defection Constraint

In this problem we will explore the conditions under which a terrorist group can ensure that its members do not defect. This problem is based on the theory of Eli Berman & David Laitin (2008), who describe this concept as the “defection constraint.”

For this setting, suppose that a terrorist group with 100 members controls a road and extorts truck drivers by charging them a hefty tax to pass. A member of the terrorist group patrols the highway and stops the truck drivers. In this game, the two players are the terrorist and the truck driver. The elements of the game are as follows:

- The truck driver chooses the value  $V$  of his cargo. He can choose to transport either a high-value cargo worth  $V = \$100,000$  or a low-value cargo worth  $V = \$10,000$ .
- When the terrorist intercepts the truck drive, he can choose to extort money or to defect. If he extorts, then he gets 90% of the truck's value and splits it with the 99 other terrorists in the group. If he defects, he takes the entire truck and keeps its value for himself.



- A defecting terrorist pays an additional cost of  $-C$ , which can be thought of as the cost of running away and avoiding capture after stealing the truck. So his total payoff is  $V - C$ . Assume  $C \geq 0$ .
- If he gets extorted, the truck driver keeps 10% of his truck's value for himself. He gets nothing if the terrorist defects.

Part a

Write the normal form of the game, treating  $C$  as an unknown variable.

		Terrorist	
		Extort	Defect
Truck Driver	High	<u>10000</u> , 900	<u>0</u> , <u>100000</u> - C 10
	Low	1000, 90	<u>0</u> , <u>10000</u> - C 0



### Part b

What are all the various Nash equilibria, depending on the value of  $C$ ? Justify your answer or show your work.

The truck driver will prefer High to Low if the terrorist extorts ( $10 > 1$ ); he is indifferent between High and Low if the terrorist defects ( $0 = 0$ ).

If  $C$  reduces the payoff <sup>to</sup> ~~by~~  $0.9\%$  of the truck's value (terrorist who extorts keeps  $90\%$  /  $100$  terrorists), then the terrorist will extort. Otherwise, the terrorist will defect. So the interesting values for  $C$  are:

$$\begin{array}{r} 9910 \\ 99100 \end{array}$$

So, the Nash equilibria are:

- $\{(High, Defect), (Low, Defect)\}$ ,  $0 \leq C \leq 9910$
- $\{(High, Defect)\}$ ,  $9910 < C < 99100$
- $\{(High, Extort), (High, Defect)\}$ ,  $C = 99100$
- $\{(High, Extort)\}$ ,  $C > 99100$



### Part c

Now consider this game from the terrorist group's point of view, which depends on extortion to raise funds. The terrorist group cannot monitor each and every member at every single truck stop. Instead, they have control over  $C$  and use that to create incentives that will keep individuals in line. Given your results from part (b), what range of values of  $C$  would be most desirable for the terrorist group? Explain your answer.

The terrorist group should try to make  $C$  as high as possible (i.e.  $> 99100$ ) such that defecting is disincentivized, even for high value trucks.

(Technically, if concerned about funds, then get  $C$  to  $99100.000000\dots 1$ ).



#### Part d

Describe one possible way in which a terrorist group could control the value of  $C$ . (You do not need to find evidence that a group has actually used such a method in the past; just explain a realistic setting in which it would work.)

It is tough to place a dollar value on the threat of capture/death to self or loved ones, but one way to raise the actual value of  $C$  is to kidnap a family member and hold them for ransom.



### Part e

Let's look at group membership from another angle. Suppose there are two terrorists who need to transport a cargo worth  $V$ . If they both remain loyal and the cargo arrives at the destination, they can split the value and each get  $\frac{V}{2}$ . But along the way, either one could defect and get a payoff of  $w$  (think of  $w$  as the outside wage they could earn by leaving the group). If someone remains loyal but their partner defects, that person loses the cargo and gets 0. Write this game in normal form.

		Terrorist 2	
		Loyal	Defect
Terrorist 1	Loyal	$\frac{V}{2}, \frac{V}{2}$	$0, w$
	Defect	$w, 0$	$w, w$



## Part f

Consider the various values  $w$  can take, relative to  $V$ . How many different games can result, depending on  $w$ ? Are any of those games the same as one of the archetypal games we discussed in class? (For example, if this could be a Prisoner's Dilemma, for what values of  $w$  would that be the case? What about other values of  $w$ ?)

The values that might possibly change the terrorists' decisions are

$$w < \frac{V}{2}, \quad w = \frac{V}{2}, \quad w > \frac{V}{2}$$

Assume  $w > 0$ . Nash eq<sup>a</sup> are:

•  $\{\text{Both Loyal}, \text{Both Defect}\}, \quad 0 < w \leq \frac{V}{2}$

↗ Both Defect

← one of  $N^*$  is not P.O.  
This is the Stag Hunt  
(if  $w < \frac{V}{2}$ ).

•  $\{\text{Both Defect}\}, \quad w > \frac{V}{2}$

← this  $N^*$  is P.O.



### Part g

Berman & Laitin find that groups that require more intense religious education, at the expense of a secular education, have lower defection rates. How does that empirically support the model in part (f)?

The model in (f) suggests that defection is more likely when  $w > \frac{1}{2}$ , because then it is the only Nash equilibrium. So, if a secular education enables terrorists to find higher-paying jobs than a religious education would, then this observation supports the model in part (f).

That is, the assumption would be that a more intense religious education lowers the terrorist's possible outside wage  $w < \frac{1}{2}$ , such that remaining loyal becomes not only a Nash eq<sup>m</sup>, but a Pareto Optimal outcome.