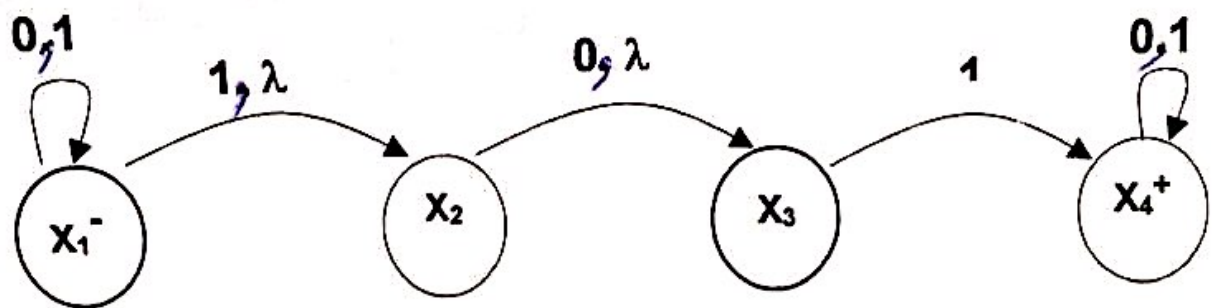


:30-13:30

2014-2015 (1)

4/20

Q1. Convert the following NFA into equivalent DFA. [5 marks]



	0	1
x1	x1	x1, x4
x2	x3	∅
x3	∅	x4
+x4	x4	x4

5- CNF has:

- A) One Nullable production
- B) One unit production
- C) One λ -production
- D) Two unit productions
- E) None of the above

6- What is the equivalent regular expression for the expression $r = a^*b$ over the $\Sigma = \{a,b\}$?

- A) $(a+b)^*b$
- B) a^*b^*b
- C) $(\lambda + a(aa)^*)(\lambda + a)b + b$
- D) $(a^*+b)^*b$
- E) $aa^*b^*b^*$

7- Which of the regular expression below gives the set of all strings with an equal number of a's and b's?

- A) $(aa+ab+ba+bb)^*$
- B) $ba(a+b)^*ab$
- C) $(a+b)^*$
- D) $(ab)^* + (ba)^*$
- E) $(ba+ab)^*$

8- Which regular expression represents the strings do not contain the substring aa over $\{a,b\}$?

- A) $(ab+ba)^*$
- B) $(ba+b)^*(\lambda+a)$
- C) $(aba)^*$
- D) $(ab)^*$
- E) $(\lambda+a)(ab+b)^*$

Question	1	2	3	4	5	6	7	8
Answer	E	D	C	E	E	C	E	D

Q2. Consider $\Sigma = \{0, 1, 2, 3\}$, construct CFG that can generate any number whose sum of its digits is even. (5 marks)

$$S \rightarrow XS$$

$$S \rightarrow SX$$

$$S \rightarrow YSY$$

$$S \rightarrow \lambda$$

$$X \rightarrow 00, 11, 22, 33$$

~~$$Y \rightarrow 01, 10, 23, 32$$~~

$$Y \rightarrow 02, 13, 20, 31$$

110 ?
310

Q2. Consider $\Sigma = \{a, b\}$, construct CFG that can generate any word that does not contain double a. (5 marks)

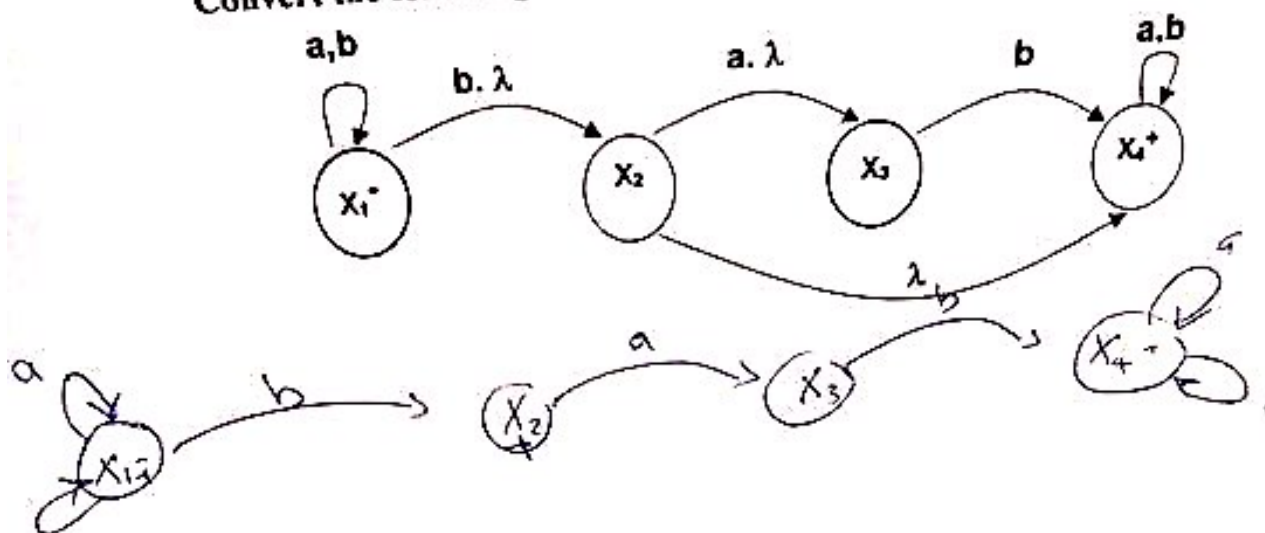
bab

$$S \rightarrow bSb$$

$$S \rightarrow a$$

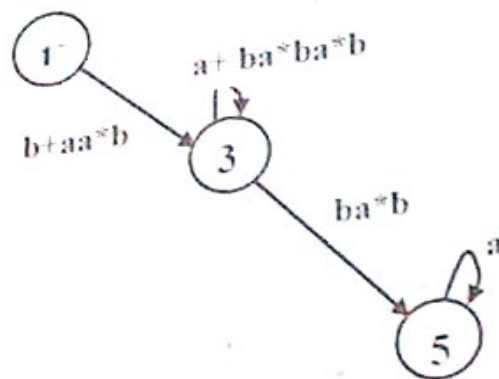
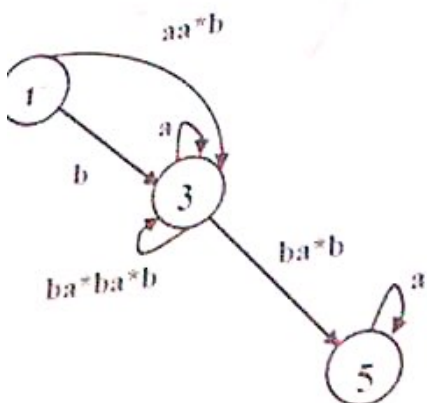
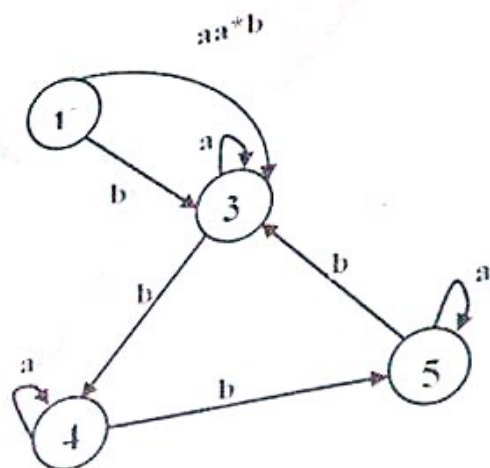
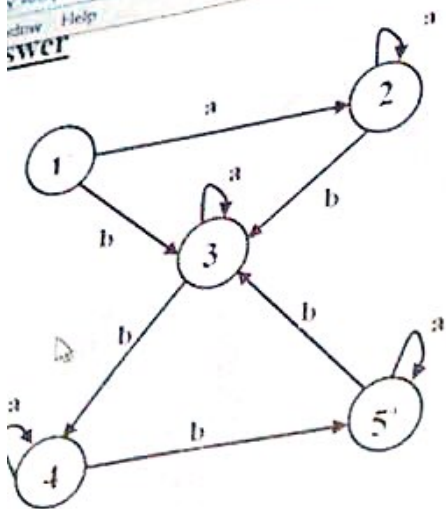
aba ?

Q2. (7 marks) Convert the following NFA into equivalent DFA



Handwritten signature/initials.

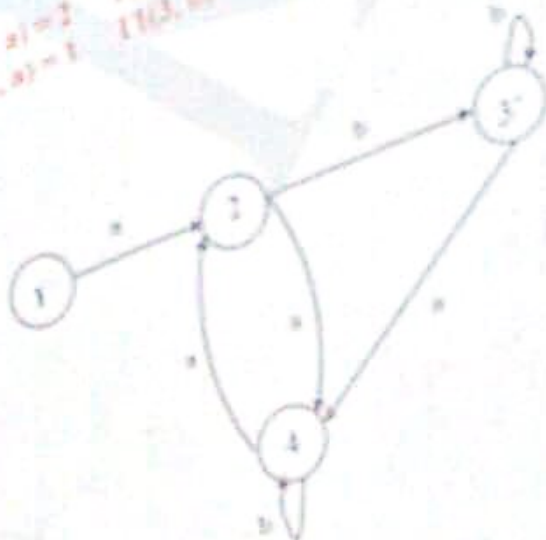
Handwritten signature/initials.

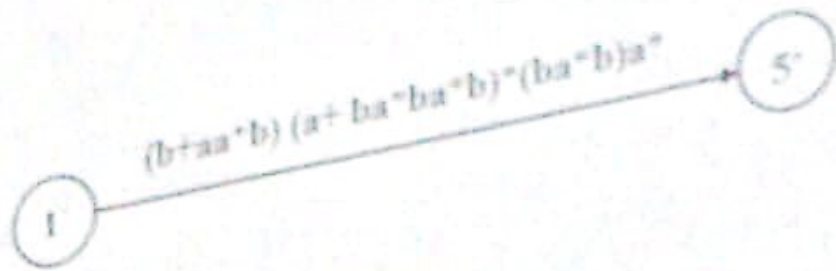


Answer

$FA = \{1, 2, 3, 4, b\}, T1, 1, 3)$

$T1(1, a) = 2$ $T1(2, a) = 1$ $T1(2, b) = 3$
 $T1(3, a) = 1$ $T1(3, b) = 3$





$$(b + aa^*b)(a + ba^*ba^*b)^*(ba^*b)a^*$$

Q3. Convert the following CFG into CNF; (7 marks)

$$S \rightarrow XS \mid Y \mid aSa$$

$$X \rightarrow aX \mid abY$$

$$Y \rightarrow bYb \mid bY \mid \lambda$$

$$S \rightarrow X^S \mid Y \mid aSa$$

$$X \rightarrow aX \mid abY$$

$$Y \rightarrow bYb \mid bY \mid \lambda$$

$$S \rightarrow X^S$$

$$S \rightarrow aSa$$

$$S \rightarrow bYb$$

$$S \rightarrow \lambda$$

$$S \rightarrow aSa$$

$$S \rightarrow aSa$$

$$S \rightarrow aX$$

$$S \rightarrow abY$$

$$S \rightarrow$$

$$S \rightarrow aSa$$

$$X \rightarrow aX$$

$$X \rightarrow abY$$

$$X \rightarrow$$

$$X \rightarrow abY$$

$$X \rightarrow abY$$

$$X \rightarrow ab$$

$$Y \rightarrow bYb$$

$$Y \rightarrow bY$$

$$Y \rightarrow$$

$$Y \rightarrow b$$

$$Y \rightarrow$$

$$S \rightarrow$$

Q1. Consider $\Sigma = \{0, 1, 2, 3\}$, construct CFG that can generate any number whose sum of its digits is even. (7 marks)

$$S \rightarrow XY \quad \text{[crossed out]} \quad A \mid B$$

$$S \rightarrow XX$$

$$S \rightarrow YY$$

$$\rightarrow XX \rightarrow \lambda$$

$$\rightarrow YY \rightarrow \lambda$$

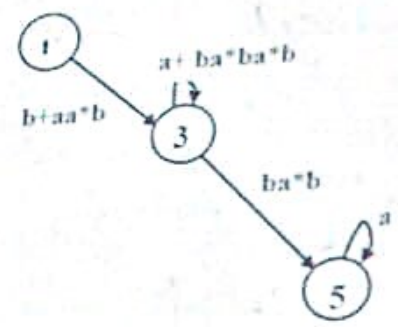
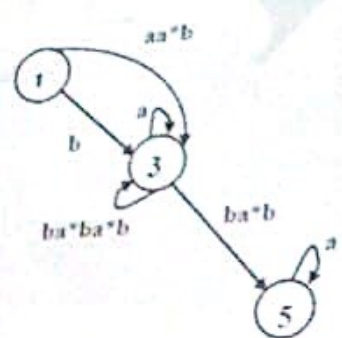
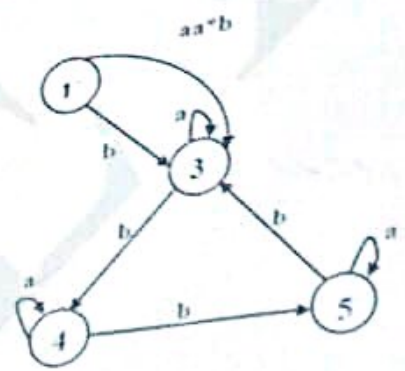
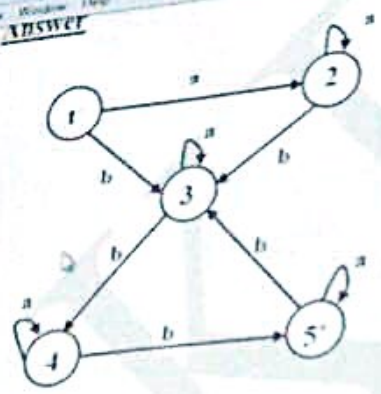
$$X \rightarrow 0 \mid 1 \mid 2 \mid 3$$

$$Y \rightarrow 0 \mid 1 \mid 2 \mid 3$$

$$S \rightarrow XX \rightarrow 0X \rightarrow 03 \quad \alpha$$

$$S \rightarrow XY \rightarrow 3Y \rightarrow 32 \quad \alpha$$

First 2021 C Theory Answer Adobe Reader
 File Edit View Window Help
ANSWER



$$(b + aa^*b)(a + ba^*ba^*b)^*(ba^*b)a^*$$

Q2. Consider $\Sigma = \{a, b\}$ find regular expression for the language that each word contains triple a. [7 Marks]

Answer

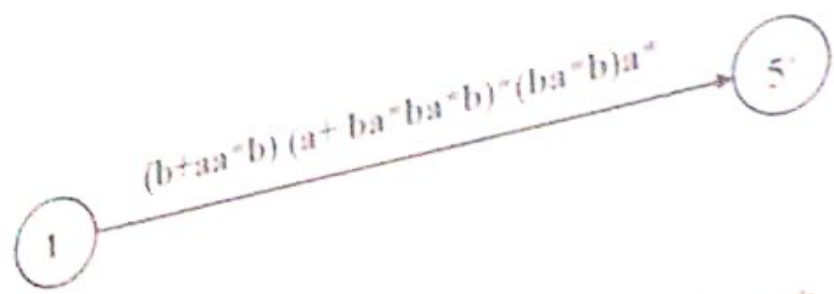
$$(b^* + \lambda)(abb^* + aabb^*)^*(\lambda + a + aa)$$

Q3. Consider $\Sigma = \{a, b\}$ construct a DFA that accepts any word that starts with a and the total number of a's are odd. [7 Marks]

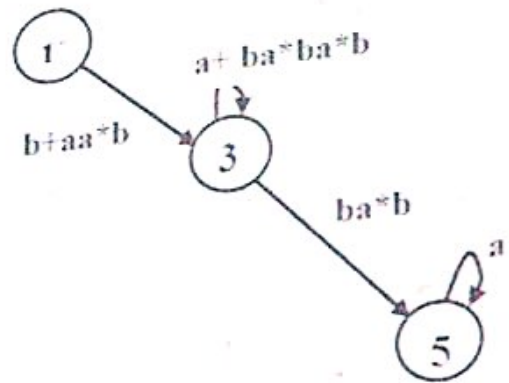
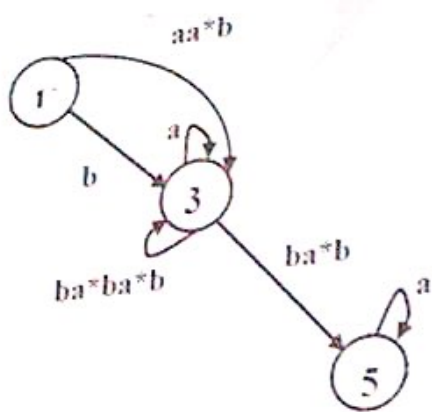
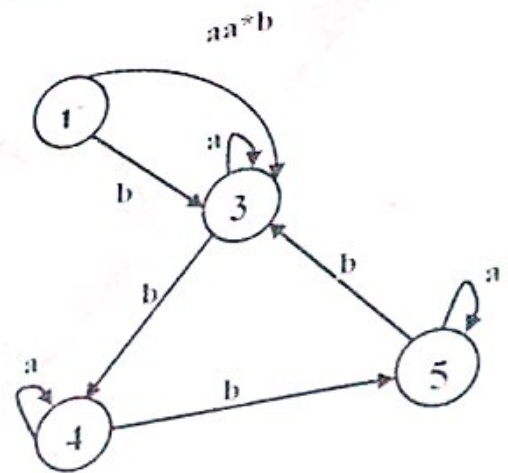
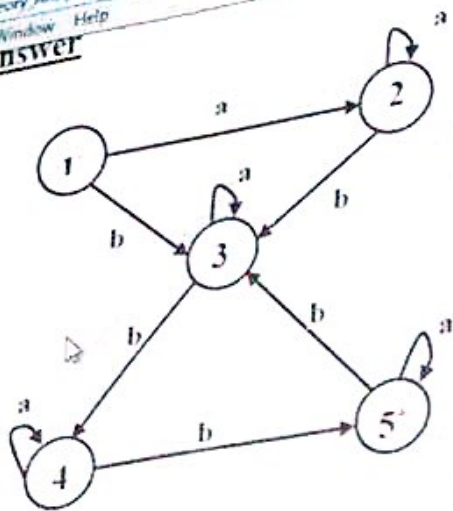
Answer

$$FA = (\{1, 2, 3\}, \{a, b\}, T1, 1, 3)$$

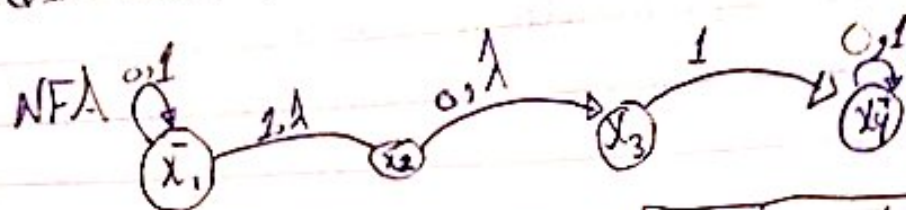
$$\begin{array}{lll} T1(1, a) = 2 & T1(2, a) = 1 & T1(2, b) = 3 \\ T1(3, a) = 1 & T1(3, b) = 3 & \end{array}$$



$$(b + aa^*b)(a + ba^*ba^*b)^*(ba^*b)a^*$$



Q1 Convert from NFA to DFA



Continue of the Answer

$$\lambda(x_1) = \{x_1, x_2, x_3\}$$

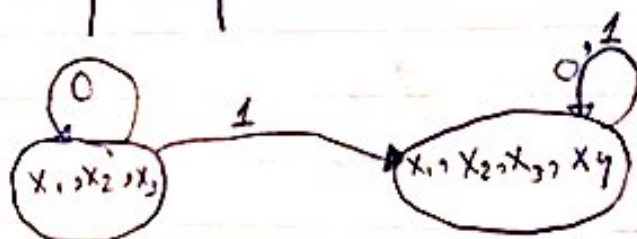
$$\lambda(x_2) = \{x_2, x_3\}$$

$$\lambda(x_3) = \{x_4\}$$

$$\lambda = \{x_4\}$$

$$\text{start } \lambda(x_1) = \{x_1, x_2, x_3\}$$

	0	1
x_1, x_2, x_3	x_1, x_2, x_3	x_1, x_2, x_3, x_4
x_1, x_2, x_3, x_4	x_1, x_2, x_3, x_4	x_1, x_2, x_3, x_4



$$TD(\{x_1, x_2, x_3\}, 1) = \{x_1, x_2, x_4\}$$

$$= (\lambda(x_1) \cup \lambda(x_2) \cup \lambda(x_3))$$

$$= x_1, x_2, x_3, x_4$$

$$TD(\{x_1, x_2, x_3\}, 0) = \{x_1, x_2, x_3\}$$

$$= (\lambda(x_1) \cup \lambda(x_2))$$

$$= x_1, x_2, x_3$$

$$TD(\{x_1, x_2, x_3, x_4\}, 1) = \{x_1, x_2, x_4\}$$

$$= (\lambda(x_1) \cup \lambda(x_2) \cup \lambda(x_3) \cup \lambda(x_4))$$

$$= x_1, x_2, x_3, x_4$$

$$TD(\{x_1, x_2, x_3, x_4\}, 0) = \{x_1, x_3, x_4\}$$

$$= (\lambda(x_1) \cup \lambda(x_3) \cup \lambda(x_4))$$

$$= x_1, x_3, x_4$$

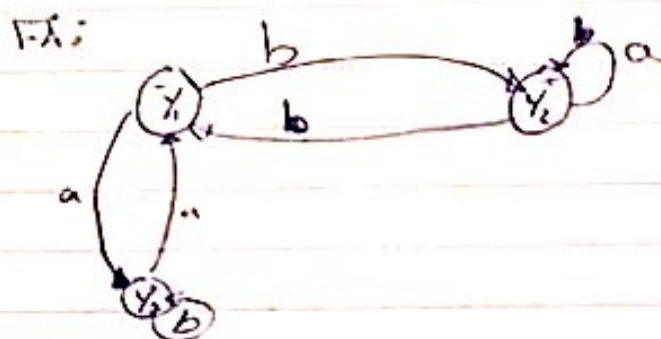
Q3 Give the regular L , that represented by the following FA:

$$FA = (\{y_1, y_2, y_3\}, \{a, b\}, \{y_1, y_2, y_3\})$$

where:

$$\begin{aligned} & \delta(y_1, a) = y_3, \quad \delta(y_1, b) = y_2, \quad \delta(y_3, a) = y_2 \\ & \delta(y_2, a) = y_3, \quad \delta(y_2, b) = y_1, \quad \delta(y_3, b) = y_1 \end{aligned}$$

Prove that L is also regular language by constructing FA



Assume

$$z_1 = y_1$$

$$\text{read } a = y_3 + y_1 \rightarrow z_2$$

$$\text{read } b = y_2 + y_1 \rightarrow z_3$$

$$z_2 = y_3 + y_1$$

$$\text{read } a = y_3 + y_1 \rightarrow z_2$$

$$\text{read } b = y_3 + y_1 + y_2 \rightarrow z_4$$

$$z_3 = y_2 + y_1$$

$$\text{read } a = y_2 + y_1 + y_3 \rightarrow z_4$$

$$\text{read } b = y_2 + y_1 \rightarrow z_3$$

$$z_4 = y_3 + y_1 + y_2$$

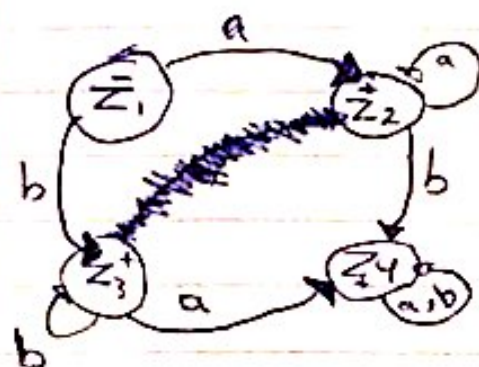
$$\text{read } a = y_1 + y_3 + y_2 \rightarrow z_4$$

$$\text{read } b = y_3 + y_2 + y_1 \rightarrow z_4$$

Continue; Create table

	a	b
z_1	z_2	z_3
z_2	z_2	z_4
z_3	z_2	z_3
z_4	z_4	z_4

* To choose start and Final we choose the first thing we started with as start, and as Final each thing had been including Finals



Q3. Give the regular L, that is represented by the following FA: (5 marks)

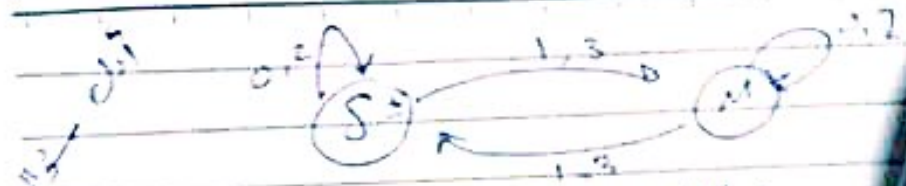
$$FA = (\{ y_1, y_2, y_3 \}, \{ a, b \}, t, y_1, \{ y_2, y_3 \})$$

Where

$$\begin{array}{lll} t(y_1, a) = y_3 & t(y_2, a) = y_2 & t(y_3, a) = y_1 \\ t(y_1, b) = y_2 & t(y_2, b) = y_1 & t(y_3, b) = y_3 \end{array}$$

Prove that L^* is also regular language by constructing FA.

$$L = (y_1 a), (y_1 b), (y_2 a), (y_2 b), (y_3 a), (y_3 b), \\ t(y_1 a), t(y_1 b), t(y_2 a), t(y_2 b), t(y_3 a), t(y_3 b), \\ y_2, y_2, y_1, y_1, y_3, y_1, y_1, y_2, y_1$$



$S \rightarrow 0S \mid 2S \mid 1M \mid 3M \mid \epsilon$
 $M \rightarrow 0M \mid 2M \mid 1S \mid 3S$

سوال ۱۰



سوال ۱۱

Remove ϵ

$S \rightarrow xz \mid y \mid aSa$
 $X \rightarrow ax \mid by \mid cb$
 $Y \rightarrow bYb \mid bY \mid bb \mid b$

Remove unit

$S \rightarrow xz \mid aSa \mid aa \mid bYb \mid bY \mid bb \mid b$
 $X \rightarrow ax \mid abY \mid ab$
 $Y \rightarrow bYb \mid bY \mid bb \mid b$

to CNF

11

$S \rightarrow XQ \mid AR_1 \mid AA \mid BR_2 \mid BY \mid \epsilon$
 $R_1 \rightarrow SA$
 $R_2 \rightarrow YB$
 $Q \rightarrow ?$
 $A \rightarrow a$
 $B \rightarrow b$
 $X \rightarrow AX \mid AR_3 \mid AB$
 $R_3 \rightarrow BY$
 $Y \rightarrow BR_2 \mid BY \mid BB \mid b$

$$(b + aa^*b)^*$$

Q2. Consider $\Sigma = \{a, b\}$ find regular expression for the language that each word contains triple a. [7 Marks]

Answer

$$(b^* + \lambda)(abb^* + aabb^*)^*(\lambda + a + aa)$$

Q3. Consider $\Sigma = \{a, b\}$ construct a DFA that accepts any word starting with a and the total number of a's are odd. [7 Marks]

Answer

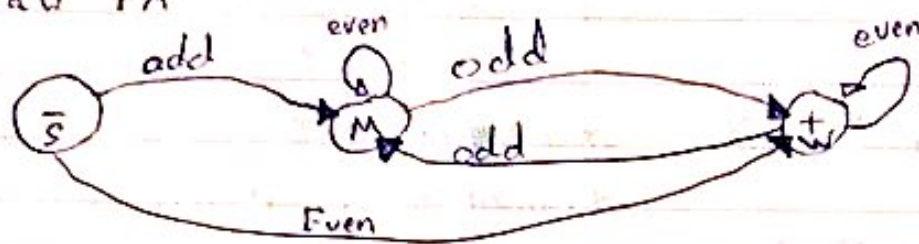
$$FA = (\{1, 2, 3\}, \{a, b\}, T1, 1, 3)$$

$$\begin{array}{lll} T1(1, a) = 2 & T1(2, a) = 1 & T1(2, b) = 3 \\ T1(3, a) = 1 & T1(3, b) = 3 & \end{array}$$

Q2 (A) Consider $\Sigma = \{0, 1, 2, 3\}$, Construct CFG that can give any number whose sum of its digits is even.

Solution

draw FA



odd = 1, 3

Even = 0, 2

CFG :

~~CFG :~~

$S \rightarrow 1M \mid 3M \mid 0W \mid 2W$

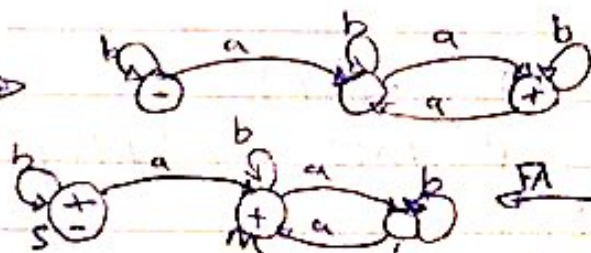
$M \rightarrow 0M \mid 2M \mid 1W \mid 3W$

$W \rightarrow 0W \mid 2W \mid 1M \mid 3M \mid \lambda$

Q2 B Consider $\Sigma = \{a, b\}$ Construct CFG that can generate any word that does not contain double a

Solution

double a FA \rightarrow



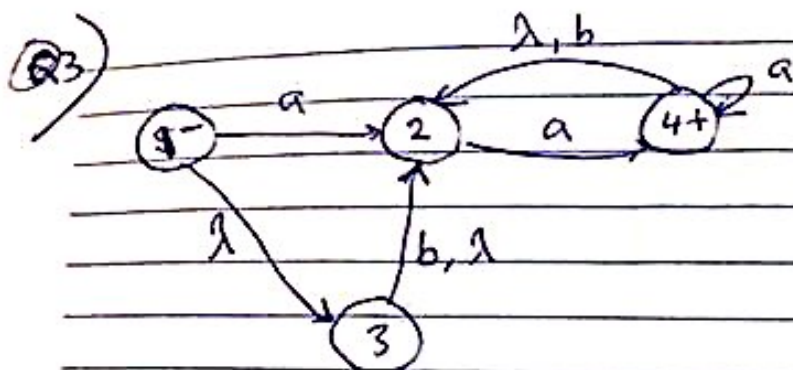
to create Not double a FA
 to Make final state non-final
 to Make non-final into final

CFG:

$S \rightarrow bS \mid aM \mid \lambda$

$M \rightarrow bM \mid \lambda$

Since this is a rejected state, I don't have to write it in my CFG



Construct the DFA

	a	b
$\lambda(1) = \{1, 2, 3\}$	$\{2, 4\}$	$\{2\}$
$\lambda(2) = \{2\}$	$\{2, 4\}$	\emptyset
$\lambda(3) = \{2, 3\}$	$\{2, 4\}$	$\{2\}$
$\lambda(4) = \{2, 4\}$		

start $\lambda(1) = \{1, 2, 3\}$

$$T_D(\{1, 2, 3\}, a) = \lambda(T_N(1, a) \cup T_N(2, a) \cup T_N(3, a))$$

$$= \lambda(2 \cup 4 \cup \emptyset) = \lambda(2, 4) = \lambda(2) \cup \lambda(4)$$

$$= \{2, 4\}$$

$$T_D(\{1, 2, 3\}, b) = \lambda(T_N(1, b) \cup T_N(2, b) \cup T_N(3, b))$$

$$= \lambda(\emptyset \cup \emptyset \cup 2) = \lambda(2) = \{2\}$$

$$T_D(\{2\}, a) = \lambda(T_N(2, a)) = \lambda(4) = \{2, 4\}$$

$$T_D(\{2\}, b) = \lambda(T_N(2, b)) = \lambda(\emptyset) = \emptyset$$

$$T_D(\{2, 4\}, a) = \lambda(T_N(2, a) \cup T_N(4, a))$$

$$= \lambda(4 \cup 4) = \lambda(4) = \{2, 4\}$$

$$T_D(\{2, 4\}, b) = \lambda(T_N(2, b) \cup T_N(4, b)) = \lambda(\emptyset \cup 2) = \{2\}$$



① RE for L

start with double a and end with at least one b and does not contain triple a.

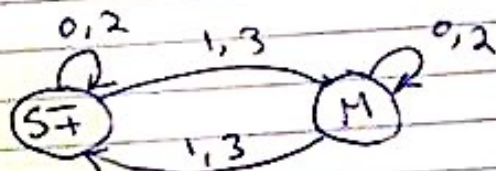
$$(aab + \lambda)(ab + aab)^+ b^+$$

② CFG for L

$$\Sigma = \{0, 1, 2, 3\}$$

sum of its digits is even

Q6



$$\begin{aligned} S &\rightarrow 0S \mid 2S \mid 1M \mid 3M \mid \lambda \\ M &\rightarrow 0M \mid 2M \mid 1S \mid 3S \end{aligned}$$

③ PDA for L

$$L = a^n b^{2n}$$

④ CFL

~~A~~ L₁: start with a and end with b
CFL L₂: contains double b.

prove

L₁L₂ is CFL -

$$\begin{aligned} L_1: S_1 &\rightarrow aM_1b \\ M_1 &\rightarrow aM_1 \mid bM_1 \mid \lambda \end{aligned}$$

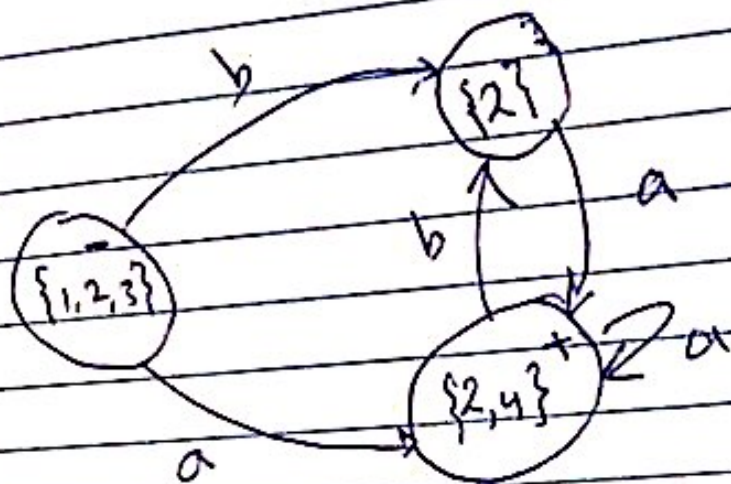
$$S \rightarrow S_1 S_2$$

L₂

$$\begin{aligned} S_2 &\rightarrow bS_2 \mid \lambda M_2 \\ M_2 &\rightarrow bS_2 \mid aF_2 \\ F_2 &\rightarrow aF_2 \mid bF_2 \mid \lambda \end{aligned}$$

Q5





$NFA = (\{1,2,3\}, \{a,b\}, t, 1, \{4\})$

$$t(1, a) = 2$$

$$t(2, a) = 4$$

$$t(1, \lambda) = 3$$

$$t(3, b) = 2$$

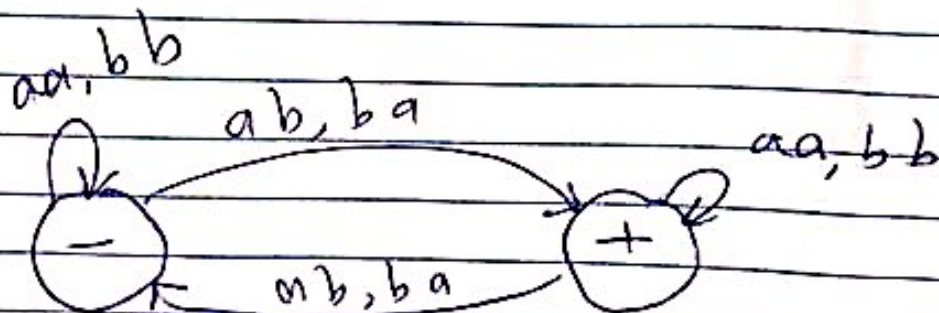
$$t(4, a) = 4$$

$$t(3, \lambda) = 2$$

$$t(4, b) = 2$$

$$t(4, \lambda) = 2$$

Q4)



Q4