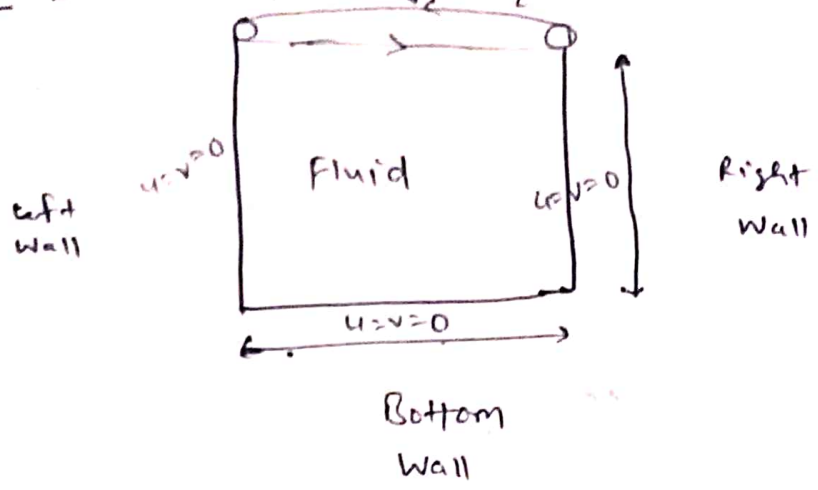
Q 1. Lid driven Cavity



BCs for ψ (streamlines)

Left wall

$u=0, \frac{\partial \psi}{\partial y}=0$

$v=0$

$\psi=C \rightarrow \psi=0$ (for easy calculation)

Right Wall

$u=0$

$\psi=C$

$\downarrow$

$\psi=0$ (By me)

Bottom Wall

$v=0$

$-\frac{\partial \psi}{\partial x}=0$

$\psi=C$

$\downarrow$

$\psi=0$ (By me)

Top Wall

$v=0$

$-\frac{\partial \psi}{\partial x}=0$

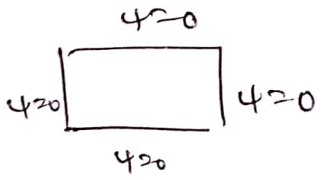
$\psi=C$

$\downarrow$

$\psi=0$ (By me)

BCs for ω (vorticity)

$$\omega = -\left(\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}\right)$$



Left Wall

$u=0, \frac{\partial \psi}{\partial y}=0, \frac{\partial^2 \psi}{\partial y^2}=0$ $\quad i=1 \quad M$, $j=1 \quad N$

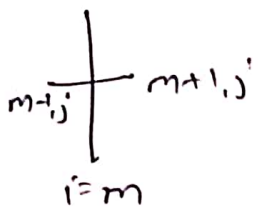
$$\omega_{i,j} = -\frac{\psi_{2,j}-2\psi_{1,j}+\psi_{0,j}}{\Delta x^2}$$

$v=0, \frac{\partial \psi}{\partial x}=0, \frac{\psi_{2,j}-\psi_{0,j}}{2\Delta x}=0 \Rightarrow \psi_{2,j}=\psi_{0,j}$

$$\omega_{i,j} = \frac{\psi_{2,j}-2\psi_{i,j}+\psi_{0,j}}{\Delta x^2} = -\frac{2}{\Delta x^2}\left(\psi_{2,j}-\psi_{i,j}\right)$$

Right Wall

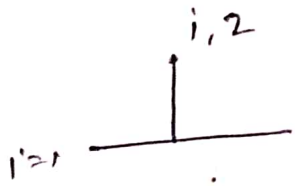
$$\omega = -\frac{\partial^2 \psi}{\partial y^2} = \frac{-\psi_{m+1,j}-2\psi_{m,j}+\psi_{m-1,j}}{\Delta x^2}$$



$\frac{\partial \psi}{\partial x}=0 \Rightarrow \psi_{m+1,j}=\psi_{m-1,j}$

$$w_{m,j} = \frac{-\psi_{m-1} - 2\psi_{m,j} + \psi_{m+1,j}}{\Delta x^2} = -\frac{2}{\Delta x^2}\left[\psi_{m-1,j} - \psi_{m,j}\right)$$

Bottom Wall



$v = 0.$

$$w_{i,1} = \frac{-\psi_{i,2} - 2\psi_{i,1} + \psi_{i,0}}{\Delta y^2}$$

$$w_{i,1} = -\frac{2}{\Delta y^2}\left(\psi_{i,2} - \psi_{i,1}\right)$$

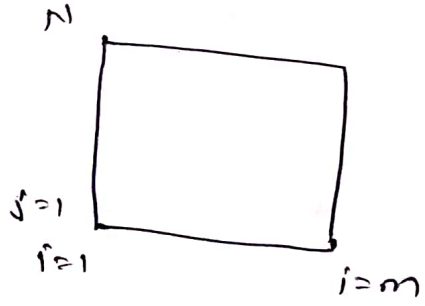
Top boundary

$$w = -\frac{\partial^2 \psi}{\partial y^2} = -\frac{\psi_{i,N+1,} - 2\psi_{i,N} + \psi_{i,N+1}}{\Delta y^2}$$

$$\psi_{1,N+1} = \psi_{1,N-1} + 2U\Delta y$$

$$w_{i,N} = \frac{-\psi_{i,N-1} + 2U\Delta y - 2\psi_{i,N-1} + \psi_{i,N-1}}{\Delta y^2}$$

$$w_{i,N} = -\frac{2}{\Delta y^2}\left\{-\psi_{i,N-1} - \psi_{i,N} + U\Delta y\right)$$



$\psi_{1,j} = 0$ Left wall

$\psi_{m,j} = 0$ Right Boundary

$\psi_{i,1} = 0$ Bottom Wall

$\psi_{1,N} = 0$ top wall

Stream function equation

$$\psi_{i,j} = \frac{1}{2(1+\beta^2)} \left\{ \psi_{i+1,j} + \psi_{i-1,j} + \beta^2 (\psi_{i,j+1} + \psi_{i,j-1}) + \Delta x^2 \omega_{i,j} \right]$$

Vorticity Equation

$$\omega_{i,j} = \frac{1}{2(1+\beta^2)} \left[\left\{ 1 - (\psi_{i,j+1} - \psi_{i,j-1}) \frac{\beta}{4\nu} \right\} \omega_{i+1,j} \right.$$

$$+ \left\{ 1 + (\psi_{i,j+1} - \psi_{i,j-1}) \frac{\beta}{4\nu} \right\} \omega_{i-1,j}$$

$$+ \left\{ 1 + (\psi_{i+1,j} - \psi_{i-1,j}) \frac{1}{4\nu\beta} \right\} \beta^2 \omega_{i,j-1}$$

$$+ \left\{ 1 - (\psi_{i+1,j} - \psi_{i-1,j}) \frac{1}{4\nu\beta} \right\} \beta^2 \omega_{i,j-1}$$

Velocity equation

$$u_{i,j} = \frac{\psi_{i,j+1} - \psi_{i,j-1}}{2\Delta y}$$

$$v_{i,j} = - \frac{\psi_{i+1,j} - \psi_{i-1,j}}{2\Delta x}$$

where,

$$\beta = \frac{\Delta x}{\Delta y}$$