Lecture 04

Multivariable linear regression

Recap

- Hypothesis

$$H(x) = Wx + b$$

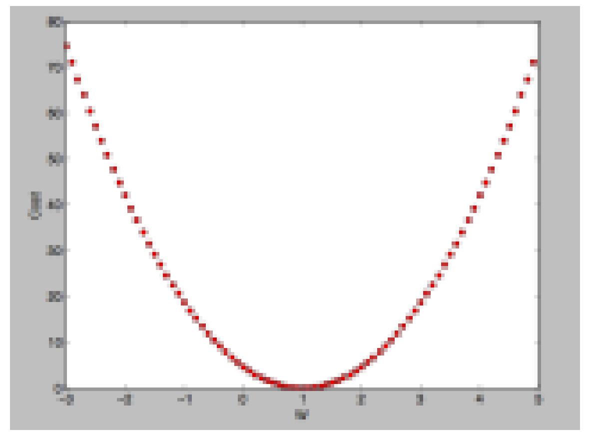
- Cost function

  = **Loss function**

prediction   true value

$$cost(W, b) = \frac{1}{m}\sum_{i=1}^{m}(H(x^{(i)}) - y^{(i)})^2$$

- Gradient descent algorithm



convex

Predicting exam score: regression using one input (x)

one-variable

one-feature

x (hours)	y (score)
10	90
9	80
3	50
2	60
11	40

Predicting exam score: regression using three inputs (x1, x2, x3)

multi-variable/feature

Test Scores for General Psychology

x_1 (quiz 1)	x_2 (quiz 2)	x_3 (midterm 1)	Y (final)
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

Hypothesis

$$H(x) = Wx + b$$

$$\boxed{H(x_1, x_2, x_3) = w_1x_1 + w_2x_2 + w_3x_3 + b}$$

Cost function

$$H(x_1, x_2, x_3) = w_1x_1 + w_2x_2 + w_3x_3 + b$$

$$\boxed{cost(W,b) = \frac{1}{m}\sum_{i=1}^{m}(H(x_1^{(i)}, x_2^{(i)}, x_3^{(i)}) - y^{(i)})^2}$$

Multi-variable

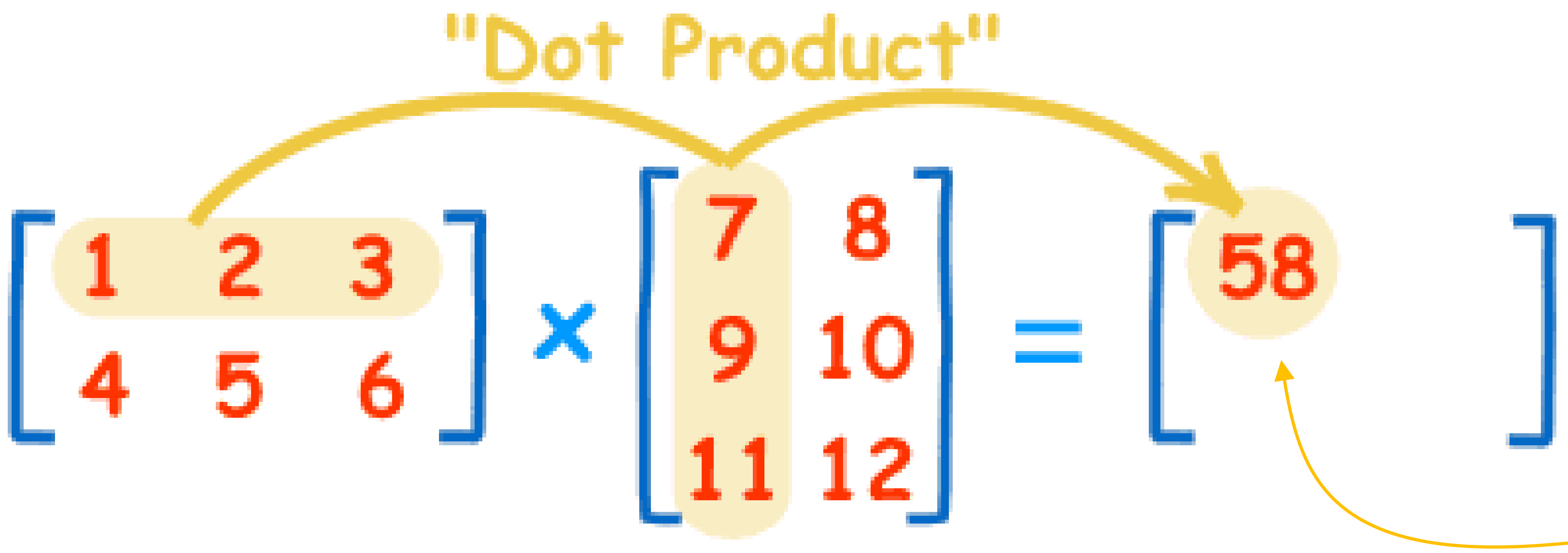
$$H(x_1, x_2, x_3) = w_1x_1 + w_2x_2 + w_3x_3 + b$$

$$H(x_1, x_2, x_3, ..., x_n) = w_1x_1 + w_2x_2 + w_3x_3 + ... + w_nx_n + b$$

Matrix

$$w_1x_1 + w_2x_2 + w_3x_3 + \ldots + w_nx_n$$

Matrix multiplication



1X 7 = 7

2X 9 = 18

3X 11 = 33

7+18+33 = **58**

Hypothesis using matrix

$$w_1x_1 + w_2x_2 + w_3x_3 + ... + w_nx_n$$

$$\boxed{\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_1w_1 + x_2w_2 + x_3w_3 \end{pmatrix}}$$

$$H(X) = XW$$

$$H(x) = Wx + b$$

Hypothesis using matrix

Test Scores for General Psychology

x_1	x_2	x_3	Y
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

$$H(x) = Wx + b$$

$$H(x_1, x_2, x_3) = x_1w_1 + x_2w_2 + x_3w_3 + b$$

Hypothesis using matrix

Test Scores for General Psychology

x_1	x_2	x_3	Y
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

$$H(x_1, x_2, x_3) = x_1w_1 + x_2w_2 + x_3w_3 \cancel{+b}$$

$$\boxed{\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_1w_1 + x_2w_2 + x_3w_3 \end{pmatrix}}$$

$$H(X) = XW$$

Many x instances

Test Scores for General Psychology

Instance : 5

x_1	x_2	x_3	Y
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

Instance

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_1w_1 + x_2w_2 + x_3w_3 \end{pmatrix}$$

Hypothesis using matrix

x_1	x_2	x_3	Y
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

$$w_1x_1 + w_2x_2 + w_3x_3 + \ldots + w_nx_n$$

Instance

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_{11}w_1 + x_{12}w_2 + x_{13}w_3 \\ x_{21}w_1 + x_{22}w_2 + x_{23}w_3 \\ x_{31}w_1 + x_{32}w_2 + x_{33}w_3 \\ x_{41}w_1 + x_{42}w_2 + x_{43}w_3 \\ x_{51}w_1 + x_{52}w_2 + x_{53}w_3 \end{pmatrix}$$

$$H(X) = XW$$

Hypothesis using matrix

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_{11}w_1 + x_{12}w_2 + x_{13}w_3 \\ x_{21}w_1 + x_{22}w_2 + x_{23}w_3 \\ x_{31}w_1 + x_{32}w_2 + x_{33}w_3 \\ x_{41}w_1 + x_{42}w_2 + x_{43}w_3 \\ x_{51}w_1 + x_{52}w_2 + x_{53}w_3 \end{pmatrix}$$

$$[\boxed{5,}\ \underbrace{3]\qquad[3,}_{=}\ \boxed{1]} \qquad\qquad [\boxed{5,}\ \boxed{1]}$$

$$H(X) = XW$$

Hypothesis using matrix

$$\left(\boxed{X}\right) \times \left(\boxed{W}\right) = \left(\boxed{H(X)}\right)$$

[5, 3] [3, 1] [5, 1]

Input

Instance : 5

X (variable value) : 3

$$H(X) = XW$$

Output

Instance : 5

Y (variable value) : 1

Hypothesis using matrix

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_{11}w_1 + x_{12}w_2 + x_{13}w_3 \\ x_{21}w_1 + x_{22}w_2 + x_{23}w_3 \\ x_{31}w_1 + x_{32}w_2 + x_{33}w_3 \\ x_{41}w_1 + x_{42}w_2 + x_{43}w_3 \\ x_{51}w_1 + x_{52}w_2 + x_{53}w_3 \end{pmatrix}$$

[n, 3] [3, 1] [n, 1]

None

$$H(X) = XW$$

None

Hypothesis using matrix (n output)

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \; ? \; = \begin{pmatrix} x_{11}w_{11}+x_{12}w_{21}+x_{13}w_{31} & x_{11}w_{12}+x_{12}w_{22}+x_{13}w_{32} \\ x_{21}w_{11}+x_{22}w_{21}+x_{23}w_{31} & x_{21}w_{12}+x_{22}w_{22}+x_{23}w_{32} \\ x_{31}w_{11}+x_{32}w_{21}+x_{33}w_{31} & x_{31}w_{12}+x_{32}w_{22}+x_{33}w_{32} \\ x_{41}w_{11}+x_{42}w_{21}+x_{43}w_{31} & x_{41}w_{12}+x_{42}w_{22}+x_{43}w_{32} \\ x_{51}w_{11}+x_{52}w_{21}+x_{53}w_{31} & x_{51}w_{12}+x_{52}w_{22}+x_{53}w_{32} \end{pmatrix}$$

[n, 3] [3, 2] [n, 2]

Output : 2

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix (n output)

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \boxed{\begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{pmatrix}} = \begin{pmatrix} x_{11}w_{11}+x_{12}w_{21}+x_{13}w_{31} & x_{11}w_{12}+x_{12}w_{22}+x_{13}w_{32} \\ x_{21}w_{11}+x_{22}w_{21}+x_{23}w_{31} & x_{21}w_{12}+x_{22}w_{22}+x_{23}w_{32} \\ x_{31}w_{11}+x_{32}w_{21}+x_{33}w_{31} & x_{31}w_{12}+x_{32}w_{22}+x_{33}w_{32} \\ x_{41}w_{11}+x_{42}w_{21}+x_{43}w_{31} & x_{41}w_{12}+x_{42}w_{22}+x_{43}w_{32} \\ x_{51}w_{11}+x_{52}w_{21}+x_{53}w_{31} & x_{51}w_{12}+x_{52}w_{22}+x_{53}w_{32} \end{pmatrix}$$

[n, 3] $\boxed{\texttt{[3, 2]}}$ [n, 2]

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

WX vs XW

- Lecture (theory):

$$H(x) = Wx + b$$

- Implementation (TensorFlow)

$$H(X) = XW$$