



Lecture 03

# How to minimize cost

# Hypothesis and Cost

$$H(x) = Wx + b$$

$$cost(W, b) = \frac{1}{m} \sum_{i=1}^m (H(x^{(i)}) - y^{(i)})^2$$

# Simplified hypothesis

$$H(x) = Wx$$
$$cost(W) = \frac{1}{m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$

# What $\text{cost}(W)$ looks like?

$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$

x	y
1	1
2	2
3	3

$$\rightarrow W = 1, \text{ cost}(W) = ? \quad 0$$

$$\frac{1}{3}((1 * 1 - 1)^2 + (1 * 2 - 2)^2 + (1 * 3 - 3)^2)$$

*0                    0                    0*

# What $\text{cost}(W)$ looks like?

$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$

x	Y
1	1
2	2
3	3

- $W=I$ ,  $\text{cost}(W)=0$

$$\frac{1}{3}((1 * 1 - 1)^2 + (1 * 2 - 2)^2 + (1 * 3 - 3)^2)$$

- $W=0$ ,  $\text{cost}(W)=4.67$

$$\frac{1}{3}((0 * 1 - 1)^2 + (0 * 2 - 2)^2 + (0 * 3 - 3)^2)$$

- $W=2$ ,  $\text{cost}(W)=?$  4.67

$$\frac{1}{3} ((2 * 1 - 1)^2 + (2 * 2 - 2)^2 + (2 * 3 - 3)^2)$$

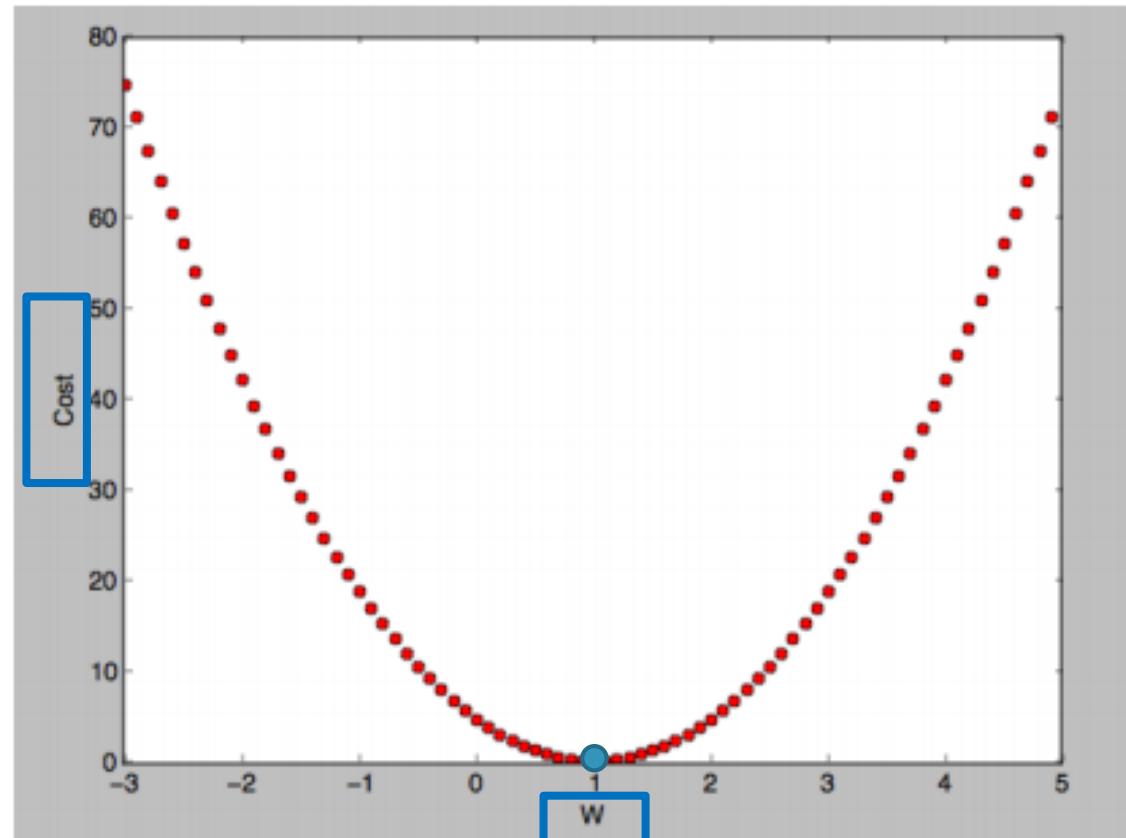
# What $\text{cost}(W)$ looks like?

- $W=1, \text{cost}(W)=0$
- $W=0, \text{cost}(W)=4.67$
- $W=2, \text{cost}(W)=4.67$

# What $\text{cost}(W)$ looks like?

$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$

- $W=1, \text{cost}(W)=0$
- $W=0, \text{cost}(W)=4.67$
- $W=2, \text{cost}(W)=4.67$

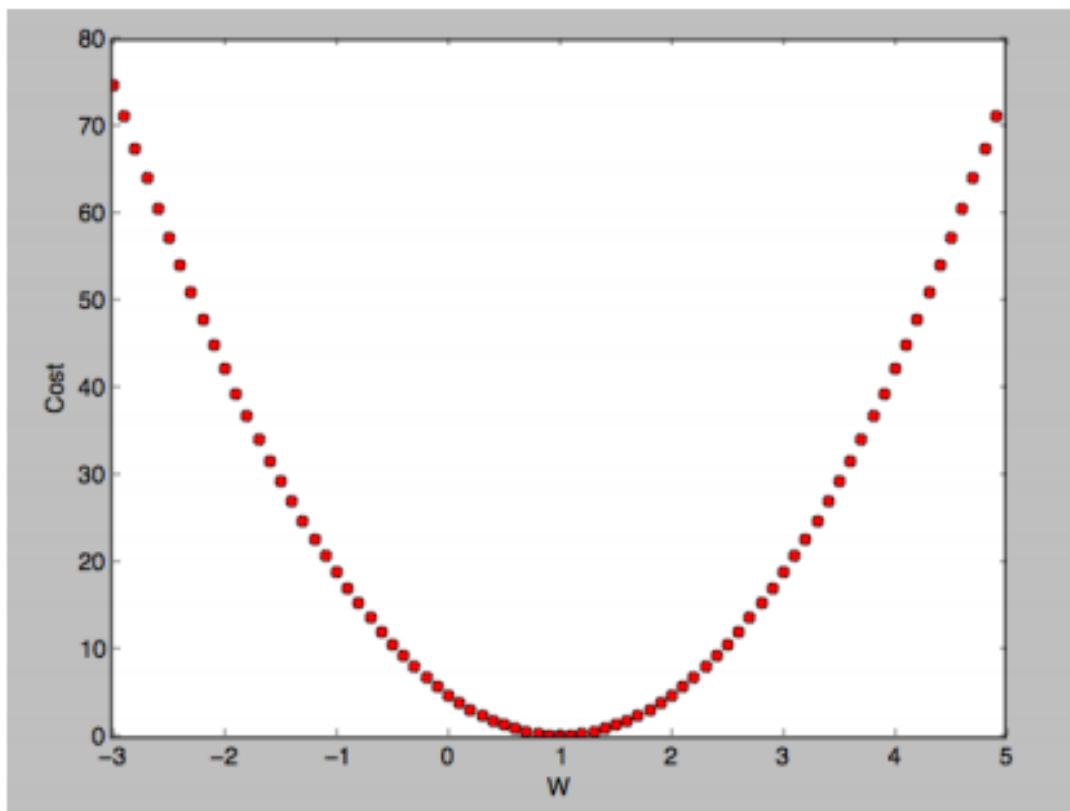


# Gradient descent algorithm

- Minimize cost function
- Gradient descent is used many **minimization problems**
- For a given **cost function**, **cost (W, b)**, it will find **W, b** to minimize cost
- It can be applied to more general function: **cost (w1, w2, w3 ...)**

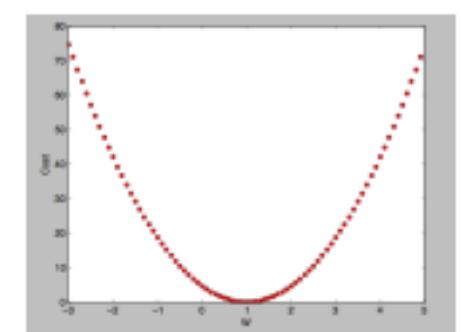
# How it works?

- How would you find the lowest point?



# How it works?

- Start with initial guesses
  - ▶ Start at  $0,0$  (or any other value)
  - ▶ Keeping changing  $W$  and  $b$  a little bit to try and reduce  $\text{cost}(W, b)$
- Each time you change the parameters, you select the gradient which reduces  $\text{cost}(W, b)$  the most possible
- Repeat
- Do so until you converge to a local minimum
- Has an interesting property
  - ▶ Where you start can determine which minimum you end up



gradient는 미분을  
이용하여 구할 수 있다.

# Formal definition

$$cost(W) = \frac{1}{m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$



$$cost(W) = \frac{1}{2m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$

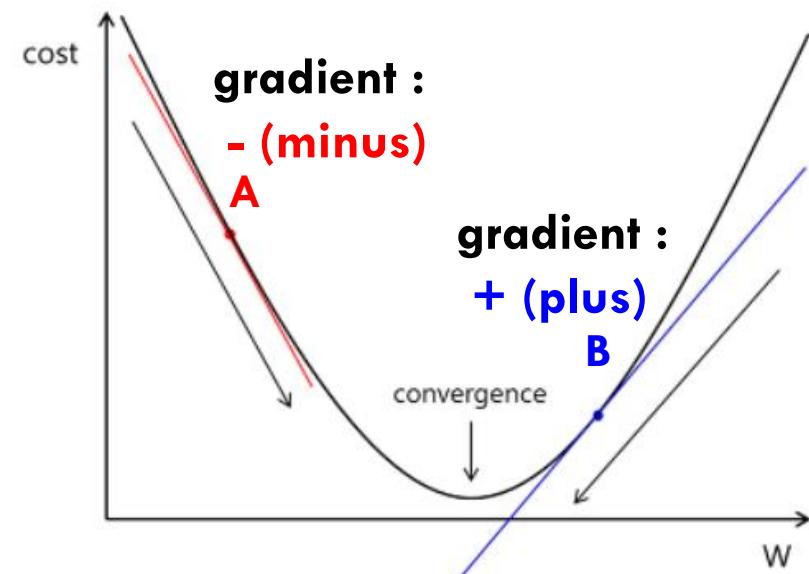
미분을 적용하기 쉽도록  
분모에 2를 곱한다.

# Formal definition

$$cost(W) = \frac{1}{2m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$

learning\_rate

$$W := W - \alpha \frac{\partial}{\partial W} cost(W)$$

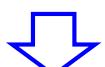


# Formal definition

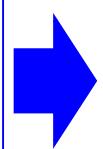
$$cost(W) = \frac{1}{m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$



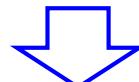
$$cost(W) = \frac{1}{2m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$



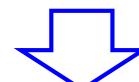
$$W := W - \alpha \frac{\partial}{\partial W} cost(W)$$



$$W := W - \alpha \frac{\partial}{\partial W} \frac{1}{2m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$



$$W := W - \alpha \frac{1}{2m} \sum_{i=1}^m 2(Wx^{(i)} - y^{(i)})x^{(i)}$$



$$W := W - \alpha \frac{1}{m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})x^{(i)}$$

# Derivative Calculator

<https://www.derivative-calculator.net/>



# Derivative Calculator

Calculate derivatives online – with steps and graphing!

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Calculadora de Derivadas en español  
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This will take a few seconds.

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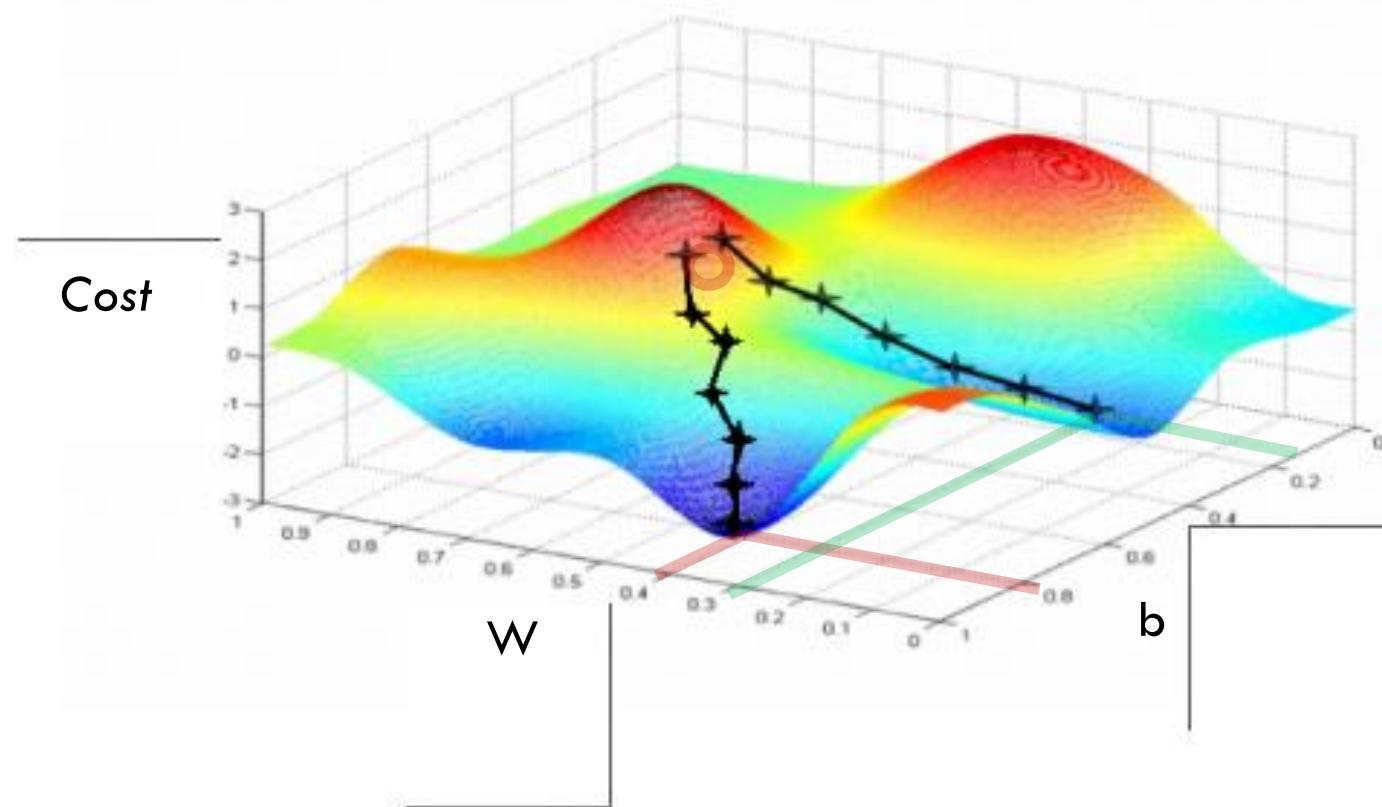


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# Gradient descent algorithm

$$\textcolor{brown}{W} := W - \alpha \frac{1}{m} \sum_{i=1}^m (\textcolor{brown}{W}x^{(i)} - \textcolor{teal}{y}^{(i)})x^{(i)}$$

# Convex function



# Convex function

$$cost(W, b) = \frac{1}{m} \sum_{i=1}^m (H(x^{(i)}) - y^{(i)})^2$$

