$A_1 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $A_2 = \begin{bmatrix} 1+j2 & 2+j3 \\ 3+j4 & 4+j5 \end{bmatrix} \Rightarrow A_1^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ (1)

$A_2^H = \begin{bmatrix} 1-j2 & 2-j3 \\ 3-j4 & 4-j5 \end{bmatrix}^T = \begin{bmatrix} 1-j2 & 3-j4 \\ 2-j3 & 4-j5 \end{bmatrix}$ $A_1 + A_2^H = \begin{bmatrix} 2-j2 & 5-j4 \\ 5-j3 & 8-j5 \end{bmatrix}$

conjugate ↓ transpose $(A_1 + A_2^H)^H = \begin{bmatrix} 2+j2 & 5+j4 \\ 5+j3 & 8+j5 \end{bmatrix}^T = \begin{bmatrix} 2+j2 & 5+j3 \\ 5+j4 & 8+j5 \end{bmatrix}$

(2)

$A+B = \begin{bmatrix} 5 & 5 & 11 \\ 1 & -7 & 11 \\ 7 & 4 & 13 \end{bmatrix}$ $A-B = \begin{bmatrix} 1 & 3 & -1 \\ 5 & -5 & 3 \\ -3 & -2 & -5 \end{bmatrix}$

$AB = \begin{bmatrix} 3 & 4 & 5 \\ 3 & -6 & 7 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 & 6 \\ -2 & -1 & 4 \\ 5 & 3 & 9 \end{bmatrix} = \begin{bmatrix} 6-8+25 & 3-4+15 & 18+16+45 \\ 6+12+35 & 3+6+21 & 18-24+63 \\ 4-2+20 & 2-1+12 & 12+4+36 \end{bmatrix}$

$AB = \begin{bmatrix} 23 & 14 & 79 \\ 53 & 30 & 57 \\ 22 & 11 & 52 \end{bmatrix}$ $B-A = \begin{bmatrix} -1 & -3 & 1 \\ -5 & 5 & -3 \\ 3 & 2 & 5 \end{bmatrix}$

$BA = \begin{bmatrix} 21 & 8 & 41 \\ -1 & 2 & -1 \\ 42 & 11 & 82 \end{bmatrix}$ $\text{Trace}(A) = 3-6+4 = 1$

$\text{Trace}(B) = 2-1+9 = 10$

$A^2B = \begin{bmatrix} 391 & \cdots & \cdots \\ \vdots & -47 & \vdots \\ \cdots & \cdots & 423 \end{bmatrix}$ $\text{Trace}(A^2B) = 391-47+423 = 767$

(3)

$A^TB^T = (BA)^T$ ✱ $(2A)^T - (3B)^T = (2A-3B)^T$

$AB = \begin{bmatrix} -6 & 2 & -5 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -3 & -4 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} -12-6-10 & \cdots \\ \cdots & 6-12+6 \end{bmatrix} = \begin{bmatrix} -28 & \cdots \\ \cdots & 0 \end{bmatrix} \Rightarrow \text{Trace}(AB) = -28$ (4)

$BA = \begin{bmatrix} 2 & 3 \\ -3 & -4 \\ 2 & -6 \end{bmatrix} \begin{bmatrix} -6 & 2 & -5 \\ 2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} -12+6 & \cdots & \cdots \\ \cdots & -6-12 & \cdots \\ \cdots & \cdots & -10+6 \end{bmatrix} = \begin{bmatrix} -6 & \cdots & \cdots \\ \cdots & -18 & \cdots \\ \cdots & \cdots & -4 \end{bmatrix} \Rightarrow \text{Trace}(BA) = -28$

(5) پاسخ تمرین جبرخطی اسری (1)

$$A = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

compute A^2, A^3, $A^n = ?$

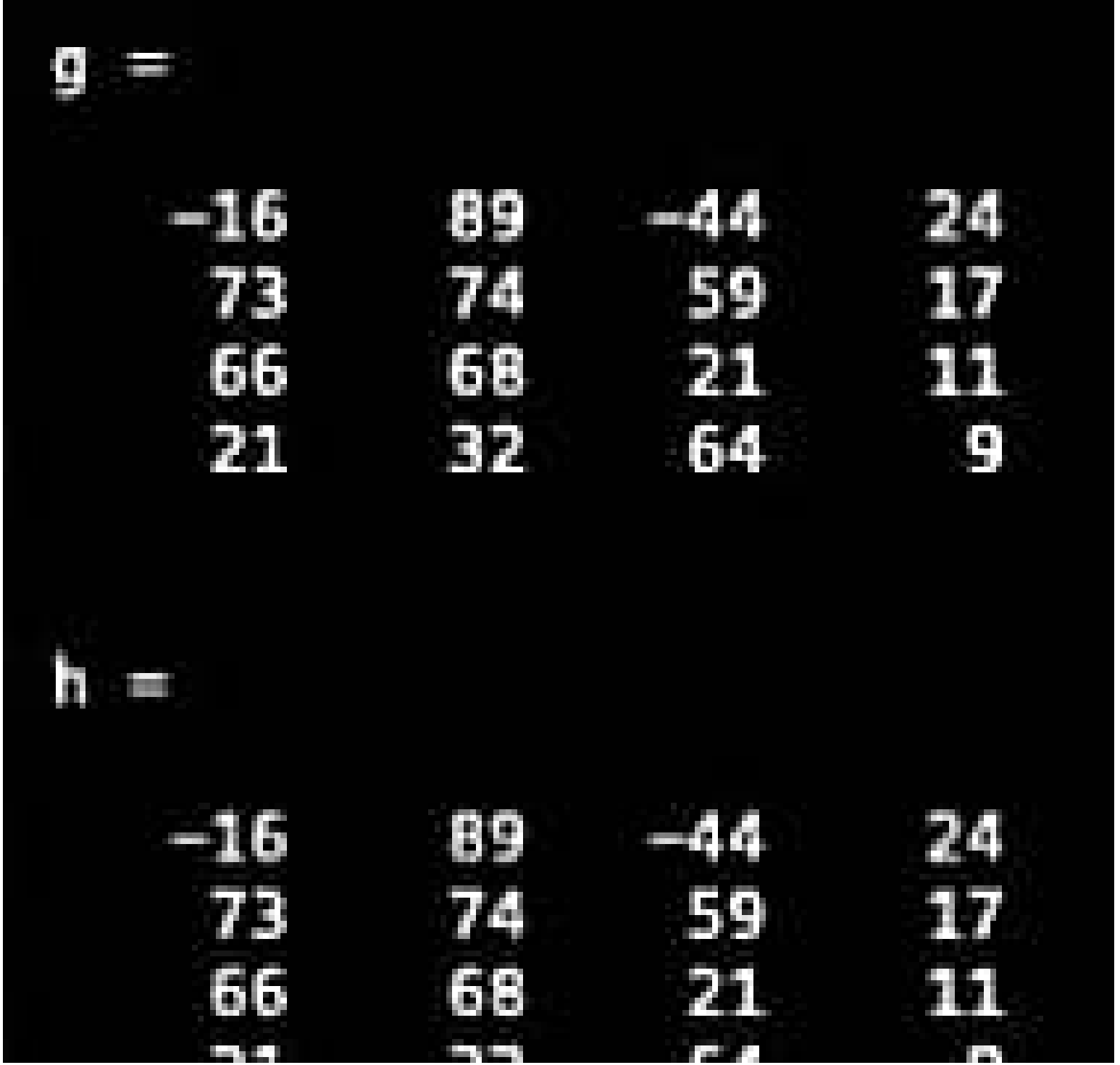
$$1/2 = a \Rightarrow A = \begin{bmatrix} a & -a \\ -a & a \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a & -a \\ -a & a \end{bmatrix}\begin{bmatrix} a & -a \\ -a & a \end{bmatrix} = \begin{bmatrix} 2a^2 & -2a^2 \\ -2a^2 & 2a^2 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 2a^2 & -2a^2 \\ -2a^2 & 2a^2 \end{bmatrix}\begin{bmatrix} a & -a \\ -a & a \end{bmatrix} = \begin{bmatrix} 4a^3 & -4a^3 \\ -4a^3 & 4a^3 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 2^{n-1}a^n & -2^{n-1}a^n \\ -2^{n-1}a^n & 2^{n-1}a^n \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

```
Q3 Ansers

a =

    16     0    -6     8
    10    10    16   -14
     6    -4    -2     0
     0    14    10     8

b =

     5     1    -2    14
     8     9    23     5
     3    -6    -9     9
     2     8     6     6

c =

    25    -3    -9   -22
     1    -2   -29   -50
     6     8    22   -27
    -6    11     7     2

d =

    25     1     6    -6
    -3    -2     8    11
    -9   -29    22     7
   -22   -50   -27     2

e =

   -16    24    36    61
   -14   -14     9   168
   -15    -1   -19    -3
    29    12    69   137
```


g =
-16 89 -44 24
73 74 59 17
66 68 21 11
21 32 64 9
h =
-16 89 -44 24
73 74 59 17
66 68 21 11