

①

$$a. \begin{bmatrix} 3 & -6 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 18 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & -6 & 9 \\ 4 & -2 & 18 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & -6 & 9 \\ 0 & 6 & 6 \end{bmatrix} \rightarrow \begin{cases} x_2 = 1 \\ x_1 = 5 \end{cases} \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$b. \begin{bmatrix} 3 & 6 & 3 \\ -1 & -1 & 2 \\ 4 & 6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 6 & 3 & 6 \\ -1 & -1 & 2 & 3 \\ 4 & 6 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 6 & 3 & 6 \\ 0 & 1 & 3 & 5 \\ 0 & -2 & -4 & -8 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 6 & 3 & 6 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 2 \end{bmatrix} \rightarrow \begin{cases} x_3 = 1 \\ x_2 = 2 \\ x_1 = -3 \end{cases} \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

$$c. \begin{bmatrix} -2 & 4 & -2 \\ -2 & 2 & 1 \\ 6 & 4 & 4 \\ -3 & 8 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 10 \\ 17 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 4 & -2 & 4 \\ -2 & 2 & 1 & 4 \\ 6 & 4 & 4 & 10 \\ -3 & 8 & 5 & 17 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 4 & -2 & 4 \\ 0 & -2 & 3 & 0 \\ 0 & 16 & -2 & 22 \\ 0 & 2 & 8 & 11 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 4 & -2 & -2 \\ 0 & -2 & 3 & 0 \\ 0 & 0 & 22 & 22 \\ 0 & 0 & 11 & 11 \end{bmatrix}$$

$$\rightarrow \begin{cases} x_3 = 1 \\ x_2 = 1.5 \\ x_1 = 0 \end{cases}$$

②

$$a. \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$b. \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -16 \end{bmatrix}$$

$$c. \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$d. \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

③

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix}$$

$$A^T = \begin{bmatrix} -a_{11} & -a_{21} & -a_{31} & \dots & -a_{n1} \\ -a_{12} & -a_{22} & -a_{32} & \dots & -a_{n2} \\ -a_{13} & -a_{23} & -a_{33} & \dots & -a_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{1n} & -a_{2n} & -a_{3n} & \dots & -a_{nn} \end{bmatrix}$$

$$\text{Tran}(A) + \text{Tran}(A^T) = (a_{11} + a_{22} + a_{33} + \dots + a_{nn}) + (-a_{11} - a_{22} - a_{33} - \dots - a_{nn}) = 0$$

④

$$B = \begin{bmatrix} 2a_{11} & a_{12}+a_{21} & a_{13}+a_{31} & \dots & a_{1n}+a_{n1} \\ a_{12}+a_{21} & 2a_{22} & a_{23}+a_{32} & \dots & a_{2n}+a_{n2} \\ a_{13}+a_{31} & a_{23}+a_{32} & 2a_{33} & \dots & a_{3n}+a_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{1n}+a_{n1} & a_{2n}+a_{n2} & a_{3n}+a_{n3} & \dots & 2a_{nn} \end{bmatrix}$$

$$a_{ij} = a_{ji} \quad i, j = 1, 2, 3, \dots, n$$

ii)

$$C = \begin{bmatrix} 0 & a_{12}-a_{21} & a_{13}-a_{31} & \dots & a_{1n}-a_{n1} \\ a_{21}-a_{12} & 0 & a_{23}-a_{32} & \dots & a_{2n}-a_{n2} \\ a_{31}-a_{13} & a_{32}-a_{23} & 0 & \dots & a_{3n}-a_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1}-a_{1n} & a_{n2}-a_{2n} & a_{n3}-a_{3n} & \dots & 0 \end{bmatrix}$$

$$a_{ij} = -a_{ji} \quad i, j = 1, 2, 3, \dots, n$$

$$④ \rightarrow B + C = 2A = \begin{bmatrix} 2a_{11} & 2a_{12} & \dots & 2a_{1n} \\ 2a_{21} & 2a_{22} & \dots & 2a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 2a_{m1} & 2a_{m2} & \dots & 2a_{mn} \end{bmatrix} \quad \text{b. } B \cdot C = 2A^T$$

$$⑤ \quad a. \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \quad b. \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad c. \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$⑥ \quad a. \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \quad b. \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad c. \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$⑦ \quad a. \begin{bmatrix} 2 & 3 & 1 \\ -2 & 1 & 1 \\ 4 & -3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 11 \\ 3 \\ 10 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 & 1 & 11 \\ -2 & 1 & 1 & 3 \\ 4 & -3 & 4 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 & 1 & 11 \\ 0 & 4 & 2 & 14 \\ 0 & -9 & 2 & -12 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 & 1 & 11 \\ 0 & 4 & 2 & 14 \\ 0 & 0 & 6.5 & 19.5 \end{bmatrix}$$

$$\rightarrow x_3 = 3, \quad x_2 = 2, \quad x_1 = 1$$

$$b. \begin{bmatrix} 2 & 3 & 4 & 1 & 24 \\ 0 & 1 & 1 & -3 & 18 \\ 0 & 0 & 4 & 5 & 10 \\ 1 & 0 & -1 & 0 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 & 4 & 1 & 24 \\ 0 & 1 & 1 & -3 & 18 \\ 0 & 0 & 4 & 5 & 10 \\ 0 & -1.5 & -3 & -0.5 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 & 4 & 1 & 24 \\ 0 & 1 & 1 & -3 & 18 \\ 0 & 0 & 4 & 5 & 10 \\ 0 & 0 & -1.5 & -5 & 22 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 2 & 3 & 4 & 1 & 24 \\ 0 & 1 & 1 & -3 & 18 \\ 0 & 0 & 4 & 5 & 10 \\ 0 & 0 & 0 & -\frac{25}{8} & \frac{206}{8} \end{bmatrix} \rightarrow x_4 = -8.24, \quad x_3 = 12.8, \quad x_2 = -19.52, \quad x_1 = 19.8$$

$$c. \begin{bmatrix} 3 & 2 & 1 & 0 \\ -2 & 1 & -1 & 2 \\ 2 & -1 & 2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 & 1 & 0 \\ 0 & \frac{7}{3} & -\frac{1}{3} & 2 \\ 0 & -\frac{7}{3} & \frac{4}{3} & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 & 1 & 0 \\ 0 & \frac{7}{3} & -\frac{1}{3} & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow x_3 = 1, \quad x_2 = 1, \quad x_1 = -1$$

$$d. \begin{bmatrix} 2 & 1 & -2 & 9 \\ 2 & 5 & 1 & 9 \\ 1 & 3 & 4 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -2 & 9 \\ 0 & 1 & 5 & -9 \\ 0 & 1 & 6 & -11 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -2 & 9 \\ 0 & 1 & 5 & -9 \\ 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow x_3 = -2, \quad x_2 = 1, \quad x_1 = 3$$

$$\begin{aligned}
 e. \left[\begin{array}{ccc|c} 3 & -4 & 4 & -15 \\ 3 & 2 & 37 & 0 \\ -4 & 6 & -5 & 25 \end{array} \right] &\rightarrow \left[\begin{array}{ccc|c} 3 & -4 & 4 & -15 \\ 0 & 6 & 33 & 15 \\ 0 & 2/3 & 1/3 & 5 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 3 & -4 & 4 & -15 \\ 0 & 6 & 33 & 15 \\ 0 & 0 & -30/9 & 30/9 \end{array} \right] \rightarrow \begin{cases} x_3 = -1 \\ x_2 = 8 \\ x_1 = 7 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 f. \left[\begin{array}{ccc|c} 3 & 1 & 1 & 0 \\ 1 & 2 & -3 & -4 \\ 0 & -4 & -6 & 26 \end{array} \right] &\rightarrow \left[\begin{array}{ccc|c} 3 & 1 & 1 & 0 \\ 0 & 5/3 & -10/3 & -4 \\ 0 & -4 & -6 & 26 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 3 & 1 & 1 & 0 \\ 0 & 5/3 & -10/3 & -4 \\ 0 & 0 & -210/15 & 82/5 \end{array} \right] \rightarrow \begin{cases} x_3 = -1.171 \\ x_2 = -4.743 \\ x_1 = 4.971 \end{cases}
 \end{aligned}$$