Notes Made By

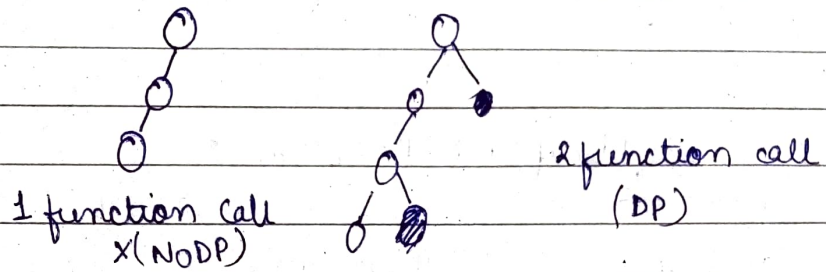
- Riti Kumari

Dynamic Playlist (Aditya Verma)

DP = enhanced recursion

How to identify DP problem

1) Where there is recursion (2 calls), DP is used (for overlapping problem)
   a) Choice



   b) Optimal - Min, max, largest

How to write DP code?

Recursive solution → memoization → Top down approach

Questions on DP.

1) 0-1 Knapsack (6)
2) Unbounded knapsack (5)
3) Fibonacci (7)
4) LCS (15) (longest common subsequence)
5) LIS (10) (longest increasing subsequence)
6) Kadane's Algorithm (6)
7) Matrix Chain Multiplication (7)
8) DP on trees (4)
9) DP on grid (14)
10) Others (5)

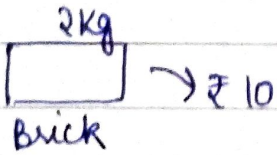
Types of Knapsack

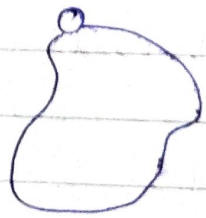
1. Subset sum
2. Equal sum partition
3. Count of subset sum
4. Minimum subset diff
5. Target sum
6.

0-1 Knapsack Problem

- Fractional Knapsack (Greedy)
- 0-1 Knapsack
- Unbounded Knapsack

0-1 Knapsack → We are given some weight & some value. Then a max weight W. pick items so that the profit is maximum. And the weight has a given bound W.

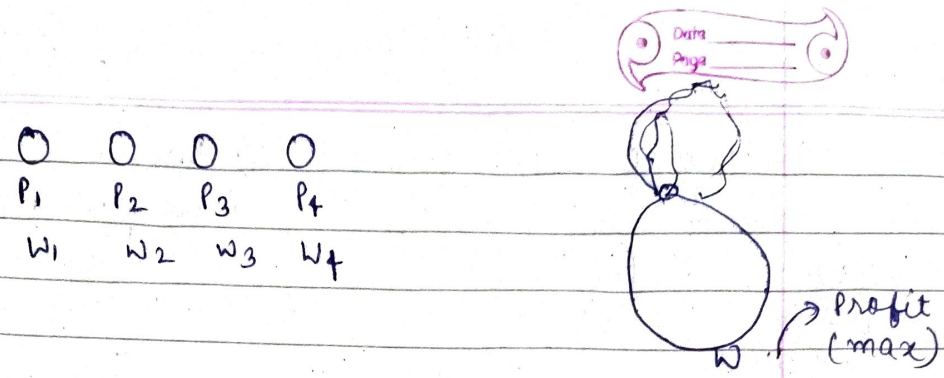




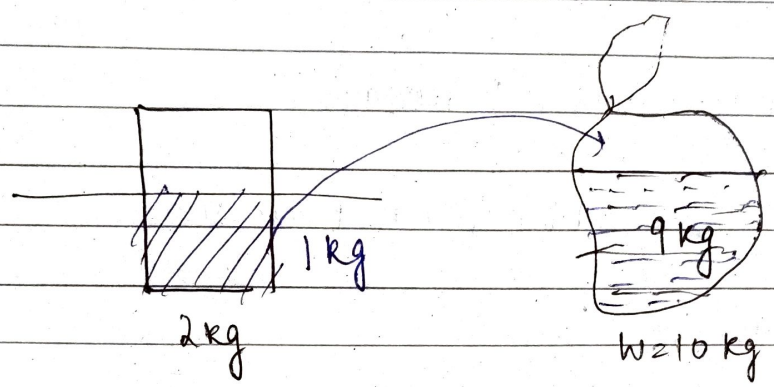
	I_1	I_2	I_3	I_4
wt[] =	1	3	4	5
val[] =	1	4	5	7

W = 7kg.

Max Profit = ?



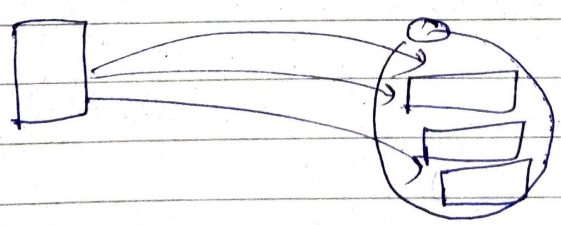
Fractional knapsack



Greedy approach

Unbounded knapsack

Unlimited supply of every item



0-1 Knapsack

1) How to identify

wt[] :	1	3	4	5	W : 7kg
val[] :	1	4	5	7	

O/p : max profit

1) Choice ↓

(either put in knapsack) (either don't put in knapsack)

2) Optimal (max profit)

DP: Recursive → Memoization (DP) → Top down (DP)

DP → recursion + storage

0-1 knapsack Recursive

Identify

DP → Recursive → DP (top down)
→ DP (memoization)

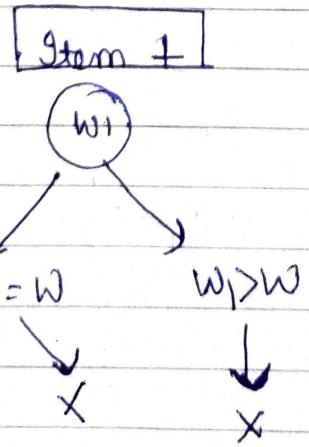
I/P wt[] = [1|3|4|5]

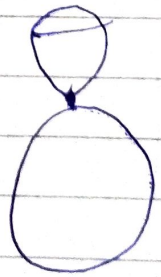
val[] = [1|4|5|7]

O/p → Max Profit

(capacity of knapsack) W = 7 kg

Choice diagram





We have to return the max profit so return type would be int.

Base cond^n → think of the smallest valid i/p.

I/P wt [ ]: [ ][ ][ ]] → (n) → 0.
val[ ]: [ ][ ]]
w : 10kg → 0 kg

max profit

```
int Knapsack ( int wt[ ], int val[ ], int w, int n) {
  // base condition
    if (n==0 || w==0)
        return 0;

  // choice diagram

    if (wt[n-1] <= w) {
        return max(val[n-1] + Knapsack (wt, val, w-wt[n-1], n-1),
                   Knapsack (wt, val, w, n-1));
    }
    else if (wt[n-1]>w) {
        return Knapsack (wt, val, w, n-1);
    }
```

weight less hain & including

n-1

weight less hai but dont include

01 Knapsack Memoization

Memoization = Recursive + 2 lines

t

n*m

func (IP/doesn't change , IP/change)

n-1

w - wt[n-1]

Val of n, m {

Knap Sack (wt[ ], val[ ], n, w)

t [n+1] [w+1]

-1	-1	-1	-1	-1
-1	-1	-1	-1	-1
-1	-1	2	-1	-1
-1	-1	-1	-1	-1
-1	-1	-1	-1	-1

n+1

w+1

if (-1) is not present then val exists so, return

initialise this matrix with -1.

```
int t[n+1][w+1]
memset (t, -1, sizeof(t))
```

Changes

Now declare the matrix globally.

```
int static t[102][1002]; constrainst
                          n<=100
memset (t, -1, sizeof t)  w<=1000

int knapsack (int wt[], int val[], int w, int n)
{
    if (n==0 || w==0)
        return 0;
    if (t[n][w] != -1)
        return t[n][w];

if (wt[n-1] <= w)
return t[n][w] = max (val[n-1] + knapsack (wt, val, w-wt[n-1], n-1
                , knapsack (wt, val, w, n-1)):
```

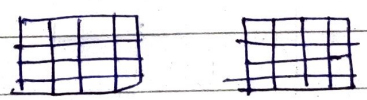
```
else if ( wt[n-1] > W)
return t[n][W]= Knapsack (wt, val, W, n-1);

}
```

The complexity of top down & memoization remains same but the problem with memoization is the stack gets full due to repeated func^n calls.

(0-1 Top Down Knapsack) → Real DP

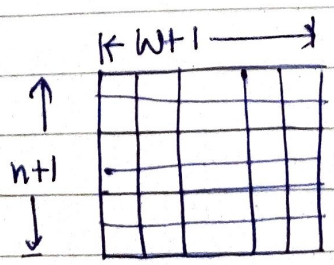
Recursive → Memoize → Top down → 6 Problems

Recursive	Memoize	Top down
BC + recursive calls	RC + table	only Table



Recursive → Memoization — Initialising matrix with -1

Recursive → Top-down (totally omit the recursive call & use the table only)



We only make table for the i/p which is changing.

2 steps to make table for top down

Step1: Initialization

Step 2: Recursive code $\xrightarrow{\text{changes}}$ Iterative code

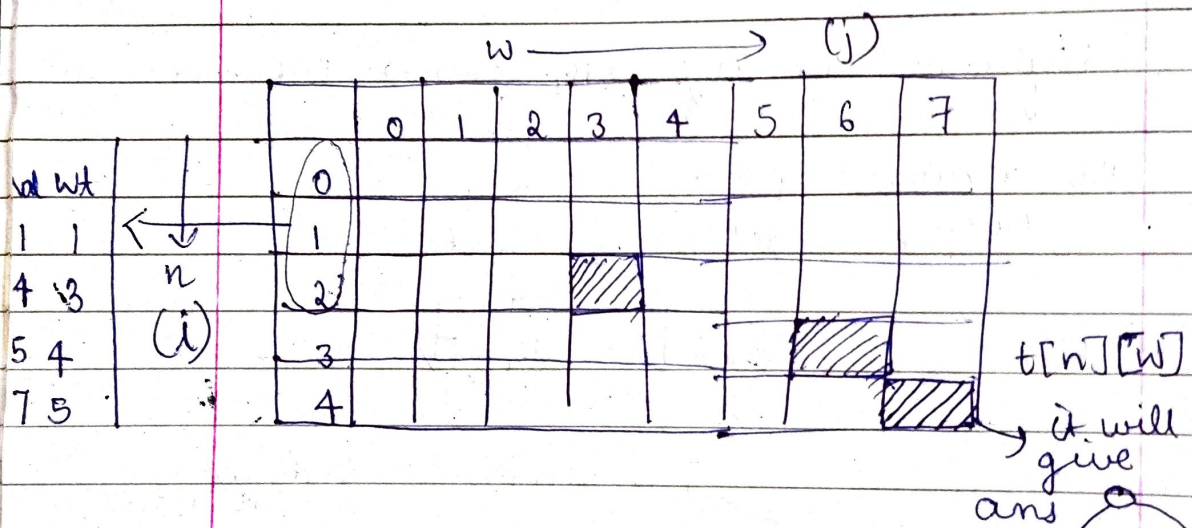
Step1: Initialize

W = 7

n = 4

wt[] = [1 3 4 5] (1 2 3)

val[] = [1 4 5 7]



wt[] = [1 3 4 5]
val[] = [1 4 5 7]

wt[] = [1 3]
val[] = [1 4]
W = 3

W=3

wt[] = [1 3 4]
val[] = [1 4 5]
W = 6

RC + table ⟶ table

Base condⁿ ⟶ Initialization

Base condⁿ

RC

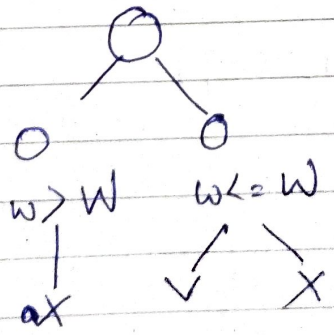
```
if(n==0 || W==0)
    return 0;
```

table

	w=0	1	2	...
1	0	0	0	0
2	0			
3	0			
4	0			

```
for (int i=0; i<n+1; i++) {
    for (int j=0; j<w+1; j++)
    {
    if (i==0 || j==0)

        t[i][j] = 0;
    }
}
```

Choice diagram



n, w ⟶ i, j

7+

dp[i][j]

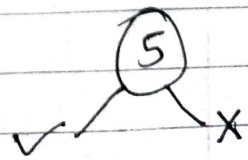
RC.

wt = 1 3 4 5

val = 1 4 5 7

W = 7

= max(val[n-1] + dp[i-1][j-wt[i-1]],
dp[i-1][j])



Recursive	Top down
if(wt[n-1] <= w)	if (wt[n-1] <= w)
return max(val[n-1] + Knapsack(wt, val, w-wt[n-1], n-1)	t[n][w] = max(val[n-1] + t[w-wt[n-1]][n-1], t[n-1][w])
Knapsack (wt, val, w, n-1)	
else if (wt[n-1] > w) return Knapsack (wt, val, w, n-1)	else t[n][w] = t[n-1][w]

Top-down approach

pseudo code

```
int t[n+1][w+1];
for (int i=1; i<n+1; i++)
    for (int j=1; j<w+1; j++)
    {
        if (wt[i-1] <= j)
            t[i][j] = max( val[i-1] + t[i-1][j-wt[i-1]],
                           t[i-1][j] )
        else
            t[i][j] = t[i-1][j]
    return t[n][w]
```

	0	1	2	3
wt =	1	3	4	5
val =	1	4	5	7

W = 7
n = 4

W = 0 1 2 3 4 5 6 7
0 1 2 3 4 5 6 7

i-1 = 0

1 → 4

1 → 7

i=1 wt[i-1] <= 1 → True

1 <= 1

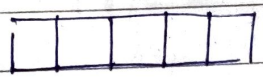
t[i][j] = max(1 + t[0][0], t[0][1])

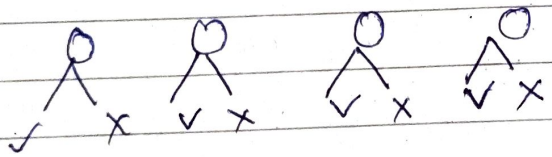
= max(1, 0)

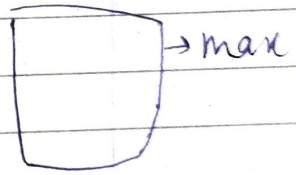
= 1

Identification of knapsack problem

1) Subset sum problem
2) Equal sum partition
3) Count of subset sum.
4) Minimum subset sum diff
5) Target sum
6) No of subset with a given diff.

Ip: Item array : 
W: Capacity





1. Subset Sum problem

arr[] : 2 3 7 8 10
sum = ...

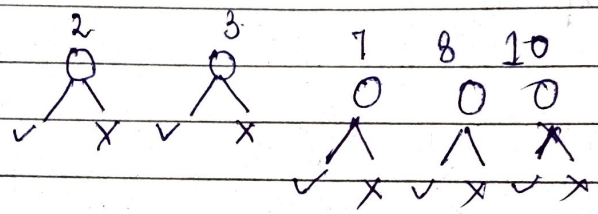
a) Problem statement
b) similarity with knapsack
c) Code variation

1) Problem statement : find if there is a subset present in an array with given sum.

arr[]: 2 3 7 8 10
Sum : 11

2) Similarity

item array[] : → 2 3 7 8 10
weight : capacity → 11



3) Code variation

t[n+1][w+1]

t[5+1][11+1]
t[6][12]

Initialization:

sum →

	0	1	2	3	4	5	6	7	8	9	10	11
0	T	F	F	F	F	F	F	F	F	F	F	F
1	T											
2	T											
3	T											
4	T											
5	T											

↓ n

q → True/false

arr[]:
sum : 0

arr[] : 1
sum = 0 '{}'

arr[] : 2
sum = 0 '{}'

arr[]:
sum = 2

when array is empty sum can't be anything

arr[]: no elements

Sum: 1 → not possible

t[n+1][sum+1]

Initialisation: for (int i = 1 . . . n+1)

for (int j = 1 . . . W+1)

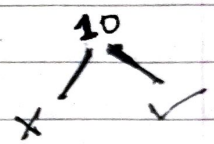
if (i==0)

t[i][j] = false.

2 3 7 8 10

if (j==0)

t[i][j] = True



knapsack → arr.

if (wt[i-1] <= j)

t[i][j] = max(val[i-1] + t[i-1][j - wt[i-1]], t[i-1][j])

there is no max in true or false.

else

t[i][j] = t[i-1][j]

Subset sum

if (arr[i-1] <= j)

t[i][j] = t[i][j - arr[i-1]]

||

t[i-1][j]

else

t[i][j] = t[i-1][j]

return t[n][sum];

i j

t[1][0] = t[1][0 - arr[0]]

||

t[0][0]

= t[1][-2] || t[0][0]

|| T

= t[i][j]

2, 3, 8

{3, 8} → True ✓

{3} → False ✗

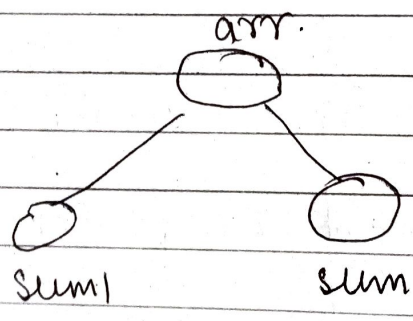
2. Equal Sum Partition Problem

1) Problem statement
2) Subset sum similarity
3) Odd / Even significance
4) Code variation

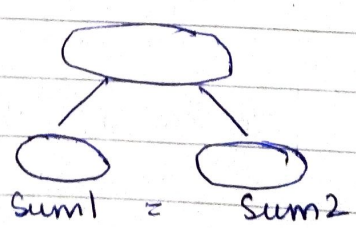
1) Problem statement

arr[] = {1, 5, 11, 5}
O/P : T/F

Is it possible to divide the array such that both the subset gives an equal sum



sum 1 = sum 2 → True / False



sum 1 = sum 2 → It is equal when the no is even. Sum of all the array elements should be even to change it into equal parts.

22

11 11

sum1 = sum2

23

11 12 ✗

sum1 ≠ sum2

I/p

arr[] → True O/P

→ False

```
for (int i = 0; i<size; i++) {
        sum = sum + arr[i];
}
```

{1, 5, 11, 5}

Subset even (sum = 22) (false)

Subset odd

Subset = 11

Subset 11

```
if (sum % 2 != 0)        (sum is odd)
        return false

else if (sum % 2 == 0)
```

* → we need to find one subset with sum 11 the next subset would automatically be 11.

```
return subsetsum(arr, sum/2);
```

3. Code:

```
i/p → arr[], n.
    int sum = 0;
for (int i = 0; i<n; i++)
        sum += arr[i];
if (sum % 2 != 0)
        return false
else
        return subsetsum(arr, sum/2);
```

3. Count of Subsets sum with a given sum.

I/p:

arr[ ] = 2 3 5 6 8 10

sum = 10.

Flow

1) Problem statement
2) Similarity to Subset sum
3) Code variation.
   - Initialisation
   - Code
4) Return Type.

1) Problem statement

arr[ ] 2 3 5 6 8 10

sum : 10 O/p = 3.

{2, 8} → Yes/True (in subset sum)

{2, 8} = 10
{5, 2, 3} = 10
{10} = 10

count = 3

2) Similarity

2, 8 | return count

Yes NO

3. Code variation

Subset sum

due to T/F. ← bool

False → 0 (no of subset 0)

True → null subset (no of subset 1)

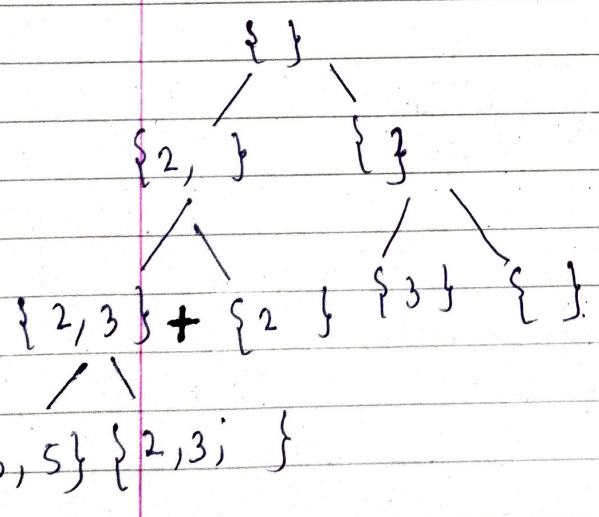
. . . . . .

if (arr[i-1] <= j) +

dp[i][j] = dp[i-1][j] + t[i-1][j-arr[i-1]]

else

dp[i][j] = dp[i-1][j]



Count

int

1	0	0	0	0	0
1					
1					

We will add all the subset so or would be changed to + . True & false case we can use ~~bool~~ or.

if (arr[i-1] <= j)

dp[i][j] = dp[i-1][j] + dp[i-1][j-arr[i-1]]

else

dp[i][j] = dp[i-1][j]

if (arr[i-1] <= j)
dp[i][j] = dp[i-1][j-arr[i-1]] + dp[i-1][j]
else
dp[i][j] = dp[i-1][j].

Date
Page

arr[] = 2 3 5 6 8 10
Sum = 10
O/p = 3

1, 3, 2, 5 5
O/p → {3, 2}
{5}

{ }
+1 / \ -1
{1} { }
+3 / \ -3 +3 / \ -3
{1,3} {1} {3} { }

arr[0] <= 1
1

i=1 j=1 2-2=0

2 <= 1

i=1 j=2

arr[0] <= 2
2 <= 2 n

dp[0][0]
dp[0][2]

dp[N+1][sum+1] = dp[6+1][10+1]
= dp[7][11]

Sum →

	0	1	2	3	4	5	6	7	8	9	10
0	1	0	0	0	0	0	0	0	0	0	0
1	1	0	1								
2	1										
3	1										
4	1										
5	1										
6	1										

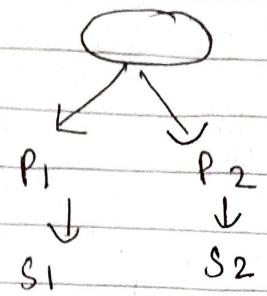
Minimum subset sum difference

1) Problem statement
2) Similarity
3) Solve using its previous concept

1) Problem statement

arr[] : 1 6 1 1 5

OP : 1



Equal sum: $S1 - S2 = 0$

Min^m Subset: $S1 - S2 = min$

$abs(S1 - S2) = min$ (should be min)

1, 6, 11, 5

{1, 6} → 7

{11, 5} → 16

16 − 7 = 9 → minimize of find could it be done in a better way.

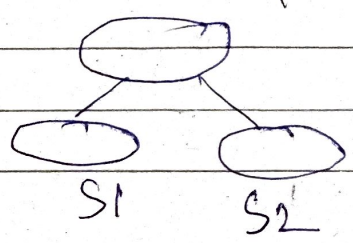
{1, 6, 5} → 12

{11} → 11

12 − 11 = 1 → Can't be minimized further → O/P.

2) Similarity

It is similar to equal sum partition.



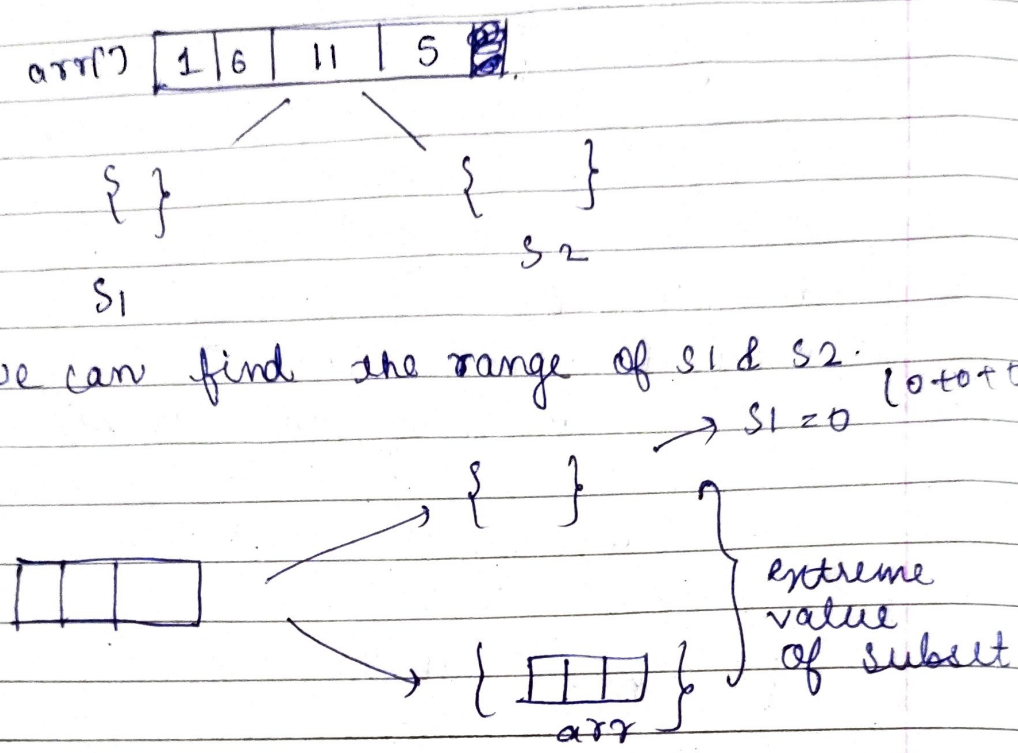
We need to find S1 & S2.

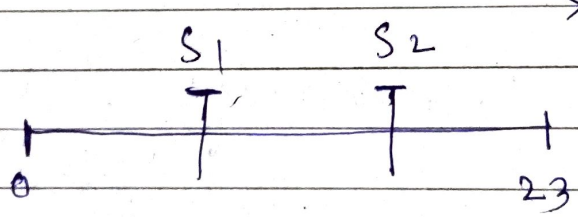
arr[] | 1 | 6 | 11 | 5 |

{ } { }

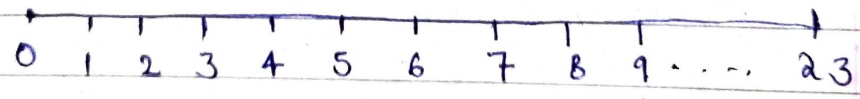
S1 S2

We can find the range of S1 & S2.

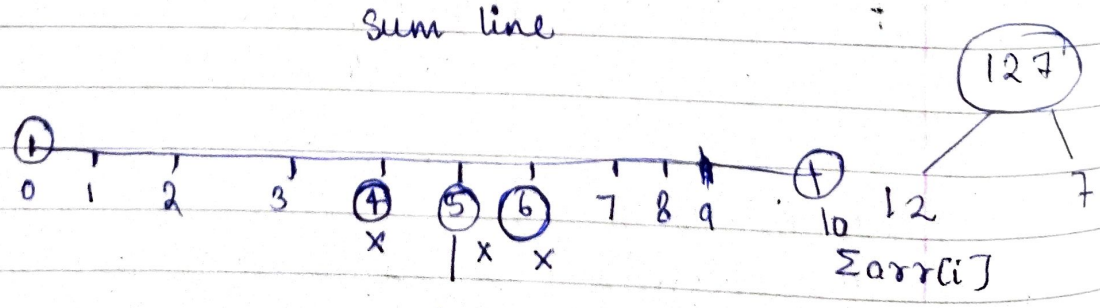




arr[]: {1, 2, 7}

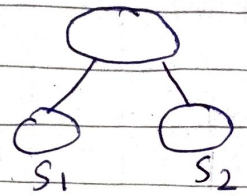


Sum line



S1/S2 can't be 5

S1/S2 = {0, 1, 2, 3, 7, 8, 9}

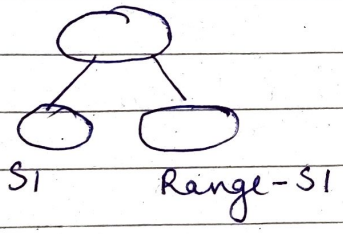


$S_1 + S_2 = \Sigma arr[i]$

$S_1 / S_2 = \{ 0, 1, 2, 3, \mid 7, 8, 9, 10 \}$

S_1 (S_1) | Range − S_1 (S_2)

$S_1 - S_2$ = minimize



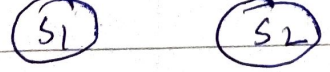
abs $(S_2 - S_1)$ or abs $(S_1 - S_2)$

abs $(S_2 - S_1)$ = minimize

abs (Range − S1 − S1) = minimize. abs(Range − 2S1) → minimize

Sum up

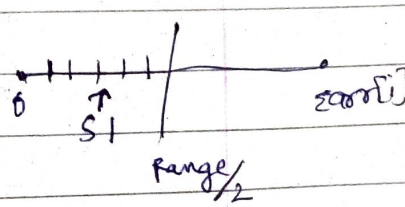
1 6 11 5



$S_1 - S_2$
$S_2 - S_1$ } minimize

↓

((Range − S1) − S1)
(Range − 2S1)



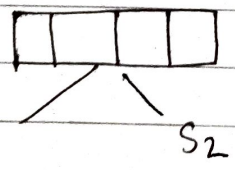
	0	1	2	3	4	5	6	7	8	9	10
0											
1											
2											
3	T	T	T	T	F	F	F	T	T	T	T

we will push the last row in vector till half way.

0	1	2	3.

```
int min = INT_MAX;
for (int i = 0; i < v.size(); i++) {
    mn = min (mn, Range - 2v[i])
}
return mn;
}
```

Concept



$(S_2 - S_1) = \min$

↓

smaller

(S_1) $(Range - S_1)$

$Range - 2S_1 \rightarrow Min^m$

Code

```
Subset sum( int arr[ ], int range)
{
last row filled in vec till halfway
| 0 | 0 | 0 |
}
```

```
int mindiff (int arr[], int n) {
  int sum = 0;
   for (int i = 0; i < n; i++) {
        sum += arr[i];
    }
    bool dp[n+1][sum+1];
     for (int i = 0; i < n+1; i++) {
         for (int j = 0; j < sum+1; j++) {
         if (i == 0)     dp[i][j] = false;
         ~~else~~ if (j == 0)  dp[i][j] = true;
         }
     }
  for (int i = 1;  i < n+1; i++) {
        for (int j = 1; j < sum+1; j++) {
           if (arr[i-1] <= j)
              dp[i][j] = dp[i-1][j-arr[i-1]] || dp[i-1][j];
           else
              dp[i][j] = dp[i-1][j];
        }
    }
   int diff = INT_MAX;
    for (int j = sum/2; j >= 0; j--) {
        if (dp[n][j] == true) {
            diff = min(sum - 2*j, diff)
        }
    return diff;
}
```

Date
Page

Count the number of subset with a given diff

1) Problem statement
2) Will try to reduce the actual statement
3) Solve it using already solved problem.

1) Problem statement

arr[] : | 1 | 1 | 2 | 3 |

Diff : 1

| 1 | 1 | 2 | 3 |

S1 S2

S1 − S2 = Diff → return the count of subset with diff^n.

| 1 | 1 | 2 | 3 |

1+3 1+2

4 3 = 1

1

| 1 | 1 | 2 | 3 |

1+3 1+2

4 3 = 1

| 1 | 1 | 2 | 3 |

1+1+2 3

4 − 3 = 1

1	1	2	3

S_1 S_2

(diff) S = sum of arr

$sum(S1) - sum(S2) = diff$
$sum(S1) + sum(S2) = S$

$2S_1 = diff + Sum(arr)$

$S_1 = \left(\frac{diff + sum(arr)}{2}\right)$

$= \frac{1+7}{2} = \frac{8}{2} = 4$

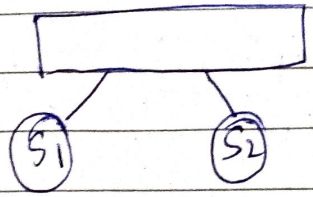
$S_1 = 4$ $S_2 = S_1 - diff$
$= 4 - 1$
$= 3.$

count = ?

- Find count when sum of S1 = 4

no of count of Subset with given diff → count of Subset sum.

Sum up.



$2S1 = diff + sum(arr)$

$S_1 = \frac{diff + sum(arr)}{2}.$

$S_1 - S2 = diff$
$S_1 + S2 = sum(arr)$

Date
Page

int sum = $\frac{diff + sum(arr)}{2}$.

return countofsubset sum(arr, sum);

int countofsubset with diff (int arr[], int n){
~~int sum=0 for(int i=0; i<n; i++){~~ int diff
~~sum+=arr[i];~~

int arrsum = 0
for (int i=0; i<n; i++){
arrsum + = arr[i];
}

int sum = 0;
Sum = $\frac{diff + arrsum}{2}$.

return countofsubset sum(arr, sum, n);
}

int countof subset sum(int arr[], int sum, int n){
int dp [n+1][sum+1];
for (int i=0; i<n+1; i++){
for (int j=0; j<sum+1; j++){

if (i==0) dp[i][j] = 0;
if (j==0) dp[i][j] = 1;
}
}

```
for (int i = 1; i < n+1; i++){
        for (int j = 1; j < sum+1; j++) {

            if (arr[i-1] <= j) {
            dp[i][j] = dp[i-1][j - arr[i-1]] + dp[i-1][j];}
            else {
            dp[i][j] = dp[i-1][j];
            }
        }
    }
    return dp[n][sum];
}
```

Target Sum.

arr : | 1 | 1 | 2 | 3 |

sum : 1

+/-

+c : [0, 0, 0, 0, 0, 0, 1]

target = 1

o/p : 3

Target value = 2

$+1 - 1 - 2 + 3 = 1$

$-1 + 1 - 2 + 3 = 1$ $\{0, 2\}$

$+1 + 1 + 2 - 3 = 1$

(+0, 2) (-0, 2)

count = 2

arr[] → | + | - | + | + |

{0, 0, 2}

{+0, +0, 2} {+0, -0, 2} {-0, +0, 2}

{-0, -0, 2}

= 4 (count

S1 S2

S1 - S2 = diff

So your answer is increased by a power of (2no of zeroes)

pow(2, no of zeroes)

+ - - +

1 1 2 3

+1+3 -1-2

(1+3) - (1+2)

S1 - S2 → count of subset with given diff^n.

~~Unbounded Knapsack~~

```
int findTargetSumWays(vector<int>& nums, int S){
    int cnt = 0, sum = 0;
    int n = nums.size();
    for(int i = 0; i < nums.size(); i++){
        sum = sum + nums[i];
        if(nums[i] == 0)
            cnt = cnt + 1;
    }
    S = abs(S);
    if(S > sum || (S+sum)%2 != 0)
        return 0;
    int s = (S+sum)/2;
    int dp[n+1][s+1];

    for(int i = 0; i < n+1; i++)
        for(int j = 0; j < sum+1; j++)
            if(i == 0) dp[i][j] = 0;
            if(j == 0) dp[i][j] = 1;
```

Cnt the no of zeroes in subset.

```
for (int i=1; i<n+1; i++){
    for (int j=1; j<s+1; j++){

        if(nums[i-1]==0)
            dp[i][j]=dp[i-1][j];

        else if (num[i-1]>j)
            dp[i][j]=dp[i-1][j];

        else
            dp[i][j]=dp[i-1][j-nums[i-1]+
                        dp[i-1][j]
        }.
    }.
    return pow(2,cnt)* dp[n][s];
}.
```

if 0 is there in the array don't take it

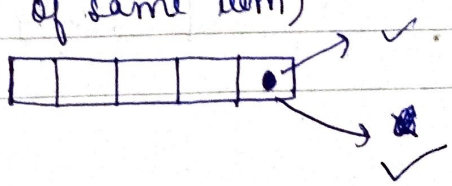
13. Unbounded Knapsack :

Related Problems

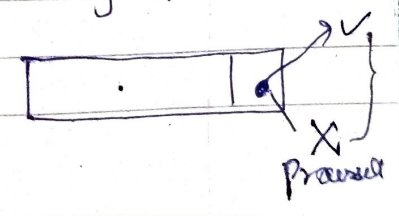
1) Road cutting
2) Coin change I (Max no of ways)
3) Coin change II (Min no of ways)
4) Max m. Ribbon cut

(Variations of unbounded knapsack)

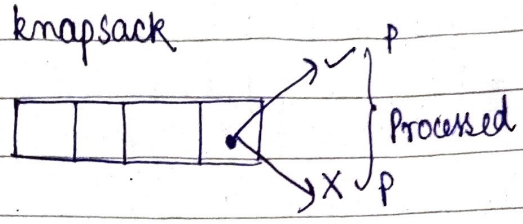
Unbounded (multiple occurences of same item)



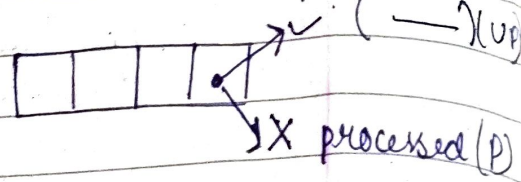
Knapsack (only one occurence)



Knapsack



Unbounded knapsack



Multiple occurrences
- ✓ (can revisit many times)
- X

Comparision betn.

Knapsack

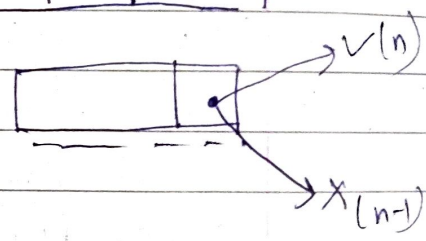
	0	1	2	3	4
0	0	0	0	0	0
1	0				
2	0				

t[n+1][w+1]

```
if (wt [i-1] <= j){
t[i][j] = max( val[i-1] + t[i-1][j-wt[i-1]],
                t[i-1][j]); }
else {
t[i][j] = t[i-1][j] ;
}.
```

Unbounded Knapsack

	0	1	2	3	4
0	0	0	0	0	0
1	0				
2	0				



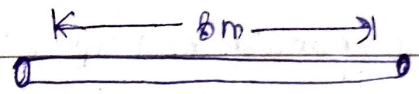

```
if (wt [i-1] <= j)
t[i][j] = max (val [i-1] + t[i][j-wt[i-1]],
                t [i-1][j] );
else
t[i][j] = t [i-1][j]
```

14. Road Cutting Problem

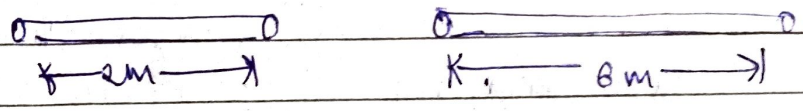
length [ ] :	1	2	3	4	5	6	7	8
price [ ] :	1	5	8	9	10	17	17	20

N : 8

1) Problem statement



length of rod is given. we need to cut it in such a way the profit is max^m.



Rs 5 Rs 17

17
5
20

Total = 5+17
= Rs 22

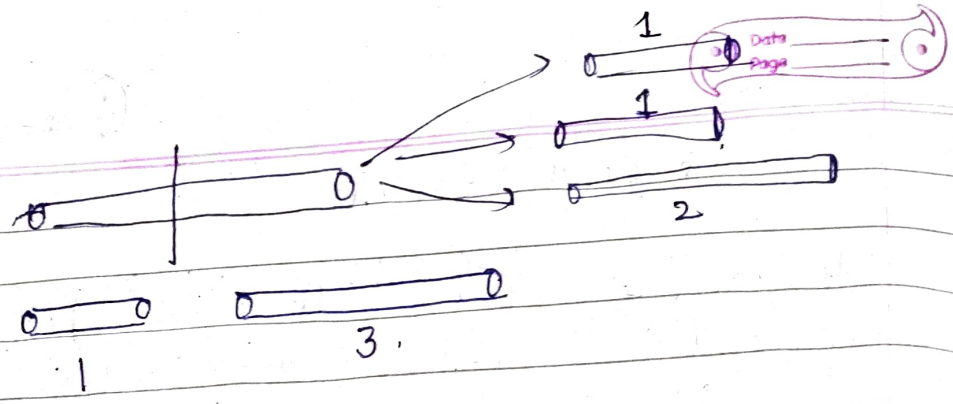
knapsack

wt
val
W.

length = 1 to N
Price = [ ] [ ] [ ] [ ] [ ]
N = 8

if length array is not give make it & fill it with 1 to N.

knapsack	Unbounded knapsack
~~price[]~~	~~value[]~~
~~wt[]~~	
val[ ]	price [ ]
wt[ ]	length [ ]
W	N



Code variation

```
t[N+1] [N+1]
if (length[i-1] <= j)
  dp[i][j] = max(price[i-1] + dp[i][j- length[i-1]],
                 dp[i-1][j])

else
dp[i][j] = dp[i-1][j];
```

Sometimes arr size changes therefore we need to find the size of array.

dp[size+1][N+1].

length →

	0	0	0	0	0	0
↓ size	0					
	0					

Coinchange Problem

Max^m no of ways

Min^m no of coins

Max^m no of ways.

Problem statement

coin [ ] : | 1 | 2 | 3 | → (unlimited)

sum : 5.

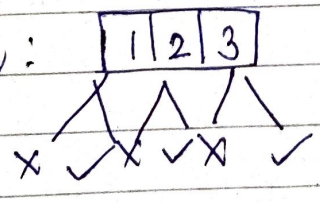
Use the coin array unlimited time & get the sum 5. Now find the max^m no of ways in which we can get 5.

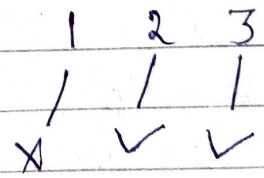
		O/P
5 ways.	2 + 3	5.
	1 + 2 + 2	5.
	1 + 1 + 3	5.
	1 + 1 + 1 + 1 + 1	5.
	1 + 1 + 1 + 2	5

Q Why knapsack?

→ Every coin has a choice to include or not

coin : | 1 | 2 | 3 |





item — wt [ ] ✓

item — val [ ] ✗ (when one array is given)

Matching

wt[] → coin[]
W → sum

1	2	3.

Sum 5 : 1 + 1 + 3

one item used many times

If taking one item many times does the sum allows then it is unbounded knapsack.

Subset sum

1	2	3	5

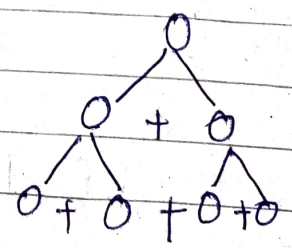
Sum : 8 → T, → F.

Subset sum.

```
if (arr[i-1]<=j){
t[i][j] = t[i-1][j] || t[i-1][j-arr[i-1]]
} else {
t[i][j] = t[i-1][j];
}
```

for count we remove || by +.

count / no of ways



We need to find maxm. no of ways.

Matching → Knapsack → 0-1
→ Unbounded

wt → coin
W → sum.

if (coin[i-1]<=j)
t[i][j] = t[i-1][j] + t[i][j-coin[i-1]].
else
t[i][j] = t[i-1][j]

sum →

size ↓	0	1	2	3	...	...
0	1	0	0	0	0	0.
1	1					
2	1					
3	1					

Coin change-II (Min^m no of coins)

coin[ ] = [1 | 2 | 3]
sum = 5

2+3 → 5 → 2 coins
1+2+2 → 5 → 3 coins
1+1+1+2 → 5 → 4 coins
1+1+3 → 5 → 3 coins
1+1+1+1+1 → 5. → 5 coins

find min^m. no. of coins to make sum as 5.

coin[ ] = [1 | 2 | 3]
sum = 5
O/p : 2 (2+3 = 5)

Initialisation : t[n+1][w+1]
↓
t[n+1][sum+1]

wt → coin[ ] = n
val → x
w → sum.

n=3
sum=5

sum (5) →

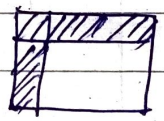
	0	1	2	3	4	5	6
0		INT_MAX -1	INT_MAX -1	INT_MAX -1	INT_MAX -1	INT_MAX -1	INT_MAX -1
1	0						
2	0						
3	0						

→ 2nd Row initialize

size (n)

sum = 3
coin[ ] = 1

↓ code

Initialisation  + Twist

coin[ ] : empty
sum : 1 } → INT_MAX (∞ coins)

coin[ ] : empty
sum : 0 } → INT_MAX - 1

coin[ ] : 1
sum : 0 } 0 coins

coin[ ] : 12
sum : 0 } 0 coins

For Quer

sum = 5.
arr = [3, 5, 2]

when
arr [3]
sum = 4

$\frac{3}{3} = 1$

$\frac{4}{3} = INT_MAX$

i \ j	0	1	2	3	4	5
0		INT_MAX	-1			
1		1				
2	0					
3						
4						

$\frac{4}{3} = INT_MAX$

$\frac{j}{arr[0]} = \frac{3}{3} = 1$

size will always remains 1 so we will use arr[0] as it is the second row

sum = 1
coin = 1

sum = 2
coin = 1 (1)

For second row

```
for(int i=1; j<sum+1; j++) {
        if (j % arr[0] == 0)
                t[i][j] = j / arr[0]
        else
                t[i][j] = INT_MAX-1
```

$\frac{j}{arr[0]}$ — $\frac{3}{3} = 1$

$\frac{4}{3} = INT_MAX$

Code variation

1st row → INT_MAX 1st col → 0

(INT_MAX-1)

if (coin[i-1] <= j)

t[i][j] = min (t[i-1][j] , t[i][j-coin[i-1]] +1)

else

t[i][j] = t[i-1][j]

include doesn't include.

```
int minCoins( int coins[], int M, int V) {
    m = arrsize   v = sum.
  int dp [M+1][ V+1];
  for (int i=0; i <M+1; i++){
      for (int j=0; j<V+1; j++) {
          if (j==0)  dp[i][j] = 0;
          if (i==0)  dp[i][j] = INT_MAX-1;
      } }

  for(int i=1, j=1; j<V+1; j++) {
          if(j % coins[0] ==0)  dp[i][j] = j/coins[0];
          else                  dp[i][j] = INT_MAX-1;

  for(int i=2; i<M+1; i++) {
      for(int j=1; j<V+1; j++) {
        if (coins [i-1]<=j)
            dp[i][j]= min (dp[i-1][j], 1+dp[i][j-coins[i-1]])
        else   dp[i][j]= dp [i-1][j];
      }}

  if(dp [M][V] == INT_MAX-1) return -1;
  return dp[M][V];
}.
```

Longest Common Subsequence

1) Longest common substring
2) Print LCS
3) Shortest common supersequence
4) Print SCS
5) Min^m no of insertion and deletion a→b
6) Largest repeating subsequence
7) Length of largest subsequence of a which is a substring is b.
8) Subsequence pattern matching
9) Count how many times a appear subsequence in b
10) Largest Palindromic subsequence
11) Largest palindromic substring
12) Count of palindromic substring
13) Minimum no of deletion in a string to make it a palindrome
14) Minimum no of insertion in a string to make it a palindrome

Longest Common Subsequence (Recursive)

Problem Statement

X : a b c d g h
Y : a b e d f h r.

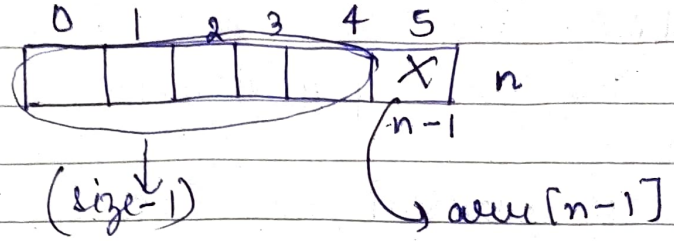
String X = abcdgh
String Y = abedfhr

g → abdh
↳ 4 (length of string)

~~Contn.~~ → longest common subsequence - abdh
contn. → longest common substring - ab

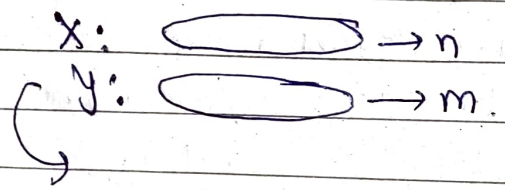
Recursive approach: Base condⁿ + Choice diagram + i/p small

(x, y)




```
func(x, y)
{
fun(x, y)
  smaller x, smaller y
}
```

Base condⁿ: Think of the smallest valid input.



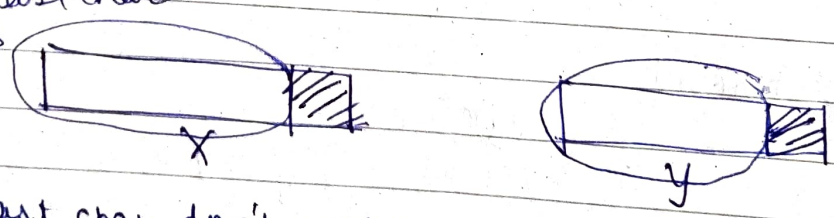
n = 0 m = 0 LCS = 0
(empty string)

```
if(n==0 || m==0)
    return 0
```

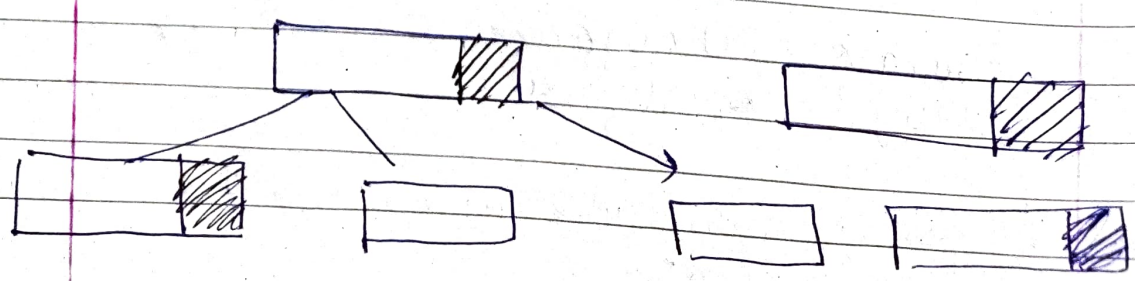
Choice diagram:

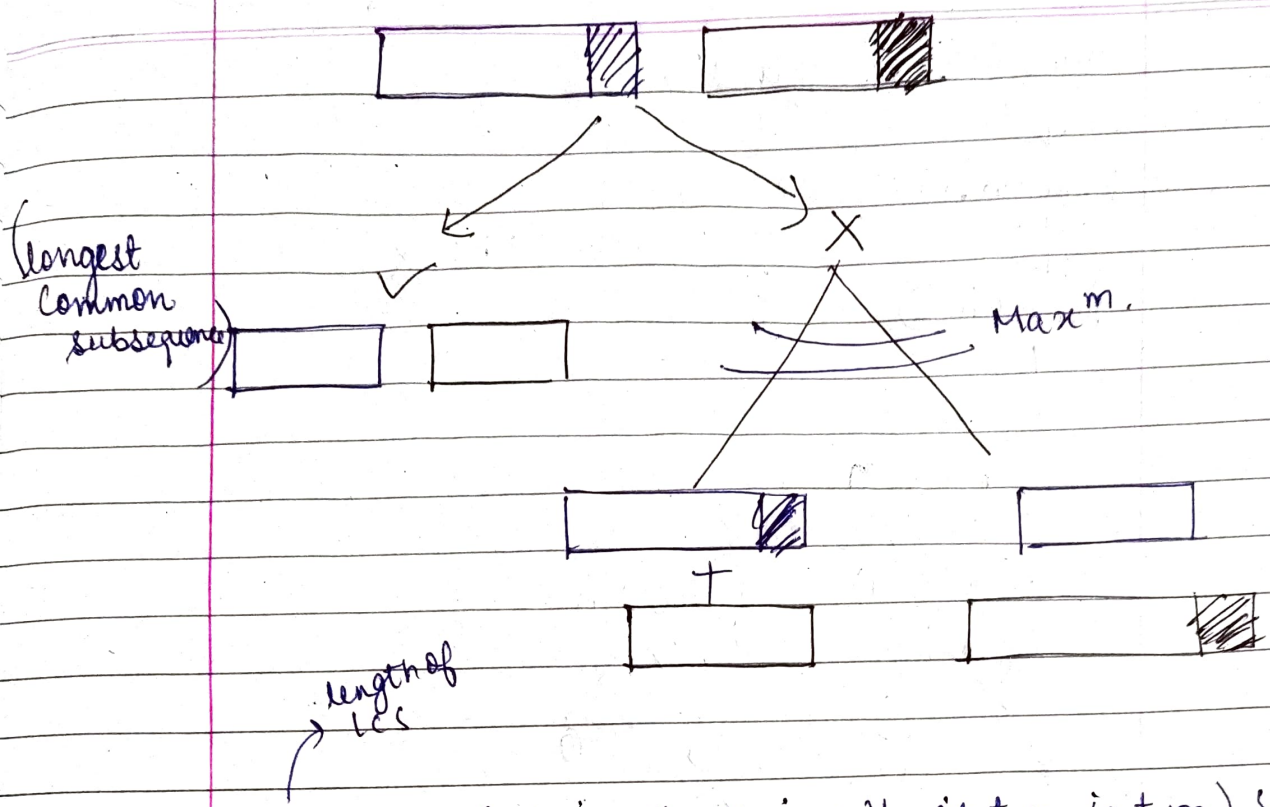
X : abcdgh
Y : abedfhr

① when last char matches



② when last char don't match






```
int LCS (string X, string Y, int n, int m) {

    if (n==0 || m==0) return 0;

    if ( X[n-1] == Y[m-1])
        return 1+LCS(X, Y, n-1, m-1);

    else
        return max (LCS(X, Y, n, m-1),
                    LCS( X, Y, n-1, m));

}
```

LCS memoization (Bottom up approach)

func(n)
↓
Func(n-1)
|
Func(n-2)
|

func( )
{
func( )
}
→ DP X

func n(n)

func(n-1) func(n-2)

n-3 → store Ans

n-3

R.C + table

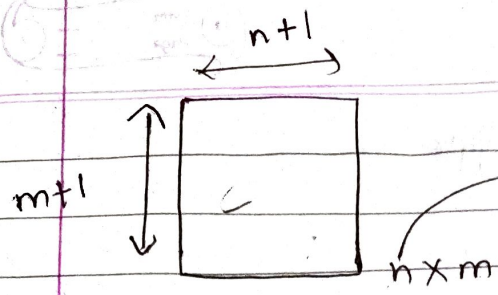
"AXyT" "AYZx"

LCS(AXY, AYZx) LCS(AXYT, AYZ)

LCS(AX AYZ) LCS(AXY, AYZ) LCS(AXY, AYZ)

Sub-Problem repeating twice

LCS(AXYT, AY)



→ R((change)
(n+1, m+1).

```
int t[1001][1001];
```

-1	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1

```
int main(){
    memset (t, -1, sizeof(t));
    LCS( )
}

int LCS( string x, string y, int m, int n){

    if (n==0 || m==0)
        return 0;
    if (t[m][n] != -1)
        return t[m][n];

    if (x[m-1] == y[n-1])
    return t[m][n]= 1+LCS(x, y, m-1, n-1)
    else
    return t[m][n] = max( LCS(x, y, m, n-1),
                          LCS(x, y, m-1, n))

}
```

exponential → $O(n^2)$

LCS Top-down Approach

x : a b c d a f
y : a c b c f

o/p → 4 (abcf)

length y →

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0					
2	0					
3	0					

length x ↓

Base ~~cond~~ n (RS) → initialization (Top down)

```
int LCS( String X, string Y, int n, int m) {
    int dp[m+1][n+1];

    for(int i=0; i<m+1; i++)
        for(int j=0; j<n+1; j++)
            if (i==0 || j==0)
                dp[i][j]=0

    for (int i=1; i<m+1; i++)
        for (int j=1; j<n+1; j++)
            if( X[i-1]==Y[j-1])
                dp[i][j] = 1+ dp[i-1][j-1]
            else
                dp[i][j]= max(dp[i][j-1], dp[i-1][j])
```

return dp[m][n];

}

Longest common substring

I/p a :→ abcde
b :→ abfce

O/p → 2 (ab) ab, c, e

longest = ab

a b c d e
a b f c e

~~1~~ 2 ~~0~~ ~~1~~ ~~0~~ 1

length = 0

m →

n ↓

0	0	0	0
0			
0			
0			

discontinuity = 0

```
if ( a[i-1] == b[j-1] )
    dp[i][j] = dp[i-1][j-1] + 1
else
    dp[i][j] = 0
```

// for max^m val.

```
int maxi = 0;
for ( i = 0 → n+1)
    for ( j = 0 → m+1)
        maxi = max(maxi, dp[i][j])
return maxi;
```

Print LCS btwⁿ 2 string

a : a c b c f
b : a b c d a f

o/p → abcf

	ϕ	a	b	c	d	a	f
a							
c							
b							
c							
f							

	ϕ	a	b	c	d	a	f
ϕ	0	0	0	0	0	0	0
a	0	1	1	1	1	1	1
c	0	1	1	2	2	2	2
b	0	1	2	2	2	2	2
c	0	1	2	3	3	3	3
f	0	1	2	3	3	3	4

Now, we need to print LCS

"f c b a"

abcf ← reverse

if equal $i, j \rightarrow (i--, j--)$

if not equal $i, j \rightarrow \max((i-1, j), (i, j-1))$

When equal

$(i--, j--)$

4

when not equal

(max bet^n a & b)

a

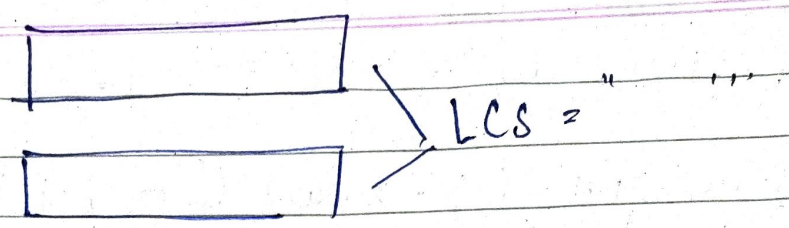
4

b

add in string

nothing to be added

```
int i = m, j = n;
String s = "";

while (i>0 && j>0)
  {
    if (a[i-1] == b[j-1])
      {
        s.push_back(t[i-1])
          i--;
          j--;
      }
    else {
          if( t[i][j-1] > t[i-1][j])
            j--;
          else
            i--;
    }

reverse(s.begin(), s.end()).
```



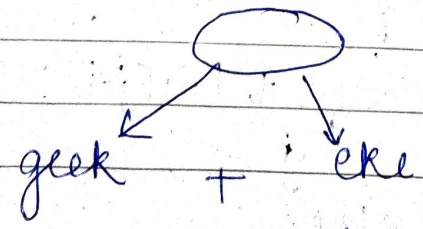

```
if (a[i-1] == b[j-1])
        S.push_back( a[i-1] )
            i--;
            j--;

    else
        move in the direc^n of max'm
        reverse (s.begin(), s.end())
```

24. Shortest Common Supersequence

a: "geek"
b: "eke"

1) Problem statement

→ 2 string are given. We need to merge the string such that we get both the subsequence.
Mix 2 sequence to make it a super sequence



Op

geekeke , geeke

find shortest

Subsequence → Sequence of events

S_1 S_2 S_3

Order should be maintained but don't need to be continuous array

a : A G G T A B
b : G X T X A Y B

Super sequence : A G G T G X A B T X A Y B

A G G X T X A Y B → shortest supersequence

A G G T A B G X T X A Y B

Give the length in o/p.

The one letter which is common write once.

A G G T A B
G X T X A Y B

↓

G T A B is common in both
i.e LCS.

Now thinking the brute force approach we can merge both the string & then subtract GTAB.

a: AGGTAB b: GXTXAYB

length of SS
a+b

Worst case: a+b = AGGTAB GXTXAYB

↓

AGGTABGXTXAYB − GTAB

↓

AGGXTXAYB

Shortest length = (m+n) − LCS

25. Minimum no of insertion & deletion to convert a string a to string b.

a : heap O/p = 1 (Insertion)
b : pea 2 (deletion)

a ⟶ b

p h e a p ⟶ pea

insertion deletion

When to use LCS.

	I/p	Q	O/p
LCS	a : ⬭ b : ⬭	LCS	int
Given Ques	a : ⬭ b : ⬭	insertion/ deletion	int

→ Pattern matching algorithm

Convert heap to pea.

p heap → pea

"ea" is the LCS of both string

heap → pea

Insertion/deletion

a → b

LCS (ea)

heap —2 deletion (hp)→ ea (LCS) —1 insertion (p)→ pea

No of deletion = a length − LCS

No of insertion = b length − LCS

26. Longest Palindromic Subsequence

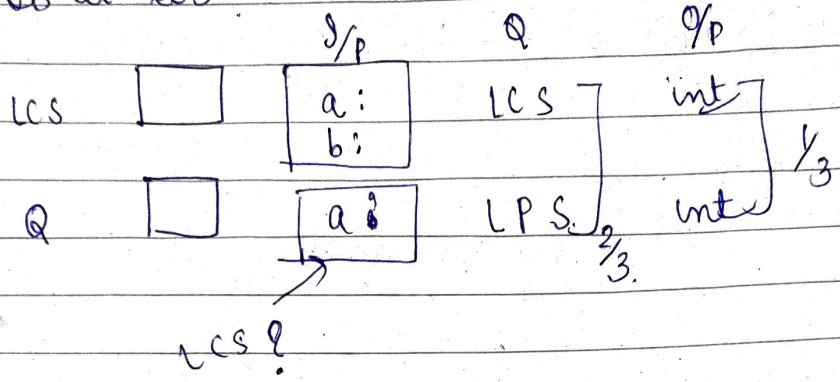
Problem Statement : S : agbcba

O/P = 5

5 ← longest [abcba ✓, bcb ✓, b ✓]

O/p → 5 (abcba)

2) Is it LCS?



LCS I/p a, b
LPS I/p a b = func (a)
(hidden string / redudent)

"agbcba"
↓
LPS
↙
a → agbcba
b → reverse(a) abcbga.
↓
LCS

(a)(b)(c)(b)(g)(a)
(a)(g)(b)(c)(b)(a) > abcba

LPS(a) ≡ LCS (a, reverse(a))
LPS(agbcba) → return(agbcba,

28. Minimum no. of deletion in a string to make it palindrome

a : "agbcba"

o/p = 1

S : agbcba

a	g	b	c	b	a

↓ No of deletions (Minimize)

→ New string (palindrome)

~~a~~g~~b~~cb~~a~~

bcb (palindrome) 3

agcba (not palindrome)

agbcba

~~a~~g~~b~~cb~~a~~ bcb (3) (LPS)

~~a~~g~~b~~c~~b~~a~~ C (5) (LPS)

agbcba abcba (1) LPS

Minm.

↑ length of LPS $\propto \frac{1}{\text{no of deletions}}$ ↓

LPS

min^m no of deletions.

→ Palindromic subsequence.

length of LPS $\propto \frac{1}{\text{no of deletions}}$ ↓

↑ (longest palindromic subsequence)

agbcba

↓

LCS(s, reverse(s))

↓

abcba (LPS)

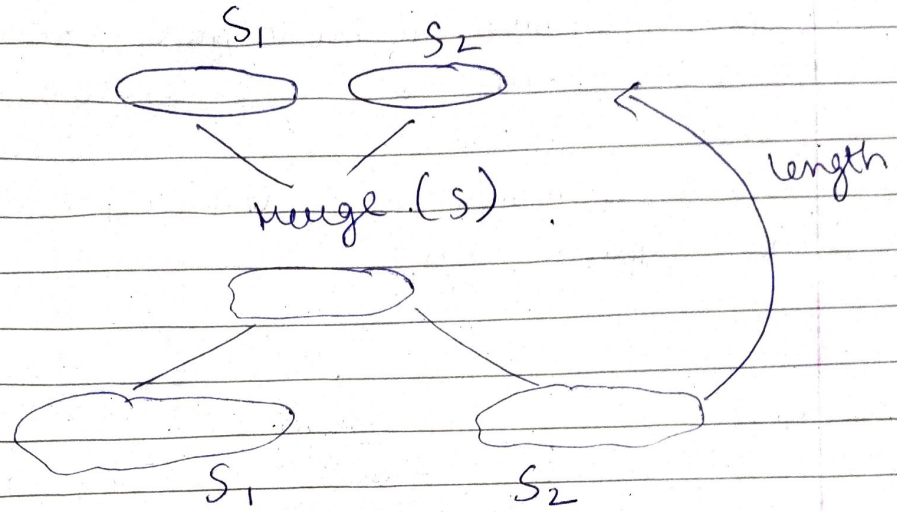
Min no of deletion = $\underset{\text{(length)}}{S}$ − LPS(length)

29. Print shortest common subsequence

i/p a: acbcf
b: abcdaf

o/p: acbcdaf

SCS



worst case

acbcf abcdaf

acbcdaf → print the whole string

m+n

acbcf abcdaf

↓

Now

m+n - LCS.

LCS $\xrightarrow{\text{(similar)}}$ SCS

Print LCS Print SCS

	ϕ	a	b	c	d	a	f
ϕ	0	0	0	0	0	0	0
a	0	1	1	1	1	1	1
c	0	1	1	2	2	2	2
b	0	1	2	2	2	2	2
c	0	1	2	3	3	3	3
f	0	1	2	3	3	3	4

→ LCS

LCS → common ni hai to move kar jao, common hai to print karlo.

SCS → common hai to ek bar print kro else print karo.

LCS.

```
String s = " ";
while (i>0 && j>0){
    if (a[i-1] == b[j-1]){
        s.push_back(a[i])
        i--;
        j--;
    }
    else{
        if(t[i][j-1] > t[i-1][j])
            j--;
        else if (t[i-1][j] > t[i][j-1])
            i--;
    }
```

SCS

```
String s = " ";
while (i>0 && j>0){
    if(a[i-1] == b[j-1]){
        s.push_back (a[i])
        i--, j--; }
    else {
        if (t[i][j-1] > t[i-1][j])
            s.pushback (b[j-1]);
            j--;
        else if ( t[i-1][j] > t[i][j-1])
            s.push_back (a[i-1]);
            i--; }
while(i>0)
    s.push_back (a[i-1]); i--;
while (j>0)
    s.push_back (b[j-1])
    j--;
```

1) Now in SCS code we include the letter on moving at i-1 or j-1.

2) Now in (i>0 && j>0) we need to change because in case of LCS we don't need to stop at the topmost but in SCS we need to.

In case of LCS

	ϕ	a	b	f
ϕ	0	0	0	0
a	0			
c	0			
b	0			
d	0			

	ϕ	a	b	f
ϕ	0	0	0	0
a	0			
f	0			
f	0			
d	0			

Some can't stop here.

a c
" "] LCS (" ")

a c
" "] SCS ~~=~~ (ab

Longest repeating subsequence

Str = "A A B E B C D D"
O/P = 3

Problem statement - Find a subsequence which is repeating

A A B E B C D D → ABD

→ order ✓
→ cont ✗

ABD

ABD (2X)
AB (2X)] longest → "ABD"
O/P = 3

S = A A B E B C D D

0 1 2 3 4 5 6 6
A A B E B C D D
A A B E B C D D

E → 3
C → 5

LCS = A AB E B C D D.
✗ ✗

A A B B D D
once ← → ans

E & C are occuring once.
letter at the same index don't take

E → 3 → i, j $i == j$ X
C → 5.

A → 0, 1
A → 0, 1

B → 2, 4
B → 2, 4

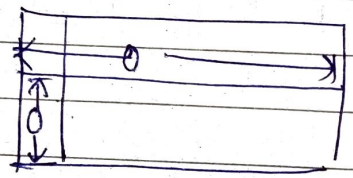
same index diff index

So don't take A of same index take from other index.

Sum up:

a = S
S
b = S.

LCS where $(i == j)$ X



We can't take same letter for both the string during LCS i.e. $(i == j)$

```
if (a[i-1] == b[j-1] && i == j)
        t[i][j] = t[i-1][j-1] + 1
else
        t[i][j] = max (t[i][j-1],
                       t[i-1][j])
```

31. Sequence Pattern Matching

a = "AXY" O/P → T/F
b = "ADXCPY"

Problem statement: Is string "A" a subsequence of "B"

a : AXY b : ADXCPY

b : A D X C P Y
a : A X Y

O/p → True.

LCS : AXY

I/p

LCS	a b	sequence	int
Q	a b	sequence	T/F

I/p

b : ADXCPY
a : AXY
LCS : AXY (LCS == a)

b : ADXCPY
a : AXYZ
LCS : AXY
(LCS != a)

```
if (LCS == A)  return true
else
    return false
```

Can it be done using length. ?

a
b
> LCS

a: AXY (3)
b: ADXCPZ(6)

m n

LCS = 0 to min(m,n)

LCS = 2

```
if (LCS == a.length())
    return true
else
    return false.
```

32. Min^m no of insertions in a string to make it palindrome

I/p: S: "aebcbda"
O/p → 2

x a e b c b d a x

d e

adebcbeda. x adebcbedax

(2) (4)

Minimum = 2

Min^m. no of deletion to make a string palindrom.

a e b e b d a

aea abcba

(4) (2)

S.length() → LPS. (deletion)

S : 'a e b c b d a'
  × ×

→ Palindromic string

[a] e [b c b] d [a]

e × } delete
d × } e & d

a e ✓ } insert e & d
d ✓ } & make their pair

no of insertions = no of deletions
= S.length − LPS.

Deletion → single (pair) → ×
Insertion → single (pair) → ✓

33. MCM (Matrix Chain Multiplication)

1) MCM
2) Printing MCM
3) Evaluate Exp. to True / Boolean Parenthesization
4) Min / Max value of an Expr.
5) Palindrome partitioning
6) Scramble string
7) Egg Dropping problem

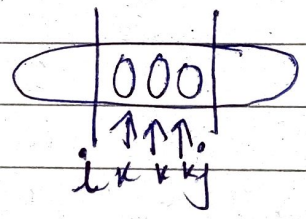
MCM format

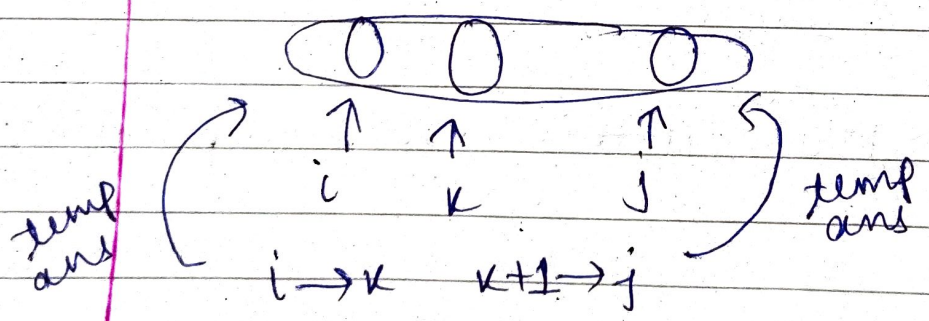
1) MCM
2) Printing MCM
3) Evaluate Exp^n to True/Boolean parenthesization
4) Min/Max value of an Expr
5) Palindrome partitioning
6) Scramble string
7) Egg dropping problem

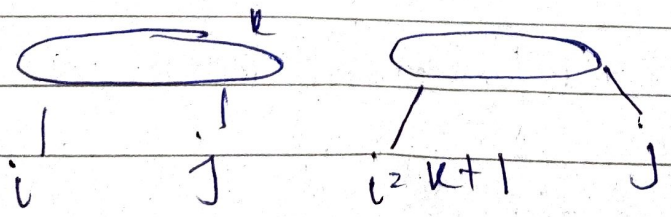
Identify → MCM → basic format

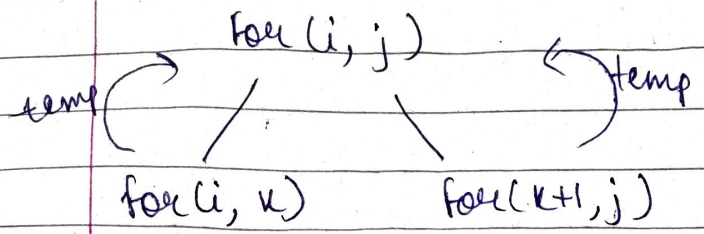
Identification + Format

MCM → storing or array

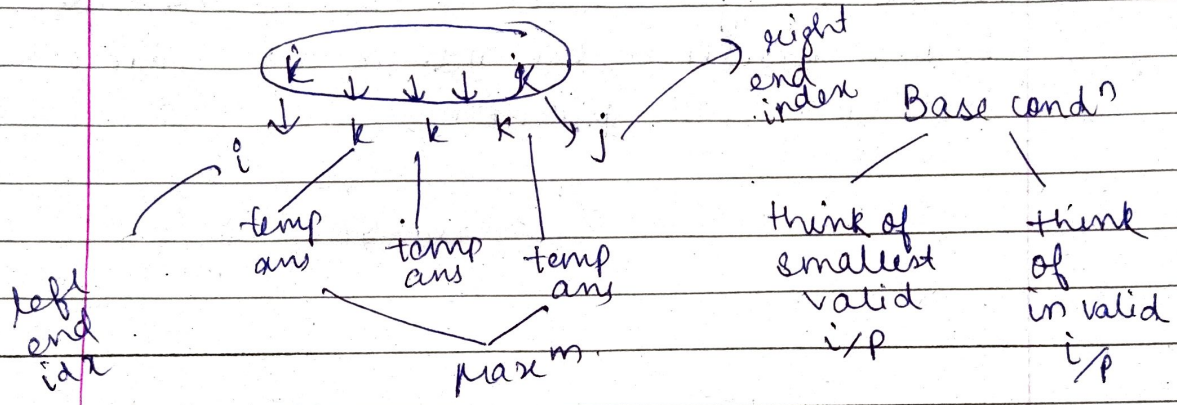








ans ← f (temp ans)




```
int solve ( int arr[], int i, int j)
{
    if (i>j)          →   this may be diff accⁿ to ques
    return 0;

for (int k=i ; k<j ; k++)
{
    // Calc. temp ans
        temp ans = solve( arr, i, k) +
                   solve (arr, k+1, j);

        ans =  func (temp ans)
}
```

MCM (Recursive)

Problem Statement : arr[] = {40, 20, 30, 10, 30}.

A_1 (m×n), A_2 (m×m), A_3 (n×n), A_4

Some matrix are given like A_1, A_2, A_3, A_4 we have to multiply the matrix to reduce the cost (no of multiplications)

b = c (then only we can multiply)

$\begin{bmatrix} & \end{bmatrix}_{2\times 3}$ a×b $\begin{bmatrix} & \end{bmatrix}_{3\times 6}$ c×d

2 × 3 3 × 6

↓ ↓ ↓

2 3 6 = 36 cost (no of multiplications)

$A_1 \; A_2 \; A_3 \; A_4$

dimension [] = {x, y, z, w}.

$A_1 \quad A_2 \quad A_3 \quad A_4$

$(A_1 (A_2 A_3)) A_4$ $\quad$ $((A_1 A_2)(A_3 A_4))$ $\quad$ $A_1 (A_2 (A_3 A_4))$

C_1 $\quad$ C_2 $\quad$ C_3

$A \rightarrow 10 * 30$
$B \rightarrow 30 * 5$
$C \rightarrow 5 * 60$

$(A B) C = 10 * 30 * 30 * 5$

$(A B) C = (10 * 30 * 30 * 5) \; 5 * 60.$

$= 10 * 5 * 30 * 5 * 60$

$= 10 * 30 * 5 * 60 * 10 * 5$

$= 4500.$

$A_1 A_2 A_3 \; A_4$

do parenthesiation in such a way cost is the least

$A_1 (A_2 A_3) A_4$ C_1
$(A_1 A_2)(A_3 A_4)$ C_2 $\quad A(B \; C)$ $\rightarrow$ min cost
$A_1 (A_2 (A_3 A_4))$
C_3

$30 * 5 \quad 5 * 60$

$B \; C \rightarrow 30 * 5 * 60$

$A * B C \rightarrow 10 * 30 \quad 30 * 60$

$10 * 30 * 60$ $\quad +$

cost

Bracket lagane par alag alag cost aa rahi hai.

Date
Page

	0	1	2	3	4
arr[ ] :	40,	20,	30,	10,	30

arr size n n-1 matrix

$A_1 \rightarrow 40 * 20$
$A_2 \rightarrow 20 * 30$
$A_3 \rightarrow 30 * 10$
$A_4 \rightarrow 10 * 30$

$A_1 * A_2 * A_3 * A_4$

Cost (minm)

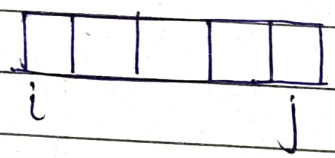
⇓

(no of multiplication should be less)

$A_i \rightarrow arr[i-1] * arr[i]$

$A[i] = arr[0] + arr[1]$

Next step → Format



(dimension)

arr[ ]: | 40 | 20 | 30 | 10 | 30 |
0 1 2 3 4

$A_1 \; A_2 \; A_3 \; A_4$

↓

$(A_1) \; (A_2 \; A_3 \; A_4)$

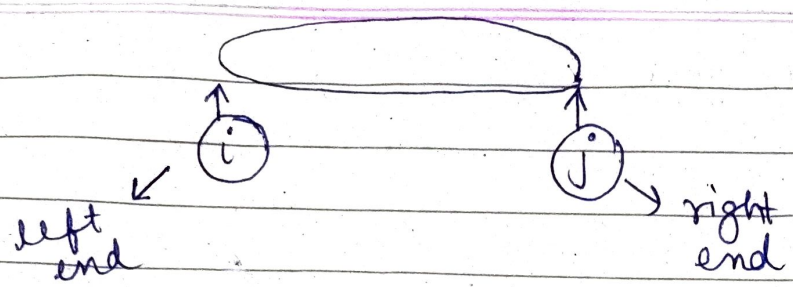
k

↓

Min cost + Min cost → temp ans

$(A_1 \; A_2 \; A_3) \; (A_4)$

put brackets on different places to get the answer
Slide k at different positions.



40	20	30	10	30
i	i			j

$A_i \rightarrow arr[i-1] * arr[i]$

when $i=0$ $\quad A_i \rightarrow arr[-1] * arr[0]$ ✗

when $i=1$ $\quad A_i \rightarrow arr[0] * arr[1]$ ✓

$A_j \rightarrow arr[j-1] * arr[j]$

$arr[3] * arr[4]$

$i > j \rightarrow size = 0$

$i == j \rightarrow size = 1$

$(i-1)$ ni milega

$i = 1$
$j = n-1$ } return cost

```
int solve ( int arr[], int i, int j)
  {
  if (i>=j)
     return 0;
     int mn = INT_MAX;
  for (int k = i ; k <= j-1 ; k++){
     int temp ans = solve (arr, i, k) +
                    solve (arr, k+1, j)
                    + arr[i-1] * arr[k] * arr[j]

  if (tempans)
     {
     mn = temp ans;
  }
```

return mn

}

A B C D

AB → A_1 CD → B_1

40×20 20*30 30*10 10*30

A*B 40*20*30 30*10*30 C*D

A_1 * B_1

40*30 30*30

+ 40*30*30 +

AB*CD

minimise

Steps for MCM

1) find i & j value
2) find right Base condⁿ.
3) Move K → i to j. (find K-loop scheme)
4) Calculate (k) cost for (i) temp ans.

40 20 30 10 30

↑ K

i to k k+1 to j

i k j

40 20 30 10 30 → empty

K (k+1) j

i to k

i j

40 20 30 10 30

k

i to k

40 * 20

20 * 30

30 * 10.

k+1 to j

10 * 30.

k = i k = j - 1 (i → k k+1 → j)

i j

40 20 30 10 30

k

i to k-1 (k to j)

2 schemes.

k = i to k = j-1 i to k k+1 to j

k = i+1 to k = j i to k-1 k to j

cost (extra)

0 1 2 3 4

40 20 30 10 30

i-1 i k k+1 j

for (i to k)

40 * 20 * 20 * 30

↓

Solve (i to k) 40 * 20 * 30.

for (k+1 to j)

30 * 10 10 * 30

↓

30 * 10 * 30

Solve (k+1 to j)

40 * 20 * 30 * 30

↓

arr[i-1] ← 40 * 30 * 30 → arr[j] arr[k]

dimension arr: [ ]

MCM Bottom Up (DP)

(dimension) arr[ ] : [ ]

```
int dp[100][100]

memset ( dp, -1 , sizeof(dp))

int solve ( int arr[ ], int i, int j)

  if (i <= j)
      return 0;

  if (dp[i][j] != -1)
      return dp[i][j];

  int mn = INT_MAX;
  for (int k = i; k<j ; k++) {

  int temp_ans = solve(arr, i, k) +
                 solve (arr, k+1, j) +
                 arr[i-1] * arr[k] * arr[j];

  if (temp_ans <mn)
      mn = temp_ans;
  }

  dp [i][j] = mn;
}
return mn;
```

~~Palindrome Partitioning~~

```
int dp [1001][1001];

class solution {
public:
    int solve(int arr[], int i, int j) {
        if (i>=j)
            return 0;

        if (dp [i][j] != -1)
            return dp[i][j];

        int mn = INT_MAX;
        for(int k = i; k<= j-1; k++){
            int temp = solve(arr, i, k) + solve(arr, k+1, j)
                        + arr[i-1]* arr[k] * arr[j]
            mn = min (temp, mn);
        }
        dp[i][j] = mn;
        return dp[i][j];
    }

int matrixmult( int N, int arr[]) {
    memset (dp, -1, sizeof (dp));
    int ans = solve (arr, 1, N-1);
    return ans;
    }.
};
```

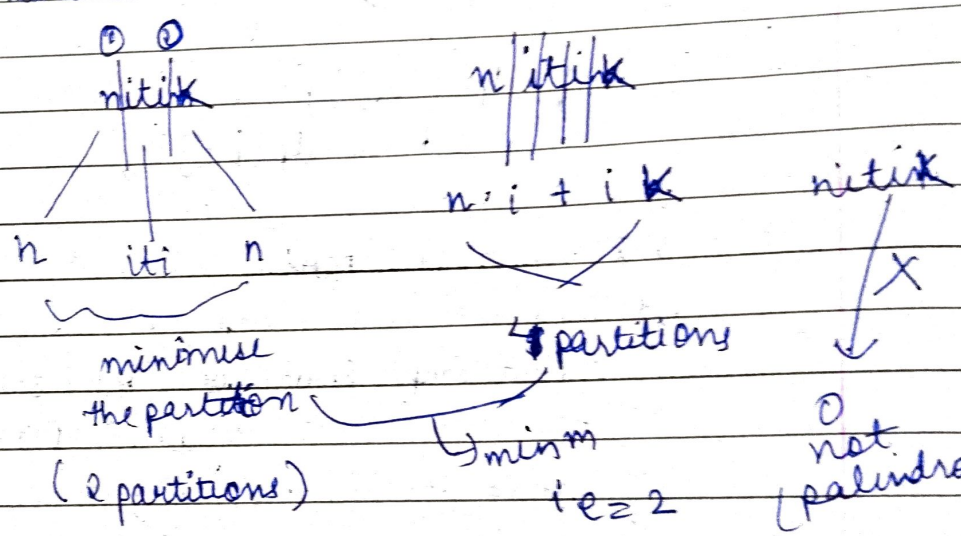
Palindrome partitioning

1) Problem statement

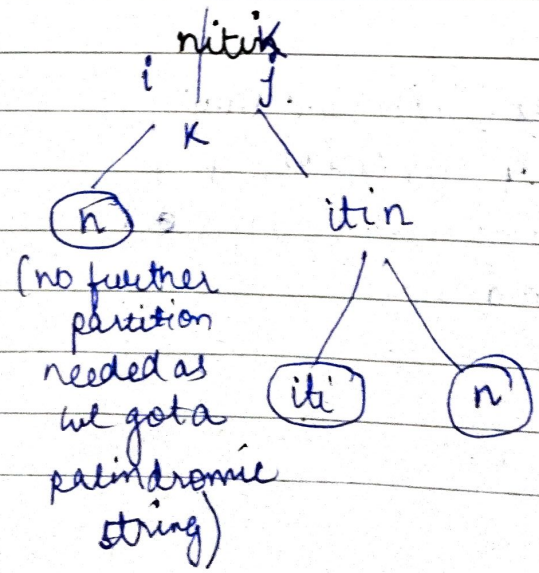
S : nitik → it is not as a whole is a palindrome

o/p : 2

divide string in such a way all the strings are palindrome.



worst case partition : n-1



n | i t i k

i to k 1 + k+1 to j

C_1 C_2

C_3

temp ans

n | i t i k

C_1 1 + C_2

temp ans.

min^m

Format

1. Find i & j
2. Find B.C.
3. Find K loop (scheme)

4. temp ans → min^m

Recursive code

n i t i k

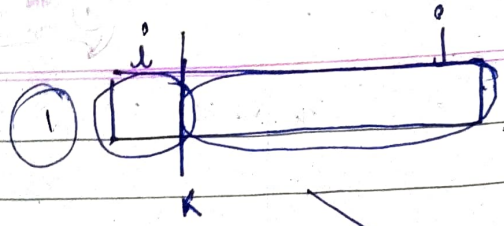
i j

i = 0 j = n-1 (no need to start i from 1 since it doesn't require i-1)

B.C i == j size = 1
i > j size = 0 } (if size is not given or 0 or equal no of partition would be 0 as string is empty)

ispalindrome (s, i, j) ← & if palindrome partition should be 0.

loop

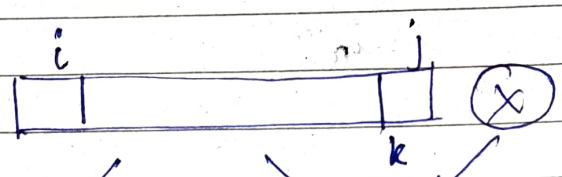


i to k k+1 to j

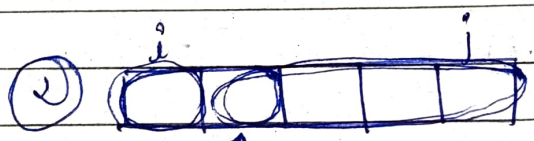
k=i k=j-1

i to k k+1 to j

k → i, i+1 k → j, j-1



i to k k+1 to j X



k (i+1)

i to k-1 k to j

k = i+1 k=j

1) k=i k=j-1 i to k k+1 to j

2) k=i+1 k=j i to k-1 k to j

```
for (int k=i ; k=j-1 ; k++)
{
    int temp = solve( s, i, k) + Solve( s, k+1, j)
                                                +1
```

i j

n | i t i n

k

i to k

"n"

k+1 to j

"itin"

$c_1 + 1 + c_2$

```
    ans = min (ans, temp)
}
return ans;
}
```

Final code

```
int solve (string s, int i, int j) {
    if (i>=j)
        return 0;

    if( is palindrome (s, i, j) == True)
        return 0;
    int mn = INT_MAX ;

    for(int k=i ; k<=j-1 ; k++) {
        int temp = 1+ solve (s, i, k) + solve(s, k+1, j);

        mn = min (mn, temp)
    }
return mn;
}
```

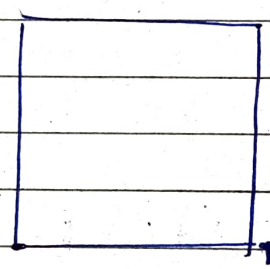
memset < -1, 0 > only allow 2 values

Date
Page

Palindrome partitioning
(bottom up) (memoization)

i/p = nitin nitik
o/p = 0 2

1) Initialise the matrix with -1
2) Check if the value is
   - -1 (value isn't evaluate
   - ≠ -1 value is evaluated so we store & return the value.



n×m
↓
variables which changes basically

i & j changes.

```
int dp [1001][1001];
memset (dp, -1, size of(dp));
int solve ( string s, int i, int j) {
        if( i>=j)
            return 0;

        if (is palindrome (s, i, j))
            return 0;
```

```
    if (dp[i][j] != -1)
        return dp[i][j];
    int mn = INT_MAX;
    for (int k = i ; k <= j-1 ; k++) {
    int temp = solve(s, i, k) + solve(s, k+1, j) + 1
    mn = min (mn, temp);
    }
    dp[i][j] = mn;
    return mn;
}

int palindromepart ( string s) {
    int n = s.length() -1;
    int ans = solve( s, 0, n);
    return ans;
}

bool ispalindrome (string s, int i, int j) {

    if (i == j)
        return True;

    if (i > j)
        return True;

    while (i<j)
    if (s[i] != s[j])
        return false
    else
        i++;
        j--;
```

Geeksforgeeks = ✓
Interviewbit = ✗

→ Further Optimization

Why the above code is not most optimized?

Since we are calling

int temp = 1 + solve(s, i, k) + solve(s, k+1, j)
RC (left) RC (right)

There is a possibility, might be one of the RC is solved or called.

The code will remain same but the diffn is

```
int temp = 1+ solve(s, i, k) + solve(s, k+1, j)

    if (t[i][k] != -1)
        left = t[i][k]
    else
        left = slove(s, i, k)
            t[i][k] = left

    if (t[k+1][j] != -1)
        right = t[k+1][j]
```

else

right = solve (s, k+1, j)

t[k+1][j] = right.

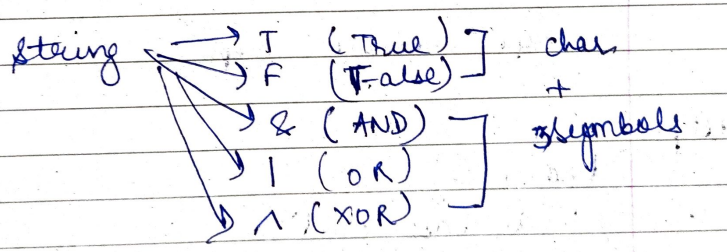
int temp = 1 + left + right;

Evaluate Expression to True
Boolean parenthesization

String : T or F and T.

o/p : 2

Problem statement : A string is given. String might have some character like T → true F → false



How to put bracket such that the expression evaluates to true.

Find the no of ways in which when bracket is put it evaluates to True.

i/p : "T | F & T ^ F"

no of ways

(( )) ( ) → T
( ) ( ) → F
( ) ( ) → T → 3 ways.
( ) ( ) → T

In MCM we put brackets for min^m cost &
in this also we do the same.

(T | F & T ^ F)

K-1 K K+1

We need to break bracket on operator

Expr^n operator Expr^n

4 steps:

1) find i & j
2) find base cond^n
3) Find K loop
4) temp ans + func^n
↓
main ans.

$$\overset{i}{T} \mid F \,\&\, T \wedge \overset{j}{F}$$

1) $i = 0$
$j = \text{strlength}() - 1$

2) BC

$$\overset{i}{(T \text{ or } F \text{ and } T)} \text{ xor } \overset{j}{(F)}$$

i to k-1   k   k+1 to j

(left) Expr^{n}1 XOR Expr^{n}2 (right)

i to k-1   k   k+1 to j

$T \wedge T$ = False
$F \wedge F$ = False
$T \wedge F$ = True
$F \wedge T$ = True

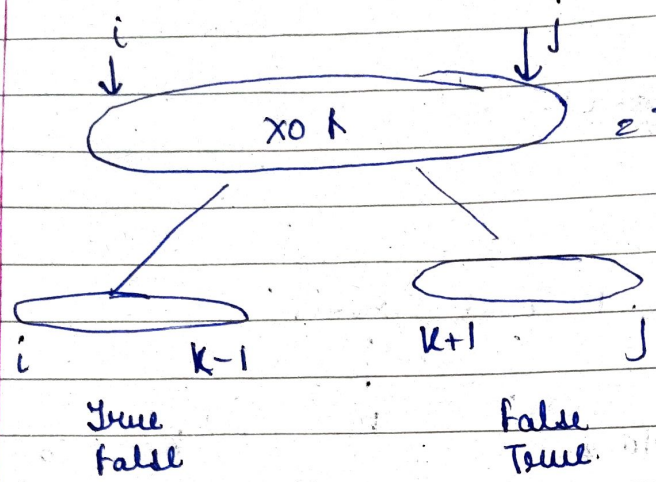
no of true ways = $rT * rF + rF * rT$

$T \wedge F = T$
$F \wedge T = T$

Expr^n. XOR Expr^n.

no of ways (True) → 2 * 1 ← no of ways false

Expr^n. XOR Expr^n.



int solve (string S, int i, int j, boolean isTrue or isFalse)

<u>Base cond^n</u>

i j

T or F and T xor F

T | F & T ^ F

boolean isTrue

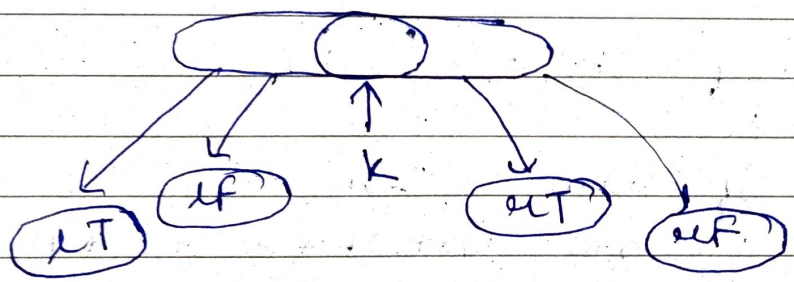
```
if (i > j) return false
if (i == j)
    if (isTrue == True)
        return S[i] == 'T'
    else
        return S[i] == 'F'
```

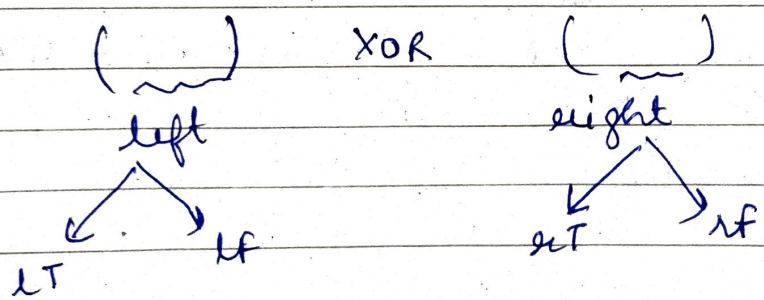
loop

i						j
T	|	F	&	T	^	F
	↑		↑		↑	
	k = i+1		k = k+2		k = j-1	

```
for (int k = i+1; k <= j-1; k = k+2)
    int ans = 0;
    int lT = (S, i, k-1, T)
    int lF = (S, i, k-1, F)
    int rT = (S, k+1, j, T)
    int rF = (S, k+1, j, T)
```

temp ans.





ans += lT * rF + lF * rT

```
if (S[k] == '&')
{
if (isTrue == True)
  ans = ans + lT * rT
else {
ans = ans + lF * rT + lT * rF + lF * rF;
}
else if (S[k] == '|') {
  if (isTrue == True)
      ans = ans + lT * rT + lT * rF +
                          lF * rT.
  else
      ans = ans + lF * rF.
}

else if (S[k] == '^')
{
  if (isTrue == True)
      ans = ans + lF * rT + lT * rF
  else
      ans = ans + lT * rT + lF * rF
}
return ans;
}
```

whole code

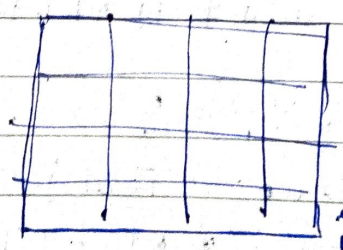
```
int solve (string s, int i, int j, bool isTrue ) {

    if (i > j) {
        return False ; }
    if (i == j) {
        if (isTrue == True)
            return s[i] == 'T';
        else
            return s[i] == 'F';
    }
for ( int k = i+1 ; k <= j-1 ; k += 2) {
    int ans = 0;
    int LT = solve (s, i, k-1, T);
    int LF = solve (s, i, k-1, F);
    int RT = solve (s, k+1, j, T);
    int RF = solve (s, k+1, j, T);

    if (s[k] == '&') {
        if (isTrue == True)
            ans = ans + LT * RT ;
        else
            ans = ans + LF * RT + LT * RF + LF * RF;
    }
    else if ( s[k] == '|') {
        if (isTrue == True)
            ans += LT * RT + LT * RF + LF * RT;
        else
            ans += LF * RF;
    }
```

```
else if ( S[k] == '^' ) {
        if (isTrue == True)
            ans += LF*RT + LT*RF;
        else
            ans += LT*RT + LF*RF;
    }
}
return ans;
}
```

Evaluate Expression to True Boolean Parenthesization - (memoization)
(Bottom Up - DP)
3D dp

Recursive → Recursive call (RC)
Top down → table
Bottom up → RC + table

PS → put brackets in such a way that the expression evaluates to true.



i*j* isTrue
(3d)

int t [1001] [1001] [2]

isTrue

True False

More better way: We can use map

map	value
i, j, isTrue	
i + " " + j + " " + isTrue	

```
unordered_map < string, int > mp;

int main(){
    mp.clear()
    Solve( )
}
```

After Base cond^n:-

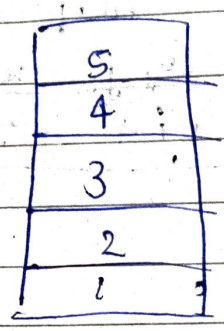
```
String temp = to_string(i);
    temp.push_back(' ');
temp.append(to_string(j));
    temp.push_back(' ');
    temp.append(to_string(isTrue));

if(mp.find(temp)!= mp.end()){
        return mp[temp];
    }
```

```
return mp[temp] = ans;
}
```

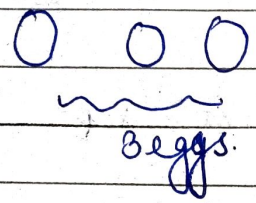
Egg Dropping Problem

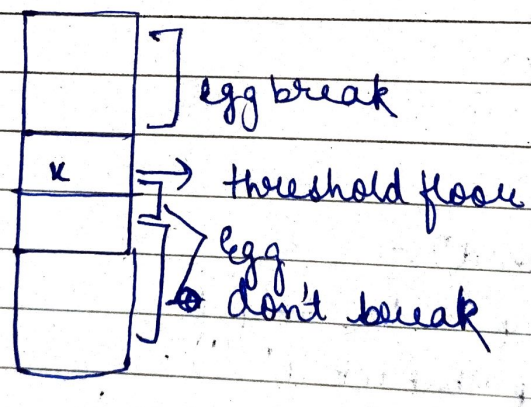
I/p : e = 3
f = 5

O/p : 3 → minmize no of attempts in worst case.

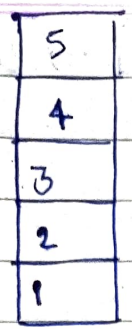


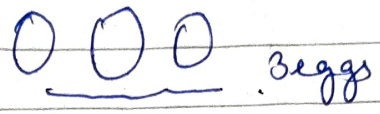
Problem statement :



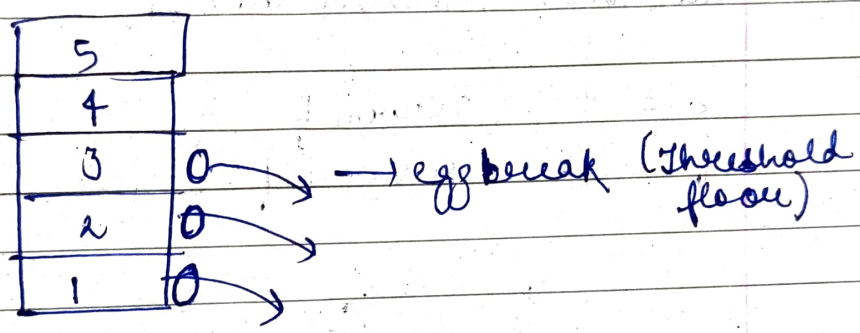


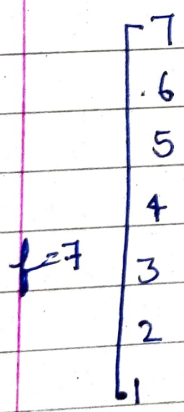
We are in a building, we need to find the minm no of attempts to find the critical floor.



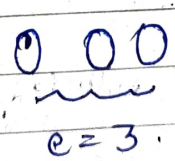


Safe strategy → worst case (1 egg). so drop from the last it won't break untill threshold.



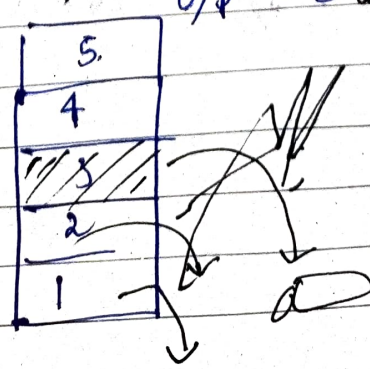


Worst-case → minimize no of attempts to find threshold floor.



I/p e = 3
f = 5

O/p = 3. attempts



(floor) f

1	2	3	4	5	6	7

i ↑ j ↑

→ Building looks like an array

from where to take k?

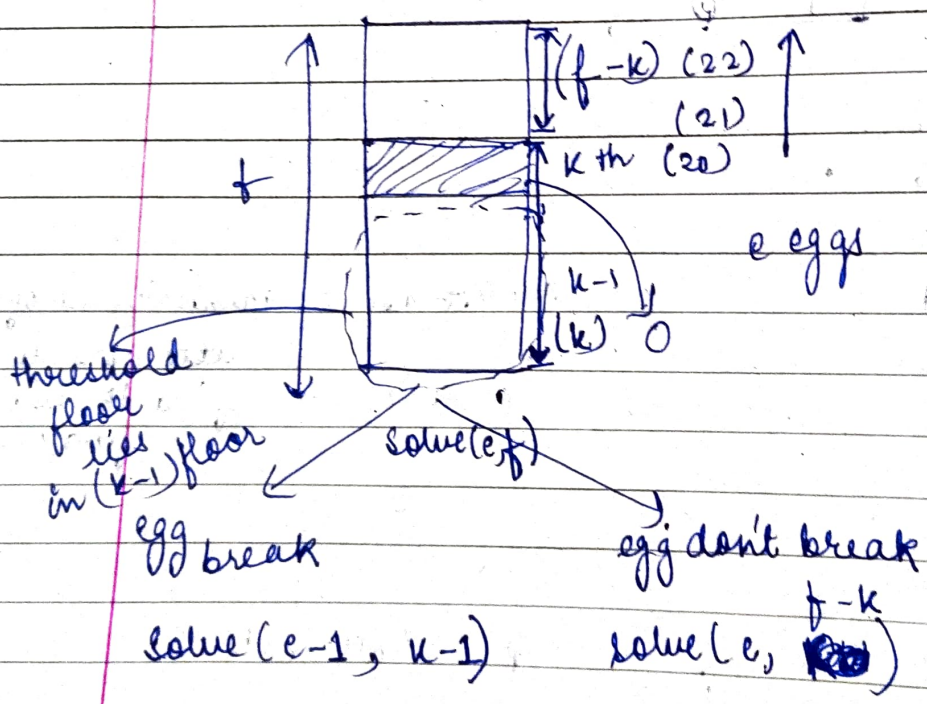
→ We will check for all k values.

for k = 1 k = f k++

Base condn. (smallest valid i/p)

e = 1 return f

f = 0/1 return f



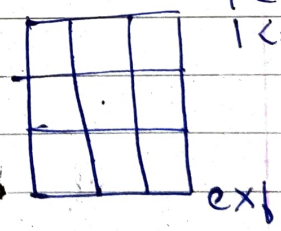
```
int solve (int e, int f)
{
    if (f == 0 || f == 1)
        return f;

    if (e == 1)
        return f;

    int mn = INT_MAX;

    for (int k = 1; k <= f; k++)
    {
        int temp = 1 + max( solve (e-1, k-1),
                            solve (e, f-k));
        mn = min (mn, temp);
    }
    return mn;
}
```

for the worst case

Egg Dropping Memoization

$1 <= T <= 30$
$1 <= e <= 1000$
$1 <= f <= 1000$

```
// globally defined int dp[1001][1001];
memset (dp, -1, sizeof(dp));
```




```
int solve ( int e, int f){
    if (e == 1)
        return f;
    if (f == 0 || f == 1)
        return f;
```

```
if (t[e][f] != -1)
    return t[e][f];
int mn = INT_MAX;
for (int k = i ; k <= f ; k++) {

    int temp = max(solve(e-1, k-1),
                   solve(e, f-k)) + 1;

    mn = min(mn, temp);
}

return t[e][f] = mn;
}
```

Further optimization

Inside the loop.

```
if (t[e-1][k-1] != -1)
    int low = t[e-1][k-1]
else
    low = solve(e-1, k-1)
    t[e-1][k-1] = low;

if (t[e][f-k] != -1)
    int high = t[e][f-k];
else
    high = solve(e, f-k);
    t[e][f-k] = high;

int temp = 1 + max(low, high);
```