

Discrete Math Notes

Propositions, Negations, Conjunctions and Disjunctions

A **proposition** is a declarative statement that is either true or false

- Eg, "The sky is blue", "The moon is made of cheese"
- Example of what is *not* a proposition, "Sit down", " $X+1 = 2$ ". The last one is not a proposition because it is not true or false

A compound proposition is comprised of propositions and one or more of the following connectives

- Negation \neg "NOT"
- Conjunction \wedge "AND"
- Disjunction \vee "or"
- Implication \rightarrow "If, then"
- Bicondition \leftrightarrow "if and only if"

Implications (Converse, inverse, contrapositive) and Biconditionals

Implication (conditional statement)

- When the hypothesis is true, the conclusion must be true for the implication to be true.

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

- Converse, if the implication is $p \rightarrow q$, then the converse is $q \rightarrow p$ (switching order)
- Inverse, if the implication is $p \rightarrow q$, then the inverse is $\neg p \rightarrow \neg q$ (negate)
- Contrapositive, if the implication is $p \rightarrow q$, then the contrapositive is $\neg q \rightarrow \neg p$ (switching the order and negate). Contrapositive will always have the same truth values as the implication

For a biconditional to be true, both propositions must share the same truth value

P	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Overall

P	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

1.1.3 Constructing a truth table for compound propositions

You need $2^{\{\text{number of propositions}\}}$ rows

You need a column for each propositional value, a column for the truth value of each expression that occurs in the compound proposition as it's built up, and lastly a column for the final result.

The order of precedence (importance/priority) for operators is as following:

- \neg
- \wedge

- \vee
- \rightarrow
- \leftrightarrow

Example

The proposition is $p \vee q \rightarrow \neg r$

- 1) first construct columns for each propositions: p, q, r
- 2) create a column for each compound proposition: $p \vee q$, $\neg r$
- 3) create a column for the final compound proposition
- 4) create $2^3 = 8$ rows

1.2.1 Translating propositional logical statements

equate each condition to a letter

the letter must always represent the **positive** outcome

Eg. " You can get a free sandwich on thursday if you buy a sandwich or a cup of soup":

- P: I buy a sandwich
- Q: I buy a cup of soup
- R: I get a free sandwich on thursday

Answer: $(P \vee Q) \rightarrow \neg R$

1.2.2 Solving Logic Puzzles

Context:

- Knights always tell the truth
- Knaves always lie

Puzzle:

- A says "B is a knight"
- B says "The two of us are opposite types"
- What are A and B?

Answer:

- P: A is a knight
- q: B is a knight
- 4 Possible outcomes
 - $p \wedge q$
 - $p \wedge \neg q$
 - $\neg p \wedge q$

- $\neg p \wedge \neg q$

Go through each one, the only one that doesn't contradict is the last one

1.2.3 Introduction to Logic circuits

- NOT gate
- OR gate
- AND gate

A Logic circuit is designed to perform complex tasks designed in terms of elementary logic functions.