

Discrete Math Notes

Propositions, Negations, Conjunctions and Disjunctions

A **proposition** is a declarative statement that is either true or false

- Eg, "The sky is blue", " The moon is made of cheese"
- Example of what is *not* a proposition, "Sit down", "X+1 = 2". The last one is not a proposition because it is not true *or* false

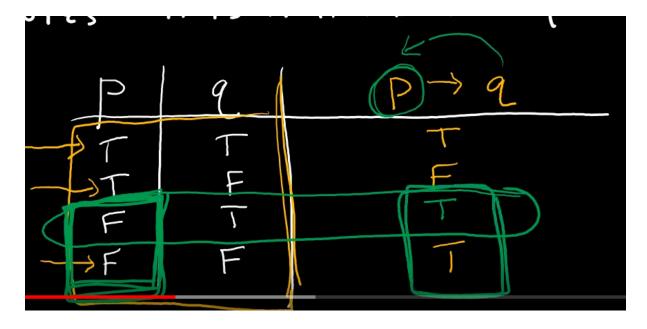
A compound proposition is comprised of propositions and one or more of the following connectives

- Negation ¬ "NOT"
- Conjuction ^ "AND"
- Disjunction v "or"
- Implication → "If, then"
- Bicondition \leftrightarrow "if and only if"

Implications (Converse, inverse, contrapositive) and Biconditionals

Implication (conditional statement)

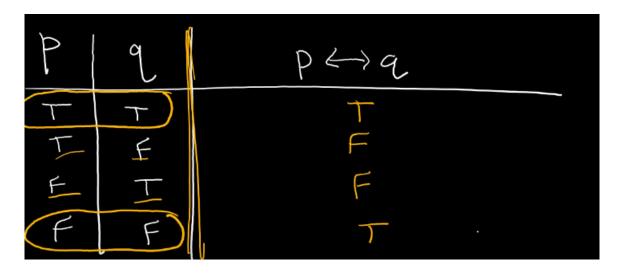
• When the hypothesis is true, the conclusion must be true for the implication to be true.



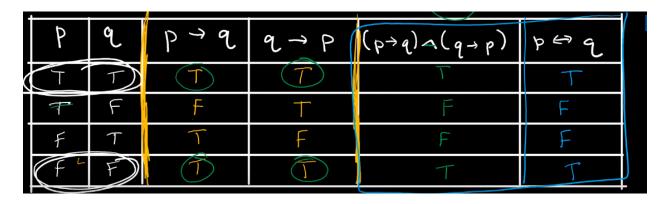
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- Converse, if the implication is $p \to q$, than the converse is $q \to p$ (switching order)
- Inverse, if the implication is $p \rightarrow q$, than the inverse is $\neg p \rightarrow \neg q$ (negate)
- Contrapositive, if the implication is p → q, then the contrapositive is ¬q → ¬p (switching the order and negate). Contrapositive will always have the same truth values as the implication

For a biconditional to be true, both propositions must share the same truth value



Overall



1.1.3 Constructing a truth table for compound propositions

You need 2^{number of propositions} rows

You need a column for each propositional value, a column for the truth value of each expression that occurs in the compound proposition as it's built up, and lastly a column for the final result.

The order of precedence (importantce/priority) for operators is as following:

- ¬
- . ^

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- V
- →
- $\bullet \leftrightarrow$

Example

The proposition is p v q $\rightarrow \neg r$

- 1) first constructi columns for each propositions: p, q, r
- 2) create a colum for each compound propisition: p v q, ¬r
- 3) create a column for the final compound proposition
- 4) create 2^3 = 8 rows

1.2.1 Translating propositional logical statements

equate each condition to a letter

the letter must always represent the **positive** outcome

Eg. " You can get a free sandwich on thursday if you buy a sandwich or a cup of soup":

- P: I buy a sandwhich
- Q: I buy a cup of soup
- R: I get a free sandwhich on thursday

Answer: $(P \lor Q) \rightarrow \neg R$

1.2.2 Solving Logic Puzzles

Context:

- · Knights always tell the truth
- Knaves always lie

Puzzle:

- · A says "B is a knight"
- B says "The two of us are opposite types"
- · What are A and B?

Answer:

- P: A is a knight
- q: B is a knight
- 4 Possible outcomes
 - p ^ q
 - p ^ ¬q
 - ¬p ^ q

• ¬p ^ ¬q

Go through each one, the only one that doesnt contradict is the last one

1.2.3 Introduction to Logic circuits

- NOT gate
- OR gate
- AND gate

A Logic circuit is designed to perform complex tasks designed in terms of elementary logic functions.