# LINEAR ALGEBRA - 2

## DOT PRODUCT &

## HYPERPLANES

# Recap.

* Machine Learning Context : training classifiers

* New terms
  (a) Feature
  (b) label
  (c) Datapoint
  (d) Dataset
  (e) Classifiers

* How to train classifier. "optimisation" (last 5 classes).

* Lines. * $y = \overset{\text{slope}}{m}x + c$ → y intercept

* $\underbrace{w_1 \overbrace{x_1}^{\text{features}} + w_2 \overbrace{x_2} + w_0}_{\text{parameters}} = 0$

## Class 1

| Height $x_2$ | Weight $y_2$ | width |
|---|---|---|
| 33 | 27 | 0 |
| 13 | 6.5 | 0 |
| 43 | 39 | 0 |
| 22.6 | 16 | 0 |
| 13 | 35 | 0 |
| 33.8 | 7.3 | 0 |
| 46 | 13 | 0 |
| 10 | 19 | 0 |
| 22 | 30 | 0 |
| 24 | 43 | 0 |

## Class 2

| height $x_1$ | weight $y_1$ | width |
|---|---|---|
| 63 | 77 | 0 |
| 67 | 97 | 0 |
| 85.7 | 60 | 0 |
| 93.8 | 90 | 0 |
| 91.4 | 105 | 0 |
| 73 | 114.8 | 0 |
| 41.7 | 66.7 | 0 |
| 50.3 | 89.5 | 0 |
| 79 | 88 | 0 |
| 56 | 104.5 | 0 |

$$w_1 x_1 + w_2 x_2 + w_0 = 0$$

$$1 x_1 + 1 x_2 + -98 = 0$$

$$\boxed{x_1 + x_2 - 98 = 0}$$ – line

1. Will I be able to separate these points using line – NO
2. What should we do – plane
3. What is the eqn of plane?

$$w_1x_1 + w_2x_2 + w_0 = 0$$

General Equation of line

2D Hyperplane

General Eq. of plane

$$w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0$$

3D Hyperplane

$$w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + \dots + w_nx_n + w_0 = 0$$

nD Hyperplane

# Linear Algebra - Vectors.

Tuples, Numpy arrays, pandas Series, list

Q1) What is a vector?

* Both Magnitude & direction (physics)
* Collection of number

Q2) How do we represent a vector?

$\bar{x}$ , $\vec{x}$ Set of all Real number in d dimen

If vector $\bar{x}$ has d dimensions we can write

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \vdots \\ x_d \end{bmatrix}$$

Column Vector

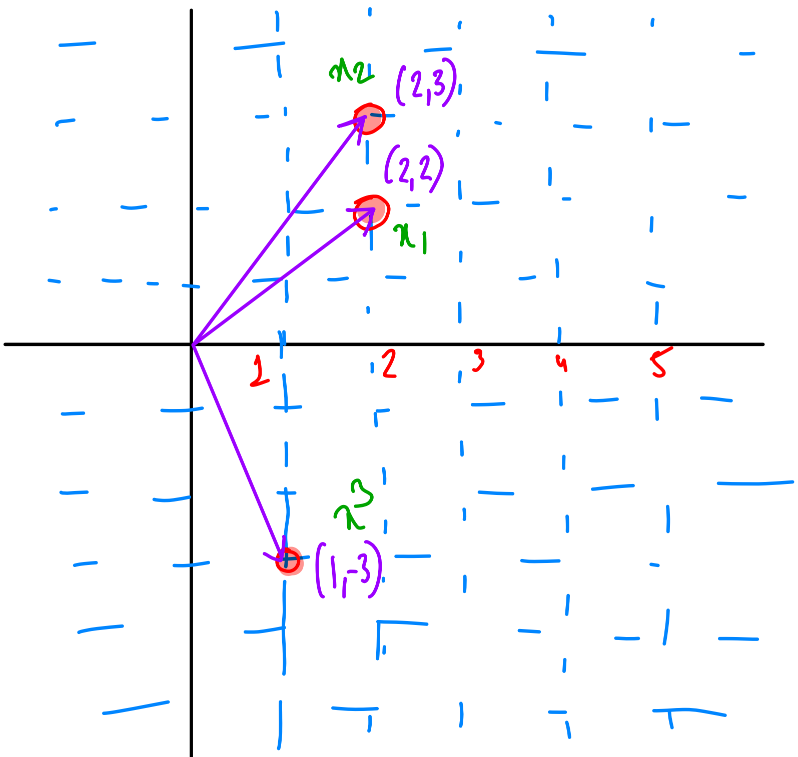
$\bar{x} = [x_1 \; x_2 \; x_3 \; x_4 \ldots x_n]$

Row vector

$x \in \mathbb{R}^d$

belongs to

# Linear Algebra - Vectors.




$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\bar{x}_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad \bar{x}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \bar{x}_3 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

Q) What are the magnitude of these vectors?

It is the length of line joining point & origin

for 2D vectors, magnitude is given by $\sqrt{x_1^2 + x_2^2}$

# Linear Algebra - Vectors.

* How to get magnitude of vector in d-dimension?

2D Vector, $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ magnitude $(\bar{x}) = \sqrt{x_1^2 + x_2^2}$

3D Vector, $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ magnitude $(\bar{x}) = \sqrt{x_1^2 + x_2^2 + x_3^2}$

dD Vector, $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_d \end{bmatrix}$, magnitude $(\bar{x}) = \sqrt{x_1^2 + x_2^2 + \quad x_d^2}$

# Linear Algebra - Vectors.

* Norm of a vector → its length
* How do we represent Norm of a vector $|| \quad ||$

norm of a vector $\bar{x}$ → $||\bar{x}||$

* $L_2$ Norm. The formula for $L_2$ Norm of $\bar{x}$

$$||\bar{x}||_2 = \sqrt{x_1^2 + x_2^2 + x_3^2 \dots x_d^2}$$

Similar to Euclidean Distance

* $L_1$ Norm: The formula for $L_1$ Norm of $\bar{x}$

$$||\bar{x}||_1 = |x_1| + |x_2| + |x_3| \dots |x_d|$$

Similar to Manhattan distance.

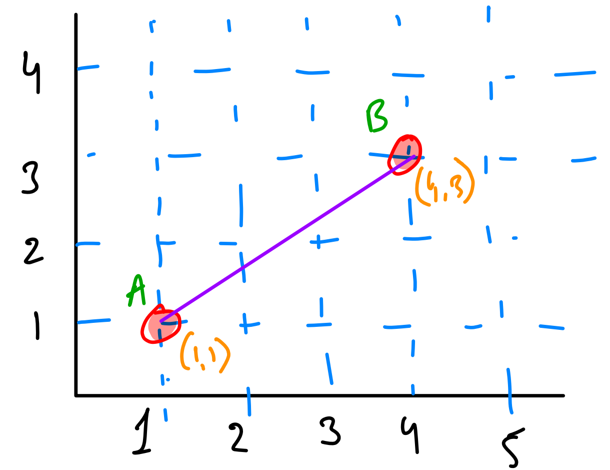
# Different Types of distances

## ① Euclidean Distance

Shortest Distance/path b/w A & B

$$\sqrt{(y_2-y_1)^2+(x_2-x_1)^2} = \sqrt{(3-1)^2+(4-1)^2}$$

$\sqrt{13}$

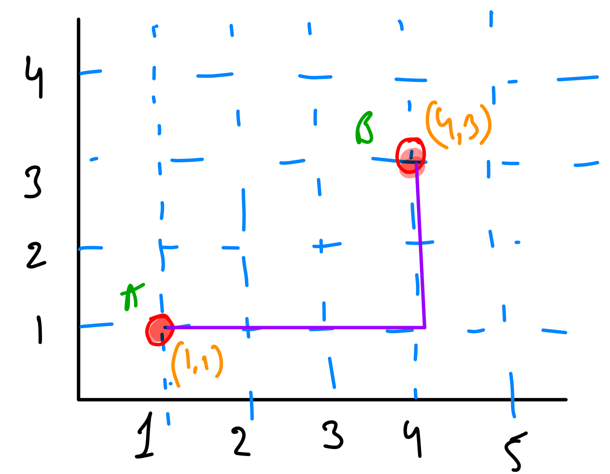



## ② Manhattan Distance.

Distance by grid shaped roads on 2D plan.

$$|y_2-y_1| + |x_2-x_1| = |4-1| + |3-1|$$

$\Rightarrow 5$



# Linear Algebra : Matrix Multiplication

$$A \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \quad B \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 1\times5 + 2\times7 & 1\times6 + 2\times8 \\ 3\times5 + 4\times7 & 3\times6 + 4\times8 \end{bmatrix}_{2\times 2}$$

Dimension = # Rows x # column.

* for Matrix Multiplication we need to check

$A_{m\times n}$ $B_{a\times b}$ $\rightarrow$ $n == a$

* Resultant Shape of matrix $\begin{bmatrix} \quad \end{bmatrix}_{m\times b}$

# Linear Algebra. Angle b/w 2 Vectors.

$\bar{x} \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2\times1}$ $\bar{y} \begin{bmatrix} 3 \\ 4 \end{bmatrix}_{2\times1}$

if $x$ is a vector

Transpose of Vector $\bar{x} = \bar{x}^T$

Transpose

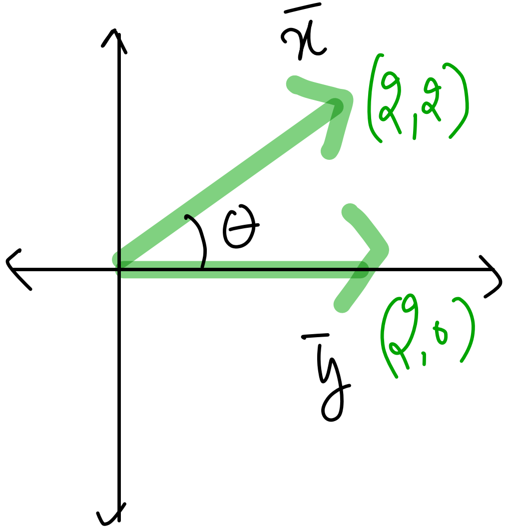
$$\bar{x}^T \begin{bmatrix} 1 & 2 \end{bmatrix}_{1\times2} \begin{bmatrix} 3 \\ 4 \end{bmatrix}_{2\times1} = \begin{bmatrix} 1\times3 + 2\times4 \end{bmatrix}_{1\times1}$$

$$= 11$$

This operation is called as DOT PRODUCT

$\bar{x}^T\bar{y}$ → Notation we use for dot product

# Connection b/w Coordinate Geometry and linear Algebra.

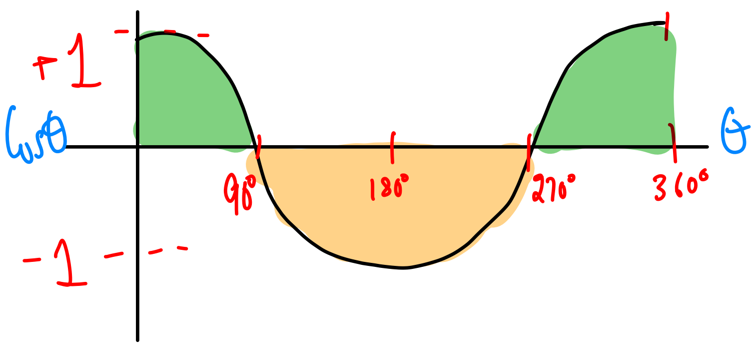



$$\cos\theta = \frac{\bar{x}^T \bar{y}}{\|\bar{x}\| \, \|\bar{y}\|}$$

$\theta$ = Angle b/w Vectors $\bar{x}$ & $\bar{y}$

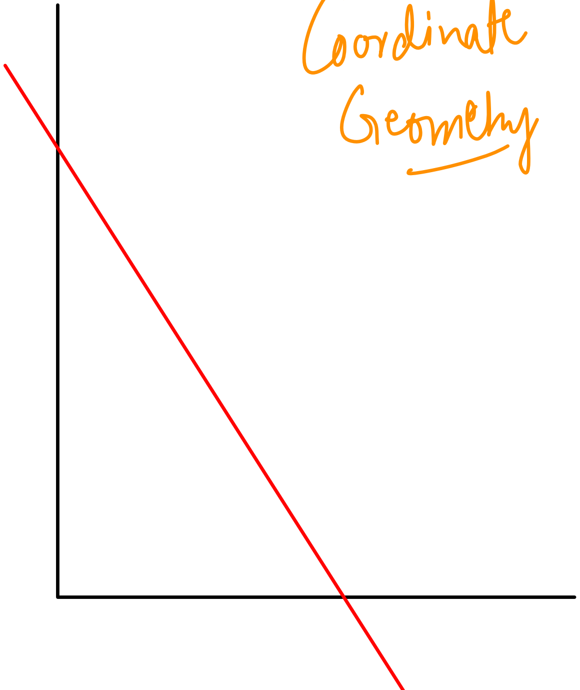
$\cos(\theta) > 0$ $\quad$ $0^\circ \le \theta < 90^\circ$

$270^\circ < \theta \le 360^\circ$




$\cos(\theta) < 0$ $\quad$ $90^\circ < \theta < 270^\circ$

## Coordinate Geometry



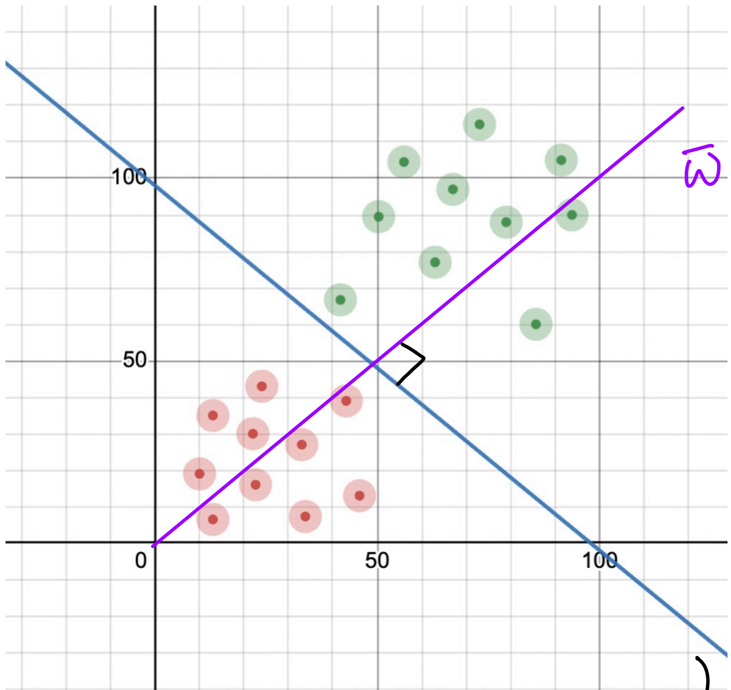
$$w_1 x_1 + w_2 x_2 + w_0 = 0$$

$$\bar{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_d \end{bmatrix} \qquad \bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_d \end{bmatrix}$$

weight vector

feature vector

$$\boxed{\bar{w}^T \bar{x} + \underbrace{w_0}_{\text{Bias}} = 0}$$




$x_1 + x_2 - 98 = 0$

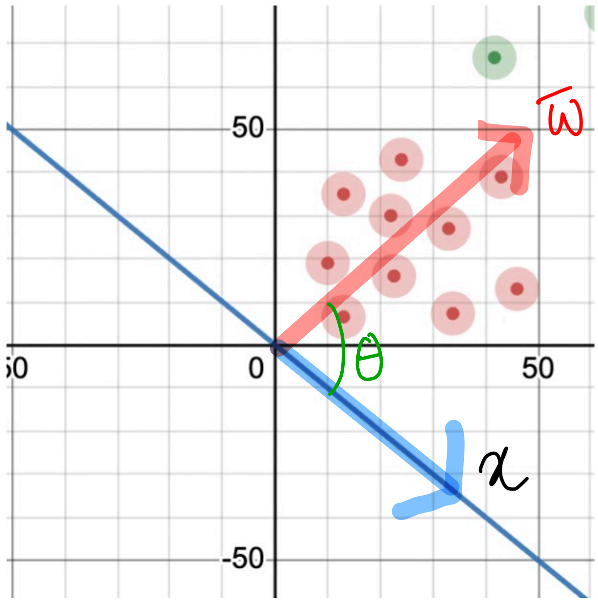
$x_1 + x_2 + w_0 = 0$

$\bar{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$w_0 = -98$

$\bar{w}^T \bar{x} + w_0 = 0$

Weight vector is $\perp$ to the hyperplane.




$$x_1 + x_2 = 0$$

$$\bar{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\boxed{\cos\theta = \frac{\bar{w}^T \bar{x}}{\|\bar{w}\| \|\bar{x}\|}}$$

$$\bar{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\boxed{\bar{w}^T \bar{x} = 0}$$

$$\cos\theta = 0$$

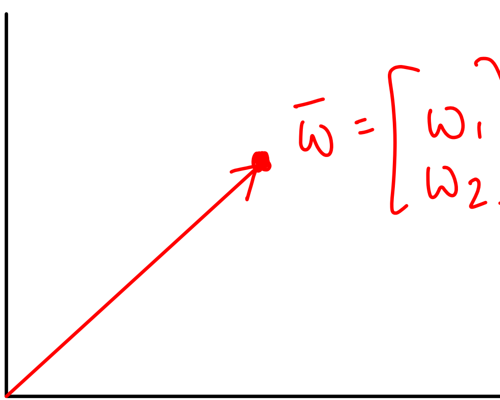
$$\theta = 90^\circ$$

# Unit Vector

"Unit Vectors represents direction in which a vector is pointing towards.

Vector with magnitude = 1 $\Rightarrow$ Unit Vector

$$\|\bar{w}\| = \sqrt{w_1^2 + w_2^2}$$



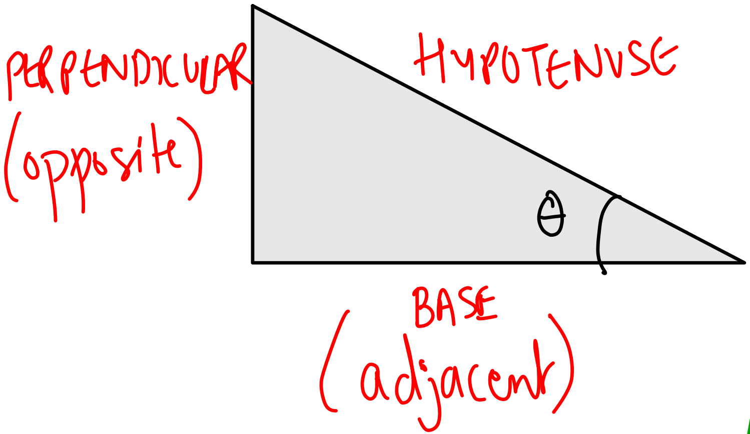
$$\bar{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$\boxed{\hat{w} = \frac{\bar{w}}{\|\bar{w}\|}} \Rightarrow \hat{w} = \begin{bmatrix} \dfrac{w_1}{\sqrt{w_1^2 + w_2^2}} \\ \dfrac{w_2}{\sqrt{w_1^2 + w_2^2}} \end{bmatrix}$$

$$\|\hat{w}\| = \sqrt{\frac{w_1^2}{w_1^2 + w_2^2} + \frac{w_2^2}{w_1^2 + w_2^2}} = \sqrt{\frac{\cancel{w_1^2 + w_2^2}}{\cancel{w_1^2 + w_2^2}}} = 1$$

# BASIC TRIGONOMETRY

$\|\hat{w}\| = 1$

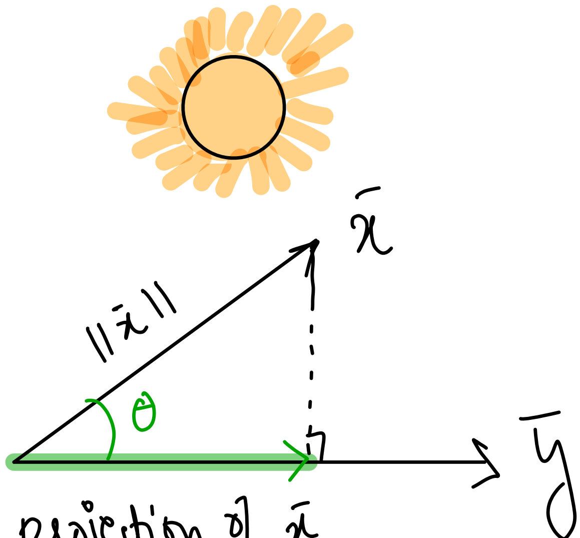



$\sin(\theta)$ = perpendicular / hypotenuse

$\cos(\theta)$ = base / hypotenuse

$\tan(\theta)$ = perpendicular / base

# Projection of a Vector




$\bar{p}$ projection of $\bar{x}$ on $\bar{y}$ (shadow)

$$\bar{x}\cdot\bar{y} = \bar{x}^T\bar{y}$$

$$\cos\theta = \frac{\bar{x}^T\bar{y}}{\|\bar{x}\|\,\|\bar{y}\|}$$ b/w $\bar{x}, \bar{y}$

$$\underbrace{\cos\theta_1}_{1} = \frac{\bar{p}^T\bar{y}}{\|\bar{p}\|\,\|\bar{y}\|} \Rightarrow \frac{\bar{p}^T\bar{y}}{\|\bar{y}\|} = \|\bar{p}\|$$

$$\cos\theta = \frac{\|\bar{p}\|}{\|\bar{x}\|}$$

$$\frac{\bar{x}^T\bar{y}}{\cancel{\|\bar{x}\|}\,\|\bar{y}\|} = \frac{\|\bar{p}\|}{\cancel{\|\bar{x}\|}}$$

$$\|\bar{p}\| = \frac{\bar{x}^T \bar{y}}{\|\bar{y}\|} = \bar{x}^T \frac{\bar{y}}{\|\bar{y}\|}$$

$$\boxed{\|\bar{p}\| = \bar{x}^T \hat{y}}$$