

* Fractional Knapsack problem using greedy approach:

— Given weights and values of n items, we need to put these items in a knapsack of capacity W to get the maximum total value in the knapsack.

— In fractional knapsack, we can break items for maximizing the total values of knapsack.

— This problem in which we can break an item is also called the fractional knapsack.

— The basic idea of the greedy approach is to calculate the ratio value/weight for each item and sort the item on the basis of the ratio.

— Then take the item with the highest ratio and add them until we get can't add the next item as a whole and at the end add the next item as much as we can.

* Algorithm:-

for $i = 1$ to n

calculate Profit/weight.

Sort objects in decreasing order of P/w Ratio

for $i = 1$ to n

if $M > 0$ and $w_i \leq M$

$M = M - w_i$

$P = P + P_i$

else

break;

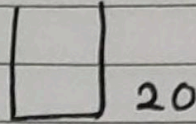
if $(M > 0)$

$P = P + P_i \left(\frac{M}{w_i} \right)$

* Example :-

objects	ob1	ob2	ob3
Profit	25	24	15
weight	18	15	10

Knapsack Capacity (M) = 20

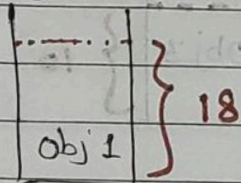


Soln \Rightarrow

1) Greedy about profit

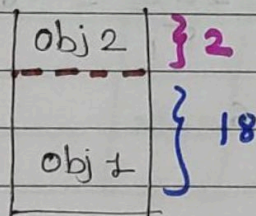
- In this we will put the object in the knapsack whose profit is maximum.

- In our e.g. obj1 has highest profit.



- But still there is 2 unit space is remaining. so we again choose an object whose profit is maximum.

- In our case now we have object2 with highest profit



There calculating profit

$$= 24 + \frac{2}{15} \times 24$$

$$= 25 + \frac{48}{15} \quad 3.2$$

$$= 25 + 3.2$$

$$\text{Overall profit} = \underline{28.2}$$

2) Greedy about weight :-

- Now we are placing objects in the knapsack whose weights are minimum so that we can place more objects into the knapsack.

- In our case obj3 has minimum weight.

	obj 1	{ 10 remaining }
10	obj 3	

- Now still 10 unit space is remaining now we again add another object whose weight is minimum

obj 2	}	10
obj 3		

obj 3	}	10
8		

Therefore calculating profit

11

$$= 15 \times 10 \times 24$$

$$= 15 + \underline{240} = 16$$

$$= 15 + 16$$

$$P = 31$$

- These two are not optimal solution because we are only considering weight and profits individually.

- The optimal solution would be that considers both weights as well as profit. Therefore we have to take ratio of P/W .

3) Greedy about the ratio of P/W:-

- for this approach we have to divide profit by weight to get ratio.

- Therefore the ratios for our given three objects are as follows.

$$1) \text{ obj 1} = \frac{25}{18} = 1.3$$

$$2) \text{ obj 2} = \frac{24}{15} = 1.6$$

$$3) \text{ obj 3} = \frac{15}{10} = 1.5$$

- After calculating the ratio we have to arrange objects in descending order according to their P/W ratio.

objects	obj 2	obj 3	obj 1
profit	24	15	25
weight	15	10	18
P/W ratio	1.6	1.5	1.3

1) Adding obj 2 as it has highest P/W ratio i.e. 1.6

-----	} 5 remaining.
obj 2	} 15

2) As there is still 5 unit space is remaining now we can add another objects whose P/W ratio is higher.

obj 3	} 5	} 20

obj 2	} 15	

$$\begin{aligned}
 &= 15 + 24 + \frac{5 \times 15}{10} \\
 &= 24 + \frac{7.5}{10} \\
 &= 24 + 7.5 \\
 &\text{optimal soln} \Rightarrow = \underline{31.5}
 \end{aligned}$$