

# Module 2



KTU Students

1. CFG
2. Derivation of string
  - a. leftmost
  - b. Rightmost
3. Derivation tree and parse tree
4. Ambiguous grammar with eg.

## Elimination of left recursion

A grammar is left recursive if it has a non-terminal/variable  $A$  such that there is a derivation  $A \xRightarrow{*} A\alpha$  for some string  $\alpha$ .

Top down parsing methods can't handle left recursive handle so, transformation that eliminates left recursion is needed.

A left recursion of production  $A \rightarrow A\alpha / \beta$  can be replaced by the non-left recursive production.

$$\begin{aligned} A &\rightarrow \beta A' \\ A' &\rightarrow \alpha A' / \epsilon \end{aligned}$$

eg. Consider the following grammar for arithmetic expression.

$$E \rightarrow E + T / T$$

$$T \rightarrow T * F / F$$

$$F \rightarrow (E) / d$$

left recursion is of the form  $A \rightarrow A\alpha / B$ .

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A'$$

$$\rightarrow \begin{matrix} E \rightarrow E + T / T \\ A \quad A \quad \alpha \quad \beta \end{matrix}$$

$$E \rightarrow TE'$$

$$E' \rightarrow TE' / \epsilon$$

$$\rightarrow \begin{matrix} T \rightarrow T * F / F \\ A \quad A \quad \alpha \quad \beta \end{matrix}$$

$$T \rightarrow FT'$$

$$T' \rightarrow +FT' / \epsilon$$

$$\rightarrow F \rightarrow (E) / d$$

Non-left recursion.

$$S \rightarrow Aa/b \quad \text{--- (1)}$$

$$A \rightarrow Ac / sd / \epsilon \quad \text{--- (2)}$$

There is an indirect left recursion (1) can be written

$$\text{as, } S \rightarrow Sca / b.$$

Eliminate  $A \rightarrow sd$  with production of  $S$

$$S \rightarrow Aa / b \quad \text{--- (1)}$$

$$A \rightarrow Ac / Aad / bd / \epsilon \quad \text{--- (2)}$$

Now, only direct recursion

$$(2) \Rightarrow \begin{matrix} A \rightarrow AC / Aad / bd / \epsilon \\ \underline{A} \quad \underline{A} \quad \underline{\alpha_1} \quad \underline{A} \quad \underline{\alpha_2} \quad \underline{\beta_1} \quad \underline{\beta_2} \end{matrix}$$

$$A \rightarrow bd A' / A'$$

$$A' \rightarrow CA' / ad A' / \epsilon$$

### ● Syntax Analysis

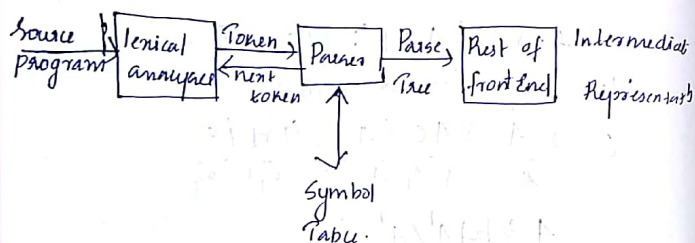
Syntax analysis / parsing is the second phase of a compiler. Syntax analyzer receives tokens from lexical analyzer and check the syntax of the given programming stmts with the help of CFG.

The syntax analyzer generates parse tree as its output. Every program language has rules that prescribe syntax structure of pgm link.

The structure of pgm language can be described by CFG / BNF (Backus - Normal Form)

## Parsing:

The system analyses follow production rule defined by CFG. The role of parser/system analyser is shown in below fig.



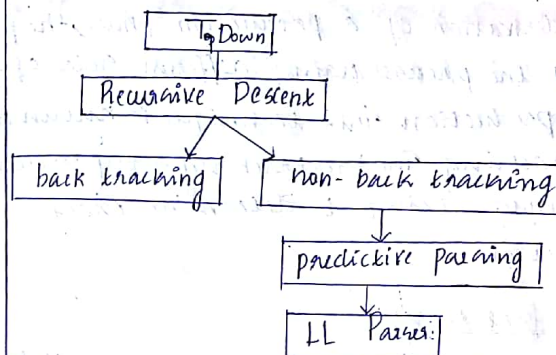
The way the production rules are applied to derive strings from grammar divides parsing into 2 types.

1) TOP-DOWN PARSING

2) BOTTOM-UP PARSING.

### TOP-DOWN PARSING

It attempts to build the parse tree from root to leaf. Top-down parser will start from the start symbol and then tries to transform the start symbol into the input stream. It follows left most derivation. In left most derivation the left most terminal is always chosen. The types of Top-down parsing are described below:



### Recursive Descent parsing:

Recursive descent is a TD parsing technique that constructs from the top and the i/p ahead from left to right. It uses procedures for every terminal and non-terminal entity. This parsing technique recursively pass into make a parse tree, which may or maynot require back tracking.

A form of recursive descent parsing that doesn't require any backtracking is known as predictive parsing.

TD parser starts from the root node and match the i/p string against the production rule.

If one derivation of a production phrase, the parser restart the process using different rules of same production. This technique is known as backtracking. The backtracking may process the input string to determine exact production.

### Left factoring:

To eliminate backtracking, left factoring is used. Left factoring is a process of factoring common prefixes of the grammar rule.

→ A grammar with common prefixes

$A \rightarrow \alpha \beta_1 / \alpha \beta_2$  [Here  $\alpha$  is the common prefix for  $\beta_1$  and  $\beta_2$ ]

$A$  can be factored as

$A \rightarrow \alpha A'$   
 $A' \rightarrow \beta_1 / \beta_2$

eg:1 Eliminate left factor in the given below grammar.

$S \rightarrow icks / ickses / a$

$C \rightarrow d$

Solu<sup>n</sup>:

Common prefix present is icks

after left factoring grammar gives

$S \rightarrow icks s' / a$

$s' \rightarrow \epsilon / es$

$C \rightarrow d$

eg:2  $E \rightarrow E \text{ Sub } E \text{ Sub } E$

$E \rightarrow E \text{ Sub } E$

$E \rightarrow \{E\}$

$E \rightarrow d$

Common prefix present in  $E$  is  $E \text{ Sub } E$

$E \rightarrow E \text{ Sub } E'$

$E' \rightarrow \text{Sub } E / \epsilon$

$E \rightarrow \{E\}$

$E \rightarrow d$

### First and follow function

#### First

First of a grammar symbol  $X$  is the first of

first terminal present in the strings derived from  $X$ . The rule to calculate the  $First$  are given below.

- 1) If  $X$  is a terminal  $a$  then  $First(X) = a$
- 2) If there is a production  $X \rightarrow E$  then  $First(X) = First(E)$



3) if there is a production  $X \rightarrow Y_1 Y_2 Y_3 \dots Y_k$  then  
 first of  $X = F(Y_1), F(Y_2), F(Y_3) \dots F(Y_k)$

eg: Find the first of symbols present in grammar.

$S \rightarrow A B$

$A \rightarrow a / \epsilon$

$B \rightarrow b$

Soln

$F(S) = \{A\} \{a, b\}$

$F(A) = \{a, \epsilon\}$

$F(B) = \{b\}$

Follow(S)

A follow of a non-terminal 'A' is a set of terminals that follow or occur to the right of A.  
 Follow(A) is calculated using following set of rules

1) if A is a start symbol then follow(A) as a dollar symbol ( $\$$ )

2) if there is a grammar rule A derives  $\alpha B \beta$ .

$X: A \rightarrow \alpha B \beta$  and  $\beta \neq \epsilon$  then follow of B is first of  $\beta$

$A \rightarrow \alpha B$

3) if there is a grammar rule A gives  $\alpha B$  or

$A \rightarrow \alpha B \beta$  and  $\beta = \epsilon$

then  $\text{Follow}(B) = \text{Follow}(A)$

eg: Find follow of grammar

$S \rightarrow A B B$

$A \rightarrow a / c$

$B \rightarrow b$

$\text{Follow}(S) = \{\$ \}$

$\text{Follow}(A) = \text{first}(B)$

$= \{b\}$

$\text{Follow}(B) = \text{Follow}(S)$

$= \{\$ \}$

eg: 2  $S \rightarrow A B C$

$F(S) \rightarrow \{a, b\}$

$F(A) \rightarrow \{a, \epsilon\}$

$F(B) \rightarrow \{b\}$

$F(C) \rightarrow \{c\}$

$\text{Follow}(S) = \{\$ \}$

$\text{Follow}(A) = \text{first}(B C)$

$= \{b, c\}$

$\text{Follow}(B) = F(C) = \{c\}$

$\text{Follow}(C) = \text{Follow}(S)$   
 $= \{\$ \}$

	FIRST	FOLLOW
$S \rightarrow ABC$	$\{a, b\}$	$\{\$, \}$
$A \rightarrow a/\epsilon$	$\{a, \epsilon\}$	$\{b\}$
$B \rightarrow b$	$\{b\}$	$\{c\}$
$C \rightarrow c$	$\{c\}$	$\{\$\}$

Find first and follow of below grammar.

$E \rightarrow TE'$   
 $E' \rightarrow +TE' / \epsilon$   
 $T \rightarrow FT'$   
 $T' \rightarrow *FT' / \epsilon$   
 $F \rightarrow id / (CE)$

$F(E) = \{E, F, id, \epsilon\}$

Variables are,  $E, E', T, T', F$

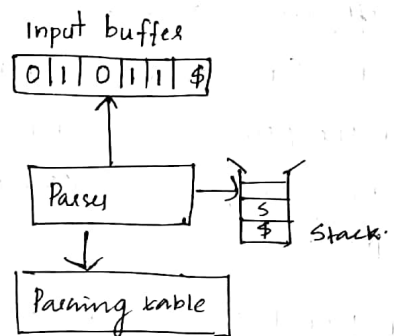
Terminals are  $\{+, *, (, ), \epsilon\}$

	First	Follow
$E \rightarrow TE'$	$\{id, \epsilon\}$	$\{\$, \}$
$E' \rightarrow +TE' / \epsilon$	$\{+, \epsilon\}$	$\{\$, \}$
$T \rightarrow FT'$	$\{id, \epsilon\}$	$\{+, \$, \}$
$T' \rightarrow *FT' / \epsilon$	$\{*, \epsilon\}$	$\{+, \$, \}$
$F \rightarrow id / (CE)$	$\{id, \epsilon\}$	$\{*, +, \$, \}$

$Follow(E') = \{\$, \}$

$Follow(T) = first(E') = \{+\}$

Non-Recursive Predictive Parser.



Steps to construct predictive parser table for a grammar  $G$  are listed below

- Step 1: Eliminate left recursion in grammar  $G$ .
- 2: Perform left factor in grammar  $G$ .
- 3: Find first & follow on the symbols in grammar  $G$ .
- 4: Construct the predictive class table.
- 5: Check if the given input string can be accepted by the parser.

Q Construct predictive parse table for the below grammar.

$S \rightarrow aABb$   
 $A \rightarrow c/\epsilon$   
 $B \rightarrow d/\epsilon$

Sol<sup>n</sup>: left recursion ~~at~~  $A \rightarrow A\alpha/\beta$ .

- 1: No left recursion. i.e. Grammar is left ~~for~~ recursion free
- 2: No common prefix, no need of left factor.
- 3:

$S \rightarrow aABb$	first $\{a\}$	follow $\{\$, \}$
$A \rightarrow c/\epsilon$	$\{c, \epsilon\}$	$\{d, b\}$
$B \rightarrow d/\epsilon$	$\{d, \epsilon\}$	$\{b\}$

4: Construct predictive class table.

	a	b	c	d	$\epsilon$	$\$$
S	$S \rightarrow aABb$					
A		$A \rightarrow \epsilon$	$A \rightarrow c$	$A \rightarrow \epsilon$		
B		$B \rightarrow \epsilon$		$B \rightarrow d$		

If first includes  $\epsilon$  then ~~for~~ consider follow.

Ex Consider an input string 'acbd' various steps in parsing is shown in below table

Input	Stack contents	Moves
acdb\$	$S$	derived using $S \rightarrow aABb$
acdh\$	$aABb$	Pop a
cdh\$	$ABb$	$A \rightarrow c$

cdh\$	c B b \$	Pop c
db\$	Bdb\$	B → d
db\$	db\$	Pop d
b\$	b\$	Pop b
\$	\$	String

Q Construct predictive parsing table (LLC1) for below grammar  $S \rightarrow (S) / \epsilon$

Sol<sup>n</sup>:  $S \rightarrow (S) / \epsilon$

Step 1: No left recursion

2. No common prefix so already left factored.

3. First & follow

$$f(S) \rightarrow \text{first}((S)) \cup \text{first}(\epsilon) \\ \{c\} \cup \{\epsilon\} = \{c, \epsilon\}$$

$$\text{Follow}(S) = \{ \$ \} \cup \text{first}())$$

$$(\$) \cup \{ ) \} = \{ \$, ) \}$$

step 4 Construct parsing table.

	(	)	\$
S	$S \rightarrow (S)$	$S \rightarrow \epsilon$	$S \rightarrow \epsilon$

Q Construct predictive table for below grammar:

Sol<sup>n</sup>:  $E \rightarrow TE'$

$$E' \rightarrow +TE' / \epsilon$$

$$T \rightarrow *FT' / \epsilon$$

$$F \rightarrow id / (E)$$

$$\text{Sol<sup>n</sup>. Follow}(E) = \text{first}(T') \cup \text{first}(T')$$

$$= \{*, \epsilon\}$$

$$\text{Follow}(E) = \text{first}()) \cup \$$$

$$= \{ \$, ) \}$$

$$\text{Follow}(E') =$$

$$\text{Follow}(T) = \text{first}(E') \cup \text{first}(E')$$

$$= \{ +, \epsilon \} \cup ($$

$$\text{Follow}(E') = \{ \$, ) \}$$



	First	Follow
E	{id, c}	{ $\phi$ , )}
E'	{+, $\epsilon$ }	{ $\phi$ , )}
T	{id, c}	{+, {, )}
T'	{*, $\epsilon$ }	{+, {, )}
F	{id, c}	{+, +, {, )}

	id	+	(	)	*	{
E	$E \rightarrow TE'$		$E \rightarrow TE'$			
E'		$E \rightarrow TE'$		$E' \rightarrow \epsilon$		$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$		$T \rightarrow FT'$			
T'		$T' \rightarrow \epsilon$		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$		$F \rightarrow ( \epsilon$			

Q  $S \rightarrow AB/\epsilon$

15/2/18  $B \rightarrow bC/\epsilon$

$C \rightarrow cS/\epsilon$

$\text{First}(S) = a$

Step1: Eliminate left recursion here this grammar doesn't contain left recursion

Step2: perform left factoring on grammar G. Here already left factored grammar

Step3:  $\text{First}(S) = \{a, \epsilon\}$

$\text{First}(B) = \{b, \epsilon\}$

$\text{First}(C) = \{c, \epsilon\}$

$\text{Follow}(S) = \phi \cup \text{follow}(C) = \{\phi\}$

$\text{follow}(B) = \text{follow}(S) = \{\phi\}$

$\text{follow}(C) = \text{follow}(B) = \{\phi\}$

	a	b	c	$\phi$
S	$S \rightarrow aB$			$S \rightarrow \epsilon$
B		$B \rightarrow bc$		$B \rightarrow \epsilon$
C			$C \rightarrow cS$	$C \rightarrow \epsilon$

	First	Follow
S	$\{a, \epsilon\}$	$\{\$ \}$
B	$\{b, \epsilon\}$	$\{\$ \}$
C	$\{c, \epsilon\}$	$\{\$ \}$

To write parsing table

Column  $\rightarrow$  Variables

rows  $\rightarrow$  terminals

To fill column S we look at the first (S)

$$\text{first}(S) = \{a, \epsilon\}$$

$$\{a, \epsilon\} = \{a, \epsilon\}$$

$$\{a, \epsilon\} = \{a, \epsilon\}$$

$$\{a, \epsilon\} = \{a, \epsilon\}$$

$$\{a\} = \{a, \epsilon\} \cup \{a\} = \{a, \epsilon\}$$

$$\{a\} = \{a, \epsilon\} \cup \{a\} = \{a, \epsilon\}$$

$$\{a\} = \{a, \epsilon\} \cup \{a\} = \{a, \epsilon\}$$

	\$	a	b	c
S				
B				
C				

LL(1)

↓ work ahead

- Positive points: are free from backtracking  
Top down parser  
Recursive.

### → Recursive Descent Parsing

Top down parsing technique

Parser construction for Recursive Descent Parsing

$E \rightarrow iE'$

$E' \rightarrow +iE' / \epsilon$

Soln. Variables -  $\{E, E'\}$

Terminals -  $\{+, i\}$

To create RDP for a particular grammar.

- Create a function for each and every variable.

For Variable E

$E()$

{ if ( $l == i$ ) //  $l$  is global look ahead

{ match( $i$ )

} }  $E'()$

For Variable  $E'$

$E'()$

{ if ( $l == +$ )

{ match( $+$ )  
match( $i$ )

}  $E'()$ ;

else  
return 0;

}

DEFINITION OF MATCH

match( $ch$ )

{

if ( $l == t$ )

$l = \text{getchar}()$ ;

else

} print

MAIN

Develop RDP code for below grammar.

main()

{  $E()$

if ( $l == \epsilon$ )

printf("Successful Parsing");

else  
printf("Error");

}

Q Develop Recursive Descent parsing for below grammar

$S \rightarrow aAb$

$S \rightarrow cd/c$

sol<sup>n</sup>: Function for S

```
S()
{
  if (l == a)
  {
    match(a);
    A();
    match(b);
  }
}
```

Function for A

```
A()
{
  if (l == c)
  {
    match(c);
    match(d);
  }
  else if (l == c)
  {
    match(c);
  }
}
```