UNIT - 1

NUMBER SYSTEMS & BOOLEAN ALGEBRA

- Introduction about digital system
- Philosophy of number systems
- Complement representation of negative numbers
- Binary arithmetic
- Binary codes
- Error detecting & error correcting codes
- Hamming codes

INTRODUCTION ABOUT DIGITAL SYSTEM

A Digital system is an interconnection of digital modules and it is a system that manipulates discrete elements of information that is represented internally in the binary form.

Now a day's digital systems are used in wide variety of industrial and consumer products such as automated industrial machinery, pocket calculators, microprocessors, digital computers, digital watches, TV games and signal processing and so on.

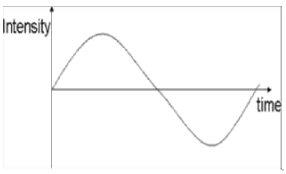
Characteristics of Digital systems

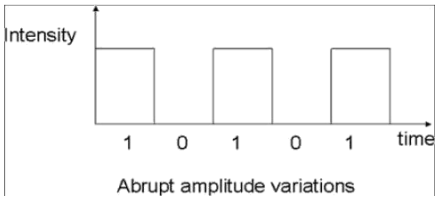
- Digital systems manipulate discrete elements of information.
- Discrete elements are nothing but the digits such as 10 decimal digits or 26 letters of alphabets and so on.
- Digital systems use physical quantities called signals to represent discrete elements.
- In digital systems, the signals have two discrete values and are therefore said to be binary.
- A signal in digital system represents one binary digit called a bit. The bit has a value either 0 or 1.

Analog systems vs Digital systems

Analog system process information that varies continuously i.e; they process time varying signals that can take on any values across a continuous range of voltage, current or any physical parameter.

Digital systems use digital circuits that can process digital signals which can take either 0 or 1 for binary system.





1. Ease of programmability

The digital systems can be used for different applications by simply changing the program without additional changes in hardware.

2. Reduction in cost of hardware

The cost of hardware gets reduced by use of digital components and this has been possible due to advances in IC technology. With ICs the number of components that can be placed in a given area of Silicon are increased which helps in cost reduction.

3.High speed

Digital processing of data ensures high speed of operation which is possible due to advances in Digital Signal Processing.

4. High Reliability

Digital systems are highly reliable one of the reasons for that is use of error correction codes.

5. Design is easy

The design of digital systems which require use of Boolean algebra and other digital techniques is easier compared to analog designing.

6. Result can be reproduced easily

Since the output of digital systems unlike analog systems is independent of temperature, noise, humidity and other characteristics of components the reproducibility of results is higher in digital systems than in analog systems.

Disadvantages of Digital Systems

- Use more energy than analog circuits to accomplish the same tasks, thus producing more heat as well.
- Digital circuits are often fragile, in that if a single piece of digital data is lost or misinterpreted the meaning of large blocks of related data can completely change.
- Digital computer manipulates discrete elements of information by means of a binary code.
- Quantization error during analog signal sampling.

Number system is a basis for counting varies items. Modern computers communicate and operate with binary numbers which use only the digits 0 &1. Basic number system used by humans is Decimal number system.

For Ex: Let us consider decimal number 18. This number is represented in binary as 10010.

We observe that binary number system take more digits to represent the decimal number. For large numbers we have to deal with very large binary strings. So this fact gave rise to three new number systems.

i) Octal number systems
ii) Hexa Decimal number system
iii) Binary Coded Decimal number(BCD) system

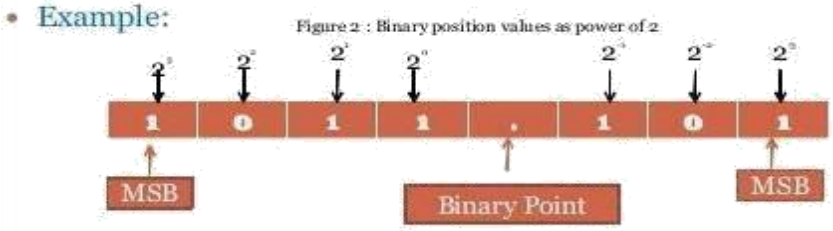
To define any number system we have to specify

- Base of the number system such as 2,8,10 or 16.
- The base decides the total number of digits available in that number system.
- First digit in the number system is always zero and last digit in the number system is always base-1.

Binary number system:

The binary number has a radix of 2. As r = 2, only two digits are needed, and these are 0 and 1. In binary system weight is expressed as power of 2.

- Example:



The left most bit, which has the greatest weight is called the Most Significant Bit (MSB). And the right most bit which has the least weight is called Least Significant Bit (LSB).

For Ex: $\mathbf{1001.01_2 = [(1) \times 2^3] + [(0) \times 2^2] + [(0) \times 2^1] + [(1) \times 2^0] + [(0) \times 2^{-1}] + [(1) \times 2^2]}$

$\mathbf{1001.01_2 = [1 \times 8] + [0 \times 4] + [0 \times 2] + [1 \times 1] + [0 \times 0.5] + [1 \times 0.25]}$

$\mathbf{1001.01_2 = 9.25_{10}}$

Decimal Number system

The decimal system has ten symbols: 0,1,2,3,4,5,6,7,8,9. In other words, it has a base of 10.

Octal Number System

Digital systems operate only on binary numbers. Since binary numbers are often very long, two shorthand notations, octal and hexadecimal, are used for representing large binary numbers. Octal systems use a base or radix of 8. It uses first eight digits of decimal number system. Thus it has digits from 0 to 7.

Hexa Decimal Number System

The hexadecimal numbering system has a base of 16. There are 16 symbols. The decimal digits 0 to 9 are used as the first ten digits as in the decimal system, followed by the letters A, B, C, D, E and F, which represent the values 10, 11,12,13,14 and 15 respectively.

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Number Base conversions

The human beings use decimal number system while computer uses binary number system. Therefore it is necessary to convert decimal number system into its equivalent binary.

i) Binary to octal number conversion
ii) Binary to hexa decimal number conversion

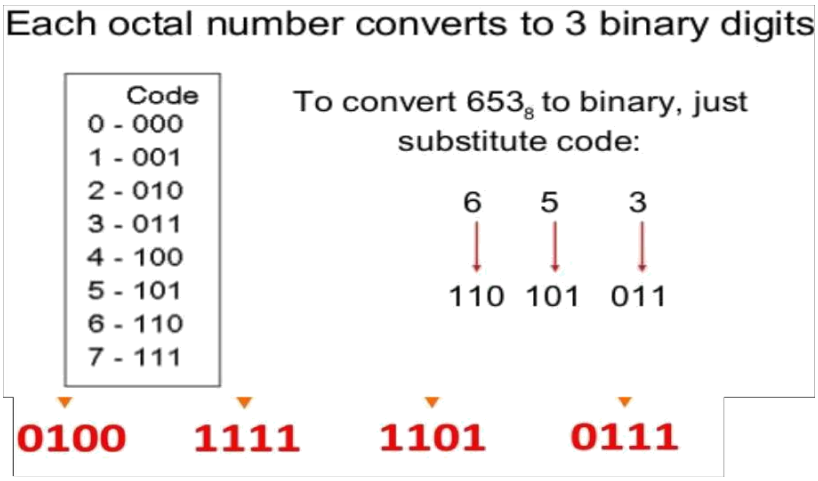
The binary number: 001 010 011 000 100 101 110 111

The octal number: 1 2 3 0 4 5 6 7

The binary number: 0001 0010 0100 1000 1001 1010 1101 1111

The hexadecimal number: 1 2 5 8 9 A D F

iii) Octal to binary Conversion



iv) Hexa to binary conversion

v) Octal to Decimal conversion

Ex: convert 4057.06_8 to octal

$=4x8^3+0x8^2+5x8^1+7x8^0+0x8^{-1}+6x8^{-2}$ $=2048+0+40+7+0+0.0937$

$=2095.0937_{10}$

vi) Decimal to Octal Conversion

Ex: convert 378.93_{10} to octal

378_{10} to octal: Successive division:

```
 8 | 378
   |___
8 |47 ---   2
   |_______
8 |5  ---   7      ↑
   |_____
   0  ---   5
```

$=572_8$

0.93_{10} to octal :

0.93x8=7.44
0.44x8=3.52 ↓
0.53x8=4.16
0.16x8=1.28

$=0.7341_8$

$378.93_{10}=572.7341_8$

vii) Hexadecimal to Decimal Conversion

Ex: $5C7_{16}$ to decimal

$=(5x16^2)+(C\ x16^1)+ (7\ x16^0)$

$=1280+192+7$

$=147_{10}$

viii) Decimal to Hexadecimal

Conversion Ex: 2598.67510

```
1 6| 2598
16 162      -6
   10       -2
```

= A26 (16)

0.67510= 0.675x16 -- 10.8

= 0.800x16 -- 12.8 ↓

=0.800x16 -- 12.8

=0.800x16 -- 12.8

=0.ACCC$_{16}$

2598.675$_{10}$ = A26.ACCC$_{16}$

ix) Octal to hexadecimal conversion:

The simplest way is to first convert the given octal no. to binary & then the binary no. to hexadecimal.

Ex: 756.603$_8$

7	5	6	.	6	0	3
111	101	110	.	110	000	011
0001	1110	1110	.	1100	0001	1000
1	E	E	.	C	1	8

x) Hexadecimal to octal conversion:

First convert the given hexadecimal no. to binary & then the binary no. to octal.

Ex: B9F.AE16

B	9	F	.	A	E		
1011	1001	1111	.	1010	1110		
101	110	011	111	.	101	011	100
5	6	3	7	.	5	3	4

=5637.534

Complements:

In digital computers to simplify the subtraction operation & for logical manipulation complements are used. There are two types of complements used in each radix system.

i) The radix complement or r's complement
ii) The diminished radix complement or (r-1)'s complement

Representation of signed no.s binary arithmetic in computers:

- Two ways of rep signed no.s
    1. Sign Magnitude form
    2. Complemented form
- Two complimented forms
    1. 1‘s compliment form
    2. 2‘s compliment form

Advantage of performing subtraction by the compliment method is reduction in the hardware.(instead of addition & subtraction only adding ckt‘s are needed.)
i. e, subtraction is also performed by adders only.

Instead of subtracting one no. from other the compliment of the subtrahend is added to minuend. In sign magnitude form, an additional bit called the sign bit is placed in front of the no. If the sign bit is 0, the no. is +ve, If it is a 1, the no is _ve.

Ex:

0	1	0	1	0	0	1

↓

Sign bit =+41 magnitude

↑

1	1	0	1	0	0	1

= -41

Note: manipulation is necessary to add a +ve no to a –ve no

Representation of signed no.s using 2’s or 1’s complement method:

If the no. is +ve, the magnitude is rep in its true binary form & a sign bit 0 is placed in front of the MSB.I f the no is _ve , the magnitude is rep in its 2‘s or 1‘s compliment form &a sign bit 1 is placed in front of the MSB.

Ex:

Given no.	Sign mag form	2‘s comp form	1‘s comp form
01101	+13	+13	+13
010111	+23	+23	+23
10111	-7	-7	-8
1101010	-42	-22	-21

Special case in 2's comp representation:

Whenever a signed no. has a 1 in the sign bit & all 0's for the magnitude bits, the decimal equivalent is -2^n , where n is the no of bits in the magnitude .
Ex: 1000= -8 & 10000=-16

Characteristics of 2's compliment no.s:

Properties:

1. There is one unique zero
2. 2's comp of 0 is 0
3. The leftmost bit can't be used to express a quantity . it is a 0 no. is +ve.
4. For an n-bit word which includes the sign bit there are $(2^{n-1}-1)$ +ve integers, 2^{n-1} –ve integers & one 0 , for a total of 2^n uniquestates.
5. Significant information is containd in the 1's of the +ve no.s & 0's of the _ve no.s
6. A _ve no. may be converted into a +ve no. by finding its 2's comp.

Signed binary numbers:

Decimal	Sign 2's comp form	Sign 1's comp form	Sign mag form
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0011	0011	0011
+0	0000	0000	0000

-0	--	1111	1000
-1	1111	1110	1001
-2	1110	1101	1010
-3	1101	1100	1011
-4	1100	1011	1100
-5	1011	1010	1101
-6	1010	1001	1110
-7	1001	1000	1111
8	1000	--	--

Methods of obtaining 2's comp of a no:

- In 3 ways
  1. By obtaining the 1's comp of the given no. (by changing all 0's to 1's & 1's to 0's) & then adding 1.
  2. By subtracting the given n bit no N from 2^n
  3. Starting at the LSB , copying down each bit upto & including the first 1 bit encountered , and complimenting the remaining bits.

  Ex: Express -45 in 8 bit 2's comp form

+45 in 8 bit form is 00101101

I method:

1's comp of 00101101 & the add 1

00101101
11010010
+1

___ – – – – – – – – – –

11010011 is 2's comp form

II method:

Subtract the given no. N from 2^n

2^n = 100000000
Subtract 45= -00101101
+1

___ – – –

11010011 is 2's comp

III method:

Original no: 00101101
Copy up to First 1 bit 1
Compliment remaining : 1101001

bits 11010011

Ex:

-73.75 in 12 bit 2‘compform

I method

01001001.1100

10110110.0011

+1

_____

10110110.0100 is 2‘s

II method:

2^8 = 100000000.0000

Sub 73.75=-01001001.1100

_____

10110110.0100 is 2‘s comp

III method :

Orginalno : 01001001.1100

Copy up to 1‘st bit 100

Comp the remaining bits: 10110110.0

_____

10110110.0100

2’s compliment Arithmetic:

- The 2‘s comp system is used to rep –ve no.s using modulus arithmetic . The word length of a computer is fixed. i.e, if a 4 bit no. is added to another 4 bit no . the result will be only of 4 bits. Carry if any , from the fourth bit will overflow called the Modulus arithmetic.
  Ex:1100+1111=1011
- In the 2‘s compl subtraction, add the 2‘s comp of the subtrahend to the minuend . If there is a carry out , ignore it , look at the sign bit I,e, MSB of the sum term .If the MSB is a 0, the result is positive.& it is in true binary form. If the MSB is a ` (carry in or no carry at all) the result is negative.& is in its 2‘s comp form. Take its 2‘s comp to find its magnitude in binary.

Ex: Subtract 14 from 46 using 8 bit 2‘s comp arithmetic:

+14 = 00001110

-14 = 11110010 2‘s comp

+46 = 00101110

-14 =+11110010 2‘s comp form of -14

-32 (1)00100000 ignore carry

Ignore carry , The MSB is 0 . so the result is +ve. & is in normal binary form. So the result is +00100000=+32.

EX: Add -75 to +26 using 8 bit 2‘s comp arithmetic

+75	= 01001011	
-75	=10110101	2‘s comp
+26	= 00011010	
-75	=+10110101	2‘s comp form of -75
-49	11001111	No carry

No carry , MSB is a 1, result is _ve & is in 2‘s comp. The magnitude is 2‘s comp of 11001111. i.e, 00110001 = 49. so result is -49

Ex: add -45.75 to +87.5 using 12 bit arithmetic

+87.5 = 01010111.1000
-45.75=+11010010.0100

-41.75 (1)00101001.1100 ignore carry
MSB is 0, result is +ve. =+41.75

1’s compliment of n number:

- It is obtained by simply complimenting each bit of the no,.& also , 1‘s comp of a no, is subtracting each bit of the no. form 1.This complemented value rep the –ve of the original no. One of the difficulties of using 1‘s comp is its rep o f zero. Both 00000000 & its 1‘s comp 11111111 rep zero.
- The 00000000 called +ve zero& 11111111 called –ve zero.

+99	=	01100011
-99	=	10011100

+77.25 = 01001101.0100
-77.25 = 10110010.1011

1’s compliment arithmetic:

In 1‘s comp subtraction, add the 1‘s comp of the subtrahend to the minuend. If there is a carryout , bring the carry around & add it to the LSB called the **end around carry.** Look at the sign bit (MSB) . If this is a 0, the result is +ve & is in true binary. If the MSB is a 1 (carry or no carry), the result is –ve & is in its is comp form .Take its 1‘s comp to get the magnitude inn binary.

Ex: Subtract 14 from 25 using 8 bit 1‘s EX: ADD -25 to +14

25 = 00011001 +14 = 00001110

-45 = 11110001 -25 =+11100110

+11 (1)00001010 -11 11110100

+1

No carry MSB =1

00001011 result=-ve=-11_{10}

MSB is a 0 so result is +ve (binary)

=+11_{10}

Binary codes

Binary codes are codes which are represented in binary system with modification from the original ones.

- Weighted Binary codes
- Non Weighted Codes

Weighted binary codes are those which obey the positional weighting principles, each position of the number represents a specific weight. The binary counting sequence is an example.

Decimal	BCD 8421	Excess-3	84-2-1	2421	5211	Bi-Quinary 5043210			5	0	4	3	2	1	0
0	0000	0011	0000	0000	0000	0100001		0		X					X
1	0001	0100	0111	0001	0001	0100010		1		X				X	
2	0010	0101	0110	0010	0011	0100100		2		X			X		
3	0011	0110	0101	0011	0101	0101000		3		X		X			
4	0100	0111	0100	0100	0111	0110000		4		X	X				
5	0101	1000	1011	1011	1000	1000001		5	X						X
6	0110	1001	1010	1100	1010	1000010		6	X					X	
7	0111	1010	1001	1101	1100	1000100		7	X				X		
8	1000	1011	1000	1110	1110	1001000		8	X			X			
9	1001	1111	1111	1111	1111	1010000		9	X		X				

Reflective Code

A code is said to be reflective when code for 9 is complement for the code for 0, and

so is for 8 and 1 codes, 7 and 2, 6 and 3, 5 and 4. Codes 2421, 5211, and excess-3 are reflective, whereas the 8421 code is not.

Sequential Codes

A code is said to be sequential when two subsequent codes, seen as numbers in binary representation, differ by one. This greatly aids mathematical manipulation of data. The 8421 and Excess-3 codes are sequential, whereas the 2421 and 5211 codes are not.

Non weighted codes

Non weighted codes are codes that are not positionally weighted. That is, each position within the binary number is not assigned a fixed value. Ex: Excess-3 code

Excess-3 Code

Excess-3 is a non weighted code used to express decimal numbers. The code derives its name from the fact that each binary code is the corresponding 8421 code plus 0011(3).

Gray Code

The gray code belongs to a class of codes called minimum change codes, in which only one bit in the code changes when moving from one code to the next. The Gray code is non-weighted code, as the position of bit does not contain any weight. The gray code is a reflective digital code which has the special property that any two subsequent numbers codes differ by only one bit. This is also called a unit- distance code. In digital Gray code has got a special place.

Decimal Number	Binary Code	Gray Code	Decimal Number	Binary Code	Gray Code
0	0000	0000	8	1000	1100
1	0001	0001	9	1001	1101
2	0010	0011	10	1010	1111
3	0011	0010	11	1011	1110
4	0100	0110	12	1100	1010
5	0101	0111	13	1101	1011
6	0110	0101	14	1110	1001
7	0111	0100	15	1111	1000

Binary to Gray Conversion

- Gray Code MSB is binary code MSB.
- Gray Code MSB-1 is the XOR of binary code MSB and MSB-1.
- MSB-2 bit of gray code is XOR of MSB-1 and MSB-2 bit of binary code.

MSB-N bit of gray code is XOR of MSB-N-1 and MSB-N bit of binary code.

8421 BCD code (Natural BCD code):

Each decimal digit 0 through 9 is coded by a 4 bit binary no. called natural binary codes. Because of the 8,4,2,1 weights attached to it. It is a weighted code & also sequential . it is useful for mathematical operations. The advantage of this code is its case of conversion to & from decimal. It is less efficient than the pure binary, it require more bits.

Ex: 14→1110 in binary

But as 0001 0100 in 8421 ode.

The disadvantage of the BCD code is that , arithmetic operations are more complex than they are in pure binary . There are 6 illegal combinations 1010,1011,1100,1101,1110,1111 in these codes, they are not part of the 8421 BCD code system . The disadvantage of 8421 code is, the rules of binary addition 8421 no, but only to the individual 4 bit groups.

BCD Addition:

It is individually adding the corresponding digits of the decimal no,s expressed in 4 bit binary groups starting from the LSD . If there is no carry & the sum term is not an illegal code , no correction is needed .If there is a carry out of one group to the next group or if the sum term is an illegal code then 6_{10}(0100) is added to the sum term of that group & the resulting carry is added to the next group.

Ex: Perform decimal additions in 8421 code

(a)25+13

In BCD 25= 0010 0101

In BCD +13 =+0001 0011

38 0011 1000

No carry , no illegal code .This is the corrected sum

(b). 679.6 + 536.8

679.6	=	0110	0111	1001	.0110 in BCD
+536.8	=	+0101	0011	0010	.1000 in BCD
1216.4		1011	1010	0110	. 1110 illegal codes
		+0110	+ 0011	+0110	. + 0110 add 0110 to each

(1)0001 (1)0000 (1)0101 . (1)0100 propagate carry

/ / / /

+1 +1 +1 +1

0001 0010 0001 0110 . 0100

1 2 1 6 . 4

BCD Subtraction:

Performed by subtracting the digits of each 4 bit group of the subtrahend the digits from the corresponding 4- bit group of the minuend in binary starting from the LSD . if there is no borrow from the next group , then 6_{10}(0110)is subtracted from the difference term of this group.

(a)38-15

In BCD 38= 0011 1000
In BCD -15 = -0001 0101

23 0010 0011

No borrow, so correct difference.

.(b) 206.7-147.8

206.7 = 0010 0000 0110 . 0111 in BCD
-147.8 = -0001 0100 0111 . 0110 in BCD

58.9 0000 1011 1110 . 1111 borrows are present
-0110 -0110 . -0110 subtract 0110

0101 1000 . 1001

Form the 9‘s & 10‘s compliment of the decimal subtrahend & encode that no. in the 8421 code . the resulting BCD no.s are then added.

EX: 305.5 – 168.8

305.5 = 305.5
-168.8= +83.1 9‘s comp of -168.8

(1)136.6
+1 end around carry
136.7 corrected difference

305.5_{10} = 0011 0000 0101 . 0101
$+831.1_{10}$ = +1000 0011 0001 . 0001 9‘s comp of 1 6 8.8 in BCD
+1011 0011 0110 . 0110 1011 is illegal code
+0110 add 0110

(1)0001 0011 0110 . 0110
+1 End around carry

0001 0011 0110 .0111
= 136.7

Excess three(xs-3)code:

It is a non-weighted BCD code .Each binary codeword is the corresponding 8421 codeword plus 0011(3).It is a sequential code & therefore , can be used for arithmetic operations..It is a self-complementing code.s o the subtraction by the method of compliment addition is more direct in xs-3 code than that in 8421 code. The xs-3 code has six invalid states 0000,0010,1101,1110,1111.. It has interesting properties when used in addition & subtraction.

Excess-3 Addition:

Add the xs-3 no.s by adding the 4 bit groups in each column starting from the LSD. If there is no carry starting from the addition of any of the 4-bit groups , subtract 0011 from the sum term of those groups (because when 2 decimal digits are added in xs-3 & there is no carry , result in xs-6). If there is a carry out, add 0011 to the sum term of those groups(because when there is a carry, the invalid states are skipped and the result is normal binary).

EX:37	0110	1010	
+28	+0101	1011	
65	1011	(1)0101	carry generated
	+1 		propagate carry
	1100	0101	add 0011 to correct 0101 &
	-0011	+0011	subtract 0011 to correct 1100
	1001	1000	$=65_{10}$

Excess -3 (XS-3) Subtraction:

Subtract the xs-3 no.s by subtracting each 4 bit group of the subtrahend from the corresponding 4 bit group of the minuend starting form the LSD .if there is no borrow from the next 4-bit group add 0011 to the difference term of such groups (because when decimal digits are subtracted in xs-3 & there is no borrow , result is normal binary). I f there is a borrow , subtract 0011 from the differenceterm(b coz taking a borrow is equivalent to adding six invalid states , result is in xs-6)

Ex: 267-175

267 = 0101 1001 1010
-175= -0100 1010 1000

0000	1111	0010
+0011	-0011	+0011
0011	1100	+0011

$=92_{10}$

Subtraction is performed by the 9‘s compliment or 10‘s compliment
Ex:687-348 The subtrahend (348) xs -3 code & its compliment are:

9‘s comp of 348 = 651
Xs-3 code of 348 = 0110 0111 1011
1‘s comp of 348 in xs-3 = 1001 1000 0100
Xs=3 code of 348 in xs=3 = 1001 1000 0100

687 687
-348 → +651 9‘s compl of 348

339 (1)338
+1 end around carry

339 corrected difference in decimal

1001	1011	1010	687 in xs-3
+1001	1000	0100	1‘s comp 348 in xs-3
_ (1)0010	(1)0011	1110	carry generated

//

+1	+1		propagate carry
(1)0011	0010	1110	
		+1	end around carry

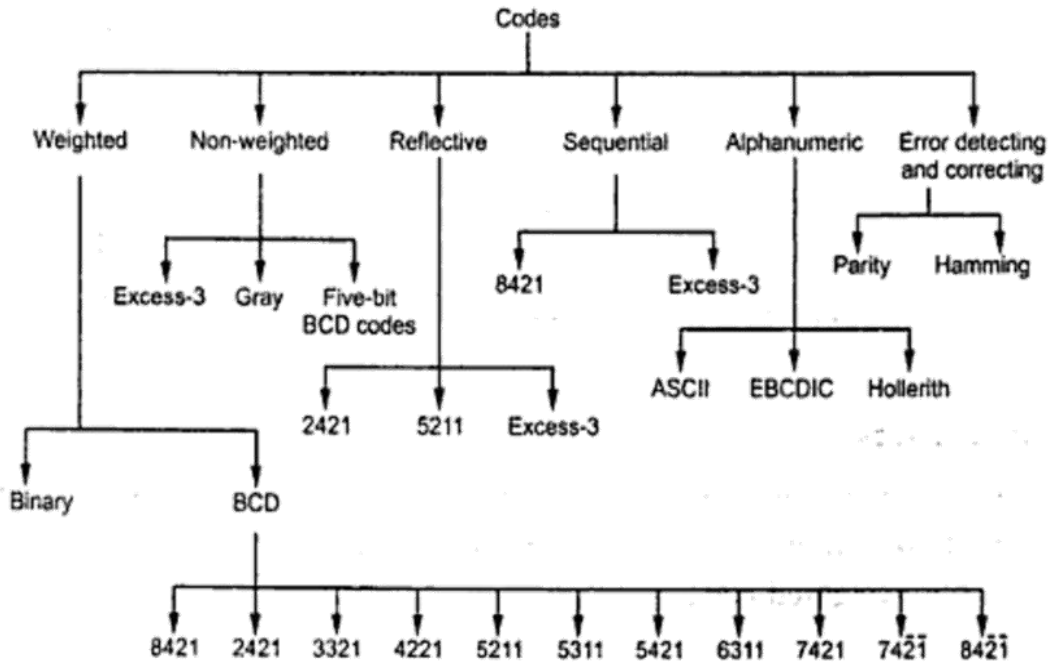
0011	0011	1111
+0011	+0011	+0011

The Gray code (reflective –code):

Gray code is a non-weighted code & is not suitable for arithmetic operations. It is not a BCD code . It is a cyclic code because successive code words in this code differ in one bit position only i.e, it is a unit distance code.Popular of the unit distance code.It is also a reflective code i.e,both reflective & unit distance. The n least significant bits for 2^n through 2^{n+1}-1 are the mirror images of thosr for 0 through 2^n-1.An N bit gray code can be obtained by reflecting an N-1 bit code about an axis at the end of the code, & putting the MSB of 0 above the axis & the MSB of 1 below the axis.

Reflection of gray codes:

Gray Code				Decimal	4 bit binary
1 bit	2 bit	3 bit	4 bit		
0	00	000	0000	0	0000
1	01	001	0001	1	0001
	11	011	0011	2	0010
	10	010	0010	3	0011
		110	0110	4	0100
		111	0111	5	0101
		101	0101	6	0110
		110	0100	7	0111
			1100	8	1000
			1101	9	1001
			1111	10	1010
			1110	11	1011
			1010	12	1100
			1011	13	1101
			1001	14	1110
			1000	15	1111



Binary codes block diagram

Error – Detecting codes: When binary data is transmitted & processed,it is susceptible to noise that can alter or distort its contents. The 1‘s may get changed to 0‘s & 1‘s .because digital systems must be accurate to the digit, error can pose a problem. Several schemes have been devised to detect the occurrence of a single bit error in a binary word, so that whenever such an error occurs the concerned binary word can be corrected & retransmitted.

Parity: The simplest techniques for detecting errors is that of adding an extra bit known as parity bit to each word being transmitted.Two types of parity**:** Oddparity**,** evenparity forodd parity, the parity bit is set to a _0‘ or a _1‘ at the transmitter such that the total no. of 1 bit in the word including the parity bit is an odd no.For even parity, the parity bit is set to a _0‘ or a _1‘ at the transmitter such that the parity bit is an even no.

Decimal	8421 code	Odd parity	Even parity
0	0000	1	0
1	0001	0	1
2	0010	0	1
3	0011	1	0
4	0100	0	1
5	0100	1	0
6	0110	1	0
7	0111	0	1
8	1000	0	1
9	1001	1	0

When the digit data is received . a parity checking circuit generates an error signal if the total no of 1‘s is even in an odd parity system or odd in an even parity system. This parity check can always detect a single bit error but cannot detect 2 or more errors with in the same word.Odd parity is used more often than even parity does not detect the situation. Where all 0‘s are created by a short ckt or some other fault condition.

Ex: Even parity scheme

(a) 10101010 (b) 11110110 (c)10111001

Ans:

(a) No. of 1‘s in the word is even is 4 so there is no error
(b) No. of 1‘s in the word is even is 6 so there is no error
(c) No. of 1‘s in the word is odd is 5 so there is error

Ex: odd parity

(a)10110111 (b) 10011010 (c)11101010

Ans:

(a) No. of 1‘s in the word is even is 6 so word has error
(b) No. of 1‘s in the word is even is 4 so word has error
(c) No. of 1‘s in the word is odd is 5 so there is no error

Checksums:

Simple parity can‘t detect two errors within the same word. To overcome this, use a sort of 2 dimensional parity. As each word is transmitted, it is added to the sum of the previously transmitted words, and the sum retained at the transmitter end. At the end of transmission, the sum called the check sum. Up to that time sent to the receiver. The receiver can check its sum with the transmitted sum. If the two sums are the same, then no errors were detected at the receiver end. If there is an error, the receiving location can ask for retransmission of the entire data, used in teleprocessing systems.

Block parity:

Block of data shown is create the row & column parity bits for the data using odd parity. The parity bit 0 or 1 is added column wise & row wise such that the total no. of 1‘s in each column & row including the data bits & parity bit is odd as

Data	Parity bit
10110	0
10001	1
10101	0
00010	0
11000	1
00000	1
11010	0

data
10110
10001
10101
00010
11000
00000
11010

Error –Correcting Codes:

A code is said to be an error –correcting code, if the code word can always be deduced from an erroneous word. For a code to be a single bit error correcting code, the minimum distance of that code must be three. The minimum distance of that code is the smallest no. of bits by which any two code words must differ. A code with minimum distance of 3 can‘t only correct single bit errors but also detect (can‘t correct) two bit errors, The key to error correction is that it must be possible to detect & locate erroneous that it must be possible to detect & locate erroneous digits. If the location of an error has been determined. Then by complementing the erroneous digit, the message can be corrected , error correcting , code is the Hamming code , In this , to each group of m information or message or data bits, K parity checking bits denoted by P1,P2,----------pk located at positions 2^{k-1} from left are added to form an (m+k) bit code word. To correct the error, k parity checks are performed on selected digits of each code word, & the position of the error bit is located by forming an error word, & the error bit is then complemented. The k bit error word is generated by putting a 0 or a 1 in the 2^{k-1}th position depending upon whether the check for parity involving the parity bit P_k is satisfied or not.Error positions & their corresponding values :

Error Position	For 15 bit code $C_4 C_3 C_2 C_1$	For 12 bit code $C_4 C_3 C_2 C_1$	For 7 bit code $C_3 C_2 C_1$
0	0 0 0 0	0 0 0 0	0 0 0
1	0 0 0 1	0 0 0 1	0 0 1
2	0 0 1 0	0 0 1 0	0 1 0
3	0 0 1 1	0 0 1 1	0 1 1
4	0 1 0 0	0 1 0 0	1 0 0
5	0 1 0 1	0 1 0 1	1 0 1
6	0 1 1 0	0 1 1 0	1 1 0
7	0 1 1 1	0 1 1 1	1 1 1
8	1 0 0 0	1 0 0 0	
9	1 0 0 1	1 0 0 1	
10	1 0 1 0	1 0 1 0	
11	1 0 1 1	1 0 1 1	
12	1 1 0 0	1 1 0 0	
13	1 1 0 1		
14	1 1 1 0		
15	1 1 1 1		

7- bit Hamming code:

To transmit four data bits, 3 parity bits located at positions 2^0 21&2^2 from left are added to make a 7 bit codeword which is then transmitted.

The word format

P_1	P_2	D_3	P_4	D_5	D_6	D_7

D—Data bits P-

Parity bits

Decimal Digit	For BCD P1P2D3P4D5D6D7	For Excess-3 P1P2D3P4D5D6D7
0	0 0 0 0 0 0 0	1 0 0 0 0 1 1
1	1 1 0 1 0 0 1	1 0 0 1 1 0 0
2	0 1 0 1 0 1 1	0 1 0 0 1 0 1
3	1 0 0 0 0 1 1	1 1 0 0 1 1 0
4	1 0 0 1 1 0 0	0 0 0 1 1 1 1
5	0 1 0 0 1 0 1	1 1 1 0 0 0 0
6	1 1 0 0 1 1 0	0 0 1 1 0 0 1
7	0 0 0 1 1 1 1	1 0 1 1 0 1 0
8	1 1 1 0 0 0 0	0 1 1 0 0 1 1
9	0 0 1 1 0 0 1	0 1 1 1 1 0 0

The bit pattern is

P1P2D3P4D5D6D7

1 1 0 1

Bits 1,3,5,7 (P_1 111) must have even parity, so P_1 =1

Bits 2, 3, 6, 7(P_2 101) must have even parity, so P_2 =0

Bits 4,5,6,7 (P_4 101)must have even parity, so P_4 =0

The final code is 1010101

EX: Code word is 1001001

Bits 1,3,5,7 (C_1 1001) →no error →put a 0 in the 1‘s position→C1=0

Bits 2, 3, 6, 7(C_2 0001)) → error →put a 1 in the 2‘s position→C2=1

Bits 4,5,6,7 (C_4 1001)) →no error →put a 0 in the 4‘s position→C3=0

15-bit Hamming Code: It transmit 11 data bits, 4 parity bits located 2^0 2^1 2^2 2^3 Word format is

P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	D10	D11	D12	D13	D14	D15

12-Bit Hamming Code: It transmit 8 data bits, 4 parity bits located at position 2^0 2^1 2^2 2^3 Word format is

P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	D10	D11	D12

Alphanumeric Codes:

These codes are used to encode the characteristics of alphabet in addition to the decimal digits. It is used for transmitting data between computers & its I/O device such as printers, keyboards & video display terminals.Popular modern alphanumeric codes are ASCII code & EBCDIC code.

Boolean functions are expressed in terms of AND, OR, and NOT operations, it is easier to implement a Boolean function with these type of gates.

Name	Graphic symbol	Algebraic function	Truth table
AND		$F = x \cdot y$	$x\ y \mid F$ $0\ 0 \mid 0$ $0\ 1 \mid 0$ $1\ 0 \mid 0$ $1\ 1 \mid 1$
OR		$F = x + y$	$x\ y \mid F$ $0\ 0 \mid 0$ $0\ 1 \mid 1$ $1\ 0 \mid 1$ $1\ 1 \mid 1$
Inverter		$F = x'$	$x \mid F$ $0 \mid 1$ $1 \mid 0$
Buffer		$F = x$	$x \mid F$ $0 \mid 0$ $1 \mid 1$
NAND		$F = (xy)'$	$x\ y \mid F$ $0\ 0 \mid 1$ $0\ 1 \mid 1$ $1\ 0 \mid 1$ $1\ 1 \mid 0$
NOR		$F = (x + y)'$	$x\ y \mid F$ $0\ 0 \mid 1$ $0\ 1 \mid 0$ $1\ 0 \mid 0$ $1\ 1 \mid 0$
Exclusive-OR (XOR)		$F = xy' + x'y$ $= x \oplus y$	$x\ y \mid F$ $0\ 0 \mid 0$ $0\ 1 \mid 1$ $1\ 0 \mid 1$ $1\ 1 \mid 0$
Exclusive-NOR or equivalence		$F = xy + x'y'$ $= (x \oplus y)'$	$x\ y \mid F$ $0\ 0 \mid 1$ $0\ 1 \mid 0$ $1\ 0 \mid 0$ $1\ 1 \mid 1$

Properties of XOR Gates

- XOR (also) : the “not-equal” function
- XOR(X,Y) = X Y = X’Y + XY’
- Identities:
  - X 0 = X
  - X 1 = X’
  - X X = 0
  - X X’ = 1
- Properties:
  - X Y = Y X
  - (X Y) W = X (Y W)

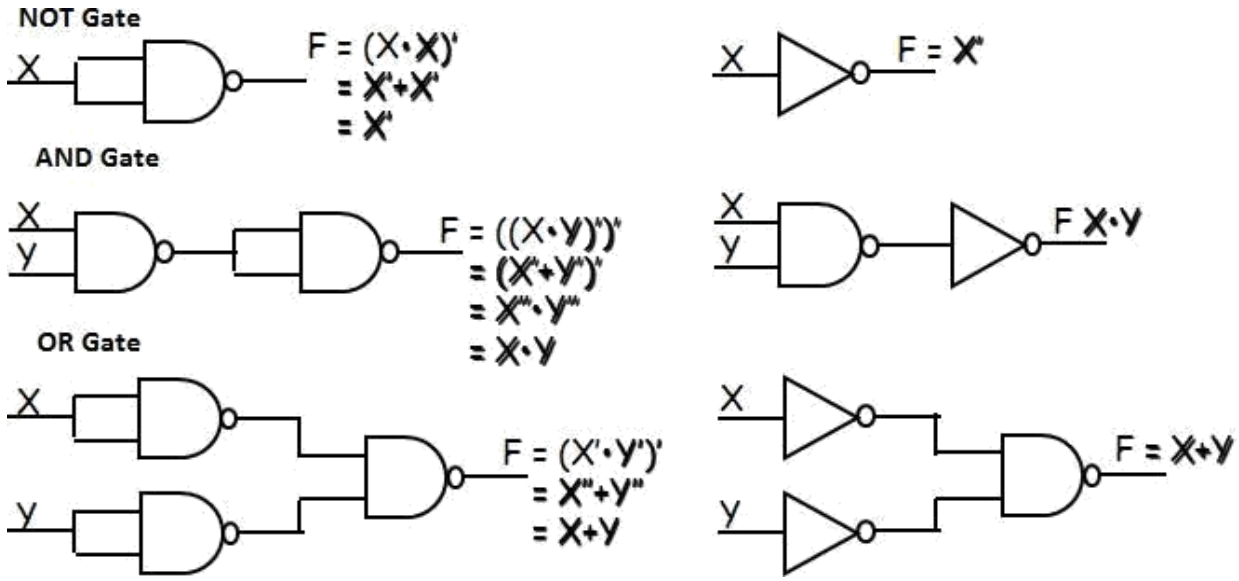
Universal Logic Gates

NAND and NOR gates are called Universal gates. All fundamental gates (NOT, AND, OR) can be realized by using either only NAND or only NOR gate. A universal gate provides flexibility and offers enormous advantage to logic designers.

NAND as a Universal Gate

NAND Known as a “universal” gate because ANY digital circuit can be implemented with NAND gates alone.

To prove the above, it suffices to show that AND, OR, and NOT can be implemented using NAND gates only.



Boolean Algebra: In 1854, George Boole developed an algebraic system now called Boolean algebra. In 1938, Claude E. Shannon introduced a two-valued Boolean algebra called switching algebra that represented the properties of bistable electrical switching circuits. For the formal definition of Boolean algebra, we shall employ the postulates formulated by E. V. Huntington in 1904.

Boolean algebra is a system of mathematical logic. It is an algebraic system consisting of the set of elements (0, 1), two binary operators called OR, AND, and one unary operator NOT. It is the basic mathematical tool in the analysis and synthesis of switching circuits. It is a way to express logic functions algebraically.

Boolean algebra, like any other deductive mathematical system, may be defined with aset of elements, a set of operators, and a number of unproved axioms or postulates. A *set* of elements is anycollection of objects having a common property. If **S** is a set and ***x*** and ***y*** are certain objects, then **x Î Sdenotes** that ***x*** is a member of the set **S**, and ***y*** Ï**S** denotes that ***y*** is not an element of **S**. A set with adenumerable number of elements is specified by braces: **A** = {1,2,3,4}, *i.e.* the elements of set **A** are thenumbers 1, 2, 3, and 4. A *binary operator* defined on a set S of elements is a rule that assigns to each pair ofelements from S a unique element from S._ Example: In *a***b*=*c*, we say that * is a binary operator if it specifies a rule for finding *c* from the pair (*a*,*b*)and also if *a*, *b*, *c* Î S.

Axioms and laws of Boolean algebra

Axioms or Postulates of Boolean algebra are a set of logical expressions that we accept without proof and upon which we can build a set of useful theorems.

	AND Operation	OR Operation	NOT Operation
Axiom1 :	0.0=0	0+0=0	$\overline{0}$=1
Axiom2:	0.1=0	0+1=1	$\overline{1}$=0
Axiom3:	1.0=0	1+0=1	
Axiom4:	1.1=1	1+1=1	

AND Law	**OR Law**
Law1: A.0=0 (Null law)	Law1: A+0=A
Law2: A.1=A (Identity law)	Law2: A+1=1
Law3: A.A=A (Impotence law)	Law3: A+A=A (Impotence law)

CLOSURE: The Boolean system is *closed* with respect to a binary operator if for every pair of Boolean values,it produces a Boolean result. For example, logical AND is closed in the Boolean system because it accepts only Boolean operands and produces only Boolean results.

_ A set *S* is closed with respect to a binary operator if, for every pair of elements of *S*, the binary operator specifies a rule for obtaining a unique element of *S*.

_ For example, the set of natural numbers N = {1, 2, 3, 4, ⋯ 9} is closed with respect to the binary operator plus (+) by the rule of arithmetic addition, since for any *a*, *b* Î N we obtain a unique *c* Î N by the operation *a* + *b* = c.

ASSOCIATIVE LAW:

A binary operator * on a set *S* is said to be associative whenever (*x* * *y*) * *z* = *x* * (*y* * *z*) for all *x*, *y*, *z* Î S, forall Boolean values x, y and z.

COMMUTATIVE LAW:

A binary operator * on a set *S* is said to be commutative whenever x * *y* = *y* * *x* for all *x*, *y*, *z* ϵ S

IDENTITY ELEMENT:

A set ***S*** is said to have an identity element with respect to a binary operation * on S if there exists an element *e* ϵ S with the property *e* * *x* = *x* * *e* = *x* for every *x* ϵ S

BASIC IDENTITIES OF BOOLEAN ALGEBRA

- *Postulate 1(Definition)*: A Boolean algebra is a closed algebraic system containing a set *K* of two or more elements and the two operators · and + which refer to logical AND and logical OR •*x + 0 = x*
- *x · 0 = 0*
- *x + 1 = 1*
- *x · 1 = 1*
- *x + x = x*
- *x · x = x*
- *x + x' = x*
- *x · x' = 0*
- *x + y = y + x*
- *xy = yx*
- *x + (y + z) = (x + y) + z*
- *x (yz) = (xy) z*
- *x (y + z) = xy + xz*
- *x + yz = (x + y)(x + z)*
- *(x + y)' = x' y'*
- *(xy)' = x' + y'*
- *(x')' = x*

DeMorgan's Theorem

(a) $(a + b)' = a'b'$

(b) $(ab)' = a' + b'$

Generalized DeMorgan's Theorem

(a) $(a + b + \ldots z)' = a'b' \ldots z'$
(b) $(a.b \ldots z)' = a' + b' + \ldots z'$

Basic Theorems and Properties of Boolean algebra Commutative law

Law1: A+B=B+A Law2: A.B=B.A

Associative law

Law1: A + (B +C) = (A +B) +C Law2: A(B.C) = (A.B)C

Distributive law

Law1: A.(B + C) = AB+ AC Law2: A + BC = (A + B).(A +C)

Absorption law

Law1: A +AB =A Law2: A(A +B) = A

Solution: A(1+B)
A

Solution: A.A+A.B
A+A.B
A(1+B)
A

Consensus Theorem

Theorem1. AB+ A'C + BC = AB + A'C Theorem2. (A+B). (A'+C).(B+C) =(A+B).(A'+C)

The BC term is called the consensus term and is redundant. The consensus term is formed from a PAIR OF TERMS in which a variable (A) and its complement (A') are present; the consensus term is formed by multiplying the two terms and leaving out the selected variable and its complement

Consensus Theorem1 Proof:

AB+A'C+BC=AB+A'C+(A+A')BC
=AB+A'C+ABC+A'BC

=AB(1+C)+A'C(1+B)

= AB+ A'C

Principle of Duality

Each postulate consists of two expressions statement one expression is transformed into the other by interchanging the operations (+) and (·) as well as the identity elements 0 and 1. Such expressions are known as duals of each other.

If some equivalence is proved, then its dual is also immediately true. If we prove: (x.x)+(x'+x')=1, then we have by duality: (x+x)·(x'.x')=0

The Huntington postulates were listed in pairs and designated by part (a) and part (b) in below table.

Table for Postulates and Theorems of Boolean algebra

Part-A	Part-B
A+0=A	A.0=0
A+1=1	A.1=A
A+A=A (Impotence law)	A.A=A (Impotence law)
	--
Commutative law: A+B=B+A	A.B=B.A
Associative law: A + (B +C) = (A +B) +C	A(B.C) = (A.B)C
Distributive law: A.(B + C) = AB+ AC	A + BC = (A + B).(A +C)
Absorption law: A +AB =A	A(A +B) = A
DeMorgan Theorem:	
	A.(A+B)=AB
Consensus Theorem: AB+ A'C + BC = AB + A'C	(A+B). (A'+C).(B+C) =(A+B).(A'+C)

Boolean Function

Boolean algebra is an algebra that deals with binary variables and logic operations.

A Boolean function described by an algebraic expression consists of binary variables, the constants 0 and 1, and the logic operation symbols.

For a given value of the binary variables, the function can be equal to either 1 or 0.

F(vars) = *expression*

Set of binary Variables

Operators (+, •, ')
Constants (0, 1)
Groupings (parenthesis)
Variables

Consider an example for the Boolean function

F1 = x + y'z

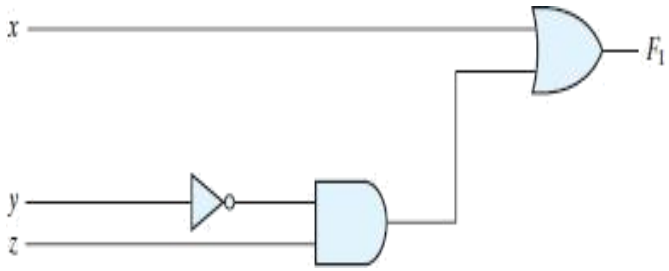
The function F1 is equal to 1 if x is equal to 1 or if both y' and z are equal to 1. F1 is equal to 0 otherwise. The complement operation dictates that when y' = 1, y = 0. Therefore, F1 = 1 if x = 1 or if y = 0 and z = 1.

A Boolean function expresses the logical relationship between binary variables and is evaluated by determining the binary value of the expression for all possible values of the variables.

A Boolean function can be represented in a truth table. The number of rows in the truth table is 2^n, where n is the number of variables in the function. The binary combinations for the truth table are obtained from the binary numbers by counting from 0 through $2^n - 1$.

Truth Table for F1

x	y	z	F_1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1



Gate Implementation of F1 = x + y'z

Note:

Q: Let a function F() depend on n variables. How many rows are there in the truth table of F() ?

A: 2^n rows, since there are 2^n possible binary patterns/combinations for the n variables.

Truth Tables

- Enumerates all possible combinations of variable values and the corresponding function value
- Truth tables for some arbitrary functions
  F1(x,y,z), F2(x,y,z), and F3(x,y,z) are shown to the below.

x	y	z	F_1	F_2	F_3
0	0	0	0	1	1
0	0	1	0	0	1
0	1	0	0	(	
0	1	1	0	1	
1	0	0	0	1	
1	0	1	0	1	
1	1	0	0	(	
1	1	1	1	(	

- Truth table: a unique representation of a Boolean function
- If two functions have identical truth tables, the functions are equivalent (and vice-versa).
- Truth tables can be used to prove equality theorems.
- However, the size of a truth table grows exponentially with the number of variables involved, hence unwieldy. This motivates the use of Boolean Algebra.

Boolean expressions-NOT unique

Unlike truth tables, expressions epresenting a Boolean function are NOT unique.

- Example:
  - F(x,y,z) = x'•y'•z' + x'•y•z' + x•y•z'
  - G(x,y,z) = x'•y'•z' + y•z'
- The corresponding truth tables for F() and G() are to the right. They are identical.
- Thus, F() = G()

x	y	z	F	G
0	0	0	1	1
0	0	1	0	0
0	1	0	1	1
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	1	1
1	1	1	0	0

Algebraic Manipulation (Minimization of Boolean function)

- Boolean algebra is a useful tool for simplifying digital circuits.
- Why do it? Simpler can mean cheaper, smaller, faster.
- Example: Simplify F = x'yz + x'yz' + xz.

  F= x'yz + x'yz' + xz
  = x'y(z+z') + xz
  = x'y•1 + xz
  = x'y + xz

- Example: Prove

  x'y'z' + x'yz' + xyz' = x'z' + yz'
- **Proof:**

  x'y'z'+ x'yz'+ xyz'
  = x'y'z' + x'yz' + x'yz' + xyz'
  = x'z'(y'+y) + yz'(x'+x)
  = x'z'•1 + yz'•1
  = x'z' + yz'

Complement of a Function

- The complement of a function is derived by interchanging (• and +), and (1 and 0), and complementing each variable.
- Otherwise, interchange 1s to 0s in the truth table column showing F.
- The *complement* of a function IS NOT THE SAME as the *dual* of a function.

Example

- Find G(x,y,z), the complement of F(x,y,z) = xy'z' + x'yz Ans: G = F' = (xy'z' + x'yz)'

  = (xy'z')' • (x'yz)' *DeMorgan*
  = (x'+y+z) • (x+y'+z') *DeMorgan* again

Note: The complement of a function can also be derived by finding the function's *dual,* and then complementing all of the literals

Canonical and Standard Forms

We need to consider formal techniques for the simplification of Boolean functions.
Identical functions will have exactly the same canonical form.

- Minterms and Maxterms
- Sum-of-Minterms and Product-of- Maxterms
- Product and Sum terms
- Sum-of-Products (SOP) and Product-of-Sums (POS)

Definitions

Literal: A variable or its complement

Product term: literals connected by •

Sum term: literals connected by +

Minterm: a product term in which all the variables appear exactly once, either complemented or uncomplemented.

Canonical form: Boolean functions expressed as a sum of Minterms or product of Maxterms are said to be in canonical form.

Minterm

- Represents exactly one combination in the truth table.
- Denoted by m_j, where j is the decimal equivalent of the minterm's corresponding binary combination (b_j).
- A variable in m_j is complemented if its value in b_j is 0, otherwise is uncomplemented.

Example: Assume 3 variables (A, B, C), and j=3. Then, b_j = 011 and its corresponding minterm is denoted by m_j = A'BC

Maxterm

- Represents exactly one combination in the truth table.
- Denoted by M_j, where j is the decimal equivalent of the maxterm's corresponding binary combination (b_j).
- A variable in M_j is complemented if its value in b_j is 1, otherwise is uncomplemented.

Example: Assume 3 variables (A, B, C), and j=3. Then, b_j = 011 and its corresponding maxterm is denoted by M_j = A+B'+C'

Truth Table notation for Minterms and Maxterms

- Minterms and Maxterms are easy to denote using a truth

table. Example: Assume 3 variables x,y,z (order is fixed)

x	y	z	Minterm	Maxterm
0	0	0	$x'y'z' = m_0$	$x+y+z = M_0$
0	0	1	$x'y'z = m_1$	$x+y+z' = M_1$
0	1	0	$x'yz' = m_2$	$x+y'+z = M_2$
0	1	1	$x'yz = m_3$	$x+y'+z'= M_3$
1	0	0	$xy'z' = m_4$	$x'+y+z = M_4$
1	0	1	$xy'z = m_5$	$x'+y+z' = M_5$
1	1	0	$xyz' = m_6$	$x'+y'+z = M_6$
1	1	1	$xyz = m_7$	$x'+y'+z' = M_7$

Canonical Forms

- Every function F() has two canonical forms:
  - Canonical Sum-Of-Products (sum of minterms)
  - Canonical Product-Of-Sums (product of maxterms)

Canonical Sum-Of-Products:

The minterms included are those m_j such that F( ) = 1 in row j of the truth table for F( ).

Canonical Product-Of-Sums:

The maxterms included are those M_j such that F( ) = 0 in row j of the truth table for F( ).

Example

Consider a Truth table for $f_1(a,b,c)$ at right
The canonical sum-of-products form for f_1
is $f_1(a,b,c) = m_1 + m_2 + m_4 + m_6$

$= a'b'c + a'bc' + ab'c' + abc'$

The canonical product-of-sums form for f_1
is $f_1(a,b,c) = M_0 \bullet M_3 \bullet M_5 \bullet M_7$

$= (a+b+c)\bullet(a+b'+c')\bullet (a'+b+c')\bullet(a'+b'+c').$

- Observe that: $m_j = M_j'$

a	b	c	f_1
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Shorthand: ∑ and ∏

- $f_1(a,b,c) = \sum m(1,2,4,6)$, where ∑ indicates that this is a sum-of-products form, and m(1,2,4,6) indicates that the minterms to be included are m_1, m_2, m_4, and m_6.
- $f_1(a,b,c) = \prod M(0,3,5,7)$, where ∏ indicates that this is a product-of-sums form, and M(0,3,5,7) indicates that the maxterms to be included are M_0, M_3, M_5, and M_7.
- Since $m_j = M_j'$ for any *j*,

  $\sum m(1,2,4,6) = \prod M(0,3,5,7) = f_1(a,b,c)$
-

Conversion between Canonical Forms

- Replace ∑ with ∏ (or *vice versa*) and replace those *j*'s that appeared in the original form with those that do not.
    - Example:

$f_1(a,b,c)$= a'b'c + a'bc' + ab'c' + abc'

= $m_1 + m_2 + m_4 + m_6$

= ∑(1,2,4,6)

= ∏(0,3,5,7)

= (a+b+c)•(a+b'+c')•(a'+b+c')•(a'+b'+c')

Standard Forms

Another way to express Boolean functions is in standard form. In this configuration, the terms that form the function may contain one, two, or any number of literals.

There are two types of standard forms: the sum of products and products of sums.

The sum of products is a Boolean expression containing AND terms, called product terms, with one or more literals each. The sum denotes the ORing of these terms. An example of a function expressed as a sum of products is

F1 = y' + xy + x'yz'

The expression has three product terms, with one, two, and three literals. Their sum is, in effect, an OR operation.

A product of sums is a Boolean expression containing OR terms, called sum terms. Each term may have any number of literals. The product denotes the ANDing of these terms. An example of a function expressed as a product of sums is

F2 = x(y' + z)(x' + y + z')

This expression has three sum terms, with one, two, and three literals. The product is an AND operation.

Example-1.

Express the Boolean function F = A + B'C as a sum of minterms.

Solution: The function has three variables: A, B, and C. The first term A is missing two variables; therefore, A = A(B + B') = AB + AB'

This function is still missing one variable, so

$A = AB(C + C') + AB' (C + C')$

$= ABC + ABC' + AB'C + AB'C'$

The second term B'C is missing one variable; hence,

$B'C = B'C(A + A') = AB'C + A'B'C$

Combining all terms, we have

$F = A + B'C$

$= ABC + ABC' + AB'C + AB'C' + A'B'C$

But AB'C appears twice, and according to theorem ($x + x = x$), it is possible to remove one of those occurrences. Rearranging the minterms in ascending order, we finally obtain

$F = A'B'C + AB'C + AB'C + ABC' + ABC$

$= m1 + m4 + m5 + m6 + m7$

When a Boolean function is in its sum-of-minterms form, it is sometimes convenient to express the function in the following brief notation:

$F(A, B, C) = \sum m (1, 4, 5, 6, 7)$

Example-2.

Express the Boolean function $F = xy + x'z$ as a product of maxterms.

Solution: First, convert the function into OR terms by using the distributive law:

$F = xy + x'z = (xy + x')(xy + z)$

$= (x + x')(y + x')(x + z)(y + z)$

$= (x' + y)(x + z)(y + z)$

The function has three variables: x, y, and z. Each OR term is missing one variable;

therefore, $x' + y = x' + y + zz' = (x' + y + z)(x' + y + z')$

$x + z = x + z + yy' = (x + y + z)(x + y' + z)$

$y + z = y + z + xx' = (x + y + z)(x' + y + z)$

Combining all the terms and removing those which appear more than once, we finally obtain $F = (x + y + z)(x + y' + z)(x' + y + z)(x' + y + z)$

$F = M0M2M4M5$

A convenient way to express this function is as follows: $F(x, y, z) = \pi M(0, 2, 4, 5)$

The product symbol, π, denotes the ANDing of maxterms; the numbers are the indices of the maxterms of the function.

Unit-II

Minimization Techniques

Two-variable k-map:

A two-variable k-map can have 2^2=4 possible combinations of the input variables A and B. Each of these combinations, , B,A ,AB(in the SOP form) is called a minterm. The minterm may be represented in terms of their decimal designations – m0 for , m1 for B,m2 for A and m3 for AB, assuming that A represents the MSB. The letter m stands for minterm and the subscript represents the decimal designation of the minterm. The presence or absence of a minterm in the expression indicates that the output of the logic circuit assumes logic 1 or logic 0 level for that combination of input variables.

The expression f= ,+ B+A +AB , it can be expressed using min

term as F= m0+m2+m3=∑m(0,2,3)

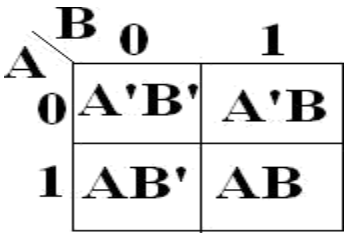
Using Truth Table:

Minterm	Inputs A	B	Output F
0	0	0	1
1	0	1	0
2	1	0	1
3	1	1	1

A 1 in the output contains that particular minterm in its sum and a 0 in that column indicates that the particular mintermdoes not appear in the expression for output . this information can also be indicated by a two-variable k-map.

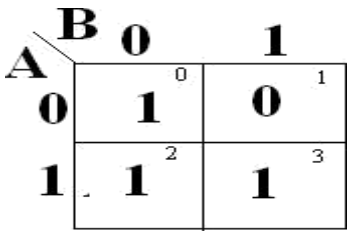
Mapping of SOP Expresions:

A two-variable k-map has 22=4 squares .These squares are called cells. Each square on the k-map represents a unique minterm. The minterm designation of the squares are placed in any square, indicates that the corresponding minterm does output expressions. And a 0 or no entry in any square indicates that the corresponding minterm does not appear in the expression for output.



The minterms of a two-variable k-map

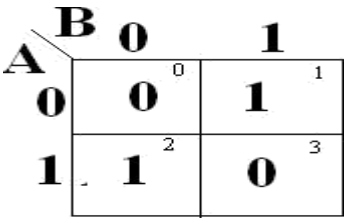
The mapping of the expressions =∑m(0,2,3)is



k-map of ∑m(0,2,3)

EX: Map the expressions f= B+A

F= m_1+m_2=∑m(1,2)The k-map is



Minimizations of SOP expressions:

To minimize Boolean expressions given in the SOP form by using the k-map, look for adjacent adjacent squares having 1‘s minterms adjacent to each other, and combine them to form larger squares to eliminate some variables. Two squares are said to be adjacent to each other, if their minterms differ in only one variable. (i.e, B & A differ only in one variable. so they may be combined to form a 2-square to eliminate the variable B.similarly all other.

The necessary condition for adjacency of minterms is that their decimal designations must differ by a power of 2. A minterm can be combined with any number of minterms adjacent to it to form larger squares. Two minterms which are adjacent to each other can be combined to form a bigger square called a 2-square or a pair. This eliminates one variable – the variable that is not common to both the minterms. For EX:

m0 and m1 can be combined to yield,

f_1 = m0+m1= + B= (B+

)= m0 and m2 can be combined to yield,

(+

f_2 = m0+m2= + =)=

m1 and m3 can be combined to yield,

f_3= m1+m3= B+AB=B(+)=B

m2 and m3 can be combined to yield,

f_4 = m2+m3=A +AB=A(B+)=A

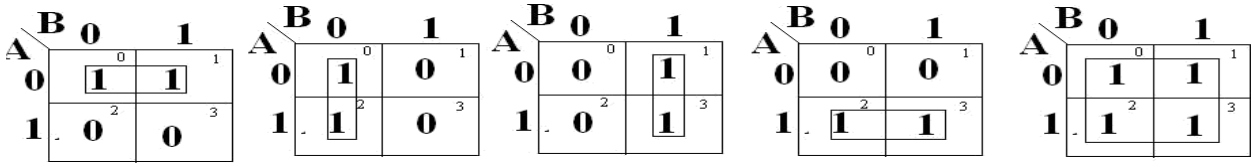
m_0 ,m_1 ,m_2 and m_3 can be combined to yield,

= ++A +AB

= (B+) +A(B+)

= +A

=1

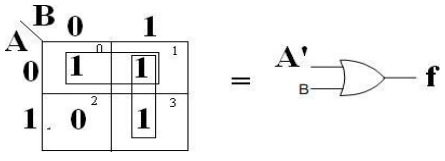


f1= f2= f3=B f4=A f5=1

The possible minterm groupings in a two-variable k-map.

Two 2-squares adjacent to each other can be combined to form a 4-square. A 4-square eliminates 2 variables. A 4-square is called a quad. To read the squares on the map after minimization, consider only those variables which remain constant through the square, and ignore the variables which are varying. Write the non complemented variable if the variable is remaining constant as a 1, and the complemented variable if the variable is remaining constant as a 0, and write the variables as a product term. In the above figure f_1 read as , because, along the square , A remains constant as a 0, that is , as , where as B is changing from 0 to 1.

EX: Reduce the minterm f= +A +AB using mapping Expressed in terms of minterms, the given expression is F=m_0+m_1+m_2+ m_3=m∑(0,1,3)& the figure shows the k-map for f and its reduction . In one 2-square, A is constant as a 0 but B varies from a 0 to a 1, and in the other 2-square, B is constant as a 1 but A varies from a 0 to a 1. So, the reduced expressions is +B.



It requires two gate inputs for realization as

f= +B (k-map in SOP form, and logic diagram.)

The main criterion in the design of a digital circuit is that its cost should be as low as possible. For that the expression used to realize that circuit must be minimal.Since the cost is proportional to number of gate inputs in the circuit in the circuit, an expression is considered minimal only if it corresponds to the least possible number of gate inputs. & there is no guarantee for that k-map in SOP is the real minimal. To obtain real minimal expression, obtain the minimal expression both in SOP & POS form form by using k-maps and take the minimal of these two minimals.

The 1‘s on the k-map indicate the presence of minterms in the output expressions, where as the 0s indicate the absence of minterms .Since the absence of a minterm in the SOP expression means the presense of the corresponding maxterm in the POS expression of the same .when a SOP expression is plotted on the k-map, 0s or no entries on the k-map represent the maxterms. To obtain the minimal expression in the POS form, consider the 0s on the k-map and follow the procedure used for combining 1s. Also, since the absence of a maxterm in the POS expression means the presence of the corresponding minterm in the SOP expression of the same , when a POS expression is plotted on the k-map, 1s or no entries on the k-map represent the minterms.

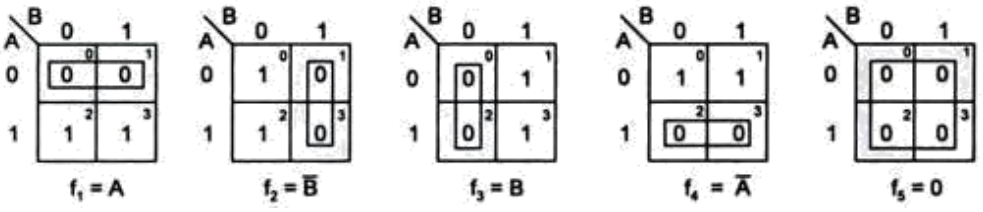
Mapping of POS expressions:

Each sum term in the standard POS expression is called a maxterm. A function in two variables (A, B) has four possible maxterms, A+B,A+ , +B, +

. They are represented as M_0, M_1, M2, and M3respectively. The uppercase letter M stands for maxterm and its subscript denotes the decimal designation of that maxterm obtained by treating the non-complemented variable as a 0 and the complemented variable as a 1 and putting them side by side for reading the decimal equivalent of the binary number so formed.

For mapping a POS expression on to the k-map, 0s are placed in the squares corresponding to the maxterms which are presented in the expression an d1s are placed in the squares corresponding to the maxterm which are not present in the expression. The decimal designation of the squares of the squares for maxterms is the same as that for the minterms. A two-variable k-map & the associated maxterms are asthe maxterms of a two-variable k-map

The possible maxterm groupings in a two-variable k-map

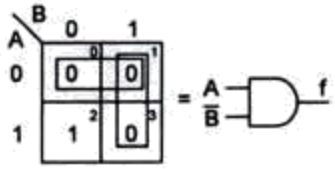


To obtain the minimal expression in POS form, map the given POS expression on to the K-map and combine the adjacent 0s into as large squares as possible. Read the squares putting the complemented variable if its value remains constant as a 1 and the non-complemented variable if its value remains constant as a 0 along the entire square (ignoring the variables which do not remain constant throughout the square) and then write them as a sum term.

Various maxterm combinations and the corresponding reduced expressions are shown in figure. In this f_1 read as A because A remains constant as a 0 throughout the square and B changes from a 0 to a 1. f_2 is read as B‘ because B remains constant along the square as a 1 and A changes from a 0 to a 1. f_5
Is read as a 0 because both the variables are changing along the square.

Ex: Reduce the expression f=(A+B)(A+B‘)(A‘+B‘) using mapping.

The given expression in terms of maxterms is f=πM(0,1,3). It requires two gates inputs for realization of the reduced expression as



F=AB‘

K-map in POS form and logic diagram

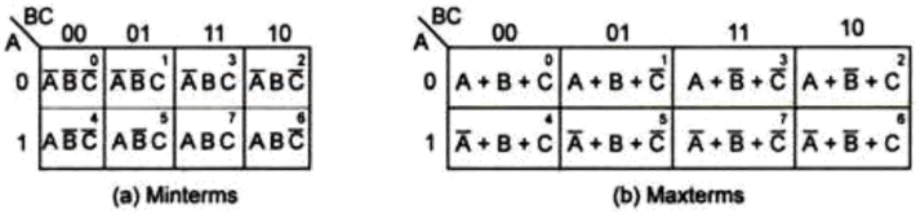
In this given expression ,the maxterm M_2 is absent. This is indicated by a 1 on the k-map. The corresponding SOP expression is $\sum m_2$ or AB‘. This realization is the same as that for the POS form.

Three-variable K-map:

A function in three variables (A, B, C) expressed in the standard SOP form can have eight possible combinations: A B C , AB C,A BC ,A BC,AB C ,AB C,ABC , and ABC. Each one of these combinations designate d by m0,m1,m2,m3,m4,m5,m6, and m7, respectively, is called a minterm. A is the MSB of the minterm designator and C is the LSB.

In the standard POS form, the eight possible combinations are:A+B+C, A+B+C , A+B +C,A+B + C ,A + B+ C,A + B + C ,A + B + C,A + B + C . Each oneof these combinations designated by M0, M1, M2, M3, M4, M5, M6, and M7respectively is called a maxterm. A is the MSB of the maxterm designator and C is the LSB.

A three-variable k-map has, therefore, $8(=2^3)$ squares or cells, and each square on the map represents a minterm or maxterm as shown in figure. The small number on the top right corner of each cell indicates the minterm or maxterm designation.



The three-variable k-map.

The binary numbers along the top of the map indicate the condition of B and C for each column. The binary number along the left side of the map against each row indicates the condition of A for that row. For example, the binary number 01 on top of the second column in fig indicates that the variable B appears in complemented form and the variable C in non-complemented form in all the minterms in that column. The binary number 0 on the left of the first row indicates that the variable A appears in complemented form in all the minterms in that row, the binary numbers along the top of the k-map are not in normal binary order. They are, infact, in the Gray code. This is to ensure that twophysically adjacent squares are really adjacent, i.e., their minterms or maxterms differ by only one variable.

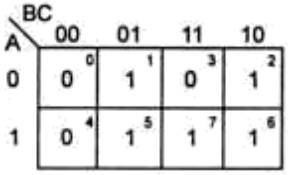
Ex: Map the expression f=: C+ + + +ABC

In the given expression , the minterms are : C=001=m_1 ; =101=m_5;

=010=m_2;

=110=m_6;ABC=111=m_7.

So the expression is f=$\sum$m(1,5,2,6,7)= $\sum$m(1,2,5,6,7). The corresponding k-map is



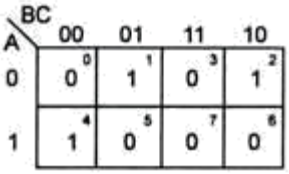
K-map in SOP form

+ +) (+ +)(A + +)(+ +)

Ex: Map the expression f= (A+B+C),(

Inthegivenexpressionthemaxterms A+B+C=000=M0; + + =101=M5; + + = 111=M7; A + + =011=M3; + + =110=M_6.

So the expression is f = π M (0,5,7,3,6)= π M (0,3,5,6,7). The mapping of the expression is are



K-map in POS form.

Minimization of SOP and POS expressions:

For reducing the Boolean expressions in SOP (POS) form plotted on the k-map, look at the 1s (0s) present on the map. These represent the minterms (maxterms). Look for the minterms (maxterms) adjacent to each other, in order to combine them into larger squares. Combining of adjacent squares in a k-map containing 1s (or 0s) for the purpose of simplification of a SOP (or POS)expression is called *looping*. Some of the minterms (maxterms) may have many adjacencies. Always start with the minterms (maxterm) with the least number of adjacencies and try to form as large as large a square as possible. The larger must form a geometric square or rectangle. They can be formed even by wrapping around, but cannot be formed by using diagonal configurations. Next consider the minterm (maxterm) with next to the least number of adjacencies and form as large a square as possible. Continue this till all the minterms (maxterms) are taken care of . A minterm (maxterm) can be part of any number of squares if it is helpful in reduction. Read the minimal expression from the k-map, corresponding to the squares formed. There can be more than one minimal expression.

Two squares are said to be adjacent to each other (since the binary designations along the top of the map and those along the left side of the map are in Gray code), if they are physically adjacent to each other, or can be made adjacent to each other by wrapping around. For squares to be combinable into bigger squares it is essential but not sufficient that their minterm designations must differ by a power of two.

General procedure to simplify the Boolean expressions:

1. Plot the k-map and place 1s(0s) corresponding to the minterms (maxterms) of the SOP (POS) expression.
2. Check the k-map for 1s(0s) which are not adjacent to any other 1(0). They are isolated minterms(maxterms) . They are to be read as they are because they cannot be combined even into a 2-square.
3. Check for those 1s(0S) which are adjacent to only one other 1(0) and make them pairs (2 squares).
4. Check for quads (4 squares) and octets (8 squares) of adjacent 1s (0s) even if they contain some 1s(0s) which have already been combined. They must geometrically form a square or a rectangle.
5. Check for any 1s(0s) that have not been combined yet and combine them into bigger squares if possible.
6. Form the minimal expression by summing (multiplying) the product the product (sum) terms of all the groups.

Reading the K-maps:

While reading the reduced k-map in SOP (POS) form, the variable which remains constant as 0 along the square is written as the complemented (non-complemented) variable and the one which remains constant as 1 along the square is written as non-complemented (complemented) variable and the term as a product (sum) term. All the product (sum) terms are added (multiplied).

Some possible combinations of minterms and the corresponding minimal expressions readfrom the k-maps are shown in fig: Here f_6 is read as 1, because along the 8-square no variable remains constant. F_5 is read as , because, along the 4-square formed by0,m_1,m_2 and m_3 , the variables B and C are changing, and A remains constant as a 0. Algebraically,

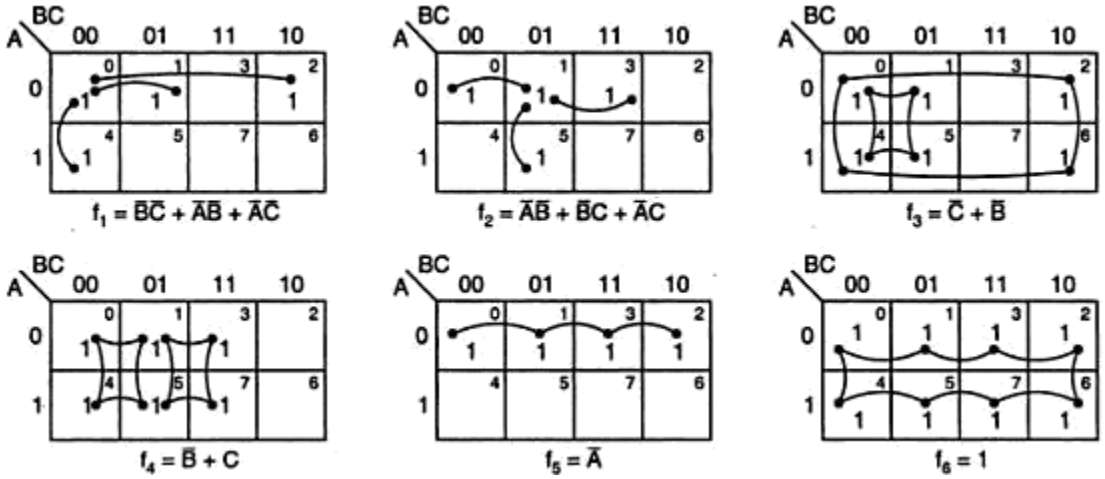
$f_5 = m_0+m_1+m_2+m_3$

$= \quad + \quad C+ \quad +$

$= \quad (\quad +C)+ \quad B(C+)$

$= \quad + B$

$= \quad (+B)=$

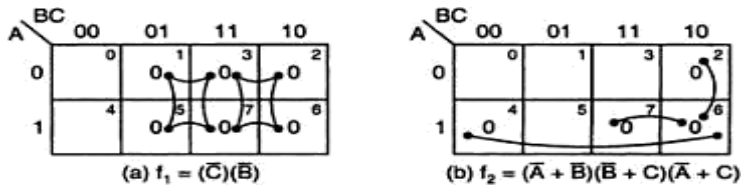


f_3 is read as + , because in the 4-square formed by m0,m2,m6, and m4, the variable A and B are changing , where as the variable C remains constant as a 0. So it is read as . In the 4-square formed by m_0, m1, m4, m_5, A and C are changing but B remains constant as a 0. So it is read as
. So, the resultant expression for f3 is the sum of these two, i.e., + .

f_1 is read as + + ,because in the 2-square formed by m_0 and m_4 , A is changing from a 0 to a 1. Whereas B and C remain constant as a 0. So it s read as . In the 2-square formed by m_0 and m_1, C is changing from a 0 to a 1, whereas A and B remain constant as a 0. So it is read as .In the 2-square formed by m_0 and m_2 , B is changing from a 0 to a 1 whereas A and C remain constant as a 0. So, it is read as . Therefore, the resultant SOP expression is

+ +

Some possible maxterm groupings and the corresponding minimal POS expressions read from the k-map are



In this figure, along the 4-square formed by M_1, M3, M7, M5, A and B are changing from a 0 to a 1, where as C remains constant as a 1. SO it is read as . Along the 4-squad formed by M_3, M2, M7, and M_6, variables A and C are changing from a 0 to a 1. But B remains constant as a 1. So it is read as . The minimal expression is the product of these two terms , i.e., f_1 = ( )( ).also in this figure, along the 2-square formed by M_4 and M6 , variable B is changing from a 0 to a 1, while variable A remains constant as a 1 and variable C remains constant as a 0. SO, read it as

+C. Similarly, the 2-square formed by M7 andM6 is read as + , while the 2-square formed by M2 and M6 is read as +C. The minimal expression is the product of these sum terms, i.e, f2 =(+)+(+)+(+C)

Ex:Reduce the expression f=∑m(0,2,3,4,5,6) using mapping and implement it in AOI logic as well as in NAND logic.The Sop k-map and its reduction , and the implementation of the minimal expression using AOI logic and the corresponding NAND logic are shown in figures below

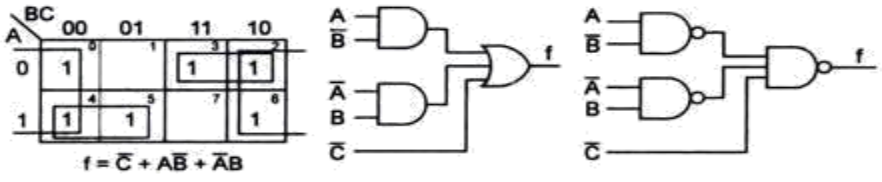
In SOP k-map, the reduction is done as:

.1 m_5 has only one adjacency m_4 , so combine m_5 and m_4 into a square. Along this 2-square A remains constant as 1 and B remains constant as 0 but C varies from 0 to 1. So read it as A .

.2 m_3 has only one adjacency m_2 , so combine m_3 and m_2 into a square. Along this 2-square A remains constant as 0 and B remains constant as 1 but C varies from 1 to 0. So read it as B.

.3 m_6 can form a 2-square with m_2 and m_4 can form a 2-square with m_0, but observe that by wrapping the map from left to right m_0, m_4 ,m_2 ,m_6 can form a 4-square. Out of these m_2 andm_4 have already been combined but they can be utilized again. So make it. Along this 4-square, A is changing from 0 to 1 and B is also changing from 0 to 1 but C is remaining constant as 0. so read it as .

4. Write all the product terms in SOP form. So the minimal SOP expression is



f_{min}=

k-map **AOI logic** **NAND logic**

Four variable k-maps:

Four variable k-map expressions can have 2^4=16 possible combinations of input variables such as , ,----------ABCD with minterm designations m0,m1 ---------- m15 respectively in SOP form & A+B+C+D, A+B+C+ ,---------- + + + with maxterms M0,M1, --------- -

-M_{15} respectively in POS form. It has 2^4=16 squares or cells.The binary number designations of rows & columns are in the gray code. Here follows 01 & 10 follows 11 called Adjacency ordering.

AB \ CD	00	01	11	10
00	0 $\bar{A}\bar{B}\bar{C}\bar{D}$	1 $\bar{A}\bar{B}\bar{C}D$	3 $\bar{A}\bar{B}CD$	2 $\bar{A}\bar{B}C\bar{D}$
01	4 $\bar{A}B\bar{C}\bar{D}$	5 $\bar{A}B\bar{C}D$	7 $\bar{A}BCD$	6 $\bar{A}BC\bar{D}$
11	12 $AB\bar{C}\bar{D}$	13 $AB\bar{C}D$	15 $ABCD$	14 $ABC\bar{D}$
10	8 $A\bar{B}\bar{C}\bar{D}$	9 $A\bar{B}\bar{C}D$	11 $A\bar{B}CD$	10 $A\bar{B}C\bar{D}$

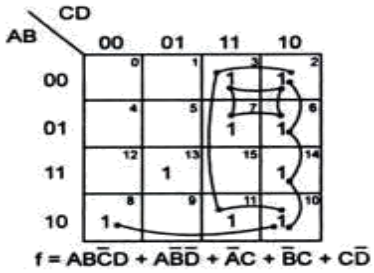
SOP form

AB \ CD	00	01	11	10
00	0 $A+B+C+D$	1 $A+B+C+\bar{D}$	3 $A+B+\bar{C}+\bar{D}$	2 $A+B+\bar{C}+D$
01	4 $A+\bar{B}+C+D$	5 $A+\bar{B}+C+\bar{D}$	7 $A+\bar{B}+\bar{C}+\bar{D}$	6 $A+\bar{B}+\bar{C}+D$
11	12 $\bar{A}+\bar{B}+C+D$	13 $\bar{A}+\bar{B}+C+\bar{D}$	15 $\bar{A}+\bar{B}+\bar{C}+\bar{D}$	14 $\bar{A}+\bar{B}+\bar{C}+D$
10	8 $\bar{A}+B+C+D$	9 $\bar{A}+B+C+\bar{D}$	11 $\bar{A}+B+\bar{C}+\bar{D}$	10 $\bar{A}+B+\bar{C}+D$

POS form

EX: Reduce using mapping the expression Σ m(2, 3, 6, 7, 8, 10, 11, 13, 14).

Start with the minterm with the least number of adjacencies. The minterm m_{13} has no adjacency. Keep it as it is. The m_8 has only one adjacency, m_{10}. Expand m_8 into a 2-square with m_{10}. The m_7 has two adjacencies, m_6 and m_3. Hence m_7 can be expanded into a 4-square with m_6, m_3 and m_2. Observe that, m_7, m_6, m_2, and m_3 form a geometric square. The m_{11} has 2 adjacencies, m_{10} and m_3. Observe that, m_{11}, m_{10}, m_3, and m_2 form a geometric square on wrapping the K-map. So expand m_{11} into a 4-square with m_{10}, m_3 and m_2. Note that, m_2 and m_3, have already become a part of the 4-square m_7, m_6, m_2, and m_3. But if m_{11} is expanded only into a 2-square with m_{10}, only one variable is eliminated. So m_2 and m_3 are used again to make another 4-square with m_{11} and m_{10} to eliminate two variables. Now only m_6 and m_{14} are left uncovered. They can form a 2-square that eliminates only one variable. Don't do that. See whether they can be expanded into a larger square. Observe that, m_2, m_6, m_{14}, and m_{10} form a rectangle. So m_6 and m_{14} can be expanded into a 4-square with m_2 and m_{10}. This eliminates two variables.



Five variable k-map:

Five variable k-map can have 2^5 =32 possible combinations of input variable as , E,--------ABCDE with minterms m_0, m_1-----m_{31} respectively in SOP & A+B+C+D+E, A+B+C+ ,---------- + + + + with maxterms M_0,M_1, ---------- M_{31} respectively in POS form. It has 2^5=32 squares or cells of the k-map are divided into 2 blocks of

16 squares each.The left block represents minterms from m_0 to m_{15} in which A is a 0, and the right block represents minterms from m_{16} to m_{31} in which A is 1.The 5-variable k-map may contain 2-squares, 4-squares , 8-squares , 16-squares or 32-squares involving these two blocks. Squares are also considered adjacent in these two blocks, if when superimposing one block on top of another, the squares coincide with one another.

Some possible 2-squares in a five-variable map are m_0, m_{16}; m_2, m_{18}; m_5, m_{21}; m_{15}, m_{31}; m_{11}, m_{27}.

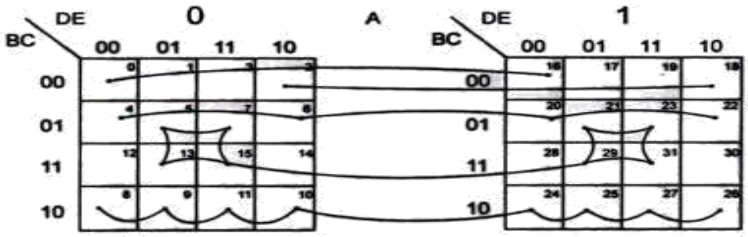
Some possible 4-squares are m_0, m_2, m_{16}, m_{18}; m_0, m_1, m_{16}, m_{17}; m_0, m_4, m_{16}, m_{20}; m_{13}, m_{15}, m_{29}, m_{31}; m_5, m_{13}, m_{21}, m_{29}.

Some possible 8-squares are m_0, m_1, m_3, m_2, m_{16}, m_{17}, m_{19}, m_{18}; m_0, m_4, m_{12}, m_8, m_{16}, m_{20}, m_{28}, m_{24}; m_5, m_7, m_{13}, m_{15}, m_{21}, m_{23}, m_{29}, m_{31}.

The squares are read by dropping out the variables which change. Some possible

Grouping s is

(a) $m_0, m_{16} = \overline{B}\overline{C}\overline{D}\overline{E}$	$M_0, M_{16} = B + C + D + E$
(b) $m_2, m_{18} = \overline{B}\overline{C}D\overline{E}$	$M_2, M_{18} = B + C + \overline{D} + E$
(c) $m_4, m_6, m_{20}, m_{22} = \overline{B}C\overline{E}$	$M_4, M_6, M_{20}, M_{22} = B + \overline{C} + E$
(d) $m_5, m_7, m_{13}, m_{15}, m_{21}, m_{23}, m_{29}, m_{31} = CE$	$M_5, M_7, M_{13}, M_{15}, M_{21}, M_{23}, M_{29}, M_{31} = \overline{C} + \overline{E}$
(e) $m_8, m_9, m_{10}, m_{11}, m_{24}, m_{25}, m_{26}, m_{27} = B\overline{C}$	$M_8, M_9, M_{10}, M_{11}, M_{24}, M_{25}, M_{26}, M_{27} = \overline{B} + C$



Ex: F=∑m(0,1,4,5,6,13,14,15,22,24,25,28,29,30,31) is SOP

POS is F=πM(2,3,7,8,9,10,11,12,16,17,18,19,20,21,23,26,27)

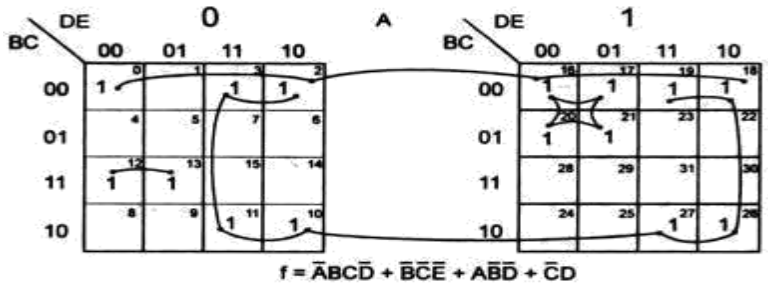
The real minimal expression is the minimal of the SOP and POS forms.

The reduction is done as

1. There is no isolated 1s
2. M_{12} can go only with m_{13}. Form a 2-square which is read as A‘BCD‘
3. M_0 can go with m_2,m_{16} and m_{18} . so form a 4-square which is read as B‘C‘E‘
4. M_{20},m_{21},m_{17} and m_{16} form a 4-square which is read as AB‘D‘
5. M2,m3,m18,m19,m10,m11,m26 and m27 form an 8-square which is read as C‘d
6. Write all the product terms in SOP form.

So the minimal expression is

F_{min}= A‘BCD‘+B‘C‘E‘+AB‘D‘+C‘D(16 inputs)



In the POS k-map ,the reduction is done as:

1. There are no isolated 0s

3. **M_1 can go only with M_5. So, make a 2-square, which is read as $(A + B + D + \bar{E})$.**
   M_4 can go with M_5, M_7, and M_6 to form a 4-square, which is read as $(A + B + \bar{C})$.

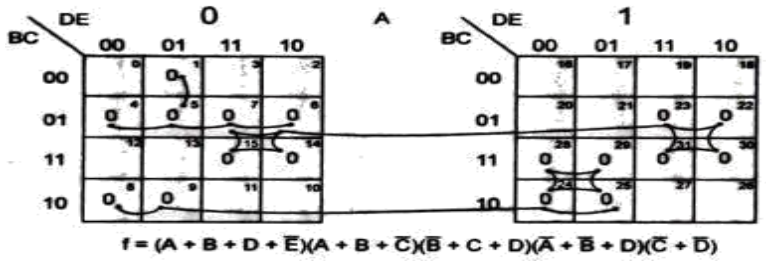
4.M_8

5. M_{28}

6.M_{30}

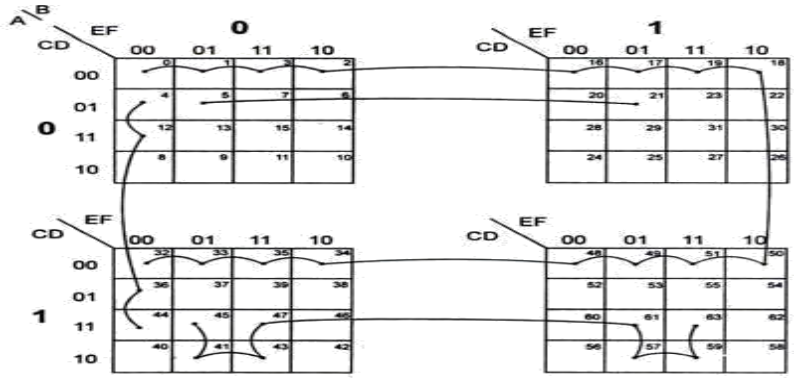
7. Sum terms in POS form. So the minimal expression in POS

   is F_{min}= A‘BcD‘+B‘C‘E‘+AB‘D‘+C‘D



Six variable k-map:

Six variable k-map can have 2^6 =64 combinations as , ,--------- ---ABCDEF with minterms m0, m1-----m63 respectively in SOP & (A+B+C+D+E+F), ---------- (+ + + + +) with maxterms M0,M1, ----------M63 respectively in POS form. It has 2^6=64 squares or cells of the k-map are divided into 4 blocks of 16 squares each.



Some possible groupings in a six variable k-map

Don't care combinations: For certain input combinations, the value of the output is unspecified either because the input combinations are invalid or because the precise value of the output is of no consequence. The combinations for which the value of experiments are not specified are called don't care combinations are invalid or because the precise value of the output is of no consequence. The combinations for which the value of expressions is not specified are called don't care combinations or Optional Combinations, such expressions stand incompletely specified. The output is a don't care for these invalid combinations.

Ex:In XS-3 code system, the binary states 0000, 0001, 0010,1101,1110,1111 are unspecified. & never occur called don't cares.

A standard SOP expression with don't cares can be converted into a standard POS form by keeping the don't cares as they are & writing the missing minterms of the SOP form as the maxterms of the POS form viceversa.

Don't cares denoted by _X' or _φ'

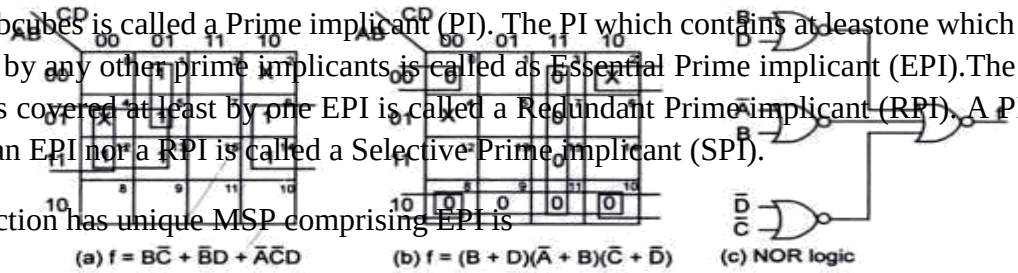
Ex:f=∑m(1,5,6,12,13,14)+d(2,4)

Or f=π M(0,3,7,9,10,11,15).πd(2,4)

SOP minimal form f_{min}= +B +
POS minimal form fmin=(B+D)(+B)(+D)
= + + + + (+

Prime implicants, Essential Prime implicants, Redundant prime implicants:

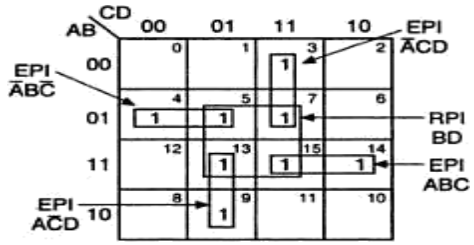
Each square or rectangle made up of the bunch of adjacent minterms is called a subcube. Each of these subcubes is called a Prime implicant (PI). The PI which contains at leastone which cannot be covered by any other prime implicants is called as Essential Prime implicant (EPI).The PI whose each 1 is covered at least by one EPI is called a Redundant Prime implicant (RPI). A PI which is neither an EPI nor a RPI is called a Selective Prime implicant (SPI).



The function has unique MSP comprising EPI is

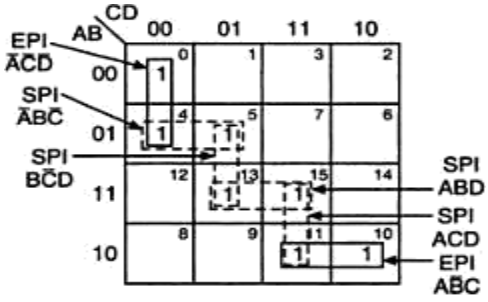
F(A,B,C,D)= CD+ABC+A D + B

The RPI _BD' may be included without changing the function but the resulting expression would not be in minimal SOP(MSP) form.



Essential and Redundant Prime Implicants

F(A,B,C,D)=∑m(0,4,5,10,11,13,15) SPI are marked by dotted squares, shows MSP form of a function need not be unique.



Essential and Selective Prime Implicants

Here, the MSP form is obtained by including two EPI's & selecting a set of SPI's to cover remaining uncovered minterms 5,13,15. & these can be covered as

(A) (4,5) &(13,15) ---------- B +ABD
(B) (5,13) & (13,15) -------- B D+ABD
(C) (5,13) & (15,11) ------- B D+ACD

F(A,B,C,D)= +A C---------EPI‘s + B +ABD

(OR) F(A,B,C,D)= +A C---------EPI‘s + B D+ABD

(OR) F(A,B,C,D)= +A C---------EPI‘s + B D+ACD

False PI’s Essential False PI’s, Redundant False PI’s & Selective False PI’s:

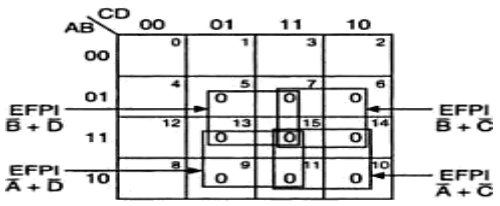
The maxterms are called falseminterms. The PI‘s is obtained by using the maxterms are called False PI‘s (FPI). The FPI which contains at least one _0‘ which can‘t be covered by only other FPI is called an Essential False Prime implicant (ESPI)

F(A,B,C,D)= ∑m(0,1,2,3,4,8,12)

=π M(5,6,7,9,10,11,13,14,15)

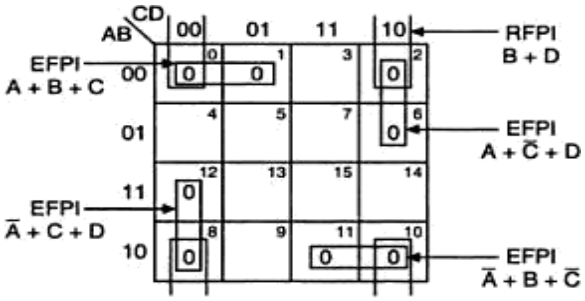
Fmin= (+)(+)(+)(+)

All the FPI, EFPI‘s as each of them contain atleast one _0‘ which can‘t be covered by any other FPI



Essential False Prime implicants

Consider Function F(A,B,C,D)= π M(0,1,2,6,8,10,11,12)



Essential and Redundant False Prime Implicants

Mapping when the function is not expressed in minterms (maxterms):

An expression in k-map must be available as a sum (product) of minterms (maxterms). However if not so expressed, it is not necessary to expand the expression algebraically into its minterms (maxterms). Instead, expansion into minterms (maxterms) can be accomplished in the process of entering the terms of the expression on the k-map.

Limitations of Karnaugh maps:

- Convenient as long as the number of variables does not exceed six.
- Manual technique, simplification process is heavily dependent on the human abilities.

Quine-Mccluskey Method:

It also known as *Tabular method.* It is more systematic method of minimizing expressions of even larger number of variables. It is suitable for hand computation as well as computation by machines i.e., programmable. . The procedure is based on repeated application of the combining theorem.

PA+P =P (P is set of literals) on all adjacent pairs of terms, yields the set of all PI‘s from which a minimal sum may be selected.

Consider expression

∑m(0,1,4,5)= + C+A +A C

First, second terms & third, fourth terms can be combined

(+)+ (C+)= +A

Reduced to

(+)=

The same result can be obtained by combining m_0& m_4 & m_1&m_5 in first step & resulting terms in the second step .

Procedure:

- Decimal Representation
- Don‘t cares
- PI chart
- EPI
- Dominating Rows & Columns
- Determination of Minimal expressions in comples cases.

Branching Method:

EXAMPLE 3.29 Obtain the set of prime implicants for the Boolean expression $f = \Sigma\, m(0, 1, 6, 7, 8, 9, 13, 14, 15)$ using the tabular method.

Solution

Group the minterms in terms of the number of 1s present in them and write their binary designations. The procedure to obtain the prime implicants is shown in Table 3.3.

Table 3.3 Example 3.29

	Column 1		Column 2		Column 3
	Minterm	**Binary designation**		**A B C D**	**A B C D**
Index 0	0	0 0 0 0 ✓	0, 1 (1)	0 0 0 – ✓	0, 1, 8, 9 (1, 8) – 0 0 – Q
Index 1	1	0 0 0 1 ✓	0, 8 (8)	– 0 0 0 ✓	... ...
	8	1 0 0 0 ✓	1, 9 (8)	– 0 0 1 ✓	... ...
Index 2	6	0 1 1 0 ✓	8, 9 (1)	1 0 0 – ✓	6, 7, 14, 15 (1, 8) – 1 1 – P
	9	1 0 0 1 ✓	6, 7 (1)	0 1 1 – ✓	
Index 3	7	0 1 1 1 ✓	6, 14 (8)	– 1 1 0 ✓	
	13	1 1 0 1 ✓	9, 13 (4)	1 – 0 1 S	
	14	1 1 1 0 ✓	7, 15 (8)	– 1 1 1 ✓	
Index 4	15	1 1 1 1 ✓	13, 15 (2)	1 1 – 1 R	
			14, 15 (1)	1 1 1 – ✓	

Comparing the terms of index 0 with the terms of index 1 of column 1, $m_0(0000)$ is combined with $m_1(0001)$ to yield 0, 1 (1), i.e. 000 –. This is recorded in column 2 and 0000 and 0001 are checked off in column 1. $m_0(0000)$ is combined with $m_8(1000)$ to yield 0, 8 (8), i.e. – 000. This is recorded in column 2 and 1000 is checked off in column 1. Note that 0000 of column 1 has already been checked off. No more combinations of terms of index 0 and index 1 are possible. So, draw a line below the last combination of these groups, i.e. below 0, 8 (8), – 000 in column 2. Now 0, 1 (1), i.e. 000 – and 0, 8 (8), i.e. – 000 are the terms in the first group of column 2.

Comparing the terms of index 1 with the terms of index 2 in column 1, $m_1(0001)$ is combined with $m_9(1001)$ to yield 1, 9 (8), i.e. – 001. This is recorded in column 2 and 1001 is checked off in column 1 because 0001 has already been checked off. $m_8(1000)$ is combined with $m_9(1001)$ to yield 8, 9 (1), i.e. 100 –. This is recorded in column 2. 1000 and 1001 of column 1 have already been checked off. So, no need to check them off again. No more combinations of terms of index 1 and index 2 are possible. So, draw a line below the last combination of these groups, i.e. 8, 9 (1), -- 001 in column 2. Now 1, 9 (8), i.e. – 001 and 8, 9 (1), i.e. 100– are the terms in the second group of column 2.

Similarly, comparing the terms of index 2 with the terms of index 3 in column 1,

$m_6(0110)$ and $m_7(0111)$ yield 6, 7 (1), i.e. 011–. Record it in column 2 and check off 6(0110) and 7(0111).

$m_6(0110)$ and $m_{14}(1110)$ yield 6, 14 (8), i.e. –110. Record it in column 2 and check off 6(0110) and 14(1110).

$m_9(1001)$ and $m_{13}(1101)$ yield 9, 13 (4), i.e. 1–01. Record it in column 2 and check off 9(1001) and 13(1101).

So, 6, 7 (1), i.e. 011–, and 6, 14 (8), i.e. –110 and 9, 13 (4), i.e. 1–01 are the terms in group 3 of column 2. Draw a line at the end of 9, 13 (4), i.e. 1–01.

Also, comparing the terms of index 3 with the terms of index 4 in column 1,

$m_7(0111)$ and $m_{15}(1111)$ yield 7, 15 (8), i.e. –111. Record it in column 2 and check off 7(0111) and 15(1111).

$m_{13}(1101)$ and $m_{15}(1111)$ yield 13, 15 (2), i.e. 11–1. Record it in column 2 and check off 13 and 15.

$m_{14}(1110)$ and $m_{15}(1111)$ yield 14, 15 (1), i.e. 111–. Record it in column 2 and check off 14 and 15.

So, 7, 15 (8), i.e. –111, and 13, 15 (2), i.e. 11–1 and 14, 15 (1), i.e. 111– are the terms in group 4 of column 2. Column 2 is completed now.

Comparing the terms of group 1 with the terms of group 2 in column 2, the terms 0, 1 (1), i.e. 000– and 8, 9 (1), i.e. 100– are combined to form 0, 1, 8, 9 (1, 8), i.e. –00–. Record it in group 1 of column 3 and check off 0, 1 (1), i.e. 000–, and 8, 9 (1), i.e. 100– of column 2. The terms 0, 8 (8), i.e. –000 and 1, 9 (8), i.e. –001 are combined to form 0, 1, 8, 9 (1, 8), i.e. –00–. This has already been recorded in column 3. So, no need to record again. Check off 0, 8 (8), i.e. –000 and 1, 9 (8), i.e. –001 of column 2. Draw a line below 0, 1, 8, 9 (1, 8), i.e. –00–. This is the only term in group 1 of column 3. No term of group 2 of column 2 can be combined with any term of group 3 of column 2. So, no entries are made in group 2 of column 2.

Comparing the terms of group 3 of column 2 with the terms of group 4 of column 2, the terms 6, 7 (1), i.e. 011–, and 14, 15 (1), i.e. 111– are combined to form 6, 7, 14, 15 (1, 8), i.e. –11–. Record it in group 3 of column 3 and check off 6, 7 (1), i.e. 011– and 14, 15 (1), i.e. 111– of column 2. The terms 6, 14 (8), i.e. –110 and 7, 15 (8), i.e. –111 are combined to form 6, 7, 14, 15 (1, 8), i.e. –11–. This has already been recorded in column 3; so, check off 6, 14 (8), i.e. –110 and 7, 15 (8), i.e. –111 of column 2.

Observe that the terms 9, 13 (4), i.e. 1–01 and 13, 15 (2), i.e. 11–1 cannot be combined with any other terms. Similarly in column 3, the terms 0, 1, 8, 9 (1, 8), i.e. –00– and 6, 7, 14, 15 (1, 8), i.e. –11– cannot also be combined with any other terms. So, these 4 terms are the prime implicants.

The terms, which cannot be combined further, are labelled as P, Q, R, and S. These form the set of prime implicants.

EX:

Obtain the minimal expression for $f = \Sigma\, m(1, 2, 3, 5, 6, 7, 8, 9, 12, 13, 15)$ using the tabular method.

Solution

The procedure to obtain the set of prime implicants is illustrated in Table 3.4.

Table 3.4 Example 3.30

Step 1		Step 2	Step 3	
Index 1	1 ✓	1, 3 (2) ✓	1, 3, 5, 7 (2, 4)	T
	2 ✓	1, 5 (4) ✓	1, 5, 9, 13 (4, 8)	S
	8 ✓	1, 9 (8) ✓	2, 3, 6, 7 (1, 4)	R
Index 2	3 ✓	2, 3 (1) ✓	8, 9, 12, 13 (1, 4)	Q
	5 ✓	2, 6 (4) ✓	5, 7, 13, 15 (2, 8)	P
	6 ✓	8, 9 (1) ✓		
	9 ✓	8, 12 (4) ✓		
	12 ✓	3, 7 (4) ✓		
Index 3	7 ✓	5, 7 (2) ✓		
	13 ✓	5, 13 (8) ✓		
Index 4	15 ✓	6, 7 (1) ✓		
		9, 13 (4) ✓		
		12, 13 (1) ✓		
		7, 15 (8) ✓		
		13, 15 (2) ✓		

The non-combinable terms P, Q, R, S and T are recorded as prime implicants.

$$P \rightarrow 5, 7, 13, 15\ (2, 8) = X\ 1\ X\ 1 = BD$$

(Literals with weights 2 and 8, i.e. C and A are deleted. The lowest minterm is $m_5(5 = 4 + 1)$. So, literals with weights 4 and 1, i.e. B and D are present in non-complemented form. So, read it as BD.)

$$Q \rightarrow 8, 9, 12, 13\ (1, 4) = 1\ X\ 0\ X = A\overline{C}$$

(Literals with weights 1 and 4, i.e. D and B are deleted. The lowest minterm is m_8. So, literal with weight 8 is present in non-complemented form and literal with weight 2 is present in complemented form. So, read it as $A\overline{C}$.)

$$R \rightarrow 2, 3, 6, 7\ (1, 4) = 0\ X\ 1\ X = \overline{A}C$$

(Literals with weights 1 and 4, i.e. D and B are deleted. The lowest minterm is m_2. So, literal with weight 2 is present in non-complemented form and literal with weight 8 is present in complemented form. So, read it as $\overline{A}C$.)

$$S \rightarrow 1, 5, 9, 13\ (4, 8) = X\ X\ 0\ 1 = \overline{C}D$$

(Literals with weights 4 and 8, i.e. B and A are deleted. The lowest minterm is m_1. So, literal with weight 1 is present in non-complemented form and literal with weight 2 is present in complemented form. So, read it as $\overline{C}D$.)

$$T \rightarrow 1, 3, 5, 7\ (2, 4) = 0\ X\ X\ 1 = \overline{A}D$$

(Literals with weights 2 and 4, i.e. C and B are deleted. The lowest minterm is 1. So, literal with weight 1 is present in non-complemented form and literal with weight 8 is present in complemented form. So, read it as $\overline{A}D$.)

The prime implicant chart of the expression

$$f = \Sigma\ m(1, 2, 3, 5, 6, 7, 8, 9, 12, 13, 15)$$

is as shown in Table 3.5. It consists of 11 columns corresponding to the number of minterms and 5 rows corresponding to the prime implicants P, Q, R, S, and T generated. Row R contains four ×s at the intersections with columns 2, 3, 6, and 7, because these minterms are covered by the prime implicant R. A row is said to cover the columns in which it has ×s. The problem now is to select a minimal subset of prime implicants, such that each column contains at least one × in the rows corresponding to the selected subset and the total number of literals in the prime implicants selected is as small as possible. These requirements guarantee that the number of unions of the selected prime implicants is equal to the original number of minterms and that, no other expression containing fewer literals can be found.

Table 3.5 Example 3.30: Prime implicant chart

		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
	1	2	3	5	6	7	8	9	12	13	15
*P → 5, 7, 13, 15 (2, 8)				×		×				×	×
*Q → 8, 9, 12, 13 (1, 4)							×	×	×	×	
*R → 2, 3, 6, 7 (1, 4)		×	×		×	×					
S → 1, 5, 9, 13 (4, 8)	×			×				×		×	
T → 1, 3, 5, 7 (2, 4)	×		×	×		×					

In the prime implicant chart of Table 3.5, m_2 and m_6 are covered by R only. So, R is an essential prime implicant. So, check off all the minterms covered by it, i.e. m_2, m_3, m_6, and m_7. Q is also an essential prime implicant because only Q covers m_8 and m_{12}. Check off all the minterms covered by it, i.e. m_8, m_9, m_{12}, and m_{13}. P is also an essential prime implicant, because m_{15} is covered only by P. So check off m_{15}, m_5, m_7, and m_{13} covered by it. Thus, only minterm 1 is not covered. Either row S or row T can cover it and both have the same number of literals. Thus, two minimal expressions are possible.

$$P + Q + R + S = BD + A\bar{C} + \bar{A}C + \bar{C}D$$

or

$$P + Q + R + T = BD + A\bar{C} + \bar{A}C + \bar{A}D$$