## Complements

Complements are used in digital computers for simplifying the subtraction operation and for logical manipulation. There are two types of complements for each base r system: the r's complement and the (r - l)'s complement.

When the value of the base r is substituted in the name, the two types are referred to as the 2's and 1's complement for binary numbers and the ID's and 9's complement for decimal numbers.

## (r - 1)'s Complement

### 9's Complement

- Given a number **N** in base r having **n** digits, the (r - I)'s complement of N is defined as $(r^n - I)$-N.
- For decimal numbers r = 10 and r - 1 = 9, so the 9's complement of N is $(10^n - 1) - N$.
- $10^n$ represents a number that consists of a single 1 followed by n 0's.
- $10^n - 1$ is a number represented by n 9's.
- For example, with n = 4 we have $10^4 = 10000$ and $10^4 - 1 = 9999$. It follows that the 9' s complement of a decimal number is obtained by subtracting each digit from 9.
- For example, the 9's complement of 546700 is 999999 - 546700 = 453299 and the 9's complement of 12389 is 99999 - 12389 = 87610.

### 1's Complement

- 1's complement of N is $(2^n - 1) - N$.
- $2^n$ is represented by a binary number that consists of a 1 followed by n 0's.
- $(2^n - 1)$ is a binary number represented by n 1's.
- For example, with n = 4, we have $2^4 = (10000)_2$ and $2^4 - 1 = (1111)_2$. Thus the 1's complement of a binary number is obtained by subtracting each digit from 1.
- However, the subtraction of a binary digit from I causes the bit to change from 0 to 1 or from 1 to 0. Therefore, the 1's complement of a binary number is formed by changing 0 to 1 and 1 to 0. For example, the 1's complement of 1011001 is 0100110.

***The (r - 1)'s complement of octal or hexadecimal numbers are obtained by subtracting each digit from 7 or F (decimal 15) respectively.***

### 7's Complement

**Example of Finding the (r - 1)'s complement of octal**

Let's take an octal number, for example, $245_8$

Step 1: Find the 7's Complement

To find the 7's complement of $245_8$, subtract each digit from 7:

- First digit: 7 - 2 = 5
- Second digit: 7 - 4 = 3
- Third digit: 7 - 5 = 2

So, the 7's complement of $245_8 = 532_8$

### 15's Complement

**Example of Finding the (r - 1)'s complement of octal**

Let's take a hexadecimal number as an example: $2A3_{16}$

**Step 1: Find the 15's Complement**

To find the 15's complement of $2A3_{16}$ subtract each digit from 15. In hexadecimal, the digits are:
0, 1, 2, ..., 9, A, B, C, D, E, F (where A = 10, B = 11, C = 12, D = 13, E = 14, F = 15).

So for each digit:

- **First digit (2):** 15−2=13. In hexadecimal, 13 is represented as D.
- **Second digit (A):** 15−10=5 So, A becomes 5.
- **Third digit (3):** 15−3=12. In hexadecimal, 12 is represented as C.

So, the 15's complement of $2A3_{16}$ is $D5C_{16}$