Machine Learning

Lecture 6: Logistic Regression

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Linear model

$$x \longrightarrow \boxed{\text{Linear}} \longrightarrow \hat{y}$$

Hours (x)	Points
1	2
2	4
3	6
4	?

Binary prediction (0 or 1) is very useful!

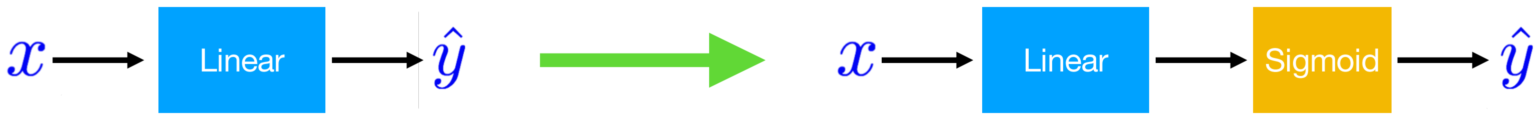
- Spent *N* hours for study, pass or fail?
- GPA and GRE scores for the HKUST PHD program, admit or not?
- Soccer game against Japan, win or lose?
- She/he looks good, propose or not?
- …

Linear to binary (pass/fail, 0/1)

$$x \longrightarrow \boxed{\text{Linear}} \longrightarrow \hat{y}$$

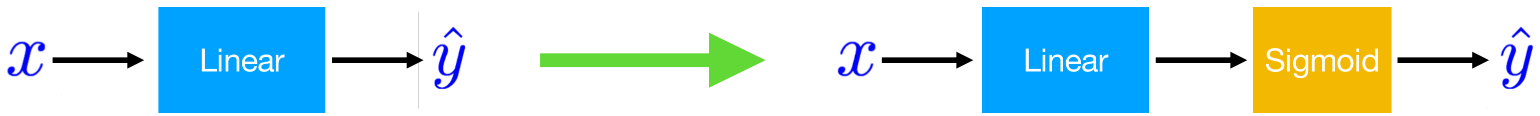
Hours (x)	Points	fail/pass
1	2	0
2	4	0
3	6	1
4	?	?

Logistic regression: pass/fail (0 or 1)

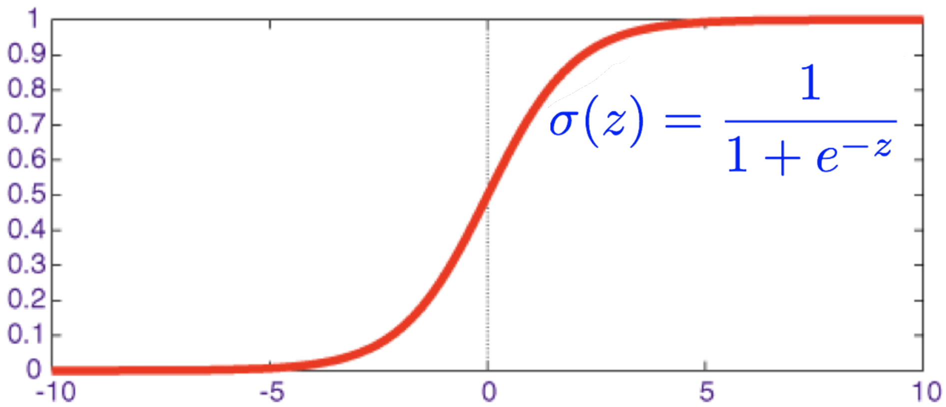


Hours (x)	Points	fail/pass
1	2	0
2	4	0
3	6	1
4	?	?

Meet Sigmoid

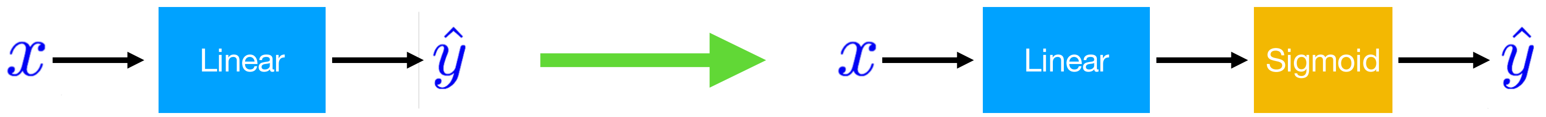


Hours (x)	Points	fail/pass
1	2	0
2	4	0
3	6	1
4	?	?

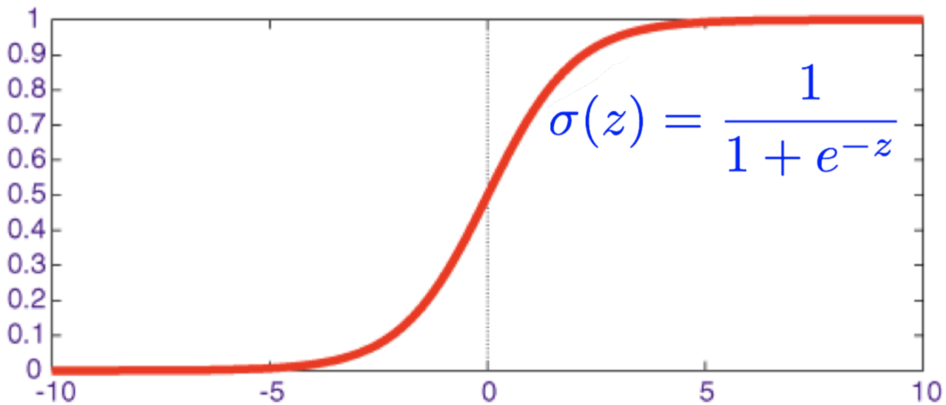


Meet Sigmoid

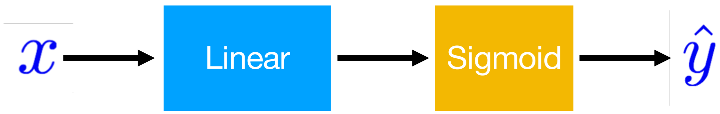
$$1 : \hat{y} > 0.5$$



Hours (x)	Points	fail/pass
1	2	0
2	4	0
3	6	1
4	?	?



Meet sigmoid

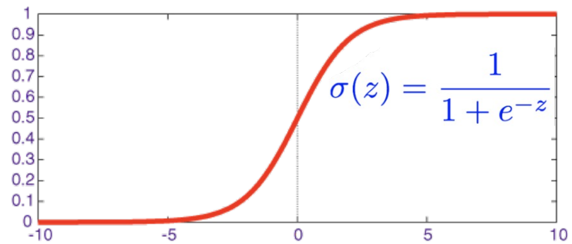


$$\hat{y} = x * w + b$$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\hat{y} = \sigma(x * w + b)$$

Hours (x)	Points	fail/pass
1	2	0
2	4	0
3	6	1
4	?	?



Meet Cross Entropy Loss

$x \rightarrow$ Linear $\rightarrow \hat{y}$

$x \rightarrow$ Linear $\rightarrow$ Sigmoid $\rightarrow \hat{y}$

$$\hat{y} = x * w + b$$

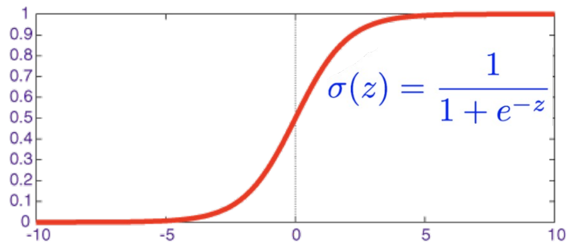
$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\hat{y} = \sigma(x * w + b)$$

$$loss = \frac{1}{N}\sum_{n=1}^{N}(\hat{y_n} - y_n)^2$$

$$loss = -\frac{1}{N}\sum_{n=1}^{N} y_n \log \hat{y}_n + (1 - y_n)\log(1 - \hat{y}_n)$$

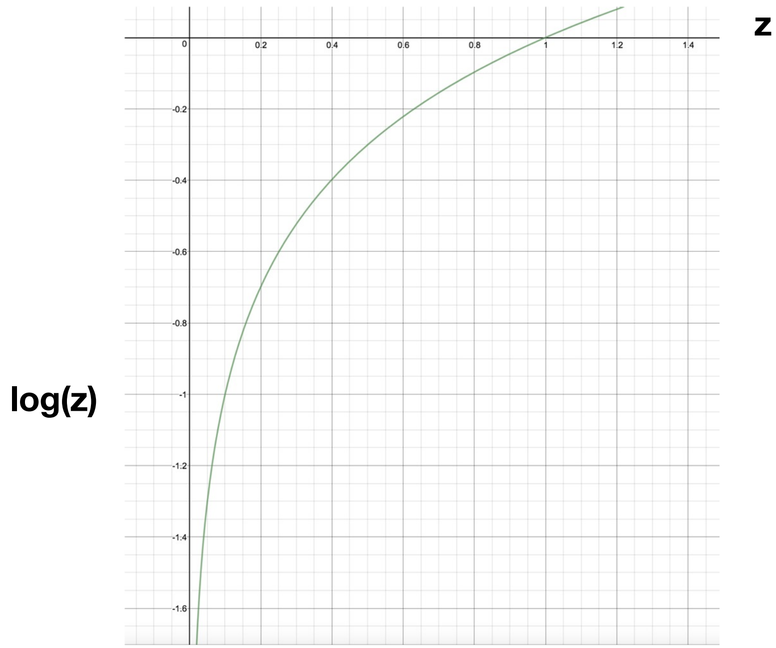
Hours (x)	Points	fail/pass
1	2	0
2	4	0
3	6	1
4	?	?



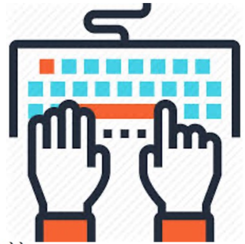
(Binary) Cross Entropy Loss

$$loss = -\frac{1}{N}\sum_{n=1}^{N} y_n \log \hat{y}_n + (1 - y_n)\log(1 - \hat{y}_n)$$

y	y_pred	loss
1	0.2	
1	0.8	
0	0.1	
0	0.9	

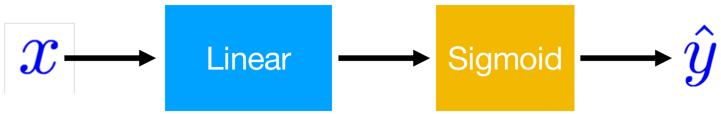


Logistic regression



`torch.nn.functional.sigmoid(input)`

Applies the element-wise function $f(x) = 1/(1 + exp(-x))$



$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\hat{y} = \sigma(x * w + b)$$

```python
import torch.nn.functional as F

class Model(torch.nn.Module):

    def __init__(self):
        super(Model, self).__init__()
        self.linear = torch.nn.Linear(1, 1)

    def forward(self, x):
        y_pred = F.sigmoid(self.linear(x))
        return y_pred
```

Logistic regression

$x \rightarrow$ Linear $\rightarrow$ Sigmoid $\rightarrow \hat{y}$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\hat{y} = \sigma(x * w + b)$$

class `torch.nn.Sigmoid` [source]

Applies the element-wise function $f(x) = 1/(1 + exp(-x))$

```
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$$loss = -\frac{1}{N}\sum_{n=1}^{N} y_n \log \hat{y}_n + (1-y_n)\log(1-\hat{y}_n)$$

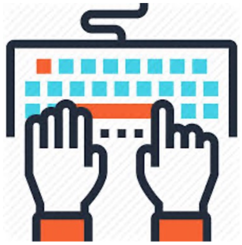
class `torch.nn.BCELoss`*(weight=None, size_average=True)* [source]

Creates a criterion that measures the Binary Cross Entropy between the target and the output:

$$loss(o, t) = -1/n \sum_i (t[i] * log(o[i]) + (1 - t[i]) * log(1 - o[i]))$$

```
criterion = torch.nn.BCELoss(size_average=True)
```

Logistic regression



```python
x_data = Variable(torch.Tensor([[1.0], [2.0], [3.0], [4.0]]))
y_data = Variable(torch.Tensor([[0.], [0.], [1.], [1.]]))

class Model(torch.nn.Module):
   def __init__(self):
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   def forward(self, x):
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# our model
model = Model()

criterion = torch.nn.BCELoss(size_average=True)
optimizer = torch.optim.SGD(model.parameters(), lr=0.01)

# Training loop
for epoch in range(1000):
       # Forward pass: Compute predicted y by passing x to the model
   y_pred = model(x_data)

   # Compute and print loss
   loss = criterion(y_pred, y_data)
   print(epoch, loss.data[0])

   # Zero gradients, perform a backward pass, and update the weights.
   optimizer.zero_grad()
   loss.backward()
   optimizer.step()

# After training
hour_var = Variable(torch.Tensor([[1.0]]))
print("predict 1 hour ", 1.0, model(hour_var).data[0][0] > 0.5)
hour_var = Variable(torch.Tensor([[7.0]]))
print("predict 7 hours", 7.0, model(hour_var).data[0][0] > 0.5)
```
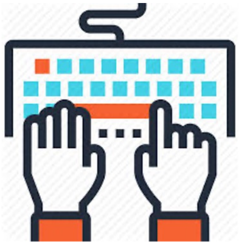
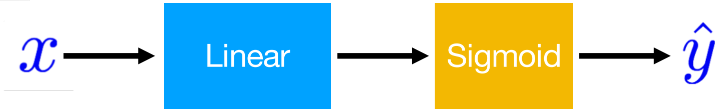

Logistic regression



1 Design your model using class

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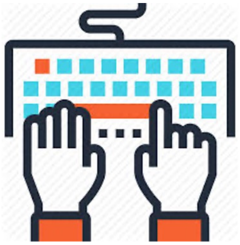
Logistic regression



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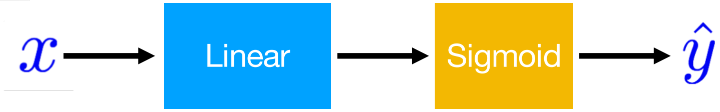
1 **Design your model using class**



2 **Construct loss and optimizer (select from PyTorch API)**

3 **Training cycle (forward, backward, update)**

```python
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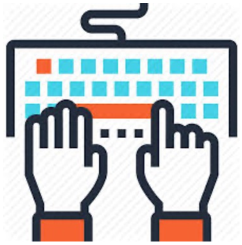
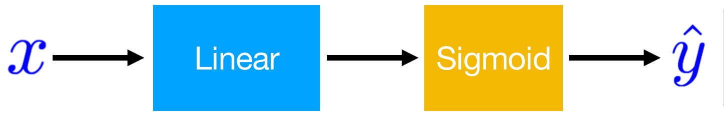
Logistic regression



```
0 1.6369143724441528
1 1.6119738817214966
2 1.5872894525527954
3 1.5628681182861328
4 1.5387169122695923
5 1.514843225479126
6 1.4912540912628174
7 1.467956781387329
8 1.4449583292007446
9 1.4222657680511475
10 1.3998862504959106
...
...
988 0.39138174057006836
989 0.39128318428993225
990 0.39118456840515137
991 0.3910861015319824
992 0.39098766446113586
993 0.3908892273902893
994 0.39079099893569946
995 0.39069271087646484
996 0.3905944228172302
997 0.39049631357192993
998 0.39039820432662964
999 0.3903001546859741
predict 1 hour  1.0 False
predict 7 hours 7.0 True
```

Exercise 6-1: Try other activation functions



Non-linear Activations

- ReLU
- ReLU6
- ELU
- SELU
- PReLU
- LeakyReLU
- Threshold
- Hardtanh
- Sigmoid
- Tanh

WHAT NEXT

Lecture 7: Wide and Deep

Backup slides

Building fun models

- Neural Net components
  - CNN
  - RNN
  - Activations
- Losses
- Optimizers

torch.nn

⊟ Convolution Layers

- Conv1d
- Conv2d
- Conv3d
- ConvTranspose1d
- ConvTranspose2d
- ConvTranspose3d

⊟ Recurrent layers

- RNN
- LSTM
- GRU
- RNNCell
- LSTMCell
- GRUCell

⊞ Containers
⊞ Convolution Layers
⊞ Pooling Layers
⊞ Padding Layers
⊞ Non-linear Activations
⊞ Normalization layers
⊞ Recurrent layers
⊞ Linear layers
⊞ Dropout layers
⊞ Sparse layers
⊞ Distance functions
⊞ Loss functions
⊞ Vision layers

⊟ Non-linear Activations

- ReLU
- ReLU6
- ELU
- SELU
- PReLU
- LeakyReLU
- Threshold
- Hardtanh
- Sigmoid
- Tanh
- LogSigmoid
- Softplus
- Softshrink
- Softsign
- Tanhshrink
- Softmin
- Softmax
- Softmax2d
- LogSoftmax

http://pytorch.org/docs/master/nn.html

⊟ **Loss functions**

- L1Loss
- MSELoss
- CrossEntropyLoss
- NLLLoss
- PoissonNLLLoss
- NLLLoss2d
- KLDivLoss
- BCELoss
- BCEWithLogitsLoss
- MarginRankingLoss
- HingeEmbeddingLoss
- MultiLabelMarginLoss
- SmoothL1Loss
- SoftMarginLoss
- MultiLabelSoftMarginLoss
- CosineEmbeddingLoss
- MultiMarginLoss
- TripletMarginLoss

http://pytorch.org/docs/master/nn.html

Loss functions

Table 1: List of losses analysed in this paper. $\mathbf{y}$ is true label as one-hot encoding, $\hat{\mathbf{y}}$ is true label as +1/-1 encoding, $\mathbf{o}$ is the output of the last layer of the network, $\cdot^{(j)}$ denotes jth dimension of a given vector, and $\sigma(\cdot)$ denotes probability estimate.

symbol	name	equation
$\mathcal{L}_1$	L_1 loss	$\|\mathbf{y}-\mathbf{o}\|_1$
$\mathcal{L}_2$	L_2 loss	$\|\mathbf{y}-\mathbf{o}\|_2^2$
$\mathcal{L}_1 \circ \sigma$	expectation loss	$\|\mathbf{y}-\sigma(\mathbf{o})\|_1$
$\mathcal{L}_2 \circ \sigma$	regularised expectation loss[1]	$\|\mathbf{y}-\sigma(\mathbf{o})\|_2^2$
$\mathcal{L}_\infty \circ \sigma$	Chebyshev loss	$\max_j \lvert\sigma(\mathbf{o})^{(j)}-\mathbf{y}^{(j)}\rvert$
hinge	hinge [13] (margin) loss	$\sum_j \max(0, \frac{1}{2}-\hat{\mathbf{y}}^{(j)}\mathbf{o}^{(j)})$
hinge^2	squared hinge (margin) loss	$\sum_j \max(0, \frac{1}{2}-\hat{\mathbf{y}}^{(j)}\mathbf{o}^{(j)})^2$
hinge^3	cubed hinge (margin) loss	$\sum_j \max(0, \frac{1}{2}-\hat{\mathbf{y}}^{(j)}\mathbf{o}^{(j)})^3$
log	log (cross entropy) loss	$-\sum_j \mathbf{y}^{(j)}\log\sigma(\mathbf{o})^{(j)}$
log^2	squared log loss	$-\sum_j [\mathbf{y}^{(j)}\log\sigma(\mathbf{o})^{(j)}]^2$
tan	Tanimoto loss	$\frac{-\sum_j \sigma(\mathbf{o})^{(j)}\mathbf{y}^{(j)}}{\|\sigma(\mathbf{o})\|_2^2+\|\mathbf{y}\|_2^2-\sum_j \sigma(\mathbf{o})^{(j)}\mathbf{y}^{(j)}}$
D_{CS}	Cauchy-Schwarz Divergence [3]	$-\log\frac{\sum_j \sigma(\mathbf{o})^{(j)}\mathbf{y}^{(j)}}{\|\sigma(\mathbf{o})\|_2\|\mathbf{y}\|_2}$

https://arxiv.org/pdf/1702.05659.pdf

torch.optim

- ***class***torch.optim.Adadelta
- ***class***torch.optim.Adagrad
- ***class***torch.optim.Adam
- ***class***torch.optim.Adamax
- ***class***torch.optim.ASGD
- ***class***torch.optim.RMSprop
- ***class***torch.optim.Rprop
- ***class***torch.optim.SGD

**http://pytorch.org/docs/master/optim.htm
l**

Three simple steps

1. Design your model using class

2. Construct loss and optimizer
(select from PyTorch API)

3. Training cycle
(forward, backward, update)

Exercise 6-1

- Try different optimizers

WHAT NEXT

Lecture 7: Wide and Deep