**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**ANS: 0.2676**

**Given *μ = 45minutes, σ = 8 minutes***

***The work begins after 10min, so the avg.time increases from 45 to 55min***

***Z = (X-μ)/6***

***= (60-55)/8***

***= 0.625***

***#Python Coding***

**P\_value = stats.norm.cdf(abs(0.625))**

**P\_value = 0.7340144**

**1-0.7340 = 0.266(approx..)**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**False**

Given,400 clerical employees, μ = 38, Standard deviation σ =6

Z-score = (Value - Mean)/SD

Z-score for 44  = (44 - 38)/6 =1=>84.13%

People above 44 age =100-84.13%=15.87%

Which is 63 out of 400

Z-score for 38=(38-38)/6=0=50%

Hence People b/w 38 and 44 age =84.13-50=34.13%

Which is 137 out of 400

**Hence we can say that the more employees lies b/w 38 and 44 rather than older than 44.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**True.**

Z-score = (Value - Mean)/SD

Z-score for 30=(30-38)/6=-1.33=36 out of 400

**Yes,The statement above mentioned is correct and it attracts about 36 employees.**

1. If *X1* ~ *N*(μ, σ2) and *X1* ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:**

Given X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2)  are two independent identically distributed random variables

According to the Central Limit Theorem, any large sum of independent, identically distributed(iid) random variables is approximately Normal

To find: 2X1

Thus, following the property of multiplication, we get

2X1 ~ N(2μ,22σ2)=> 2X1~N(2μ,4σ2)

And following property of addition

X1+X2~N(μ+ μ, σ2+ σ2)~N(2μ,2σ2)

And the difference between the two is given by

2X1-( X1+X2)~ N(2μ-2μ,2σ2+4σ2)~N(0,6σ2)

**The mean of 2X1 and X1+X2 is same but the variance of 2X1 is 2 times more than variance of X1+X2**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

ANS**: D.48.5, 151.5**

Ans: Given: P(a<x<b) = 0.99 ,mean =100,Standard Deviation = 20

#Using Python

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

Z(0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

Z(99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

a = -(20\*2.576) + 100= 48.5

b = (20\*2.576)+100= 151.5

**Two values symmetric about mean for the given standard normal distribution are[48.5,151.5]**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

*Ans:* Given $1=Rs. 45

*a*). 95% of the probability lies between 1.96 standard deviations of the mean.

=(12-1.96\*5, 1.96\*5)

=($2.2M,$22.8M)

**=(Rs.99M,Rs.1026M)**

b). 5th percentile of profit (in Rupees) for the company

=P(Z<=(p-12/5))=0.05

*From p values of z score table, we get*

*(p-12)/5=-1.644*

*P=12-8.22=3.78*

**Thus at 5th percentile of profit is Rs. 170.1 Million**

c). Loss is when profit < 0

Thus: p < 0

**The first division of company, thus have larger probability of making a loss in a given year**