

Maths

Part - A

1:

(MATHS) PART-A (2 Marks)

) convert decimal no $(12.25)_{10}$ to an equivalent binary no.

PART-A

) convert decimal no $(12.25)_{10}$ to an equivalent binary number

$(12.25)_{10} = 12, 0.25$

solution

$$\begin{array}{r} 12 \\ \times 2 \\ \hline 6 \\ \end{array}$$

$$\begin{array}{r} 0.25 \\ \times 2 \\ \hline 0.5 \\ \end{array}$$

$$\begin{array}{r} 6 \\ \times 2 \\ \hline 3 \\ \end{array}$$

$$\begin{array}{r} 0.5 \\ \times 2 \\ \hline 1 \\ \end{array}$$

$$\begin{array}{r} 3 \\ \times 2 \\ \hline 1 \\ \end{array}$$

$$\begin{array}{r} 1 \\ \times 2 \\ \hline 0 \\ \end{array}$$

$$\begin{array}{r} 1 \\ \times 2 \\ \hline 0 \\ \end{array}$$

$= 1100.01$

$(12.25)_{10} = (1100.01)_2$

PART - A

1) $(12.25)_{10} = 12, 0.25$

$$\begin{array}{r} 12 \\ \times 2 \\ \hline 6 \\ \times 2 \\ \hline 3 \\ \times 2 \\ \hline 1 \\ \hline 0-1 \end{array}$$

$0.25 \times 2 = 0.5$
 $0.5 \times 2 = 1.0$

$$= 1100.01$$

$$(12.25)_{10} = (1100.01)_2$$

2:

2) Let p, q & r be the propositions

p : you have the flu

q : miss the final examination

r : pass the proposition

$(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$ as an English sentence.

② Let p, q, r be the propositions

p : you have the flu

q : you miss the final examination

r : you pass the proposition

$(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$ as an English sentence

you have the flu, then you fail the proposition

or you miss the final examination, then you fail the proposition.

2) You have the flu, then you fail the proposition or you miss the final examination, then you fail the proposition.

3:

$$③ \text{ find } x, y \text{ & } z \text{ of } \begin{bmatrix} x+3 & 3x-2y \\ 3x-z & x+y+z \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -7+2y \\ y+4 & 2x \end{bmatrix}$$

$$③ \text{ find } x, y \text{ & } z \text{ if } \begin{bmatrix} x+3 & 3x-2y \\ -3x-z & x+y+z \end{bmatrix} = \begin{bmatrix} 2 & -7+2y \\ y+4 & 2x \end{bmatrix}$$

$$x+3 = 2 \Rightarrow x = 2-3 \Rightarrow x = -1$$

↙ $3x-2y = -7+2y \Rightarrow 3(-1)-2y = -4+2y$
 $\Rightarrow -3+4 = 2y+2y \Rightarrow 1 = 4y$
 $\Rightarrow y = \boxed{1}$

$$x+y+z = 2x \Rightarrow -1+1+z = 2(-1) \Rightarrow z = \boxed{-2}$$

$$3) \quad x + 3 = 2$$

$$x = 2 - 3$$

$$\boxed{x = -1}$$

$$3x - 2y = -7 + 2y$$

$$3(-1) - 2y = -7 + 2y$$

$$-3 + 7 = 2y + 2y$$

$$4 = 4y$$

$$\boxed{y=1} \quad \rightarrow$$

$$x + y + z = 2x$$

$$-1 + 1 + z = 2(-1)$$

$$\boxed{z = -2}$$

4:

4) find sum of square of eigen values

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

(4) Find sum of square of eigen values $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$

The given matrix is upper triangular Matrix

By using property :-

The eigen values of triangular matrix are just the diagonal elements of matrix.

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

The eigen values of Matrix A is 1, 2 and 3

$$\begin{aligned} \text{Sum of square of eigen values} &= 1^2 + 2^2 + 3^2 \\ &= 1 + 4 + 9 \\ &= 14 \end{aligned}$$

4) The given matrix is upper triangular matrix by using property ³:

The eigen values of triangular matrix are just the diagonal elements of matrix

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

The eigen values of Matrix A is 1, 2 and 3

$$\begin{aligned} \text{sum of square of eigen values} &= 1^2 + \\ &\quad 2^2 + 3^2 \\ &= 1 + 4 + 9 \\ &= 14 \end{aligned}$$

5:

5) Equivalence relation & eg:

5. Define equivalence relation & example.

A relation in a set A which is
(i) reflexive (ii) symmetric & (iii) transitive is called
an equivalence relation

eg:

Let R be the relation ($>$) from $A = \{1, 2, 3, 4\}$,
 $B = \{1, 3, 5\}$, then

$$R = \{(2, 1), (3, 1), (4, 1), (4, 3)\}$$

5) A relation in a set A which is
(i) reflexive (ii) symmetric (iii) transitive
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Eg: Let R be the relation ($>$) from
 $A = \{1, 2, 3, 4\}$, $B = \{1, 3, 5\}$, then

$$R = \{(2, 1), (3, 1), (4, 1), (4, 3)\}$$

6:

6) Write the following statement
in symbolic form

"if either Ravi takes calculus or
Kamal takes sociology then Sam will
take English"

6. write the following statement in symbolic form
"if either Ravi takes calculus or Kamel takes
sociology then Sam will take English"

P : Ravi takes calculus

Q : Kamal takes Sociology

R : Sam takes English

$$\therefore (P \vee Q) \rightarrow R$$

7:

③ By the pigeonhole principle, if 10 coins are distributed among 7 boxes then find minimum no. of coins required to be placed in same box.

④ By the pigeonhole principle, if 10 coins are distributed among 7 boxes then find minimum no. of coins required to be placed in same box.

$$10 \text{ coins} = n$$

$$7 \text{ boxes} = m$$

Number of pigeon holes (m) < Number of pigeons (n)

$$= \left[\frac{n-1}{m} \right] + 1$$

$$= \left[\frac{10-1}{7} \right] + 1$$

$$\therefore 5+1 = 6$$

so, the minimum number of coins required to be placed in the same box would be 6.

→ 40 coins = n

7 boxes = m

Number of pigeon holes (m) < Number of pigeons (n)

$$= \left\lceil \frac{n-1}{m} \right\rceil + 1$$

$$= \left\lceil \frac{40-1}{7} \right\rceil + 1$$

$$= 5 + 1 = 6$$

so, the minimum number of coins required to be placed in the same box would be 6.

8:

$$8) A \cup B = A \cup C \wedge A \cap B = A \cap C \\ \text{then prove } B = C$$

⑧ Given: $A \cup B = A \cup C$ & $A \cap B = A \cap C$

Given:

$$A \cup B = A \cup C \quad \text{&} \quad A \cap B = A \cap C$$

By using De Morgan's laws:

~~$A \cup B = (A \cap C) \cup (B \cap C)$~~

$$\Rightarrow (A \cup B)^c = (A \cup C)^c$$

$$\Rightarrow (A^c \cap B^c) \cup (B^c \cap C^c) = C$$

$$\Rightarrow (A \cap B) \cup (B \cap C) = C \rightarrow ①$$

~~Again, $A \cup B = A \cup C$~~



$$\Rightarrow (A \cup B) \cap B = (A \cup C) \cap B$$

$$\rightarrow B = (A \cap B) \cup (C \cap B)$$

$$\rightarrow B = (A \cap B) \cup (B \cap C) \rightarrow ②$$

from ① & ② we get

$B = C$ Hence proved

$$8) A \cup B = A \cup C \wedge A \cap B = A \cap C$$

$$\Rightarrow (A \cup B) \cap C = (A \cup C) \cap C$$

$$\Rightarrow (A \cap C) \cup (B \cap C) = C$$

$$\Rightarrow (A \cap B) \cup (B \cap C) = C \rightarrow \textcircled{1}$$

Again, $A \cup B = A \cup C$

$$\Rightarrow (A \cup B) \cap B = (A \cup C) \cap B \rightarrow$$

$$\Rightarrow B = (A \cap B) \cup (C \cap B)$$

$$\Rightarrow B = (A \cap B) \cup (B \cap C) \rightarrow \textcircled{2}$$

from $\textcircled{1} \wedge \textcircled{2}$ we get

$$B = C$$

\therefore Hence proved

9:

q) $A = \{1, 2, 3, 4, 5\}$ and relation
set $R = \{(a, b) \mid a+b \text{ is even}\}$.
Draw graph associated to the given
relation by taking A as the vertex
set and R as edge set.

What is Graph Theory?

1. Graph Theory is a branch of Mathematics in which we study graphs.
2. Graphs are mathematical structures which consists of a set V of vertices and set E of edges.
3. Vertices(also called nodes)

Origins of Graph Theory:

1. The 'feeble glance' which Leonhard Euler (1707–1783) directed towards the geometry of position consists of a single paper now considered to be the starting point of modern graph theory.
2. The paper published by Euler appeared in Commentarii Academiae Scientiarum Imperialis Petropolitanae in 1736 on the Seven Bridges of Königsberg is regarded as the first paper in the history of graph theory.
3. He created first graph to simulate a real time place and situation to solve a problem which was then considered one of the toughest problems.

10:

10) short notes on Origin of
Graph Theory

Part - B

11:

(10 marks)

i) a) i) Prove by mathematical induction $1+3+5+\dots+(2n-1) = n^2$

ii) Buying a new car there are two body types Sedan and

i)

16 Marks

ii) Mathematical Induction

i) $1+3+5+\dots+(2n-1) = n^2$

Step 1 Prove that the statement is true for $n=1$

$$(2(1)-1) = 1^2$$

$$2-1 = 1$$

$$1 = 1$$

$$\text{LHS} = \text{RHS}$$

$p(1)$ is true

Step 2 Assume that the proposition is true for $n=k$

$$1+3+5+\dots+(2k-1) = k^2 \text{ is true}$$

Step 3 P.T the proposition is true for $n = k+1$

$$1+3+5\dots(2(k+1)-1) = k^2$$

$$1+3+5\dots2k+2-1 = k^2$$

$$1+3+5\dots(2k-1)$$

$$1+3+5\dots+(2k-1)+(2k+1) = (k+1)^2$$

$$k^2 + 2k+1 = (k+1)^2$$

$$(k+1)^2 = (k+1)^2$$



Hence Proved

ii)

ii) Car

How many total choices are there

Let 'a' be the body styles of car

'b' be the colours of the car

'c' be the models of the car

$$\text{Total choices} = a \times b \times c$$

$$= 2 \times 5 \times 3$$

$$= 30$$



There are 30 choices

12:

[PART-B]

12) (P.) Prove $a \rightarrow (b \rightarrow c)$, $a \rightarrow (b \wedge \neg c)$ and $(a \wedge d)$ are inconsistent (8M)

ii) Using truth table prove

$$P \rightarrow (Q \rightarrow R) \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R) \quad (8M)$$

- i) $a \rightarrow (b \rightarrow c)$, $(d \rightarrow (b \wedge \neg c))$ and $(a \wedge d)$ is invalid:

(1) Premises are $(P \rightarrow Q)$

$$A \rightarrow (B \rightarrow C)$$

$$D \rightarrow (B \wedge \neg C)$$

$$A \wedge D$$

\therefore Contraria
positive
 $\Rightarrow P \rightarrow Q \equiv$
 $\neg A \rightarrow \neg B \rightarrow \neg P$

Conclusion Eq.

F.

$A \rightarrow (B \rightarrow C)$, $D \rightarrow (B \wedge \neg C)$ and $(A \wedge D)$.

S.no	Statement	Rule	Reason.
1.	$A \wedge D$	Rule P	Given Premise.
2.	A	Rule T	Simplification, 1
3.	D	Rule T	Simplification, 1
4.	$A \rightarrow (B \rightarrow C)$	Rule P	Given premise.
5.	$B \rightarrow C$	Rule T	Modus Ponens, 2, 3
6.	$\neg B \vee C$	Rule T	Law of Conditional, 5.
7.	$D \rightarrow (B \wedge \neg C)$	Rule P	Given premise
8.	$\neg(B \wedge C) \rightarrow \neg D$	Rule T	Contraria Positive, 7
9.	$\neg(\neg B \vee C) \rightarrow \neg D$	Rule T	Negate Statement, 8
10.	$\neg D$	Rule T	Modus Ponens, 6, 9
11.	$D \wedge \neg D$	Rule T	Conjunction of 3, 10
12	F	Rule T	Negation Law, 11.



You

Prove $a \rightarrow (b \rightarrow c)$, $(d \rightarrow (b \wedge \neg c))$ and $(a \wedge d)$ are invalid:



ChatGPT

To prove that the statement $a \rightarrow (b \rightarrow c)$, $(d \rightarrow (b \wedge \neg c))$ and $(a \wedge d)$ is invalid, we need to find a counterexample where the premises are true, but the conclusion is false.

Let's construct a truth table to evaluate the validity of the statement:

a	b	c	$a \rightarrow (b \rightarrow c)$	$d \rightarrow (b \wedge \neg c)$	$a \wedge d$	Conclusion
T	T	T	T	F	T	T
T	T	F	F	T	T	T
T	F	T	T	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	F	F
F	T	F	T	T	F	F
F	F	T	T	T	F	F
F	F	F	T	T	F	F

In the truth table, when a is true and d is true, but the conclusion $(a \wedge d)$ is false, indicating that there is a counterexample where the premises are true, but the conclusion is false. Therefore, the statement is invalid.



ii) Using Truth table: P. T.: - $P \rightarrow (Q \rightarrow R) \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)$



You

Using Truth table: P. T.:

$P \rightarrow (Q \rightarrow R) \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)$



ChatGPT

To evaluate the validity of the statement $P \rightarrow (Q \rightarrow R) \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)$, we will construct a truth table for all possible truth values of P , Q , and R , and then evaluate the truth value of the statement.

Let's construct the truth table:

P	Q	R	$Q \rightarrow R$	$P \rightarrow (Q \rightarrow R)$	$P \rightarrow Q$	$P \rightarrow R$	$(P \rightarrow Q) \rightarrow (P \rightarrow R)$	Concl
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T
T	F	T	T	T	F	T	T	T
T	F	F	T	T	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

In the truth table, we can see that the conclusion column always matches the $(P \rightarrow (Q \rightarrow R)) \Rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$ column, indicating that the statement is valid for all possible truth values of P , Q , and R .

Therefore, using the truth table, we have proved that the statement $P \rightarrow (Q \rightarrow R) \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)$ is a tautology (always true).

◻ ⊙ ♦

↓

13:

13) (i) $f(x)$ find eigen values
A = $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ & eigen vectors
(16M)

13

Find the Eigen value & Eigen Vector

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 1 Frame the characteristic equation

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

 S_1 = Sum of diagonal number

$$S_1 = 2+2+1 = 5$$

 S_2 = Sum of minors of diagonal number

$$S_2 = \left| \begin{array}{cc} 2 & 1 \\ 0 & 1 \end{array} \right| + \left| \begin{array}{cc} 2 & 1 \\ 0 & 1 \end{array} \right| + \left| \begin{array}{cc} 2 & 1 \\ 1 & 2 \end{array} \right|$$

$$S_2 = (2-0) + (2-0) + (4-1)$$

$$S_2 = 2+2+3 = 7$$

 S_3 = Determinant of Diagonal number Matrix

$$S_3 = 2 \left| \begin{array}{cc} 2 & 1 \\ 0 & 1 \end{array} \right| - 1 \left| \begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array} \right| + 1 \left| \begin{array}{cc} 1 & 2 \\ 0 & 0 \end{array} \right|$$

$$S_3 = 2(2-0) - 1(1-0) + 1(0)$$

$$S_3 = 4 - 1$$

$$\boxed{S_3 = 3}$$

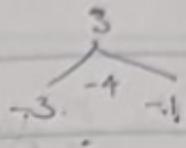
The value of $S_1 = 5, S_2 = 7, S_3 = 3$

\therefore The characteristic eqn. is

$$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$$

Step 2 Find the Eigen Values

$$1 \left| \begin{array}{cccc} 1 & -5 & 7 & -3 \\ 0 & 1 & -4 & 3 \\ \hline 1 & -4 & 3 & | 0 \end{array} \right.$$



Quadratic equation $\rightarrow \lambda^2 - 4\lambda + 3 = 0$

$$(\lambda - 3)(\lambda - 1) = 0$$



$$\boxed{\lambda = 3, \lambda = 1}$$

\therefore The Eigen Values are 1, 3, 1

Step 3 Find the Eigen Vector

$$[A - \lambda I][x] = 0$$

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x = 0$$

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} x = 0$$

$$\begin{bmatrix} 2-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

case (i) $\lambda = 1$

Let's assume $x_3 = 0$,

$$x_1 + x_2 = 0$$

Holding $x_2 = 1$, we get

$$x_1 = x_2$$

$$x_1 = -1$$

Thus $x_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

case (ii) since 1 is repeated

$$\lambda = 1$$

Let's assume $x_2 = 0$

$$x_1 + x_3 = 0$$

Holding $x_3 = 1$, we get

$$x_1 = -x_3$$

$$x_1 = -1$$

Thus, $x_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

Part-B

16 Marks

1A) Let R be the following relation on $A = \{1, 2, 3, 4\}$

$$R = \{(1, 3), (1, 4), (3, 2), (3, 3), (3, 4)\}$$

i) Matrix of relation R

ii) domain and range of R

iii) composition of relation R

iv) draw a directed graph of R

1B) Let R be the following relation on A

$$A = \{1, 2, 3, 4\}$$

$$R = \{(1, 3), (1, 4), (3, 2), (3, 3), (3, 4)\}$$

i) Matrix of relation R .

	1	2	3	4		1	1	1
1	0	0	1	1		1	1	1
2	0	0	0	0		0	1	0
3	0	1	1	1		0	0	1
4	0	0	0	0		0	0	0

ii) Domain and range of R

Domain of R = {1, 3} The set of all first element in the ordered pairs.

Range of R = {2, 3, 4} The set of all second element in the ordered pairs.

iii) composition of relation R.

$R \circ R = \{(1, 2), (1, 3), (1, 4), (3, 2), (3, 3), (3, 4)\}$ calculated by taking each ordered pair.

iv) draw a directed graph of R

1 → 3

→ 4

3 → 2

→ 3

→ 4

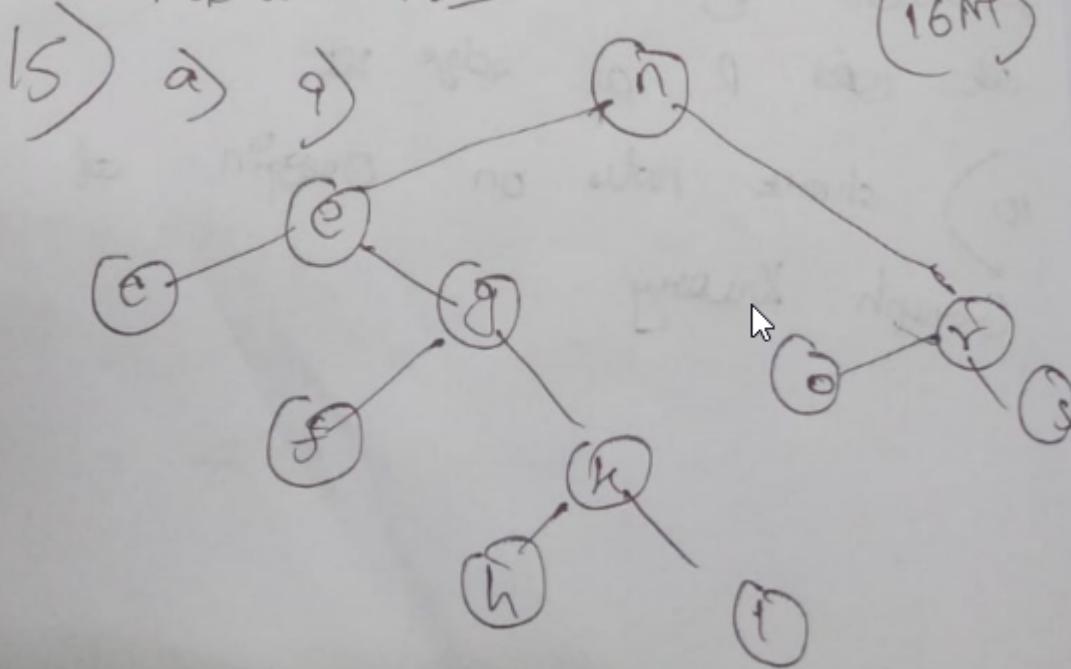
Part - C (i guess this comes under part - c)

15:

Pre order tree
Post order ~~postorder~~ order

PART - C

(16 M)



Know the concept and DIY

<https://www.javatpoint.com/tree-traversal>

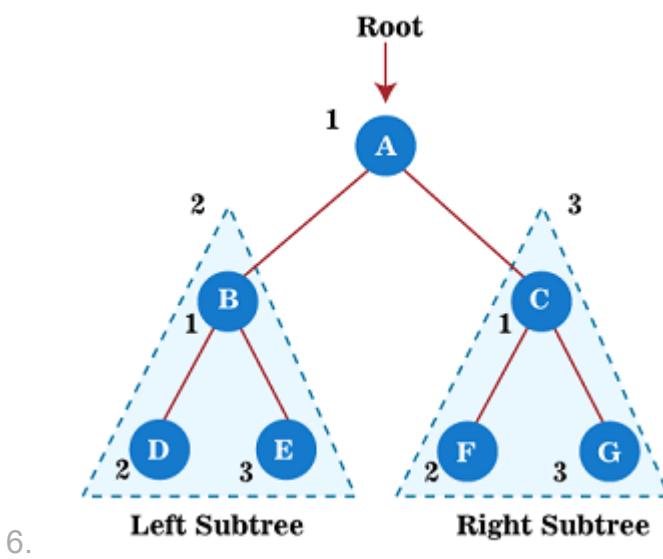
i)

1. Preorder traversal:

1. This technique follows the '**root left right**' policy.
2. It means that, first root node is visited after that the left subtree is traversed recursively, and finally, right subtree is recursively traversed.
3. As the root node is traversed before (or pre) the left and right subtree, it is called preorder traversal.

4. Algorithm:

1. Until all nodes of the tree are not visited
 1. Step 1 - Visit the root node
 2. Step 2 - Traverse the left subtree recursively.
 3. Step 3 - Traverse the right subtree recursively.
5. Output: A → B → D → E → C → F → G



6.

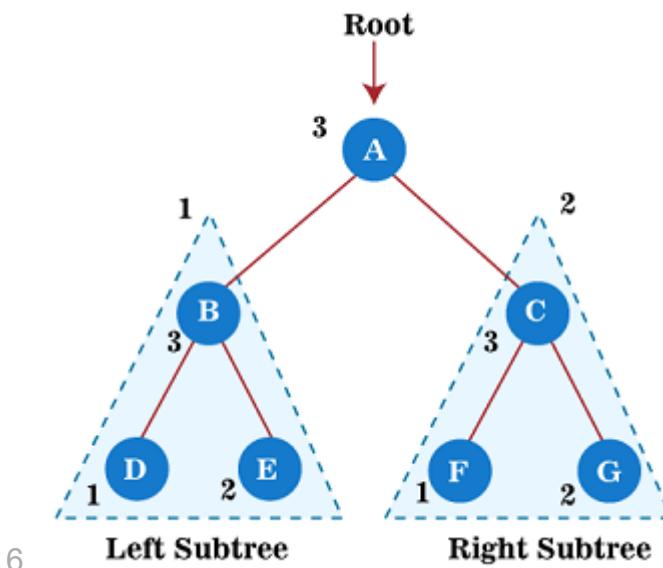
2. Postorder traversal:

1. This technique follows the '**left-right root**' policy.
2. It means that the first left subtree of the root node is traversed, after that recursively traverses the right subtree, and finally, the root node is traversed.
3. As the root node is traversed after (or post) the left and right subtree, it is called postorder traversal.

4. Algorithm:

1. Until all nodes of the tree are not visited
 1. Step 1 - Traverse the left subtree recursively.
 2. Step 2 - Traverse the right subtree recursively.
 3. Step 3 - Visit the root node.

5. Output: **D → E → B → F → G → C → A**



6.

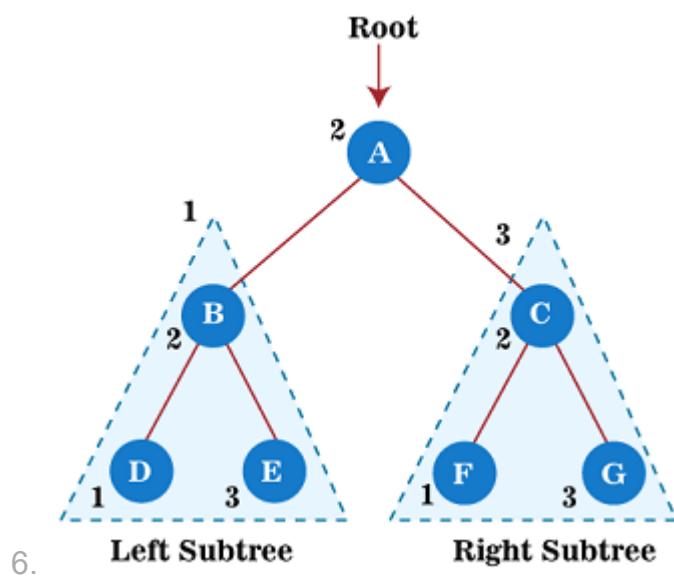
3. Inorder traversal:

1. This technique follows the '**left-right root**' policy.
2. It means that first left subtree is visited after that root node is traversed, and finally, the right subtree is traversed.
3. As the root node is traversed between the left and right subtree, it is named inorder traversal.

4. Algorithm:

1. Until all nodes of the tree are not visited
 1. Step 1 - Traverse the left subtree recursively.

2. Step 2 - Visit the root node.
3. Step 3 - Traverse the right subtree recursively.
5. Output: $D \rightarrow B \rightarrow E \rightarrow A \rightarrow F \rightarrow C \rightarrow G$



ii)

PART - C: ii) For the graph of Figure 1.20, give an example of each of the following or explain why no such an example exists.

1. An $x - y$ walk of length 6:

1. An example of an $x - y$ walk of length 6 in the graph would be: $x - u - v - w - z - t - y$
2. This walk consists of 6 edges.

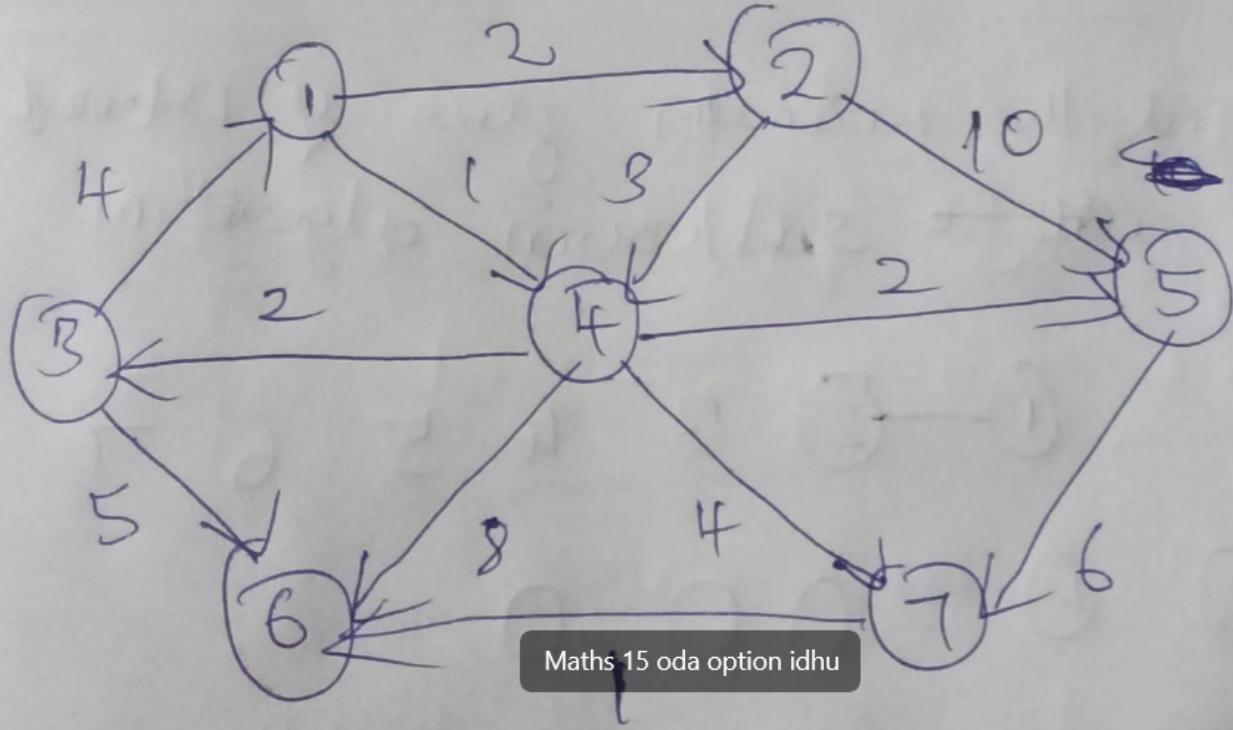
2. A $v - w$ trail that is not a $v - w$ path:

1. In the graph, a $v - w$ trail that is not a $v - w$ path could be: $v - u - w$.
2. This trail visits the vertex u twice, which makes it a trail but not a path.

Additional:

15) Find the maximum flow of network
using salkerson's algorithm

selected graph



(14) If $\{A, B, C\}$
both
closely and
geographically
prove
 $A - (B \cap C)$

$$= (A - B) \cup (A - C)$$

8M

Let $S = (A, B, C)$
 $T = (B, C, A)$
 $\omega = (A, D)$

content & re

true diagram

Maths 14 question oda option idhu

END :)