Detection of Stochastic Gravitational Wave Background Based on Topological Data Analysis

M.Sc. Thesis defense

Ali Salehi

Computational Cosmology Group
Department of Physics
Shahid Beheshti University

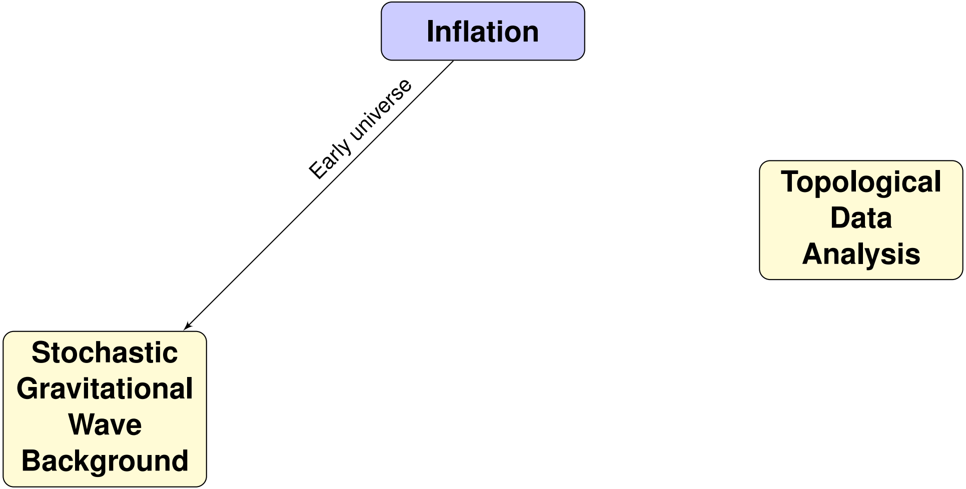
28 Shahrivar 2023

Overview

Topological Data Analysis

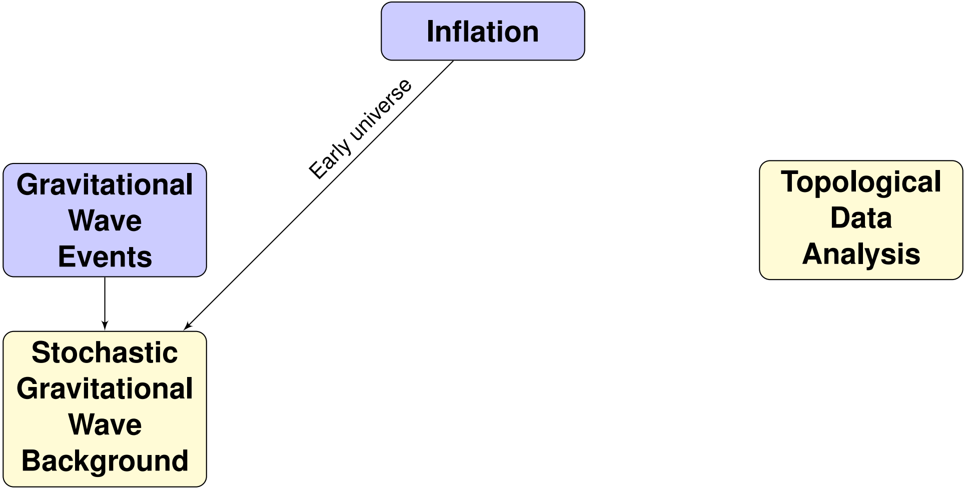
Stochastic Gravitational Wave Background

Overview



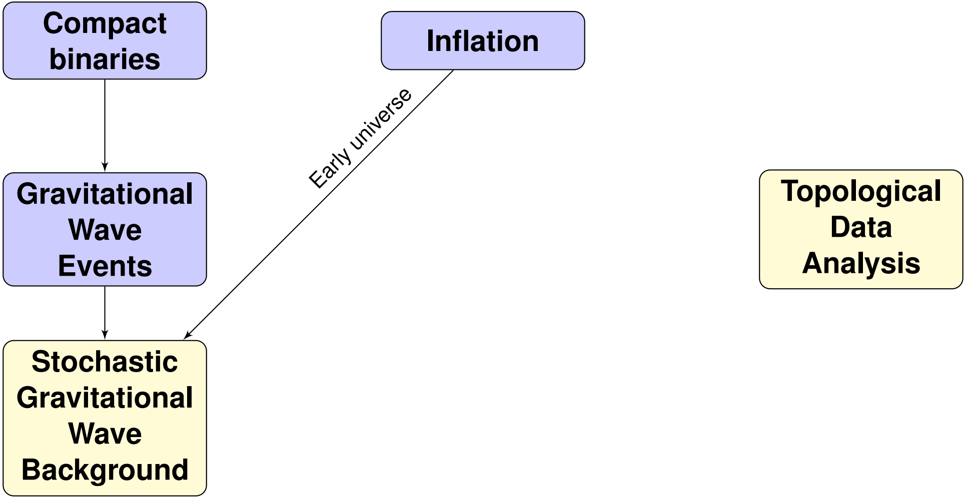
Inflation
Early universe
Topological Data Analysis
Stochastic Gravitational Wave Background

Overview

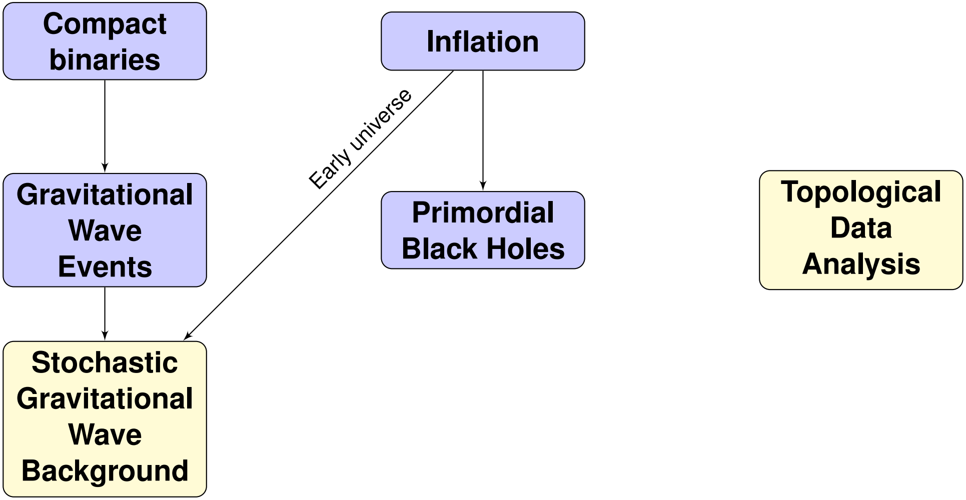


Inflation
Early universe
Gravitational Wave Events
Stochastic Gravitational Wave Background
Topological Data Analysis

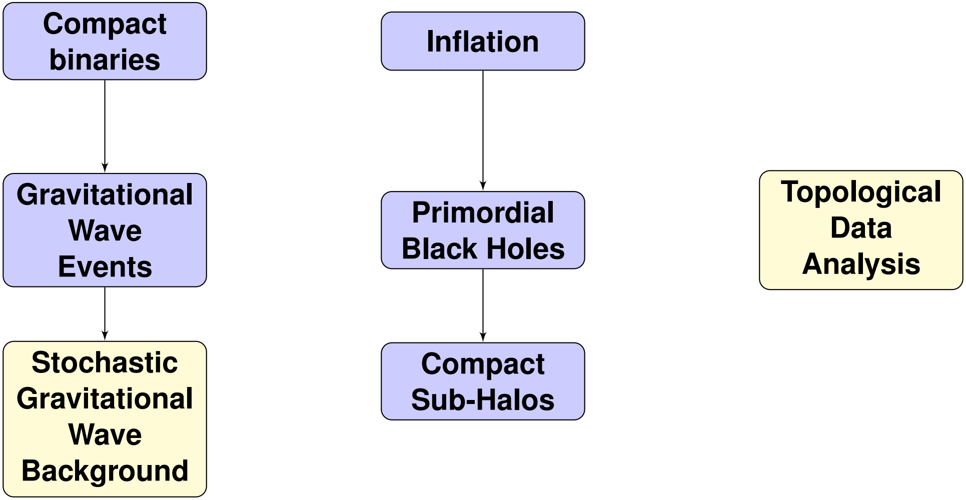
Overview


Compact binaries
Inflation
Early universe
Gravitational Wave Events
Topological Data Analysis
Stochastic Gravitational Wave Background

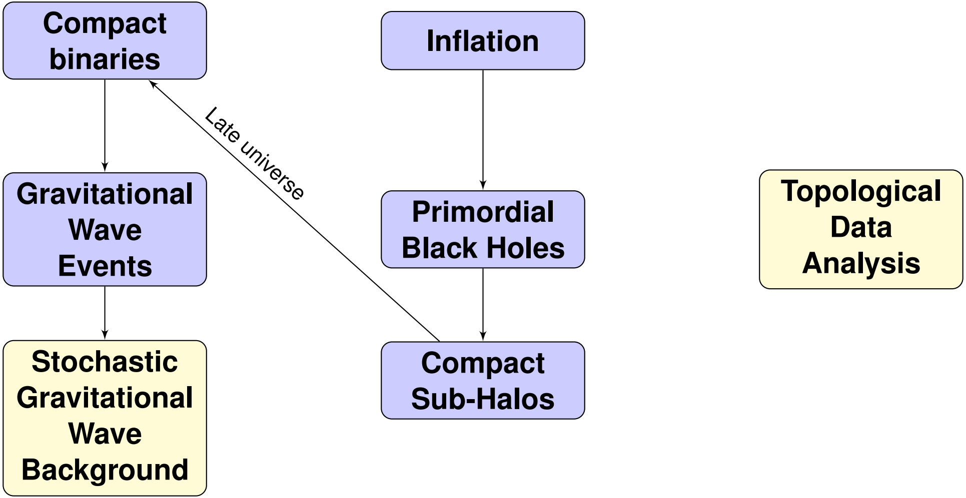
Overview


Compact binaries
Gravitational Wave Events
Stochastic Gravitational Wave Background
Inflation
Early universe
Primordial Black Holes
Topological Data Analysis

Overview

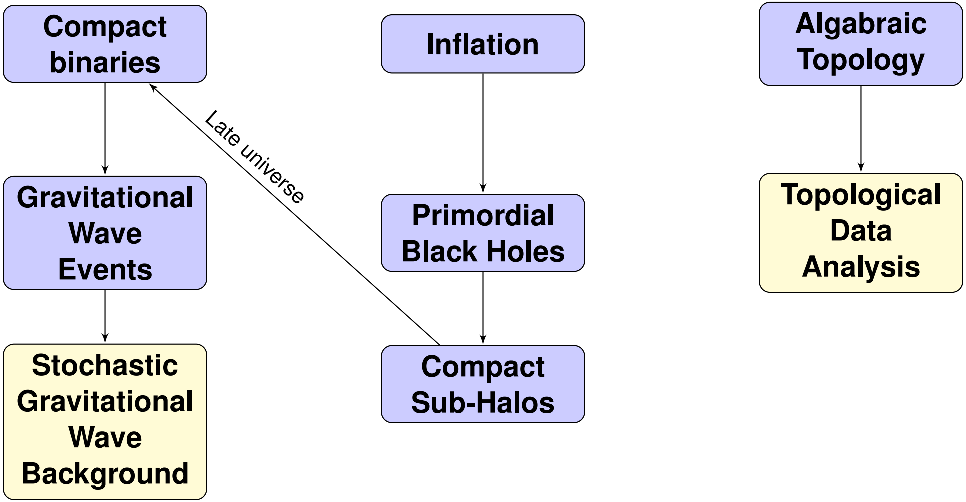

Compact binaries
Gravitational Wave Events
Stochastic Gravitational Wave Background
Inflation
Primordial Black Holes
Compact Sub-Halos
Topological Data Analysis

Overview

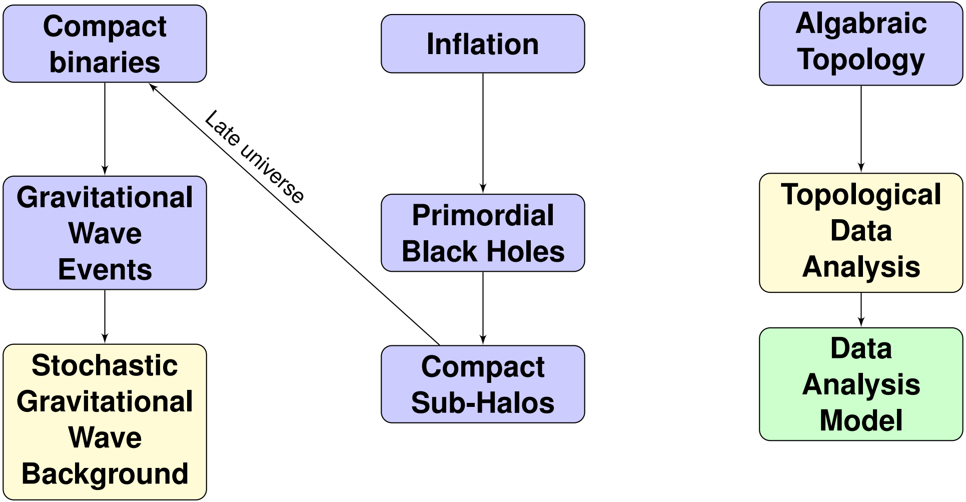


Compact binaries
Inflation
Late universe
Gravitational Wave Events
Primordial Black Holes
Topological Data Analysis
Stochastic Gravitational Wave Background
Compact Sub-Halos

Overview

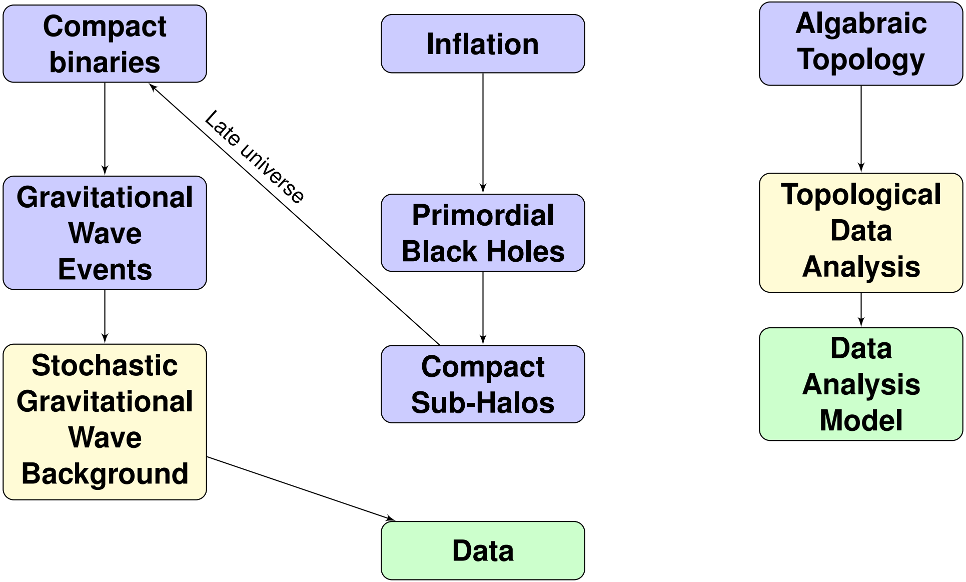

Compact binaries
Inflation
Algabraic Topology
Late universe
Gravitational Wave Events
Primordial Black Holes
Topological Data Analysis
Stochastic Gravitational Wave Background
Compact Sub-Halos

Overview

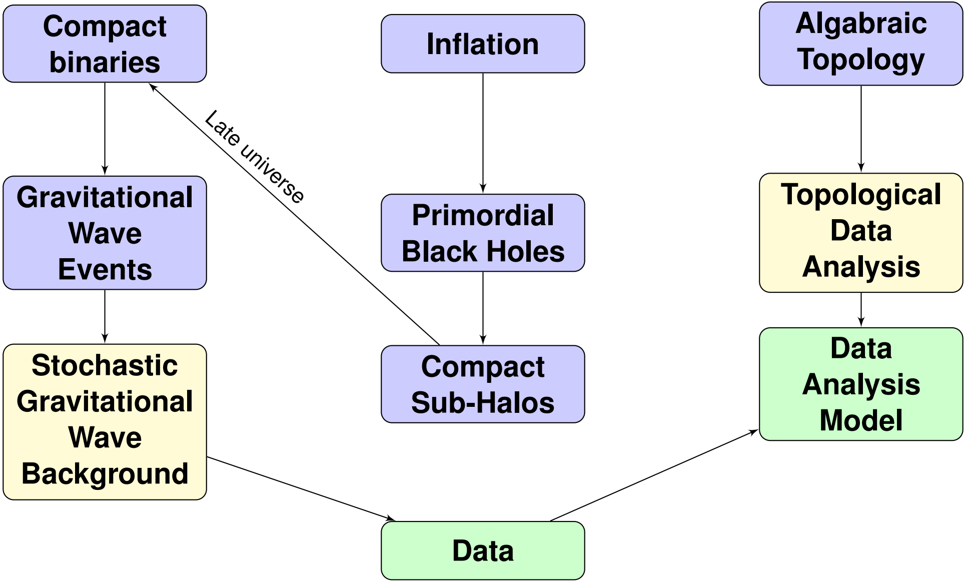


Compact binaries
Gravitational Wave Events
Stochastic Gravitational Wave Background
Late universe
Inflation
Primordial Black Holes
Compact Sub-Halos
Algabraic Topology
Topological Data Analysis
Data Analysis Model

Overview

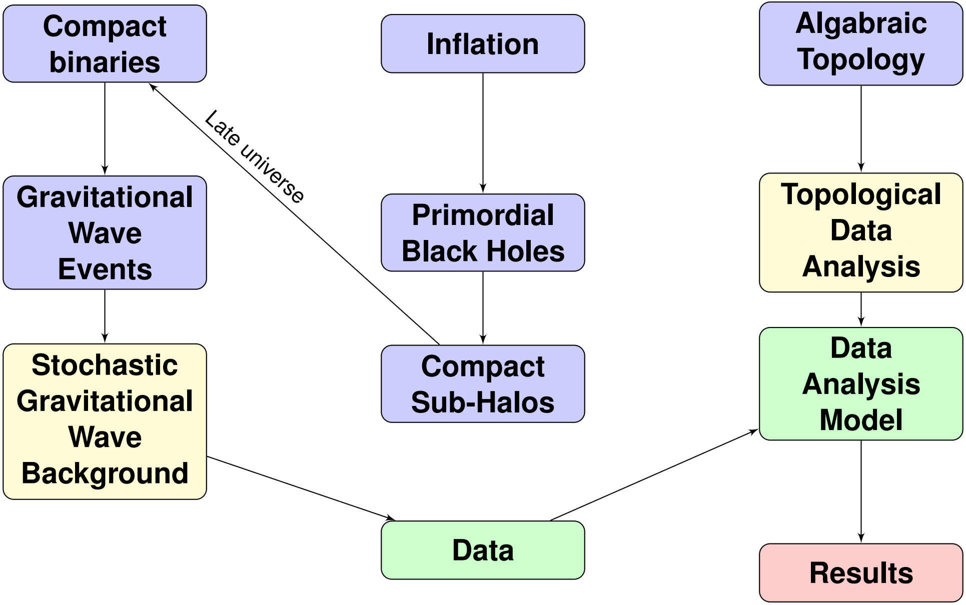

Compact binaries
Inflation
Algabraic Topology
Late universe
Gravitational Wave Events
Primordial Black Holes
Topological Data Analysis
Stochastic Gravitational Wave Background
Compact Sub-Halos
Data Analysis Model
Data

Overview



Compact binaries
Inflation
Algabraic Topology
Late universe
Gravitational Wave Events
Primordial Black Holes
Topological Data Analysis
Stochastic Gravitational Wave Background
Compact Sub-Halos
Data Analysis Model
Data

Overview



Compact binaries
Inflation
Algebraic Topology
Late universe
Gravitational Wave Events
Primordial Black Holes
Topological Data Analysis
Stochastic Gravitational Wave Background
Compact Sub-Halos
Data Analysis Model
Data
Results

Outline

Section 1

Gravitational Waves

Where do the gravitational waves come from?

Einstein Equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Where do the gravitational waves come from?

Einstein Equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$
$$G_{\mu\nu} = G_{\mu\nu}(g_{\mu\nu}, \dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu})$$

Where do the gravitational waves come from?

Einstein Equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$G_{\mu\nu} = G_{\mu\nu}(g_{\mu\nu}, \dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu})$$

$$= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

Where do the gravitational waves come from?

Einstein Equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\begin{aligned} G_{\mu\nu} &= G_{\mu\nu}(g_{\mu\nu}, \dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \\ &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \end{aligned}$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

Where do the gravitational waves come from?

Einstein Equations:

$$
\begin{aligned}
G_{\mu\nu} &= \frac{8\pi G}{c^4} T_{\mu\nu} \\
G_{\mu\nu} &= G_{\mu\nu}(g_{\mu\nu}, \dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \\
&= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R
\end{aligned}
$$

$$
\begin{aligned}
R &= g^{\mu\nu} R_{\mu\nu} \\
R_{\mu\nu} &= R^{\sigma}{}_{\mu\sigma\nu}
\end{aligned}
$$

Where do the gravitational waves come from?

Einstein Equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\begin{aligned} G_{\mu\nu} &= G_{\mu\nu}(g_{\mu\nu}, \dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \\ &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \end{aligned}$$

$$\begin{aligned} R &= g^{\mu\nu} R_{\mu\nu} \\ R_{\mu\nu} &= R^{\sigma}{}_{\mu\sigma\nu} \\ R^{\sigma}{}_{\mu\rho\nu} &= R^{\sigma}{}_{\mu\rho\nu}(\dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \end{aligned}$$

Where do the gravitational waves come from?

Einstein Equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$G_{\mu\nu} = G_{\mu\nu}(g_{\mu\nu}, \dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu})$$

$$= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$R_{\mu\nu} = R^{\sigma}{}_{\mu\sigma\nu}$$

$$R^{\sigma}{}_{\mu\rho\nu} = R^{\sigma}{}_{\mu\rho\nu}(\dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu})$$

Einstein Field Equations:

Where do the gravitational waves come from?

Einstein Equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\begin{aligned} G_{\mu\nu} &= G_{\mu\nu}(g_{\mu\nu}, \dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \\ &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \end{aligned}$$

$$\begin{aligned} R &= g^{\mu\nu} R_{\mu\nu} \\ R_{\mu\nu} &= R^{\sigma}{}_{\mu\sigma\nu} \\ R^{\sigma}{}_{\mu\rho\nu} &= R^{\sigma}{}_{\mu\rho\nu}(\dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \end{aligned}$$

Einstein Field Equations:

$$G_{\mu\nu} = 0$$

Where do the gravitational waves come from?

Einstein Equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\begin{aligned} G_{\mu\nu} &= G_{\mu\nu}(g_{\mu\nu}, \dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \\ &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \end{aligned}$$

$$\begin{aligned} R &= g^{\mu\nu} R_{\mu\nu} \\ R_{\mu\nu} &= R^{\sigma}{}_{\mu\sigma\nu} \\ R^{\sigma}{}_{\mu\rho\nu} &= R^{\sigma}{}_{\mu\rho\nu}(\dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \end{aligned}$$

Einstein Field Equations:

$$G_{\mu\nu} = 0$$

if: $g_{\mu\nu} = \eta_{\mu\nu}$ (everywhere)

Where do the gravitational waves come from?

Einstein Equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\begin{aligned} G_{\mu\nu} &= G_{\mu\nu}(g_{\mu\nu}, \dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \\ &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \end{aligned}$$

$$\begin{aligned} R &= g^{\mu\nu} R_{\mu\nu} \\ R_{\mu\nu} &= R^{\sigma}{}_{\mu\sigma\nu} \\ R^{\sigma}{}_{\mu\rho\nu} &= R^{\sigma}{}_{\mu\rho\nu}(\dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \end{aligned}$$

Einstein Field Equations:

$$G_{\mu\nu} = 0$$

if: $g_{\mu\nu} = \eta_{\mu\nu}$ (everywhere)

$\therefore$ $R_{\mu\nu} = 0 \implies G_{\mu\nu} = 0$

Where do the gravitational waves come from?

Einstein Equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$G_{\mu\nu} = G_{\mu\nu}(g_{\mu\nu}, \dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu})$$
$$= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$R = g^{\mu\nu} R_{\mu\nu}$$
$$R_{\mu\nu} = R^{\sigma}{}_{\mu\sigma\nu}$$
$$R^{\sigma}{}_{\mu\rho\nu} = R^{\sigma}{}_{\mu\rho\nu}(\dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu})$$

Einstein Field Equations:

$$G_{\mu\nu} = 0$$

if: $g_{\mu\nu} = \eta_{\mu\nu}$ (everywhere)

$\therefore$ $R_{\mu\nu} = 0 \implies G_{\mu\nu} = 0$

First order variations:

Where do the gravitational waves come from?

Einstein Equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\begin{aligned} G_{\mu\nu} &= G_{\mu\nu}(g_{\mu\nu}, \dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \\ &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \end{aligned}$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$R_{\mu\nu} = R^{\sigma}{}_{\mu\sigma\nu}$$

$$R^{\sigma}{}_{\mu\rho\nu} = R^{\sigma}{}_{\mu\rho\nu}(\dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu})$$

Einstein Field Equations:

$$G_{\mu\nu} = 0$$

$$\text{if:} \quad g_{\mu\nu} = \eta_{\mu\nu} \quad \text{(everywhere)}$$

$$\therefore \quad R_{\mu\nu} = 0 \implies G_{\mu\nu} = 0$$

First order variations:

$$\text{if:} \quad g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon h_{\mu\nu}$$

Where do the gravitational waves come from?

Einstein Equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\begin{aligned} G_{\mu\nu} &= G_{\mu\nu}(g_{\mu\nu}, \dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \\ &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \end{aligned}$$

$$\begin{aligned} R &= g^{\mu\nu} R_{\mu\nu} \\ R_{\mu\nu} &= R^{\sigma}{}_{\mu\sigma\nu} \\ R^{\sigma}{}_{\mu\rho\nu} &= R^{\sigma}{}_{\mu\rho\nu}(\dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \end{aligned}$$

Einstein Field Equations:

$$G_{\mu\nu} = 0$$

$$\begin{aligned} \text{if:} \quad & g_{\mu\nu} = \eta_{\mu\nu} \quad \text{(everywhere)} \\ \therefore \quad & R_{\mu\nu} = 0 \implies G_{\mu\nu} = 0 \end{aligned}$$

First order variations:

$$\text{if:} \quad g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon h_{\mu\nu}$$

$$\vdots$$

Where do the gravitational waves come from?

Einstein Equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\begin{aligned} G_{\mu\nu} &= G_{\mu\nu}(g_{\mu\nu}, \dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \\ &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \end{aligned}$$

$$\begin{aligned} R &= g^{\mu\nu} R_{\mu\nu} \\ R_{\mu\nu} &= R^{\sigma}{}_{\mu\sigma\nu} \\ R^{\sigma}{}_{\mu\rho\nu} &= R^{\sigma}{}_{\mu\rho\nu}(\dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu}) \end{aligned}$$

Einstein Field Equations:

$$G_{\mu\nu} = 0$$

$$\begin{aligned} &\text{if:} \quad g_{\mu\nu} = \eta_{\mu\nu} \quad \text{(everywhere)} \\ &\therefore \quad R_{\mu\nu} = 0 \implies G_{\mu\nu} = 0 \end{aligned}$$

First order variations:

$$\text{if:} \quad g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon h_{\mu\nu}$$

$$\vdots$$

$$\therefore \quad \boxed{\Box h_{\mu\nu} = 0}$$

But what are gravitational waves?

$$\Box h_{\mu\nu} = 0$$

But what are gravitational waves?

$$\Box h_{\mu\nu} = 0$$

$$\rightarrow \quad \frac{1}{c^2}\frac{\partial^2}{\partial t^2} h_A = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) h_A$$

$(T{=}+,\times)$

But what are gravitational waves?

$$\Box h_{\mu\nu} = 0$$

$$\rightarrow \quad \frac{1}{c^2}\frac{\partial^2}{\partial t^2} h_A = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) h_A$$

$(T{=}+,\times)$

Image credit: Youtube: minutephysics

But what are gravitational waves?

$$\Box h_{\mu\nu} = 0$$

$$\rightarrow \quad \frac{1}{c^2}\frac{\partial^2}{\partial t^2} h_A = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) h_A$$

$$(T{=}+,\times)$$

In Transverse-Traceless Guage:

Image credit: Youtube: minutephysics

But what are gravitational waves?

$$\Box h_{\mu\nu} = 0$$

$$\rightarrow \quad \frac{1}{c^2}\frac{\partial^2}{\partial t^2}h_A = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) h_A$$

$(T{=}+,\times)$

In Transverse-Traceless Guage:

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Image credit: Youtube: minutephysics

Polarizations are the signature of gravitational waves

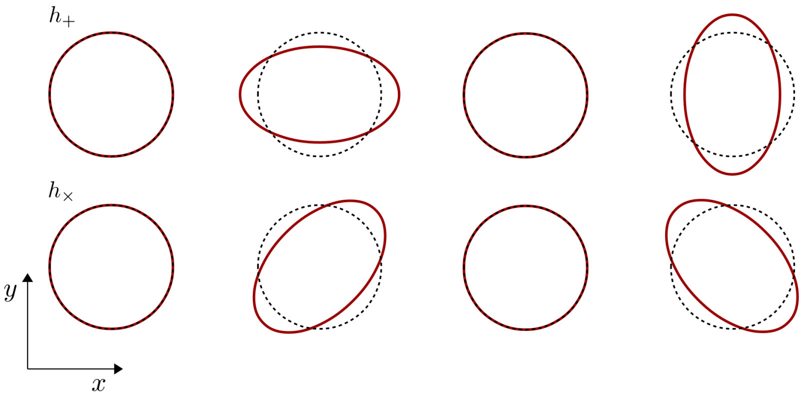


Image credit:

S. Dodelson, F. Schmidt, Modern Cosmology

Polarizations are the signature of gravitational waves

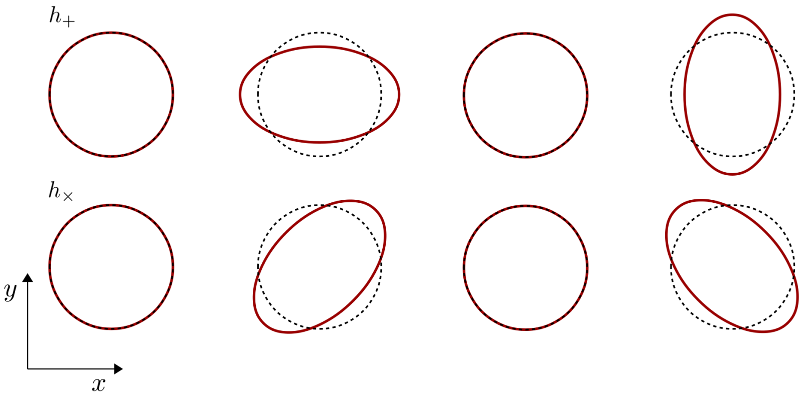


Image credit:

S. Dodelson, F. Schmidt, Modern Cosmology

Other Gravitational wave features:

Polarizations are the signature of gravitational waves

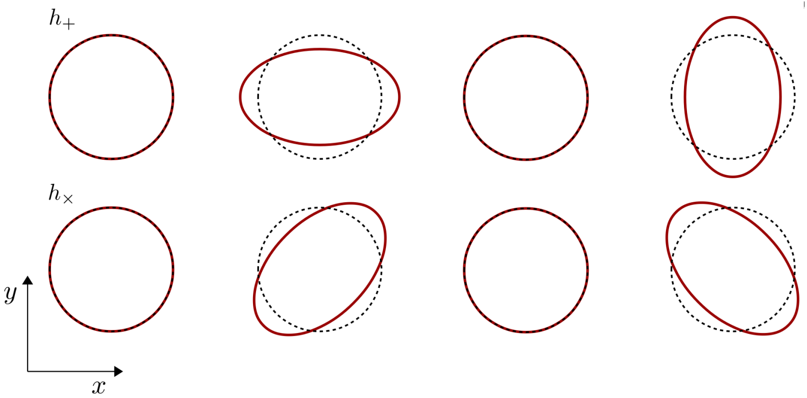


Image credit:

S. Dodelson, F. Schmidt, Modern Cosmology

Other Gravitational wave features:

- They can not be detected locally (Because of equivalance principal)

Polarizations are the signature of gravitational waves

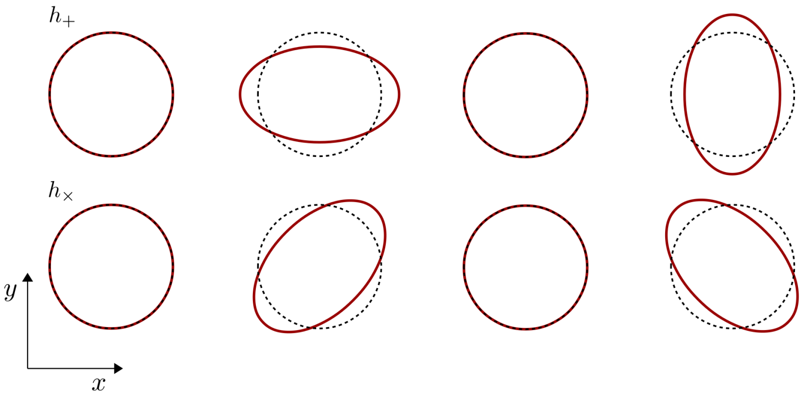


Image credit:

S. Dodelson, F. Schmidt, Modern Cosmology

Other Gravitational wave features:

- They can not be detected locally (Because of equivalance principal)
- They travel at the speed of light

Sources of Gravitational Waves

Burst:

Sources of Gravitational Waves

Burst:

- Supernovae

Sources of Gravitational Waves

Burst:

- Supernovae

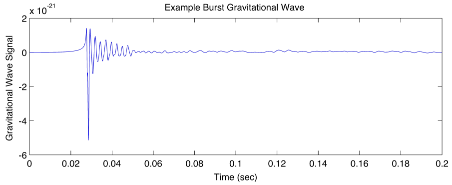
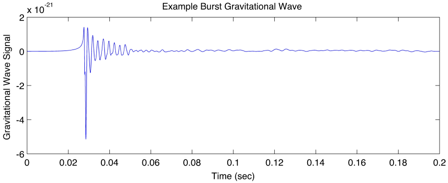


Image credit: LIGO Scientific Collaboration

Sources of Gravitational Waves



Burst:

- Supernovae

Continuous and Inspiral:

Image credit: LIGO Scientific Collaboration

Sources of Gravitational Waves

Burst:

- Supernovae

Continuous and Inspiral:

- Binary mergers (NS, SBH, PBH, SMBH, ...)

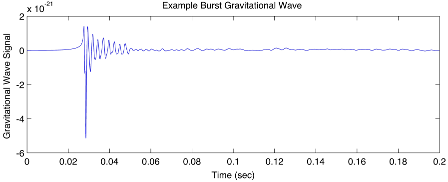
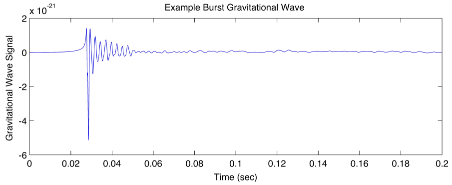


Image credit: LIGO Scientific Collaboration

Sources of Gravitational Waves



Burst:

- Supernovae

Continuous and Inspiral:

- Binary mergers (NS, SBH, PBH, SMBH, ...)
- Neutron stars with deformities

Image credit: LIGO Scientific Collaboration

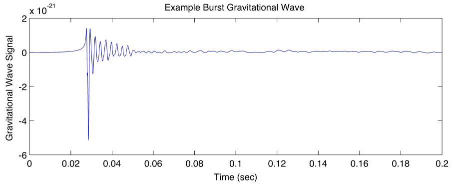
Sources of Gravitational Waves

Burst:

- Supernovae

Continuous and Inspiral:

- Binary mergers (NS, SBH, PBH, SMBH, ...)
- Neutron stars with deformities



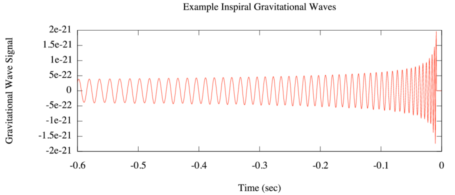


Image credit: LIGO Scientific Collaboration

Sources of Gravitational Waves

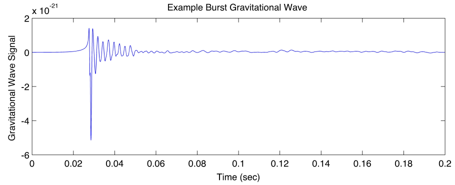
Burst:

- Supernovae

Continuous and Inspiral:

- Binary mergers (NS, SBH, PBH, SMBH, ...)
- Neutron stars with deformities

Stochastic:



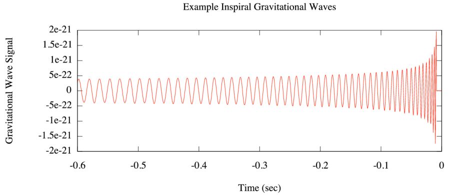


Image credit: LIGO Scientific Collaboration

Sources of Gravitational Waves

Burst:

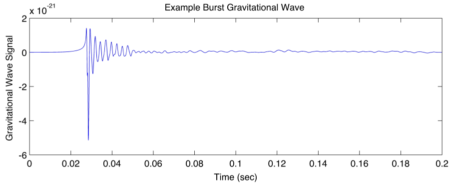
- Supernovae

Continuous and Inspiral:

- Binary mergers (NS, SBH, PBH, SMBH, ...)
- Neutron stars with deformities

Stochastic:

- Primordial tensor perturbations



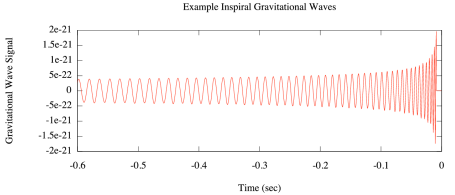


Image credit: LIGO Scientific Collaboration

Sources of Gravitational Waves

Burst:

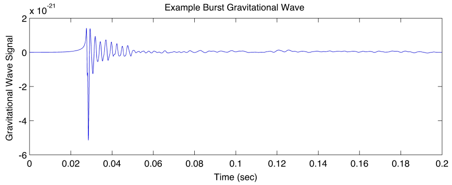
- Supernovae

Continuous and Inspiral:

- Binary mergers (NS, SBH, PBH, SMBH, ...)
- Neutron stars with deformities

Stochastic:

- Primordial tensor perturbations
- Secondary tensor perturbations



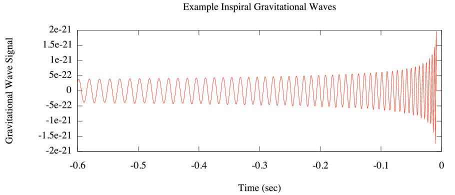


Image credit: LIGO Scientific Collaboration

Sources of Gravitational Waves

Burst:

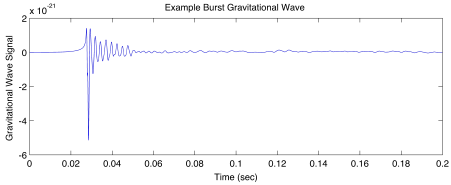
- Supernovae

Continuous and Inspiral:

- Binary mergers (NS, SBH, PBH, SMBH, ...)
- Neutron stars with deformities

Stochastic:

- Primordial tensor perturbations
- Secondary tensor perturbations
- Topological defects (e.g. cosmic strings)



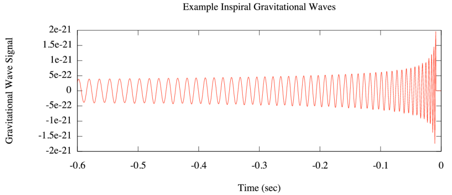


Image credit: LIGO Scientific Collaboration

Sources of Gravitational Waves

Burst:

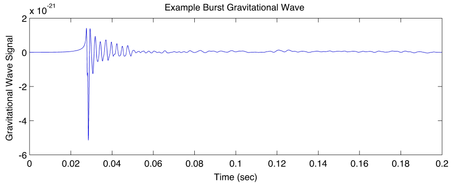
- Supernovae

Continuous and Inspiral:

- Binary mergers (NS, SBH, PBH, SMBH, ...)
- Neutron stars with deformities

Stochastic:

- Primordial tensor perturbations
- Secondary tensor perturbations
- Topological defects (e.g. cosmic strings)
- Superposition of the other types



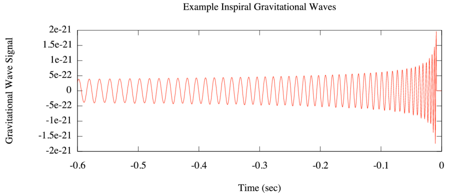


Image credit: LIGO Scientific Collaboration

Sources of Gravitational Waves

Burst:

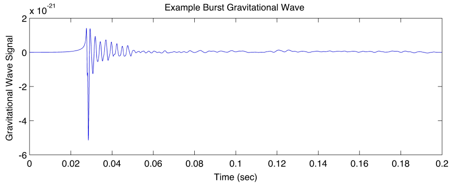
- Supernovae

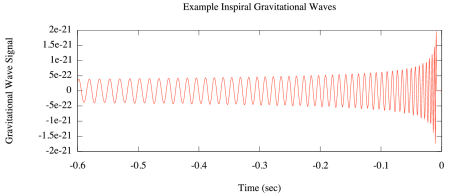
Continuous and Inspiral:

- Binary mergers (NS, SBH, PBH, SMBH, ...)
- Neutron stars with deformities

Stochastic:

- Primordial tensor perturbations
- Secondary tensor perturbations
- Topological defects (e.g. cosmic strings)
- Superposition of the other types





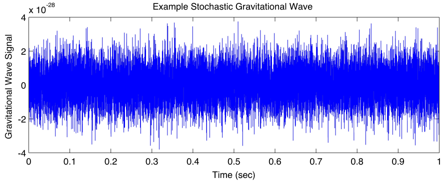


Image credit: LIGO Scientific Collaboration

How to detect gravitational waves?

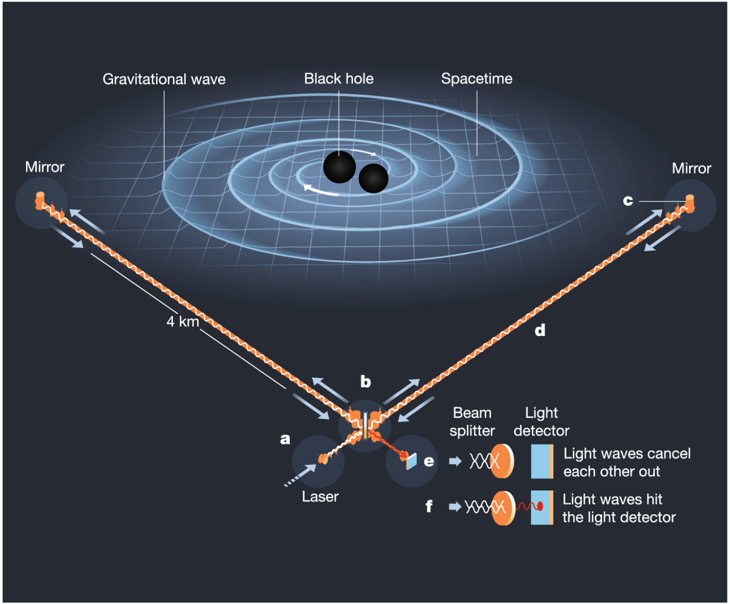
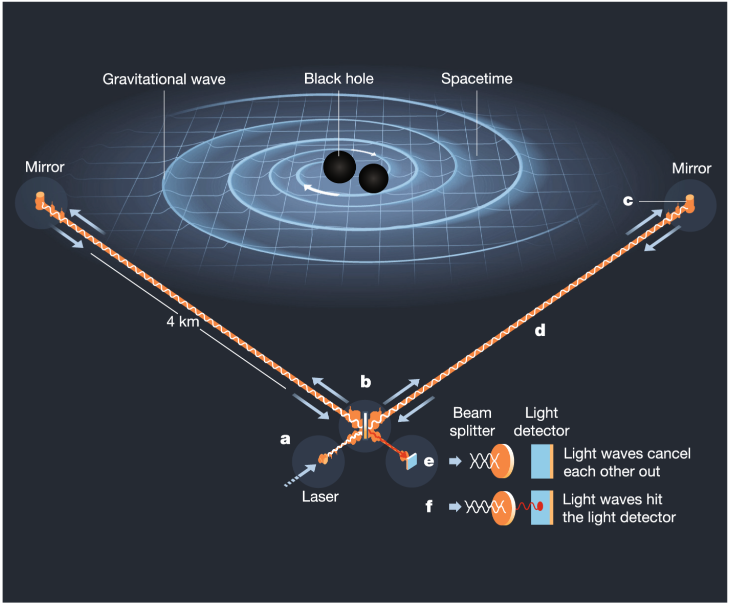


Image credit: The Royal Swedish Academy of Sciences

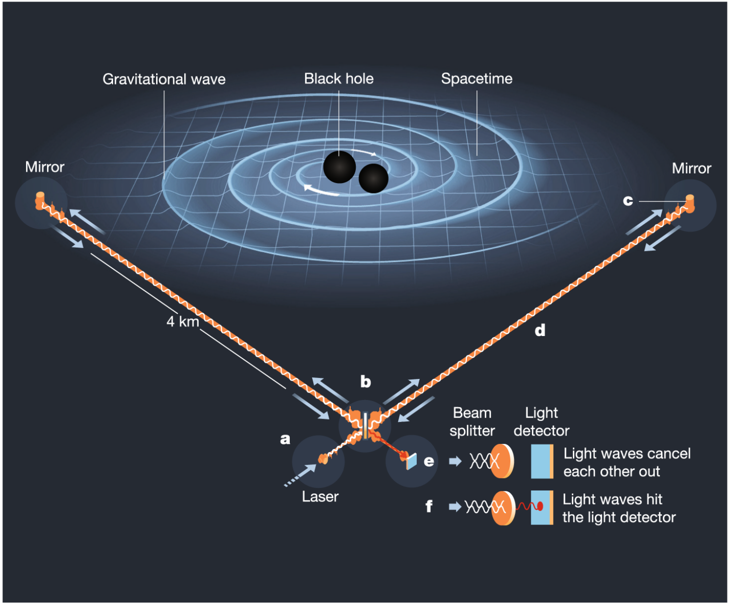
How to detect gravitational waves?



Other detection methods:

Image credit: The Royal Swedish Academy of Sciences

How to detect gravitational waves?

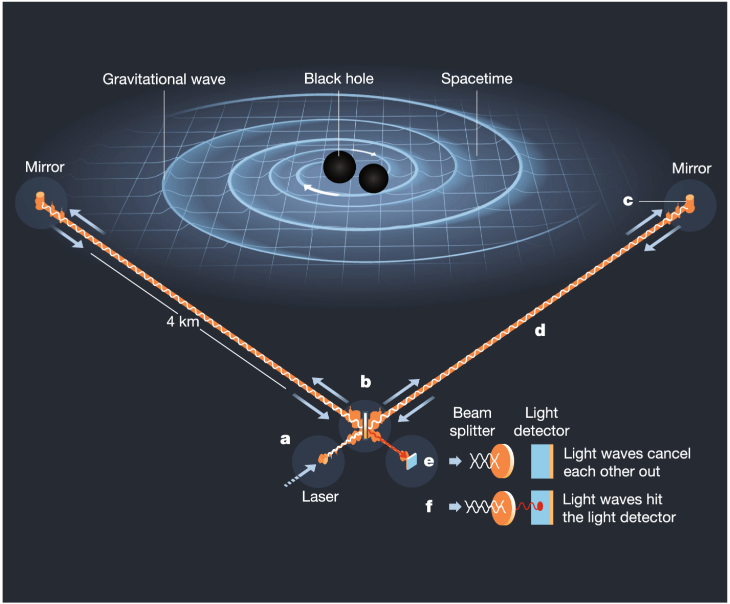


Other detection methods:

- Weber bar

Image credit: The Royal Swedish Academy of Sciences

How to detect gravitational waves?

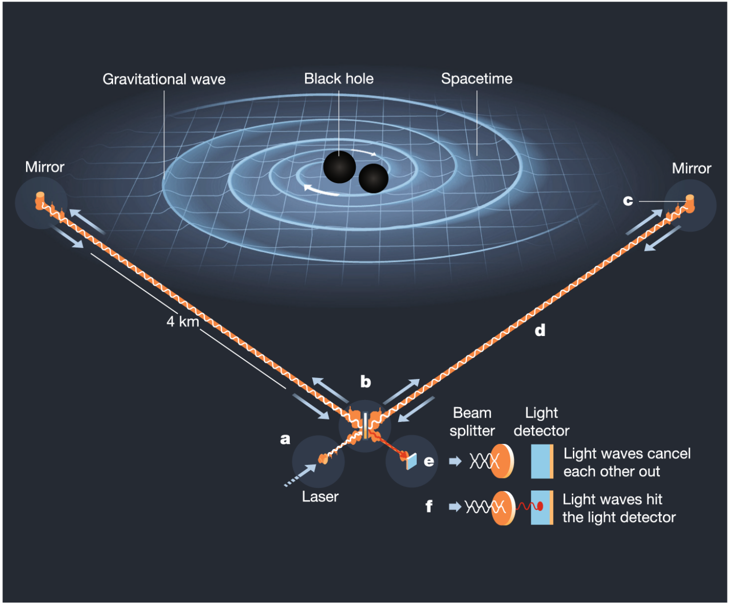


Other detection methods:

- Weber bar
- Pulsar timing arrays

Image credit: The Royal Swedish Academy of Sciences

How to detect gravitational waves?

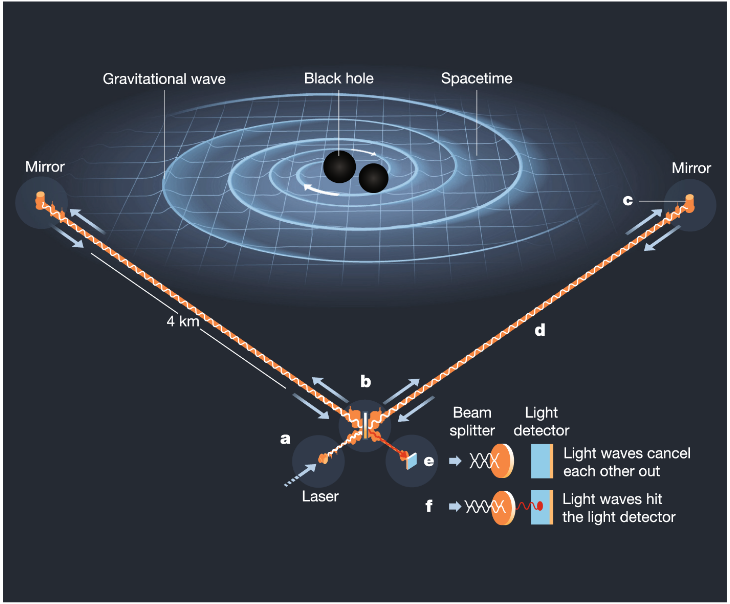


Other detection methods:

- Weber bar
- Pulsar timing arrays
- CMB B-mode

Image credit: The Royal Swedish Academy of Sciences

How to detect gravitational waves?



Other detection methods:

- Weber bar
- Pulsar timing arrays
- CMB B-mode
- Other non-direct ways

Image credit: The Royal Swedish Academy of Sciences

Gravitational waves from black hole mergers

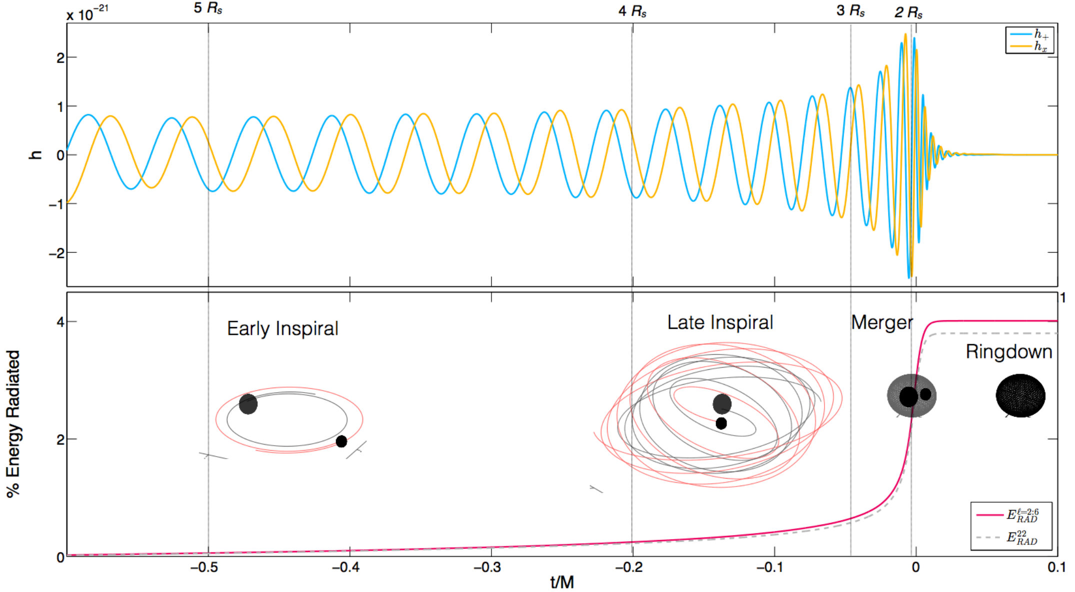


Image credit: Einstein at Georgia Tech

Stochastic gravitational wave background

Image credit: Megna pools

"We are like an insect sitting at the edge of a pool, trying to figure out who jumped in where and when and what's happening all over the pool."

- Richard Feynman

Stochastic gravitational wave background

Image credit: Megna pools

"We are like an insect sitting at the edge of a pool, trying to figure out who jumped in where and when and what's happening all over the pool."

- Richard Feynman

$$\Omega_{SGWB}(f) = \frac{1}{\rho_c} \frac{\mathrm{d}\rho_{GW}(f)}{\mathrm{d}\ln f}$$

Stochastic gravitational wave background

Image credit: Megna pools

"We are like an insect sitting at the edge of a pool, trying to figure out who jumped in where and when and what's happening all over the pool."

- Richard Feynman

$$\Omega_{SGWB}(f) = \frac{1}{\rho_c} \frac{\mathrm{d}\rho_{GW}(f)}{\mathrm{d}\ln f}$$

$$h_{ab}(t, \vec{x}) = \sum_{A=+,\times} \int_{-\infty}^{\infty} \mathrm{d}f \int_{S^2} \mathrm{d}\hat{\Omega} h_A(f, \hat{\Omega}) e^{2\pi i f(t - \hat{\Omega}\cdot\vec{x})} \hat{e}^A_{ab}(\hat{\Omega})$$

Stochastic gravitational wave background

"We are like an insect sitting at the edge of a pool, trying to figure out who jumped in where and when and what's happening all over the pool."

- Richard Feynman

Image credit: Megna pools

$$\Omega_{SGWB}(f) = \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{GW}(f)}{\mathrm{d}\ln f}$$

$$\langle h_A(f,\hat{\Omega})h^*_{A'}(f',\hat{\Omega}')\rangle_{\text{ens}} = \frac{\delta_{AA'}}{2}\frac{\delta^{(2)}(\hat{\Omega},\hat{\Omega}')}{4\pi}\frac{\delta(f-f')}{2}S_h(f)$$

$$h_{ab}(t,\vec{x}) = \sum_{A=+,\times}\int_{-\infty}^{\infty}\mathrm{d}f\int_{S^2}\mathrm{d}\hat{\Omega}\, h_A(f,\hat{\Omega})e^{2\pi i f(t-\hat{\Omega}\cdot\vec{x})}\hat{e}^A_{ab}(\hat{\Omega})$$

Stochastic gravitational wave background

"We are like an insect sitting at the edge of a pool, trying to figure out who jumped in where and when and what's happening all over the pool."

- Richard Feynman

Image credit: Megna pools

$$\Omega_{SGWB}(f) = \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{GW}(f)}{\mathrm{d}\ln f}$$

$$\langle h_A(f,\hat{\Omega})h^*_{A'}(f',\hat{\Omega}')\rangle_{\text{ens}} = \frac{\delta_{AA'}}{2}\frac{\delta^{(2)}(\hat{\Omega},\hat{\Omega}')}{4\pi}\frac{\delta(f-f')}{2}S_h(f)$$

$$h_{ab}(t,\vec{x}) = \sum_{A=+,\times}\int_{-\infty}^{\infty}\mathrm{d}f\int_{S^2}\mathrm{d}\hat{\Omega}h_A(f,\hat{\Omega})e^{2\pi i f(t-\hat{\Omega}\cdot\vec{x})}\hat{e}^A_{ab}(\hat{\Omega})$$

$$\boxed{\Omega_{SGWB}(f) = \frac{2\pi^2c^4}{3H_0^2}f^3S_h(f)}$$

Recap

We want to detect:

Recap

We want to detect:

- A stochastic gravitational wave background

Recap

We want to detect:

- A stochastic gravitational wave background
- Coming from binary black hole mergers

Recap

We want to detect:

- A stochastic gravitational wave background
- Coming from binary black hole mergers
- Our chosen black holes to study have a primordial origin

Recap

We want to detect:

- A stochastic gravitational wave background
- Coming from binary black hole mergers
- Our chosen black holes to study have a primordial origin

To achieve these, we need:

Recap

We want to detect:

- A stochastic gravitational wave background
- Coming from binary black hole mergers
- Our chosen black holes to study have a primordial origin

To achieve these, we need:

1. Learn about primordial black holes and their population statistics

Recap

We want to detect:

- A stochastic gravitational wave background
- Coming from binary black hole mergers
- Our chosen black holes to study have a primordial origin

To achieve these, we need:

1. Learn about primordial black holes and their population statistics
2. Build a model for their merger rates based on the population statistics

Recap

We want to detect:

- A stochastic gravitational wave background
- Coming from binary black hole mergers
- Our chosen black holes to study have a primordial origin

To achieve these, we need:

1. Learn about primordial black holes and their population statistics
2. Build a model for their merger rates based on the population statistics
3. Simulate a stochastic gravitational wave background

Recap

We want to detect:

- A stochastic gravitational wave background
- Coming from binary black hole mergers
- Our chosen black holes to study have a primordial origin

To achieve these, we need:

1. Learn about primordial black holes and their population statistics
2. Build a model for their merger rates based on the population statistics
3. Simulate a stochastic gravitational wave background
4. Build a model to analyse the data

Recap

We want to detect:

- A stochastic gravitational wave background
- Coming from binary black hole mergers
- Our chosen black holes to study have a primordial origin

To achieve these, we need:

1. Learn about primordial black holes and their population statistics
2. Build a model for their merger rates based on the population statistics
3. Simulate a stochastic gravitational wave background
4. Build a model to analyse the data
5. Run the analyses

Section 2

Primordial Black Holes

Primordial black hole formation

In the early universe ($\sim$ before the first second), if some part of the cosmos that has the same density as a black hole becomes causaly connected (a.k.a. the corresponding mode enteres the horizon) that part of the cosmos collapses to a black hole.

Let's calculate the mass:

Primordial black hole formation

In the early universe ($\sim$ before the first second), if some part of the cosmos that has the same density as a black hole becomes causaly connected (a.k.a. the corresponding mode enteres the horizon) that part of the cosmos collapses to a black hole.
Let's calculate the mass:

$$H^2(t) = \frac{\pi G}{3}\rho(t), \qquad H(t) \equiv \frac{1}{2}\frac{\mathrm{d}a}{\mathrm{d}t}$$

Primordial black hole formation

In the early universe ($\sim$ before the first second), if some part of the cosmos that has the same density as a black hole becomes causaly connected (a.k.a. the corresponding mode enteres the horizon) that part of the cosmos collapses to a black hole.
Let's calculate the mass:

$$H^2(t) = \frac{\pi G}{3}\rho(t), \qquad H(t) \equiv \frac{1}{2}\frac{\mathrm{d}a}{\mathrm{d}t}$$

$$\rho(t) = \frac{3}{32\pi G}t^{-2} \sim \frac{1}{G}t^{-2}$$

Primordial black hole formation

In the early universe ($\sim$ before the first second), if some part of the cosmos that has the same density as a black hole becomes causaly connected (a.k.a. the corresponding mode enteres the horizon) that part of the cosmos collapses to a black hole.
Let's calculate the mass:

$$H^2(t) = \frac{\pi G}{3}\rho(t), \qquad H(t) \equiv \frac{1}{2}\frac{\mathrm{d}a}{\mathrm{d}t}$$

$$\rho_{BH} = \frac{M_{BH}}{V_{BH}} = \frac{M_{BH}}{(4/3)\pi r_s^3}$$

$$\rho(t) = \frac{3}{32\pi G}t^{-2} \sim \frac{1}{G}t^{-2}$$

Primordial black hole formation

In the early universe ($\sim$ before the first second), if some part of the cosmos that has the same density as a black hole becomes causaly connected (a.k.a. the corresponding mode enteres the horizon) that part of the cosmos collapses to a black hole.
Let's calculate the mass:

$$H^2(t) = \frac{\pi G}{3}\rho(t), \qquad H(t) \equiv \frac{1}{2}\frac{\mathrm{d}a}{\mathrm{d}t}$$

$$\rho_{BH} = \frac{M_{BH}}{V_{BH}} = \frac{M_{BH}}{(4/3)\pi r_s^3}$$

$$\rho(t) = \frac{3}{32\pi G}t^{-2} \sim \frac{1}{G}t^{-2}$$

$$r_s = \frac{3c^6}{32\pi G^3 M_{BH}^2} \sim \frac{c^3}{G^3 M_{BH}^2}$$

Primordial black hole formation

In the early universe ($\sim$ before the first second), if some part of the cosmos that has the same density as a black hole becomes causaly connected (a.k.a. the corresponding mode enteres the horizon) that part of the cosmos collapses to a black hole.
Let's calculate the mass:

$$H^2(t) = \frac{\pi G}{3}\rho(t), \qquad H(t) \equiv \frac{1}{2}\frac{\mathrm{d}a}{\mathrm{d}t}$$

$$\rho_{BH} = \frac{M_{BH}}{V_{BH}} = \frac{M_{BH}}{(4/3)\pi r_s^3}$$

$$\rho(t) = \frac{3}{32\pi G}t^{-2} \sim \frac{1}{G}t^{-2}$$

$$r_s = \frac{3c^6}{32\pi G^3 M_{BH}^2} \sim \frac{c^3}{G^3 M_{BH}^2}$$

Condition for collapse: $\rho_{PBH} = \rho(t)$

Primordial black hole formation

In the early universe ($\sim$ before the first second), if some part of the cosmos that has the same density as a black hole becomes causaly connected (a.k.a. the corresponding mode enteres the horizon) that part of the cosmos collapses to a black hole.
Let's calculate the mass:

$$H^2(t) = \frac{\pi G}{3}\rho(t), \qquad H(t) \equiv \frac{1}{2}\frac{\mathrm{d}a}{\mathrm{d}t}$$

$$\rho_{BH} = \frac{M_{BH}}{V_{BH}} = \frac{M_{BH}}{(4/3)\pi r_s^3}$$

$$\rho(t) = \frac{3}{32\pi G}t^{-2} \sim \frac{1}{G}t^{-2}$$

$$r_s = \frac{3c^6}{32\pi G^3 M_{BH}^2} \sim \frac{c^3}{G^3 M_{BH}^2}$$

Condition for collapse: $\rho_{PBH} = \rho(t)$

$$M_{PBH} \sim \frac{c^3 t}{G} \sim 10^{-15}\left(\frac{t}{10^{-23}\ s}\right)\ gr$$

Clustering of primordial black holes

$$\delta(\vec{x}) \equiv \delta_L(\vec{x}) + \delta_s(\vec{x})$$

Clustering of primordial black holes

$$\delta(\vec{x}) \equiv \delta_L(\vec{x}) + \delta_s(\vec{x}), \quad \delta(\vec{x}) = \frac{\rho(\vec{x}) - \rho_c}{\rho_c}$$

Clustering of primordial black holes

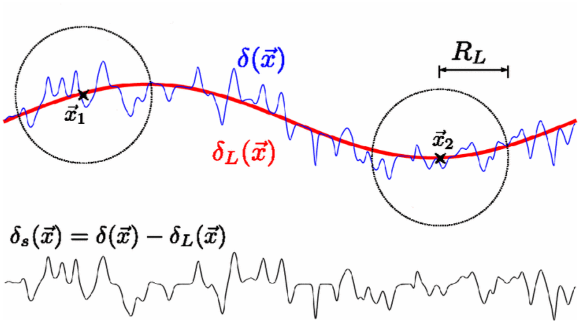


Image credit: F. Schmidt, et al.

$$\delta(\vec{x}) \equiv \delta_L(\vec{x}) + \delta_s(\vec{x}), \quad \delta(\vec{x}) = \frac{\rho(\vec{x}) - \rho_c}{\rho_c}$$

Clustering of primordial black holes

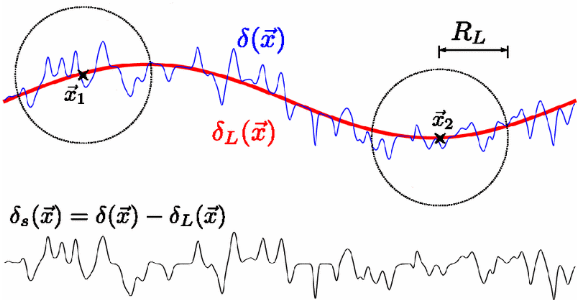


Image credit: F. Schmidt, et al.

$$\delta(\vec{x}) \equiv \delta_L(\vec{x}) + \delta_s(\vec{x}), \quad \delta(\vec{x}) = \frac{\rho(\vec{x}) - \rho_c}{\rho_c}$$

$$\delta_L(\vec{x}; R_L) = \int \mathrm{d}^2\vec{x}'\delta(\vec{x})W_L(\vec{x} + \vec{x}'; R_L)$$

Clustering of primordial black holes

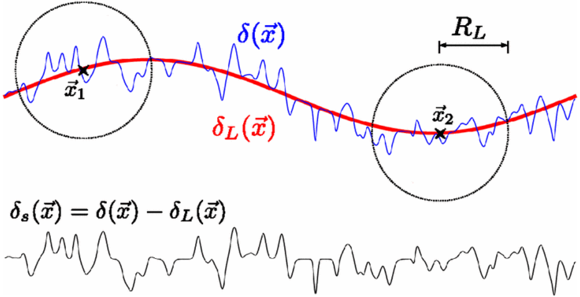


Image credit: F. Schmidt, et al.

$$\delta(\vec{x}) \equiv \delta_L(\vec{x}) + \delta_s(\vec{x}), \qquad \delta(\vec{x}) = \frac{\rho(\vec{x}) - \rho_c}{\rho_c}$$

$$\delta_L(\vec{x}; R_L) = \int \mathrm{d}^2\vec{x}'\delta(\vec{x})W_L(\vec{x}+\vec{x}'; R_L)$$

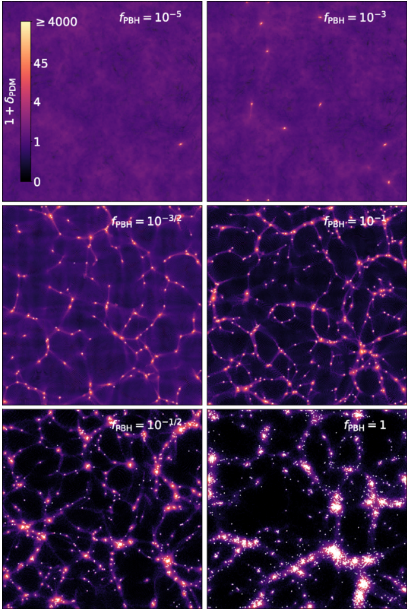


Image credit: D. Inman, et al.

Section 3

Simulations

Spectral energy density

$$\Omega_{SGWB} = \Omega_{SGWB}(f, \{\Theta_{\text{source}}, \Theta_{\text{astrophysics}}, \Theta_{\text{cosmology}}, \cdots\})$$

Spectral energy density

$$\Omega_{SGWB} = \Omega_{SGWB}(f, \{\Theta_{\text{source}}, \Theta_{\text{astrophysics}}, \Theta_{\text{cosmology}}, \cdots\})$$

Depends on:

Spectral energy density

$$\Omega_{SGWB} = \Omega_{SGWB}(f, \{\Theta_{\text{source}}, \Theta_{\text{astrophysics}}, \Theta_{\text{cosmology}}, \cdots\})$$

Depends on:

- Source type, Mass scales, ...

Spectral energy density

$$\Omega_{SGWB} = \Omega_{SGWB}(f, \{\Theta_{\text{source}}, \Theta_{\text{astrophysics}}, \Theta_{\text{cosmology}}, \cdots\})$$

Depends on:

- Source type, Mass scales, ...
- Location of sources, Dynamical enviornments, ...

Spectral energy density

$$\Omega_{SGWB} = \Omega_{SGWB}(f, \{\Theta_{\text{source}}, \Theta_{\text{astrophysics}}, \Theta_{\text{cosmology}}, \cdots\})$$

Depends on:

- Source type, Mass scales, ...
- Location of sources, Dynamical enviornments, ...
- Cosmology model parameters, Mass function, ...

Spectral energy density

$$\Omega_{SGWB} = \Omega_{SGWB}(f, \{\Theta_{\text{source}}, \Theta_{\text{astrophysics}}, \Theta_{\text{cosmology}}, \cdots\})$$

Depends on:

- Source type, Mass scales, ...
- Location of sources, Dynamical enviornments, ...
- Cosmology model parameters, Mass function, ...
- Other observational or Intermediary effects

Spectral energy density

$$\Omega_{SGWB} = \Omega_{SGWB}(f, \{\Theta_{\text{source}}, \Theta_{\text{astrophysics}}, \Theta_{\text{cosmology}}, \cdots\})$$

Depends on:

- Source type, Mass scales, ...
- Location of sources, Dynamical enviornments, ...
- Cosmology model parameters, Mass function, ...
- Other observational or Intermediary effects

Ways to calculate Ω_{gw}:

Spectral energy density

$$\Omega_{SGWB} = \Omega_{SGWB}(f, \{\Theta_{\text{source}}, \Theta_{\text{astrophysics}}, \Theta_{\text{cosmology}}, \dots\})$$

Depends on:

- Source type, Mass scales, ...
- Location of sources, Dynamical enviornments, ...
- Cosmology model parameters, Mass function, ...
- Other observational or Intermediary effects

Ways to calculate Ω_{gw}:

1. **Analytical approach**: For relatively simple models (e.g. inflationary gravitational waves in ΛCDM [Dodelson, Schmidt])

Spectral energy density

$$\Omega_{SGWB} = \Omega_{SGWB}(f, \{\Theta_{\text{source}}, \Theta_{\text{astrophysics}}, \Theta_{\text{cosmology}}, \cdots\})$$

Depends on:

- Source type, Mass scales, ...
- Location of sources, Dynamical enviornments, ...
- Cosmology model parameters, Mass function, ...
- Other observational or Intermediary effects

Ways to calculate Ω_{gw}:

1. **Analytical approach**: For relatively simple models (e.g. inflationary gravitational waves in ΛCDM [Dodelson, Schmidt])
2. **Simulation**: For Complicated models with high accuracy (e.g. [Clesse, et al.], [Jenkins, et al.])

Spectral energy density

$$\Omega_{SGWB} = \Omega_{SGWB}(f, \{\Theta_{\text{source}}, \Theta_{\text{astrophysics}}, \Theta_{\text{cosmology}}, \cdots\})$$

Depends on:

- Source type, Mass scales, ...
- Location of sources, Dynamical enviornments, ...
- Cosmology model parameters, Mass function, ...
- Other observational or Intermediary effects

Ways to calculate Ω_{gw}:

1. **Analytical approach**: For relatively simple models (e.g. inflationary gravitational waves in ΛCDM [Dodelson, Schmidt])
2. **Simulation**: For Complicated models with high accuracy (e.g. [Clesse, et al.], [Jenkins, et al.])
3. **Semi-analytocal approach**: For quickly testing various complicated models whithout using too much computational resource (e.g. [Braglia, et al.], [Durrer], [Mukherjee, et al.])

Semi-Analytical approach

$$\Omega_{SGWB}(f, \{\Theta\}_{i=0}^{n}) =$$

Semi-Analytical approach

$$\Omega_{SGWB}(f, \{\Theta\}_{i=0}^{n}) =$$

$$\frac{f}{\rho_c c^2} \int \mathrm{d}^n\Theta_i \int \mathrm{d}z$$

Semi-Analytical approach

$$\Omega_{SGWB}(f, \{\Theta\}_{i=0}^{n}) =$$

$$\frac{f}{\rho_c c^2} \int \mathrm{d}^n\Theta_i \int \mathrm{d}z \quad \overbrace{\frac{\mathrm{d}t_r}{\mathrm{d}z}}^{\text{cosmology}}$$

Semi-Analytical approach

$$\Omega_{SGWB}(f, \{\Theta\}_{i=0}^{n}) =$$

$$\frac{f}{\rho_c c^2} \int \mathrm{d}^n\Theta_i \int \mathrm{d}z \quad \overbrace{\frac{\mathrm{d}t_r}{\mathrm{d}z}}^{\text{cosmology}} \quad \underbrace{\frac{\mathrm{d}^m \tau_{\text{merg}}(z, \Theta_j)}{\mathrm{d}\Theta_0 \mathrm{d}\Theta_1 \ldots \mathrm{d}\Theta_m}}_{\substack{\text{astrophysics} \\ \text{\& cosmology}}}$$

Semi-Analytical approach

$$\Omega_{SGWB}(f, \{\Theta\}_{i=0}^{n}) =$$

$$\frac{f}{\rho_c c^2} \int \mathrm{d}^n\Theta_i \int \mathrm{d}z \quad \overbrace{\frac{\mathrm{d}t_r}{\mathrm{d}z}}^{\text{cosmology}} \quad \underbrace{\frac{\mathrm{d}^m \tau_{\text{merg}}(z, \Theta_j)}{\mathrm{d}\Theta_0 \mathrm{d}\Theta_1 \ldots \mathrm{d}\Theta_m}}_{\substack{\text{astrophysics} \\ \text{\& cosmology}}} \overbrace{\frac{\mathrm{d}E_{\mathrm{GW}}(f_r, \Theta_k)}{\mathrm{d}f_r}}^{\text{GW source}}$$

Semi-Analytical approach

$$\Omega_{SGWB}(f, \{\Theta\}_{i=0}^{n}) =$$

$$\frac{f}{\rho_c c^2} \int \mathrm{d}^n\Theta_i \int \mathrm{d}z \quad \overbrace{\frac{\mathrm{d}t_r}{\mathrm{d}z}}^{\text{cosmology}} \quad \underbrace{\frac{\mathrm{d}^m \tau_{\mathrm{merg}}(z, \Theta_j)}{\mathrm{d}\Theta_0 \mathrm{d}\Theta_1 \ldots \mathrm{d}\Theta_m}}_{\substack{\text{astrophysics} \\ \text{\& cosmology}}} \overbrace{\frac{\mathrm{d}E_{\mathrm{GW}}(f_r, \Theta_k)}{\mathrm{d}f_r}}^{\text{GW source}}$$

$$f_r = (1 + z)f$$

Cosmology part

The cosmology is encoded in the Hubble parameter which appears in number of events which occur in a comoving volume between the cosmic times $t_r(z)$ and $t_r(z + \mathrm{d}z)$

$$\frac{\mathrm{d}t_r}{\mathrm{d}z}$$

Cosmology part

The cosmology is encoded in the Hubble parameter which appears in number of events which occur in a comoving volume between the cosmic times $t_r(z)$ and $t_r(z + \mathrm{d}z)$

$$\frac{\mathrm{d}N}{\mathrm{d}z} = \dot{N}\frac{\mathrm{d}t_r}{\mathrm{d}z} = \frac{\dot{N}}{H(z)(1+z)}$$

Cosmology part

The cosmology is encoded in the Hubble parameter which appears in number of events which occur in a comoving volume between the cosmic times $t_r(z)$ and $t_r(z + \mathrm{d}z)$

$$\frac{\mathrm{d}N}{\mathrm{d}z} = \dot{N}\frac{\mathrm{d}t_r}{\mathrm{d}z} = \frac{\dot{N}}{H(z)(1+z)}$$

$$H^2(z) = H_0^2\left[\Omega_{\mathrm{r}}(1+z)^4 + (\Omega_{\mathrm{DM}} + \Omega_{\mathrm{b}})(1+z)^3 + \Omega_{\mathrm{k}}(1+z)^2 + \Omega_\Lambda\right]$$

Astrophysics/Cosmology part

Following the pictrue in [M. Braglia, J. Garcia-Bellido, and S. Kuroyanagi, Testing primordial black holes with multi-band observations of the stochastic gravitational wave background, *Journal of Cosmology and Astroparticle Physics*, vol.2021, no.12, p.012, 2021.]

$\dot{N}$

Astrophysics/Cosmology part

Following the pictrue in [M. Braglia, J. Garcia-Bellido, and S. Kuroyanagi, Testing primordial black holes with multi-band observations of the stochastic gravitational wave background, *Journal of Cosmology and Astroparticle Physics*, vol.2021, no.12, p.012, 2021.]

$$\dot{N} = \frac{\mathrm{d}^2 \tau_{\text{merg}}(z, m_1, m_2, \{\alpha_z, \alpha_c\})}{\mathrm{d}\log_{10} m_1 \mathrm{d}\log_{10} m_2} =$$

Astrophysics/Cosmology part

Following the pictrue in [M. Braglia, J. Garcia-Bellido, and S. Kuroyanagi, Testing primordial black holes with multi-band observations of the stochastic gravitational wave background, *Journal of Cosmology and Astroparticle Physics*, vol.2021, no.12, p.012, 2021.]

$$\dot{N} = \frac{\mathrm{d}^2 \tau_{\text{merg}}(z, m_1, m_2, \{\alpha_z, \alpha_c\})}{\mathrm{d} \log_{10} m_1 \mathrm{d} \log_{10} m_2} =$$

$$\underbrace{R_{\text{clust}} \frac{(m_1 + m_2)^{10/7}}{(m_1 m_2)^{5/7}} \frac{(1+z)^{\alpha_z}}{[1 + (M_{tot}/M_\star)]^{\alpha_c}}}_{\text{PBH clustering/binary formation}}$$

Astrophysics/Cosmology part

Following the pictrue in [M. Braglia, J. Garcia-Bellido, and S. Kuroyanagi, Testing primordial black holes with multi-band observations of the stochastic gravitational wave background, *Journal of Cosmology and Astroparticle Physics*, vol.2021, no.12, p.012, 2021.]

$$\dot{N} = \frac{\mathrm{d}^2\tau_{\mathrm{merg}}(z, m_1, m_2, \{\alpha_z, \alpha_c\})}{\mathrm{d}\log_{10} m_1 \mathrm{d}\log_{10} m_2} =$$

$$\underbrace{R_{\mathrm{clust}} \frac{(m_1 + m_2)^{10/7}}{(m_1 m_2)^{5/7}} \frac{(1+z)^{\alpha_z}}{[1 + (M_{tot}/M_\star)]^{\alpha_c}}}_{\mathrm{PBH\ clustering/binary\ formation}} \overbrace{f_{PBH}(m_1) f_{PBH}(m_2)}^{primordial/thermal\ \ evolution}$$

Merger rate Calculation an PBH mass function

$$\frac{\mathrm{d}^2\tau_{\mathrm{merg}}^{\text{our model}}(z, m_1, m_2)}{\mathrm{d}\log_{10} m_1 \mathrm{d}\log_{10} m_2} = R_{\mathrm{clust}} \frac{(m_1 + m_2)^{10/7}}{(m_1 m_2)^{5/7}} f_{PBH}(m_1) f_{PBH}(m_2)$$

Merger rate Calculation an PBH mass function

$$\frac{\mathrm{d}^2\tau_{\text{merg}}^{\text{our model}}(z, m_1, m_2)}{\mathrm{d}\log_{10} m_1 \mathrm{d}\log_{10} m_2} = R_{\text{clust}} \frac{(m_1 + m_2)^{10/7}}{(m_1 m_2)^{5/7}} f_{PBH}(m_1) f_{PBH}(m_2)$$

To fix R_{clust} :

$$\int \mathrm{d}m_1 \int \mathrm{d}m_2 \frac{\mathrm{d}^2\tau_{\text{merg}}^{\text{our model}}(0, m_1, m_2)}{\mathrm{d}\log_{10} m_1 \mathrm{d}\log_{10} m_2} = 38\ yr^{-1} Gpc^{-1}$$

Merger rate Calculation an PBH mass function

$$\frac{\mathrm{d}^2\tau_{\mathrm{merg}}^{\text{our model}}(z,m_1,m_2)}{\mathrm{d}\log_{10}m_1\,\mathrm{d}\log_{10}m_2} = R_{\mathrm{clust}}\frac{(m_1+m_2)^{10/7}}{(m_1 m_2)^{5/7}} f_{PBH}(m_1) f_{PBH}(m_2)$$

To fix R_{clust} :

$$\int \mathrm{d}m_1 \int \mathrm{d}m_2 \frac{\mathrm{d}^2\tau_{\mathrm{merg}}^{\text{our model}}(0,m_1,m_2)}{\mathrm{d}\log_{10}m_1\,\mathrm{d}\log_{10}m_2} = 38\ yr^{-1}\,Gpc^{-1}$$

$$f_{PBH}(m,\{\boxed{\sigma_{PBH}},\mu\}) = F_0\frac{1}{\sqrt{2\pi}\sigma_{PBH}m}\exp\left[-\frac{(\log_{10}\frac{m}{\mu})^2}{2\sigma_{PBH}^2}\right]$$

Source part

$$\frac{\mathrm{d}E_{\mathrm{GW}}(\Theta_i)}{\mathrm{d}f_r} = \frac{(G\pi)^{2/3}\mathcal{M}_c^{3/5}}{3}\mathcal{G}(f_r, \Theta)$$

Source part

$$\frac{\mathrm{d}E_{\mathrm{GW}}(\Theta_i)}{\mathrm{d}f_r} = \frac{(G\pi)^{2/3}\mathcal{M}_c^{3/5}}{3}\mathcal{G}(f_r, \Theta)$$

$$\mathcal{M}_c = \frac{(m_1 + m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

Source part

$$\frac{\mathrm{d}E_{\mathrm{GW}}(\Theta_i)}{\mathrm{d}f_r} = \frac{(G\pi)^{2/3}\mathcal{M}_c^{3/5}}{3}\mathcal{G}(f_r,\Theta)$$

$$\mathcal{M}_c = \frac{(m_1 + m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$\mathcal{G}(f_r) = \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right.$$

Source part

$$\frac{\mathrm{d}E_{\mathrm{GW}}(\Theta_i)}{\mathrm{d}f_r} = \frac{(G\pi)^{2/3}\mathcal{M}_c^{3/5}}{3}\mathcal{G}(f_r, \Theta)$$

$$\mathcal{M}_c = \frac{(m_1 + m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$\mathcal{G}(f_r) = \begin{cases} f_r^{-1/3} & f_r < f_{\mathrm{merg}} \\ & \\ & \end{cases}$$

Source part

$$\frac{\mathrm{d}E_{\mathrm{GW}}(\Theta_i)}{\mathrm{d}f_r} = \frac{(G\pi)^{2/3}\mathcal{M}_c^{3/5}}{3}\mathcal{G}(f_r, \Theta)$$

$$\mathcal{M}_c = \frac{(m_1 + m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$\mathcal{G}(f_r) = \begin{cases} f_r^{-1/3} & f_r < f_{\mathrm{merg}} \\ \frac{f_r^{2/3}}{f_{\mathrm{merg}}} & f_{\mathrm{merg}} \leq f_r < f_{\mathrm{ring}} \\ & \\ & \end{cases}$$

Source part

$$\frac{\mathrm{d}E_{\mathrm{GW}}(\Theta_i)}{\mathrm{d}f_r} = \frac{(G\pi)^{2/3}\mathcal{M}_c^{3/5}}{3}\mathcal{G}(f_r,\Theta)$$

$$\mathcal{M}_c = \frac{(m_1+m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

$$\mathcal{G}(f_r) = \begin{cases} f_r^{-1/3} & f_r < f_{\mathrm{merg}} \\ \frac{f_r^{2/3}}{f_{\mathrm{merg}}} & f_{\mathrm{merg}} \leq f_r < f_{\mathrm{ring}} \\ \frac{1}{f_{\mathrm{merg}} f_{\mathrm{ring}}^{4/3}} \left(\frac{f_r}{1+\left(\frac{f_r - f_{\mathrm{ring}}}{f_w/2}\right)^2} \right)^2 & f_{\mathrm{ring}} \leq f_r < f_{\mathrm{cut}} \end{cases}$$

Source part

$$\frac{\mathrm{d}E_{\mathrm{GW}}(\Theta_i)}{\mathrm{d}f_r} = \frac{(G\pi)^{2/3}\mathcal{M}_c^{3/5}}{3}\mathcal{G}(f_r,\Theta)$$

$$f_x = \frac{c^3}{\pi G M_{tot}}\left(a_1\eta^2 + a_2\eta + a_3\right)$$

$$\mathcal{M}_c = \frac{(m_1+m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

$$\mathcal{G}(f_r) = \begin{cases} f_r^{-1/3} & f_r < f_{\mathrm{merg}} \\ \frac{f_r^{2/3}}{f_{\mathrm{merg}}} & f_{\mathrm{merg}} \leq f_r < f_{\mathrm{ring}} \\ \frac{1}{f_{\mathrm{merg}} f_{\mathrm{ring}}^{4/3}}\left(\frac{f_r}{1+\left(\frac{f_r - f_{\mathrm{ring}}}{f_w/2}\right)^2}\right)^2 & f_{\mathrm{ring}} \leq f_r < f_{\mathrm{cut}} \end{cases}$$

Source part

$$\frac{\mathrm{d}E_{\mathrm{GW}}(\Theta_i)}{\mathrm{d}f_r} = \frac{(G\pi)^{2/3}\mathcal{M}_c^{3/5}}{3}\mathcal{G}(f_r, \Theta)$$

$$f_x = \frac{c^3}{\pi G M_{tot}}\left(a_1\eta^2 + a_2\eta + a_3\right)$$

$$\mathcal{M}_c = \frac{(m_1 + m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$M_{tot} = m_1 + m_2$$

$$\mathcal{G}(f_r) = \begin{cases} f_r^{-1/3} & f_r < f_{\mathrm{merg}} \\ \frac{f_r^{2/3}}{f_{\mathrm{merg}}} & f_{\mathrm{merg}} \le f_r < f_{\mathrm{ring}} \\ \frac{1}{f_{\mathrm{merg}} f_{\mathrm{ring}}^{4/3}}\left(\frac{f_r}{1+\left(\frac{f_r - f_{\mathrm{ring}}}{f_w/2}\right)^2}\right)^2 & f_{\mathrm{ring}} \le f_r < f_{\mathrm{cut}} \end{cases}$$

Source part

$$\frac{\mathrm{d}E_{\rm GW}(\Theta_i)}{\mathrm{d}f_r} = \frac{(G\pi)^{2/3}\mathcal{M}_c^{3/5}}{3}\mathcal{G}(f_r,\Theta)$$

$$\mathcal{M}_c = \frac{(m_1+m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

$$\mathcal{G}(f_r) = \begin{cases} f_r^{-1/3} & f_r < f_{\rm merg} \\ \frac{f_r^{2/3}}{f_{\rm merg}} & f_{\rm merg} \leq f_r < f_{\rm ring} \\ \frac{1}{f_{\rm merg}f_{\rm ring}^{4/3}}\left(\frac{f_r}{1+\left(\frac{f_r-f_{\rm ring}}{f_w/2}\right)^2}\right)^2 & f_{\rm ring} \leq f_r < f_{\rm cut} \end{cases}$$

$$f_x = \frac{c^3}{\pi G M_{tot}}\left(a_1\eta^2 + a_2\eta + a_3\right)$$

$$M_{tot} = m_1 + m_2$$

$$\eta = \frac{m_1+m_2}{M_{tot}^2}$$

From spectral energy density to wave forms

page 11

Stochastic gravitational wave background

"We are like an insect sitting at the edge of a pool, trying to figure out who jumped in where and when and what's happening all over the pool."

\- Richard Feynman

Image credit: Megna pools

$$\Omega_{SGWB}(f) = \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{GW}(f)}{\mathrm{d}\ln f}$$

$$\langle h_A(f,\hat{\Omega})h^*_{A'}(f',\hat{\Omega}')\rangle_{\text{ens}} = \frac{\delta_{AA'}}{2}\frac{\delta^{(2)}(\hat{\Omega},\hat{\Omega}')}{4\pi}\frac{\delta(f-f')}{2}S_h(f)$$

$$h_{ab}(t,\vec{x}) = \sum_{A=+,\times}\int_{-\infty}^{\infty}\mathrm{d}f\int_{S^2}\mathrm{d}\hat{\Omega}\, h_A(f,\hat{\Omega})e^{2\pi i f(t-\hat{\Omega}\cdot\vec{x})}\hat{e}^A_{ab}(\hat{\Omega})$$

$$\boxed{\Omega_{SGWB}(f) = \frac{2\pi^2c^4}{3H_0^2}f^3S_h(f)}$$

A. Salehi CCG SBU SGWB detection with TDA 2023/06 11 / 23

From spectral energy density to wave forms

$$h(t) = \int_{-\infty}^{\infty} \mathrm{d}f h(f) e^{2\pi i f(t-\phi)}$$

page 11

Stochastic gravitational wave background

"We are like an insect sitting at the edge of a pool, trying to figure out who jumped in where and when and what's happening all over the pool."

- Richard Feynman

Image credit: Megna pools

$$\Omega_{SGWB}(f) = \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{GW}(f)}{\mathrm{d}\ln f}$$

$$\langle h_A(f,\hat{\Omega}) h^*_{A'}(f',\hat{\Omega}')\rangle_{\text{ens}} = \frac{\delta_{AA'}}{2}\frac{\delta^{(2)}(\hat{\Omega},\hat{\Omega}')}{4\pi}\frac{\delta(f-f')}{2} S_h(f)$$

$$h_{ab}(t,\vec{x}) = \sum_{A=+,\times}\int_{-\infty}^{\infty}\mathrm{d}f \int_{S^2}\mathrm{d}\hat{\Omega} h_A(f,\hat{\Omega}) e^{2\pi i f(t-\hat{\Omega}\cdot\vec{x})}\hat{e}^A_{ab}(\hat{\Omega})$$

$$\boxed{\Omega_{SGWB}(f) = \frac{2\pi^2 c^4}{3H_0^2} f^3 S_h(f)}$$

A. Salehi CCG SBU SGWB detection with TDA 2023/06 11 / 23

From spectral energy density to wave forms

page 11

Stochastic gravitational wave background

"We are like an insect sitting at the edge of a pool, trying to figure out who jumped in where and when and what's happening all over the pool."

- Richard Feynman

Image credit: Megna pools

$$\Omega_{SGWB}(f) = \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{GW}(f)}{\mathrm{d}\ln f}$$

$$\langle h_A(f,\hat{\Omega}) h^*_{A'}(f',\hat{\Omega}')\rangle_{\text{ens}} = \frac{\delta_{AA'}}{2}\frac{\delta^{(2)}(\hat{\Omega},\hat{\Omega}')}{4\pi}\frac{\delta(f-f')}{2}S_h(f)$$

$$h_{ab}(t,\vec{x}) = \sum_{A=+,\times}\int_{-\infty}^{\infty}\mathrm{d}f\int_{S^2}\mathrm{d}\hat{\Omega}\, h_A(f,\hat{\Omega})e^{2\pi i f(t-\hat{\Omega}\cdot\vec{x})}\hat{e}^A_{ab}(\hat{\Omega})$$

$$\boxed{\Omega_{SGWB}(f) = \frac{2\pi^2 c^4}{3H_0^2}f^3 S_h(f)}$$

A. Salehi CCG SBU SGWB detection with TDA 2023/06 11 / 23

$$h(t) = \int_{-\infty}^{\infty}\mathrm{d}f h(f)e^{2\pi i f(t-\phi)}$$

$$\langle h(f)h^\star(f')\rangle = \frac{\delta(f-f')}{2}S_h(f)$$

From spectral energy density to wave forms

page 11

Stochastic gravitational wave background

"We are like an insect sitting at the edge of a pool, trying to figure out who jumped in where and when and what's happening all over the pool."

- Richard Feynman

Image credit: Megna pools

$$\Omega_{SGWB}(f) = \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{GW}(f)}{\mathrm{d}\ln f}$$

$$\langle h_A(f,\hat{\Omega})h^*_{A'}(f',\hat{\Omega}')\rangle_{\text{ens}} = \frac{\delta_{AA'}}{2}\frac{\delta^{(2)}(\hat{\Omega},\hat{\Omega}')}{4\pi}\frac{\delta(f-f')}{2}S_h(f)$$

$$h_{ab}(t,\vec{x}) = \sum_{A=+,\times}\int_{-\infty}^{\infty}\mathrm{d}f\int_{S^2}\mathrm{d}\hat{\Omega}\,h_A(f,\hat{\Omega})e^{2\pi i f(t-\hat{\Omega}\cdot\vec{x})}\hat{e}^A_{ab}(\hat{\Omega})$$

$$\boxed{\Omega_{SGWB}(f) = \frac{2\pi^2c^4}{3H_0^2}f^3S_h(f)}$$

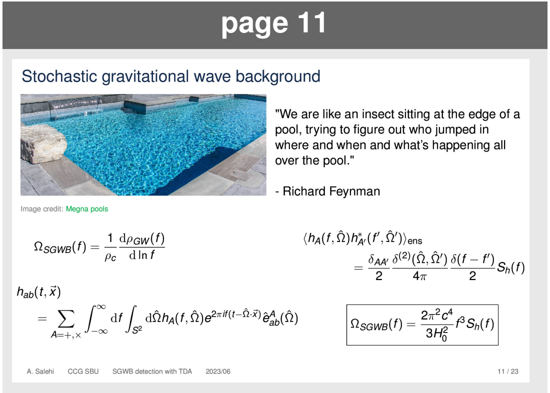
A. Salehi CCG SBU SGWB detection with TDA 2023/06 11 / 23

$$h(t) = \int_{-\infty}^{\infty}\mathrm{d}f h(f)e^{2\pi i f(t-\phi)}$$

$$\langle h(f)h^\star(f')\rangle = \frac{\delta(f-f')}{2}S_h(f)$$

To achieve this we used *LALSuite* package with a few added features:

From spectral energy density to wave forms



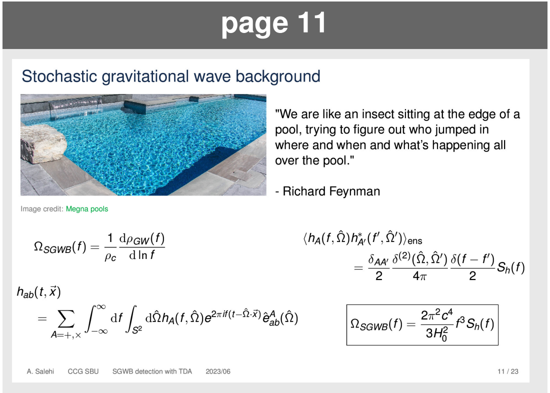
$$h(t) = \int_{-\infty}^{\infty}\mathrm{d}f h(f)e^{2\pi i f(t-\phi)}$$

$$\langle h(f)h^\star(f')\rangle = \frac{\delta(f-f')}{2}S_h(f)$$

To achieve this we used *LALSuite* package with a few added features:

- A new data structure to read the numerical spectrum

From spectral energy density to wave forms



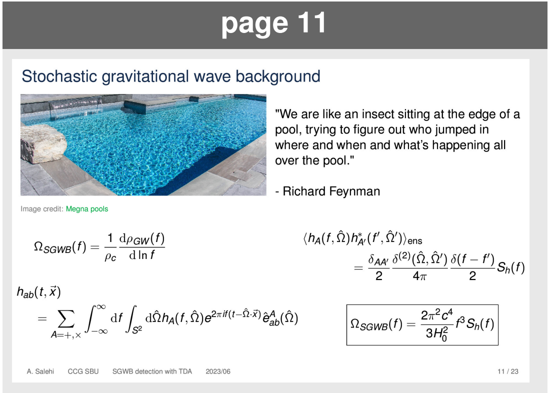
$$h(t) = \int_{-\infty}^{\infty}\mathrm{d}f h(f)e^{2\pi i f(t-\phi)}$$

$$\langle h(f)h^\star(f')\rangle = \frac{\delta(f-f')}{2}S_h(f)$$

To achieve this we used *LALSuite* package with a few added features:

- A new data structure to read the numerical spectrum
- New functions to simulate SGWB for pawr-law and numerical spectra

From spectral energy density to wave forms



page 11

Stochastic gravitational wave background

"We are like an insect sitting at the edge of a pool, trying to figure out who jumped in where and when and what's happening all over the pool."

- Richard Feynman

Image credit: Megna pools

$$\Omega_{SGWB}(f) = \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{GW}(f)}{\mathrm{d}\ln f}$$

$$\langle h_A(f,\hat{\Omega})h^*_{A'}(f',\hat{\Omega}')\rangle_{\text{ens}} = \frac{\delta_{AA'}}{2}\frac{\delta^{(2)}(\hat{\Omega},\hat{\Omega}')}{4\pi}\frac{\delta(f-f')}{2}S_h(f)$$

$$h_{ab}(t,\vec{x}) = \sum_{A=+,\times}\int_{-\infty}^{\infty}\mathrm{d}f\int_{S^2}\mathrm{d}\hat{\Omega}\, h_A(f,\hat{\Omega})e^{2\pi i f(t-\hat{\Omega}\cdot\vec{x})}\hat{e}^A_{ab}(\hat{\Omega})$$

$$\boxed{\Omega_{SGWB}(f) = \frac{2\pi^2 c^4}{3H_0^2}f^3 S_h(f)}$$

A. Salehi CCG SBU SGWB detection with TDA 2023/06 11 / 23

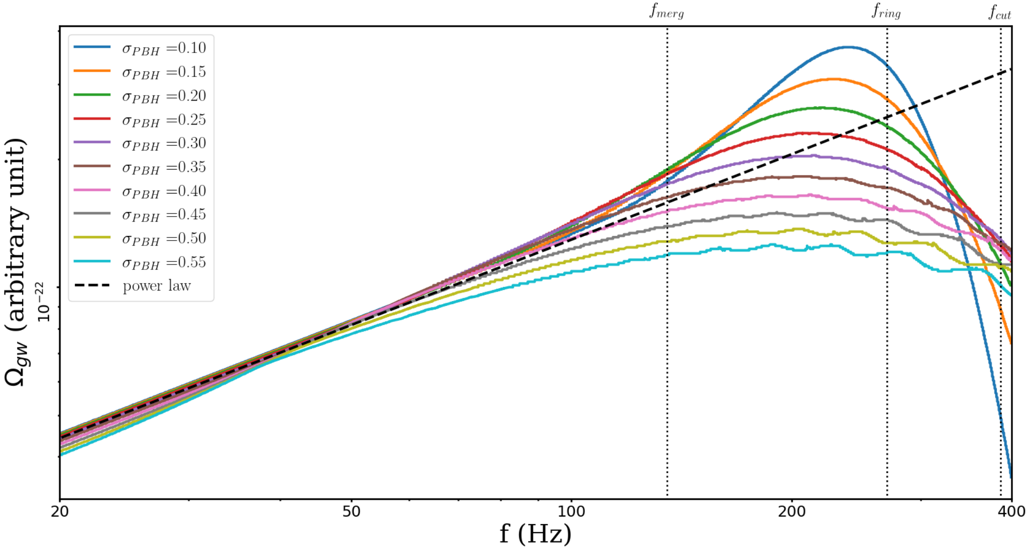
$$h(t) = \int_{-\infty}^{\infty}\mathrm{d}f h(f)e^{2\pi i f(t-\phi)}$$

$$\langle h(f)h^\star(f')\rangle = \frac{\delta(f-f')}{2}S_h(f)$$

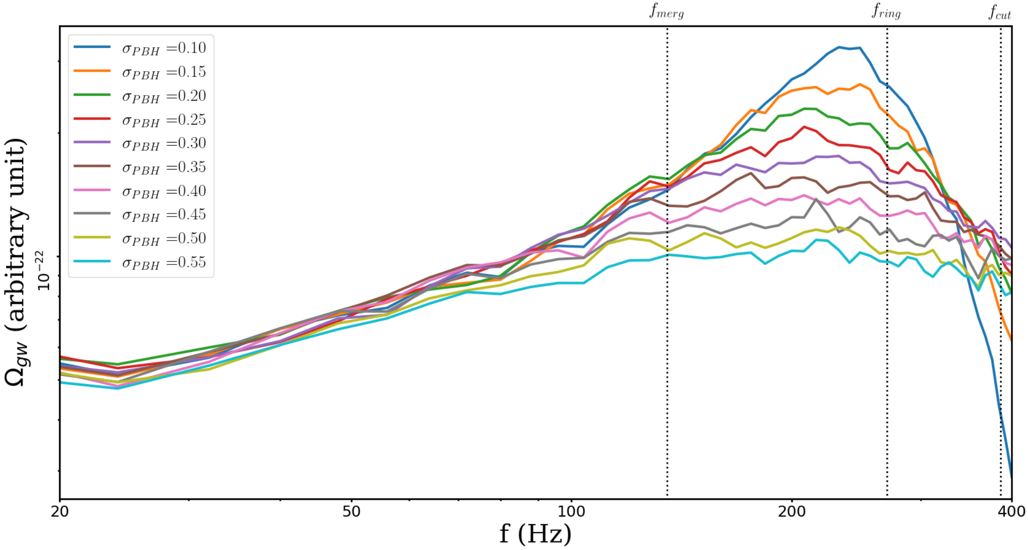
To achieve this we used *LALSuite* package with a few added features:

- A new data structure to read the numerical spectrum
- New functions to simulate SGWB for pawr-law and numerical spectra
- Options to access these new functionalities with an app

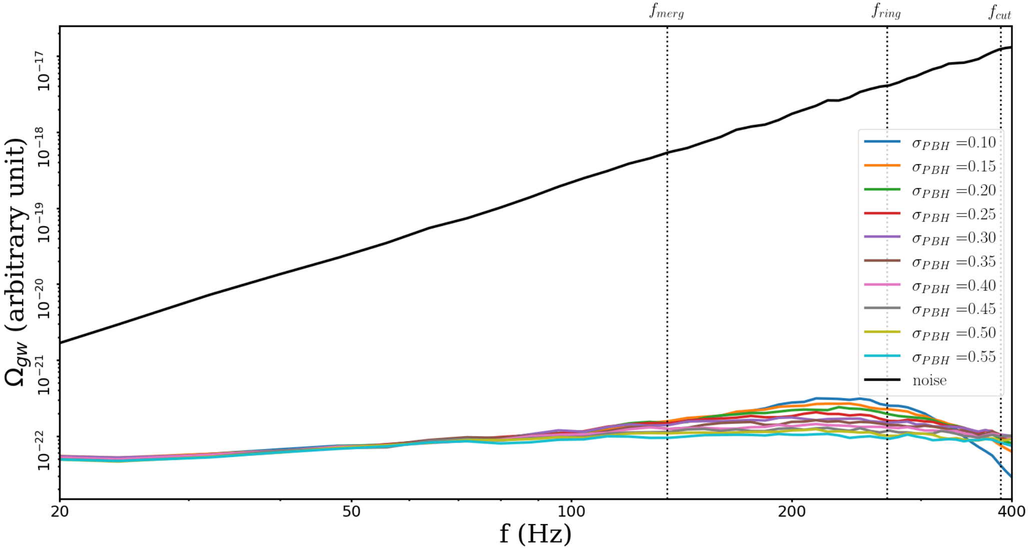
Spectral energy density plots (theoretical)



Spectral energy density plots (numerical)



Spectral energy density plots (noisy)



Recap

We want to detect:

- A stochastic gravitational wave background
- Coming from binary black hole mergers
- Our chosen black holes to study have a primordial origin

To achieve these, we need:

1. ~~Learn about primordial black holes and their population statistics~~
2. ~~Build a model for their merger rates based on the population statistics~~
3. ~~Simulate a stochastic gravitational wave background~~
4. Build a model to analyse the data
5. Run the analyses

Section 4

Topological Data Analysis

What is topology?

Image credit: Physics world

What is topology?

Image credit: Physics world

Topology:

What is topology?

Image credit: Physics world

Topology:

- The study of holes

What is topology?

Image credit: Physics world

Topology:

- The study of holes
- Rubber-sheet geometry

What is topology?

Image credit: Physics world

Topology:

- The study of holes
- Rubber-sheet geometry

Branches of topology:

What is topology?

Image credit: Physics world

Topology:

- The study of holes
- Rubber-sheet geometry

Branches of topology:

- General Topology or Point Set Topology

What is topology?

Image credit: Physics world

Topology:

- The study of holes
- Rubber-sheet geometry

Branches of topology:

- General Topology or Point Set Topology
- Combinatorial Topology

What is topology?

Image credit: Physics world

Topology:

- The study of holes
- Rubber-sheet geometry

Branches of topology:

- General Topology or Point Set Topology
- Combinatorial Topology
- Differential Topology

What is topology?

Image credit: Physics world

Topology:

- The study of holes
- Rubber-sheet geometry

Branches of topology:

- General Topology or Point Set Topology
- Combinatorial Topology
- Differential Topology
- *AlgebraicTopology*

Homeomorphism

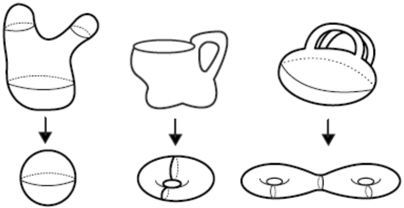


Image credit: Annenberg Learner

Homeomorphism

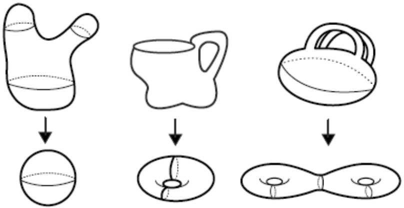


Image credit: Annenberg Learner

Image source: Reddit

Homeomorphism

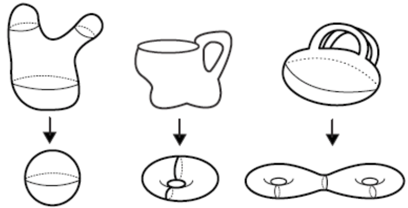


Image credit: Annenberg Learner

$\beta_k(\psi)$

Image source: Reddit

Homeomorphism

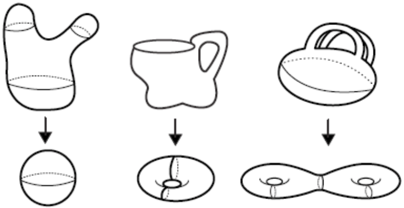


Image credit: Annenberg Learner

$$\beta_k(\psi) = \dim(H_k(\psi))$$

Image source: Reddit

Homeomorphism

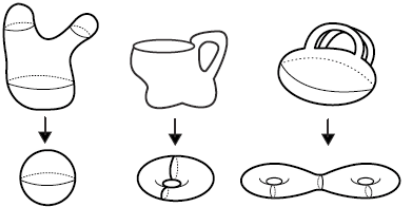


Image credit: Annenberg Learner

$$\beta_k(\psi) = \dim(H_k(\psi))$$

Topological space / Betti number						
β_0	1	1	1	1	1	1
β_1	0	0	1	0	2	4
β_2	0	0	0	1	1	1

Image credit: Masoomy, et al.

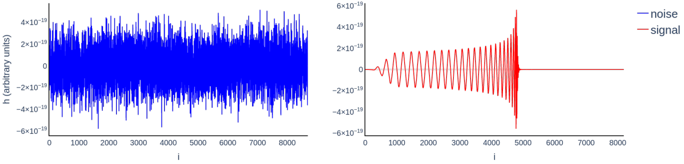
Me: Mom can we get donuts?

Mom (Topologist): We have donuts at home.

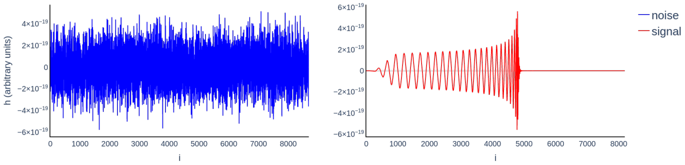
Donuts at home:

Image source: Reddit

Topology of time series

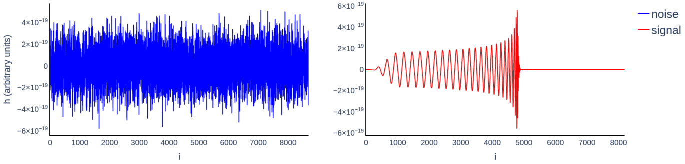


Topology of time series



Taken's embedding:

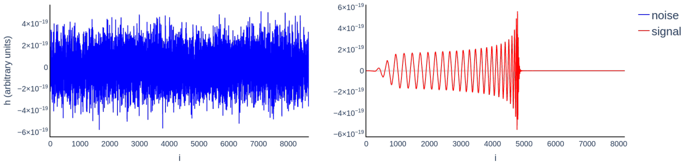
Topology of time series



Taken's embedding:

$$x(t) = (x_1, x_2, \ldots, x_i, x_{i+1}, \ldots, x_n)$$

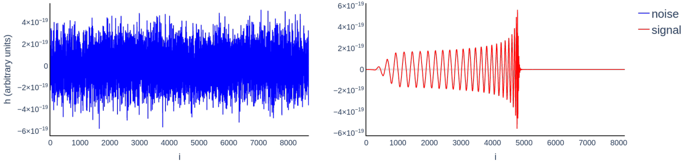
Topology of time series



Taken's embedding:

$$x(t) = (x_1, x_2, \ldots, x_i, x_{i+1}, \ldots, x_n)$$
$$t_i = i\Delta t$$

Topology of time series

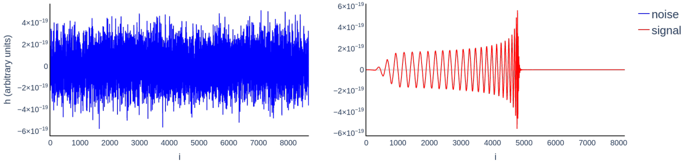


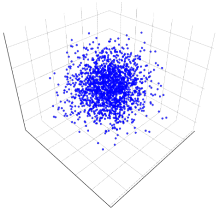
Taken's embedding:

$$x(t) = (x_1, x_2, \ldots, x_i, x_{i+1}, \ldots, x_n)$$
$$t_i = i\Delta t$$

$$x(t) \mapsto \begin{pmatrix} x_i \\ x_{i+\tau} \\ \vdots \\ x_{i+(d-1)\tau} \end{pmatrix}$$

Topology of time series



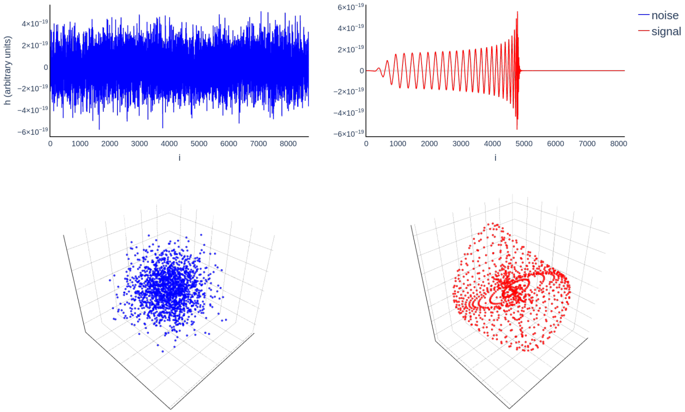


Taken's embedding:

$$x(t) = (x_1, x_2, \ldots, x_i, x_{i+1}, \ldots, x_n)$$
$$t_i = i\Delta t$$

$$x(t) \mapsto \begin{pmatrix} x_i \\ x_{i+\tau} \\ \vdots \\ x_{i+(d-1)\tau} \end{pmatrix}$$

Topology of time series



Taken's embedding:

$$x(t) = (x_1, x_2, \dots, x_i, x_{i+1}, \dots, x_n)$$
$$t_i = i\Delta t$$

$$x(t) \mapsto \begin{pmatrix} x_i \\ x_{i+\tau} \\ \vdots \\ x_{i+(d-1)\tau} \end{pmatrix}$$

Homology theory

In simple terms, homology theory tries to connect the dots in a topological space together and figure out what shape they make

Homology theory

In simple terms, homology theory tries to connect the dots in a topological space together and figure out what shape they make

To achieve this, homology theory uses *Simplexes* and *Simplical complexes*

Homology theory

In simple terms, homology theory tries to connect the dots in a topological space together and figure out what shape they make

To achieve this, homology theory uses *Simplexes* and *Simplical complexes*

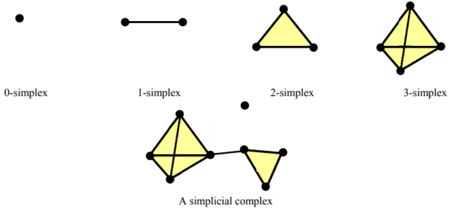


Image credit: Zulkepli, et al.

Homology theory

In simple terms, homology theory tries to connect the dots in a topological space together and figure out what shape they make

To achieve this, homology theory uses *Simplexes* and *Simplical complexes*

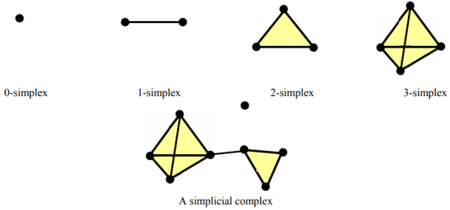


Image credit: Zulkepli, et al.

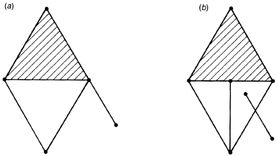


Image credit: Nakahara

Homology theory

In simple terms, homology theory tries to connect the dots in a topological space together and figure out what shape they make

To achieve this, homology theory uses *Simplexes* and *Simplical complexes*

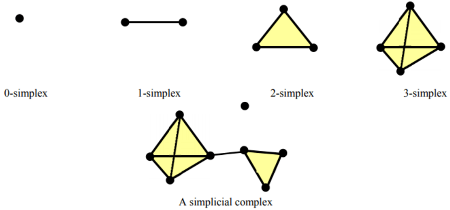


Image credit: Zulkepli, et al.

Vietoris-Rips complex

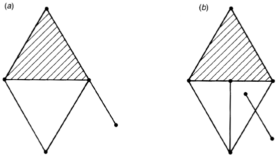


Image credit: Nakahara

Homology theory

In simple terms, homology theory tries to connect the dots in a topological space together and figure out what shape they make

To achieve this, homology theory uses *Simplexes* and *Simplical complexes*

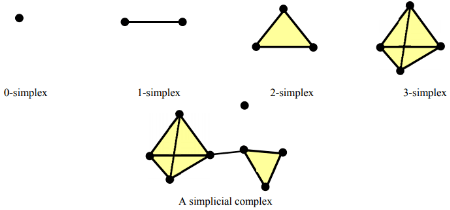


Image credit: Zulkepli, et al.

Vietoris-Rips complex

$$VR_s(\mathbb{X}) = \left\{ \sigma^r = \langle p_0 \dots p_r \rangle \;\middle|\; \forall p_i, p_j \in \sigma^r : \; l^2(p_i, p_j) \leq s \right\}$$

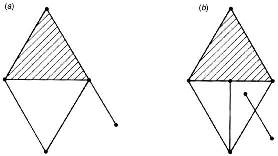


Image credit: Nakahara

Homology theory

In simple terms, homology theory tries to connect the dots in a topological space together and figure out what shape they make

To achieve this, homology theory uses *Simplexes* and *Simplical complexes*

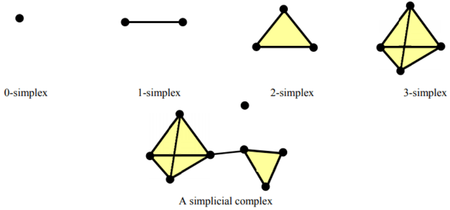


Image credit: Zulkepli, et al.

Vietoris-Rips complex

$$VR_s(\mathbb{X}) = \left\{ \sigma^r = \langle p_0 \dots p_r \rangle \;\middle|\; \forall p_i, p_j \in \sigma^r : \; l^2(p_i, p_j) \leq s \right\}$$

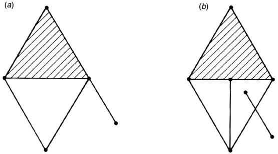


Image credit: Nakahara

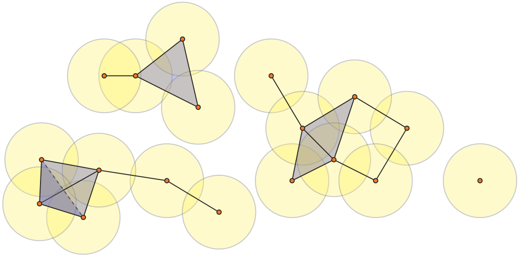
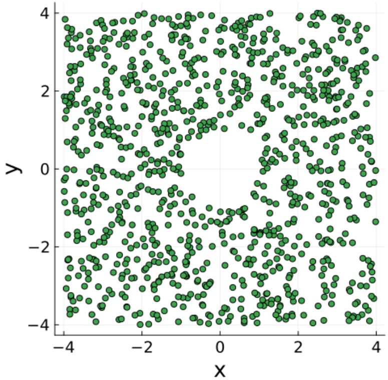
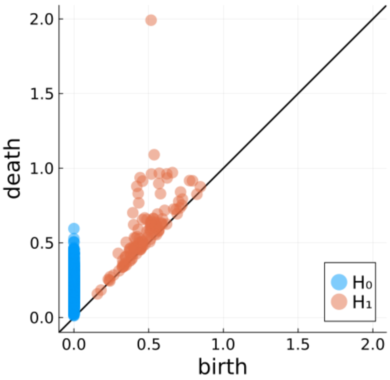


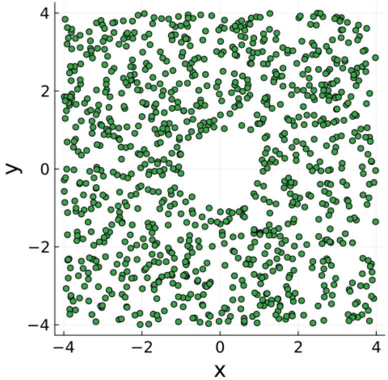
Image credit: Topaz, et al.

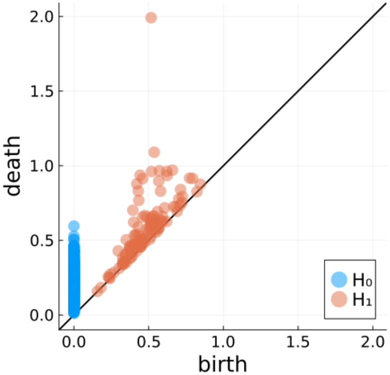
Persistance homology





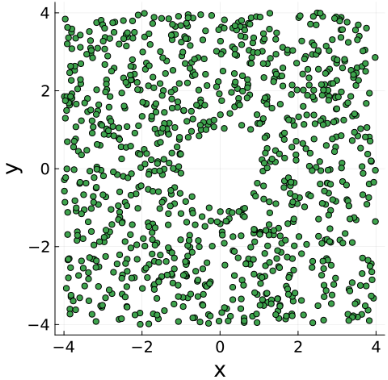
Persistance homology

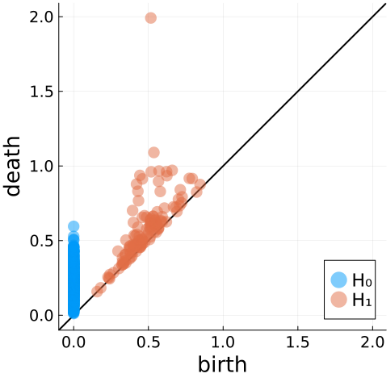




$$\mathbb{X} = \{\mathbb{D}_0, \ldots, \mathbb{D}_N\}$$

Persistance homology





$$\mathbb{X} = \{\mathbb{D}_0, \ldots, \mathbb{D}_N\}$$

$$\mathbb{D} = \{(b, d) \mid 0 \leq b < d \in \mathbb{R}\}$$

Vectorizations and amplitudes

$$\phi : \ \mathbb{X} \to \mathbb{V}$$

Vectorizations and amplitudes

$$\phi : \mathbb{X} \to \mathbb{V}$$

$$A : \mathbb{X} \to \mathbb{R}$$

Vectorizations and amplitudes

$$\phi : \mathbb{X} \to \mathbb{V}$$

$$A : \mathbb{X} \to \mathbb{R}$$

$$A(s) = ||\phi(s)||_p$$

Vectorizations and amplitudes

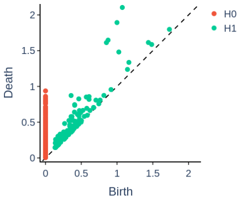
$$\phi: \ \mathbb{X} \rightarrow \mathbb{V}$$

$$A: \ \mathbb{X} \rightarrow \mathbb{R}$$

$$A(s) = ||\phi(s)||_p$$

$$||x||_p = (x_1^p + \cdots + x_2^p)^{1/p}$$

Vectorizations and amplitudes



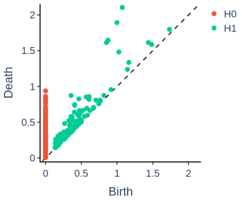
$$\phi : \ \mathbb{X} \rightarrow \mathbb{V}$$

$$A : \ \mathbb{X} \rightarrow \mathbb{R}$$

$$A(s) = ||\phi(s)||_p$$

$$||x||_p = (x_1^p + \cdots + x_2^p)^{1/p}$$

Vectorizations and amplitudes

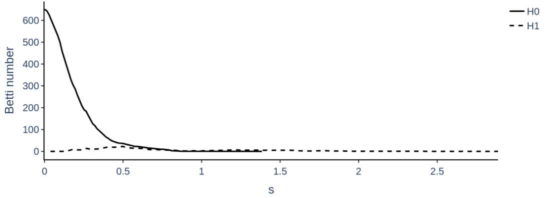


$$\phi : \mathbb{X} \to \mathbb{V}$$

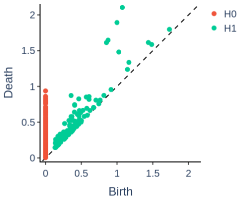
$$A : \mathbb{X} \to \mathbb{R}$$

$$A(s) = ||\phi(s)||_p$$

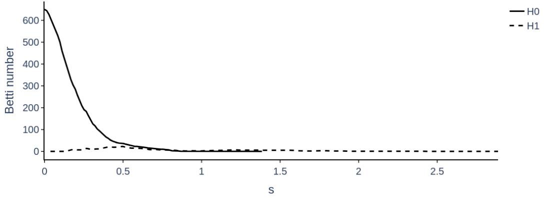
$$||x||_p = (x_1^p + \cdots + x_2^p)^{1/p}$$



Vectorizations and amplitudes



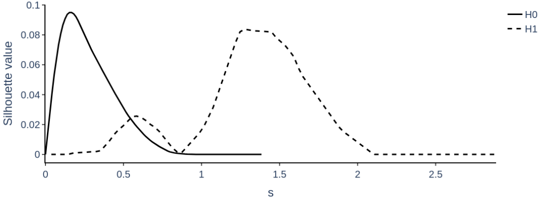
$$\phi: \mathbb{X} \to \mathbb{V}$$



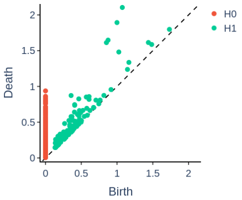
$$A: \mathbb{X} \to \mathbb{R}$$

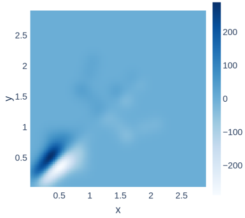
$$A(s) = ||\phi(s)||_p$$

$$||x||_p = (x_1^p + \cdots + x_2^p)^{1/p}$$

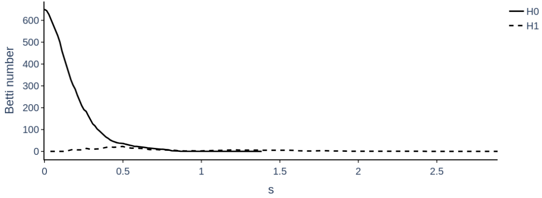


Vectorizations and amplitudes



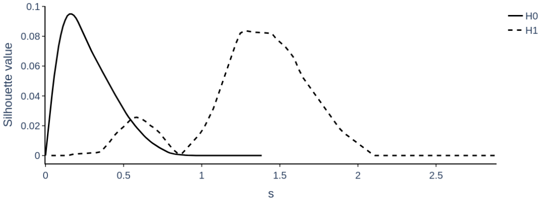


$$\phi : \mathbb{X} \to \mathbb{V}$$



$$A : \mathbb{X} \to \mathbb{R}$$

$$A(s) = ||\phi(s)||_p$$



$$||x||_p = (x_1^p + \cdots + x_2^p)^{1/p}$$

Recap

What we have by now:

Recap

What we have by now:

- A model for primordial black hole binary mergers

Recap

What we have by now:

- A model for primordial black hole binary mergers
- A way to simulate a stochastic background based on our model for different population statistics (σ_{PBH})

Recap

What we have by now:

- A model for primordial black hole binary mergers
- A way to simulate a stochastic background based on our model for different population statistics (σ_{PBH})
- A data analysis method wich has a promise of being resiliant to noise

Recap

What we have by now:

- A model for primordial black hole binary mergers
- A way to simulate a stochastic background based on our model for different population statistics (σ_{PBH})
- A data analysis method wich has a promise of being resiliant to noise

Now we are ready to build a data analysis model

Recap

What we have by now:

- A model for primordial black hole binary mergers
- A way to simulate a stochastic background based on our model for different population statistics (σ_{PBH})
- A data analysis method wich has a promise of being resiliant to noise

Now we are ready to build a data analysis model
We need to:

Recap

What we have by now:

- A model for primordial black hole binary mergers
- A way to simulate a stochastic background based on our model for different population statistics (σ_{PBH})
- A data analysis method wich has a promise of being resiliant to noise

Now we are ready to build a data analysis model
We need to:

- Find topological features to detect a signal amidst different levels of noise

Recap

What we have by now:

- A model for primordial black hole binary mergers
- A way to simulate a stochastic background based on our model for different population statistics (σ_{PBH})
- A data analysis method wich has a promise of being resiliant to noise

Now we are ready to build a data analysis model
We need to:

- Find topological features to detect a signal amidst different levels of noise
- Find topological features to discriminate between different values of σ_{PBH}

Recap

What we have by now:

- A model for primordial black hole binary mergers
- A way to simulate a stochastic background based on our model for different population statistics (σ_{PBH})
- A data analysis method wich has a promise of being resiliant to noise

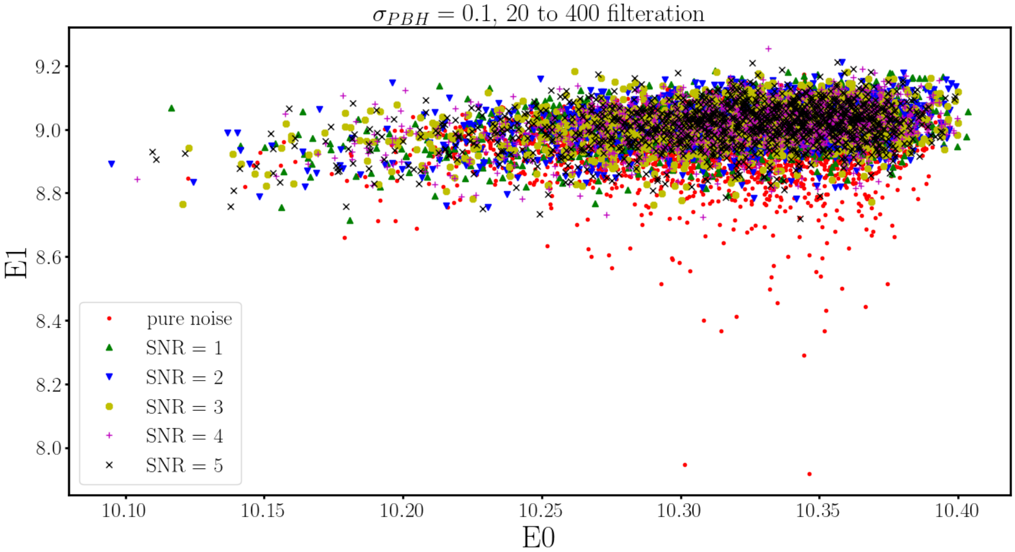
Now we are ready to build a data analysis model
We need to:

- Find topological features to detect a signal amidst different levels of noise
- Find topological features to discriminate between different values of σ_{PBH}
- A classification method to classify signals with different values of σ_{PBH}

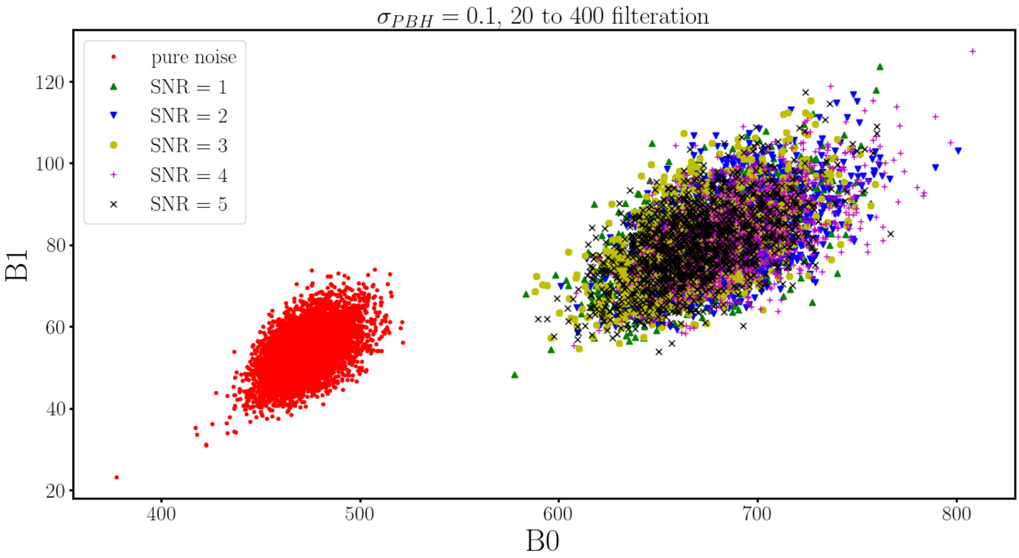
Section 5

Results

Entropy measures



Distance measures



Distance measures

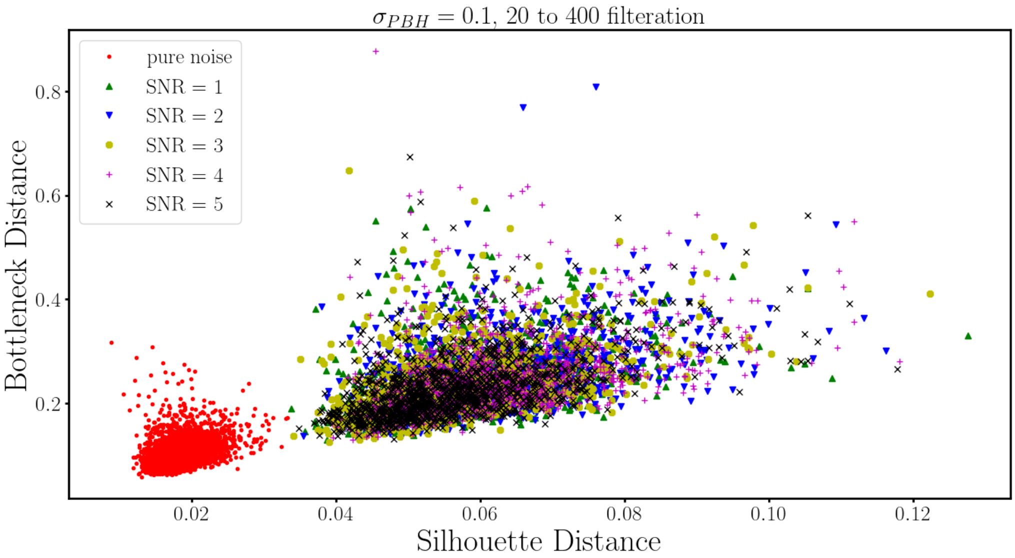
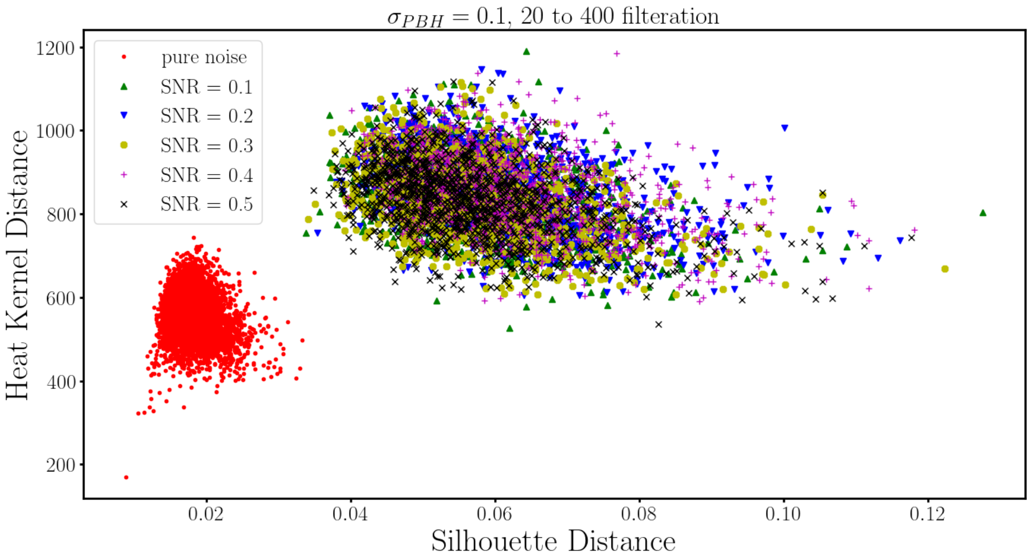
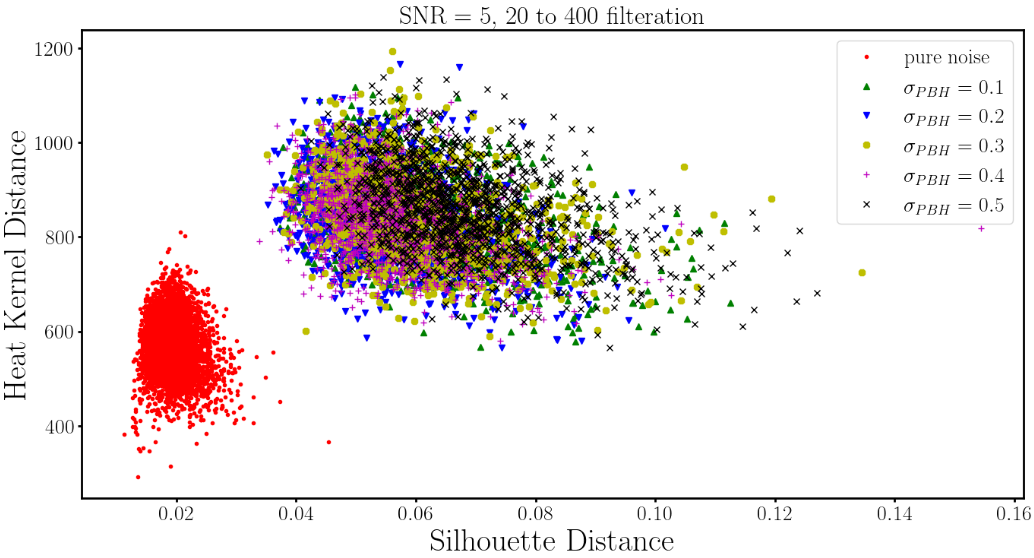


Image measures



First attempt to classification



What did we learn?

1. In contrast to the result from Bresten and Jung, entropy measures are not good for detecting a stochastic background.

What did we learn?

1. In contrast to the result from Bresten and Jung, entropy measures are not good for detecting a stochastic background.

2. Betti, Silhoutte and Heat Kernel amplitudes when combined together two by two, are very resiliant to noise.

What did we learn?

1. In contrast to the result from Bresten and Jung, entropy measures are not good for detecting a stochastic background.

2. Betti, Silhoutte and Heat Kernel amplitudes when combined together two by two, are very resiliant to noise.

3. The best topological measure to use for detecting the presence of a signal is Betti distance beacuse it's simpler to calculate.

What did we learn?

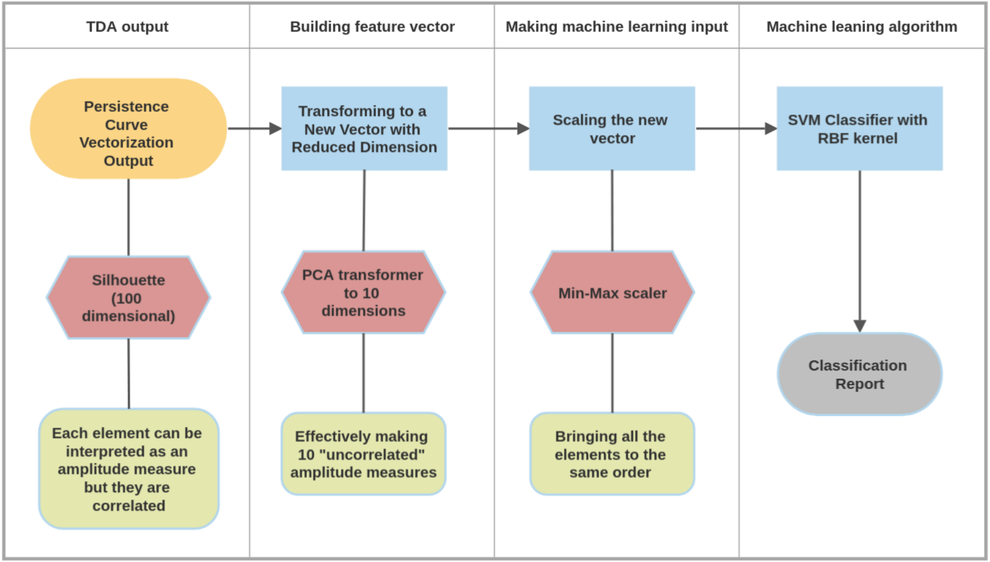
1. In contrast to the result from Bresten and Jung, entropy measures are not good for detecting a stochastic background.
2. Betti, Silhoutte and Heat Kernel amplitudes when combined together two by two, are very resiliant to noise.
3. The best topological measure to use for detecting the presence of a signal is Betti distance beacuse it's simpler to calculate.
4. Scaler topological measures can not classify the signals based on their physical parameters.

What did we learn?

1. In contrast to the result from Bresten and Jung, entropy measures are not good for detecting a stochastic background.
2. Betti, Silhoutte and Heat Kernel amplitudes when combined together two by two, are very resiliant to noise.
3. The best topological measure to use for detecting the presence of a signal is Betti distance beacuse it's simpler to calculate.
4. Scaler topological measures can not classify the signals based on their physical parameters.

Hence to build a classification pipeline, we need to use more complicated topological measures.

A classification pipeline



Classification report

Physical parameter	precision (%)	recall (%)	f1-score	support
$\sigma_{PBH} = 0.10$	91	85	0.88	379
$\sigma_{PBH} = 0.15$	81	86	0.83	345
$\sigma_{PBH} = 0.20$	93	93	0.93	357
$\sigma_{PBH} = 0.25$	88	94	0.91	375
$\sigma_{PBH} = 0.30$	85	93	0.89	328
$\sigma_{PBH} = 0.35$	95	91	0.93	373
$\sigma_{PBH} = 0.40$	99	97	0.98	347
$\sigma_{PBH} = 0.45$	94	81	0.87	366
$\sigma_{PBH} = 0.50$	89	93	0.91	367
$\sigma_{PBH} = 0.55$	85	87	0.86	367
accuracy			0.9	3604
macro avrage	90	90	0.9	3604

Classification report

Physical parameter	precision (%)	recall (%)	f1-score	support
σ_{PBH} = 0.10	91	85	0.88	379
σ_{PBH} = 0.15	81	86	0.83	345
σ_{PBH} = 0.20	93	93	0.93	357
σ_{PBH} = 0.25	88	94	0.91	375
σ_{PBH} = 0.30	85	93	0.89	328
σ_{PBH} = 0.35	95	91	0.93	373
σ_{PBH} = 0.40	99	97	0.98	347
σ_{PBH} = 0.45	94	81	0.87	366
σ_{PBH} = 0.50	89	93	0.91	367
σ_{PBH} = 0.55	85	87	0.86	367
accuracy			0.9	3604
macro avrage	90	90	0.9	3604

$$\text{precision} = \frac{\text{TP}}{\text{TP} + \text{FP}},$$

Classification report

Physical parameter	precision (%)	recall (%)	f1-score	support
$\sigma_{PBH} = 0.10$	91	85	0.88	379
$\sigma_{PBH} = 0.15$	81	86	0.83	345
$\sigma_{PBH} = 0.20$	93	93	0.93	357
$\sigma_{PBH} = 0.25$	88	94	0.91	375
$\sigma_{PBH} = 0.30$	85	93	0.89	328
$\sigma_{PBH} = 0.35$	95	91	0.93	373
$\sigma_{PBH} = 0.40$	99	97	0.98	347
$\sigma_{PBH} = 0.45$	94	81	0.87	366
$\sigma_{PBH} = 0.50$	89	93	0.91	367
$\sigma_{PBH} = 0.55$	85	87	0.86	367
accuracy			0.9	3604
macro avrage	90	90	0.9	3604

$$\text{precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}, \qquad \text{recall} = \frac{\text{TP}}{\text{TP} + \text{FN}},$$

Classification report

Physical parameter	precision (%)	recall (%)	f1-score	support
$\sigma_{PBH} = 0.10$	91	85	0.88	379
$\sigma_{PBH} = 0.15$	81	86	0.83	345
$\sigma_{PBH} = 0.20$	93	93	0.93	357
$\sigma_{PBH} = 0.25$	88	94	0.91	375
$\sigma_{PBH} = 0.30$	85	93	0.89	328
$\sigma_{PBH} = 0.35$	95	91	0.93	373
$\sigma_{PBH} = 0.40$	99	97	0.98	347
$\sigma_{PBH} = 0.45$	94	81	0.87	366
$\sigma_{PBH} = 0.50$	89	93	0.91	367
$\sigma_{PBH} = 0.55$	85	87	0.86	367
accuracy			0.9	3604
macro avrage	90	90	0.9	3604

$$\mathrm{precision} = \frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FP}}, \qquad \mathrm{recall} = \frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FN}}, \qquad \mathrm{f1-score} = 2\frac{\mathrm{recall}\times\mathrm{precision}}{\mathrm{recall}+\mathrm{precision}}$$

Section 6

Conclusion and Outlook

Our work in perspective

Model selection

Our work in perspective

Model selection

↓

Spectral energy density calculation

Our work in perspective

Model selection

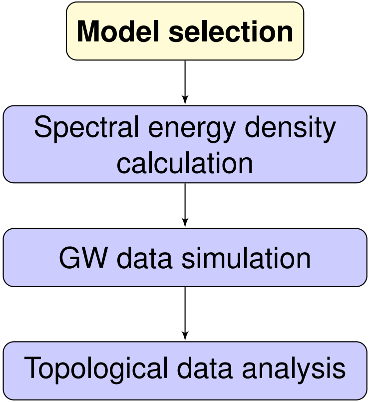
↓

Spectral energy density calculation

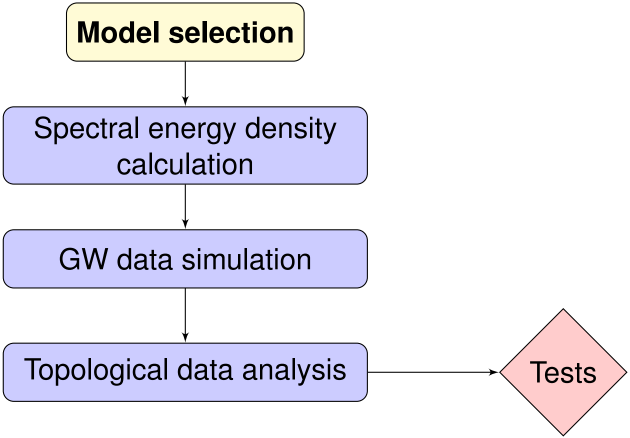
↓

GW data simulation

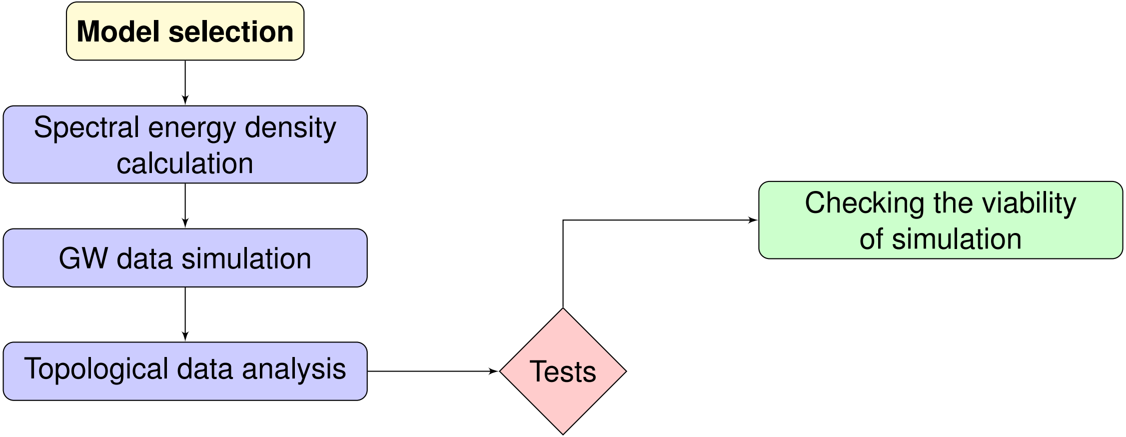
Our work in perspective



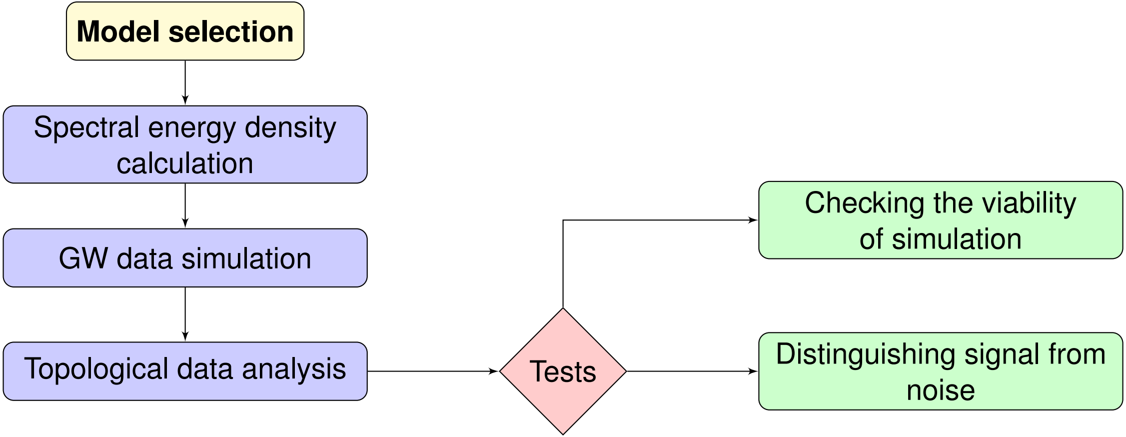
Our work in perspective



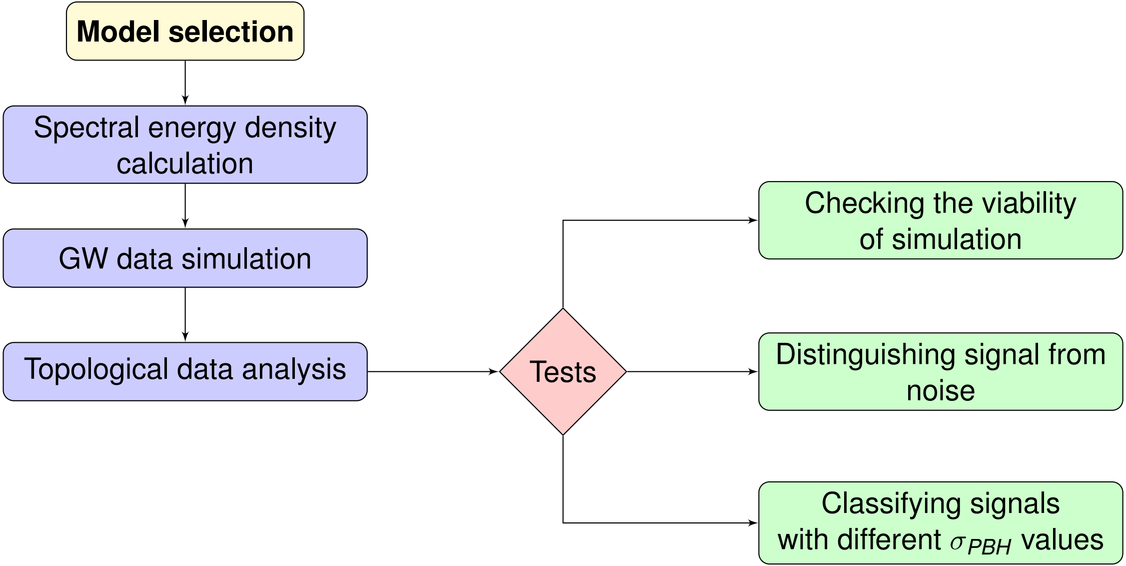
Our work in perspective



Our work in perspective



Our work in perspective



General statistics

For this work:

General statistics

For this work:

- 3600 signals for 10 different physics were simulated.

General statistics

For this work:

- 3600 signals for 10 different physics were simulated.
- 6 different noise amplitudes were added.

General statistics

For this work:

- 3600 signals for 10 different physics were simulated.
- 6 different noise amplitudes were added.
- Tens of embedding dimensions and time delays were tested.

General statistics

For this work:

- 3600 signals for 10 different physics were simulated.
- 6 different noise amplitudes were added.
- Tens of embedding dimensions and time delays were tested.
- Tens of topological vectorization and amplitude measures were tested.

General statistics

For this work:

- 3600 signals for 10 different physics were simulated.
- 6 different noise amplitudes were added.
- Tens of embedding dimensions and time delays were tested.
- Tens of topological vectorization and amplitude measures were tested.
- A 10 dimensional feature vector was selected.

General statistics

For this work:

- 3600 signals for 10 different physics were simulated.
- 6 different noise amplitudes were added.
- Tens of embedding dimensions and time delays were tested.
- Tens of topological vectorization and amplitude measures were tested.
- A 10 dimensional feature vector was selected.
- A 90% classification accuracy was achieved.

Innovations

Three important innovations were made in this research:

Innovations

Three important innovations were made in this research:

- A code was written to calculate a spectral energy density using the semi-analytical formula; which is expandable to other astrophysical, cosmological and dynamical scenarios.

Innovations

Three important innovations were made in this research:

- A code was written to calculate a spectral energy density using the semi-analytical formula; which is expandable to other astrophysical, cosmological and dynamical scenarios.
- A number of functions and a data structure were added to the *LALSuite* package which enable it to simulate a SGWB from a custom Ω_{SGWB}.

Innovations

Three important innovations were made in this research:

- A code was written to calculate a spectral energy density using the semi-analytical formula; which is expandable to other astrophysical, cosmological and dynamical scenarios.
- A number of functions and a data structure were added to the *LALSuite* package which enable it to simulate a SGWB from a custom Ω_{SGWB}.
- A novel topological measure was introduced which can classify different SGWBs based on their physical parameters.

Suggestions

In the future we can expand on this research by doing:

Suggestions

In the future we can expand on this research by doing:

- A Fischer forcast to evaluate the detection probability of PBH-induced SGWBs via different ovservatories.

Suggestions

In the future we can expand on this research by doing:

- A Fischer forcast to evaluate the detection probability of PBH-induced SGWBs via different ovservatories.
- Comparing our classification results with the results from more classical algorithms.

Suggestions

In the future we can expand on this research by doing:

- A Fischer forcast to evaluate the detection probability of PBH-induced SGWBs via different ovservatories.
- Comparing our classification results with the results from more classical algorithms.
- Considering other PBH formation scenarios and simulating their SGWBs.

Suggestions

In the future we can expand on this research by doing:

- A Fischer forcast to evaluate the detection probability of PBH-induced SGWBs via different ovservatories.
- Comparing our classification results with the results from more classical algorithms.
- Considering other PBH formation scenarios and simulating their SGWBs.
- Changing various astrophysical and cosmological parameters to investigate our models ability to constrain those parameters.

Accessibility

In the end, if you are interested in my research, you can sownload my thesis and see my contact information on my page at physics department's computational cosmology group website:

ccg.sbu.ac.ir/people/ali-salehi


Thank you for your attention!