Electron endpoint energy in beta decay with a massive neutrino

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Abstract

We show that if neutrino has a non-zero rest mass m_ν, it can be proven that for neutron beta decay, the endpoint energy of electron is given $E_{e,max} = \frac{m_n^2+m_e^2-(m_p+m_{\bar{\nu}})^2}{2m_n}$. So by knowing the endpoint energy, the neutrino rest mass can be calculated.

The Karlsruhe Tritium Neutrino Experiment (KATRIN), tries to calculate mass of neutrino by studying cross sections of the beta decay. By now, it has been able to put an upper limit of 1.1 eV on it.

In neutron decay: n $\rightarrow$ p + e + $\bar{\nu}$, conservation of momentum can be written as follows:

$$P_n = P_p + P_{\bar{\nu}} + P_e \,. \tag{1}$$

Here $P = (\frac{E}{c}, \vec{p})$ is the momentum four-vector. c is the speed of light and energy and momentum are defined as follows

$$E = \gamma m c^2 \tag{2}$$

$$\vec{p} = \gamma m \vec{v} \tag{3}$$

and the Minkowski metric is $\eta = diag(1, -1, -1, -1)$. We can change Eq.(1) to

$$(P_n - P_e)^2 = (P_p + P_{\bar{\nu}})^2 \tag{4}$$

and by writing momentum four-vectors in neutron's rest frame, with a little algebra, we get

$$2m_n E_e = (m_n^2 + m_e^2 - m_p^2 - m_{\bar{\nu}}^2)c^2 - \frac{2E_p E_{\bar{\nu}}}{c^2} + 2\vec{p}_p \cdot \vec{p}_{\bar{\nu}} \,. \tag{5}$$

By replacing E and $\vec{p}$ from Eq.(2 & 3)

$$E_e = \frac{m_n^2 + m_e^2 - m_p^2 - m_{\bar{\nu}}^2 - 2\gamma_p \gamma_{\bar{\nu}} m_p m_{\bar{\nu}} (1 - \frac{\vec{v}_p \cdot \vec{v}_{\bar{\nu}}}{c^2})}{2m_n} \,. \tag{6}$$

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All the masses are constant, so to calculate $E_{e,max}$ we need to minimize the last term of Eq.(6), first we define

$$X = \gamma_p \gamma_{\bar{\nu}} (1 - \frac{\vec{v}_p \cdot \vec{v}_{\bar{\nu}}}{c^2}) = \gamma_p \gamma_{\bar{\nu}} (1 - \frac{v_p v_{\bar{\nu}} \cos(\theta)}{c^2}) \,. \tag{7}$$

By taking first and second derivative of X with respect to θ, we see that $\theta = 0$ is a minimum. So

$$X = \gamma_p \gamma_{\bar{\nu}} (1 - \frac{v_p v_{\bar{\nu}}}{c^2}) = \frac{c^2 - v_p v_{\bar{\nu}}}{\sqrt{(c^2 - v_p^2)(c^2 - v_{\bar{\nu}}^2)}} \,, \tag{8}$$

and

$$X^2 = \frac{c^4 + v_p^2 v_{\bar{\nu}}^2 - 2 v_p v_{\bar{\nu}} c^2}{c^4 + v_p^2 v_{\bar{\nu}}^2 - (v_p^2 + v_{\bar{\nu}}^2) c^2} = \frac{a - 2xy}{a - (x^2 + y^2)} \geq 1 \,. \tag{9}$$

Now it is proved that the minimum value of X equals to 1 and

$$E_{e,max} = \frac{m_n^2 + m_e^2 - (m_p + m_{\bar{\nu}})^2}{2 m_n} \,. \tag{10}$$

CONCLUSION

Proton, neutron and electron masses are known to very good precision, so by obtaining electron endpoint energy in beta decays, neutrino's mass can be calculated:

$$m_{\bar{\nu}} = \sqrt{m_n^2 + m_e^2 - 2 m_n E_{e,max}} - m_p \,. \tag{11}$$