

REVIEW OF CO-ORDINATES AND DIRECTION COSINES

Some Formulae

- (1) Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- (2) Coordinate of the point that divides the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) be,

- (i) Internally in the ratio $m_1:m_2$ then,

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \quad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \quad z = \frac{m_1z_2 + m_2z_1}{m_1 + m_2}$$

- (ii) Externally in the ratio $m_1:m_2$ then

$$x = \frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \quad y = \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \quad z = \frac{m_1z_2 - m_2z_1}{m_1 - m_2}$$

- (3) If a, b, c are direction ratios of a line then the direction cosines of the line are,

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, \quad m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \quad n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

- (4) If l_1, m_1, n_1 are direction cosines of a line and l_2, m_2, n_2 are direction cosines of second line and

- (i) if θ be the angle between the lines, then

$$\cos \theta = l_1l_2 + m_1m_2 + n_1n_2$$

Or, equivalently, if a_1, b_1, c_1 are direction ratios of a line and a_2, b_2, c_2 are direction ratios of second line and if θ be the angle between the lines, then

$$\cos \theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

- (ii) if the lines are perpendicular to each other then

$$l_1l_2 + m_1m_2 + n_1n_2 = 0 \quad \text{OR, equivalently, } a_1a_2 + b_1b_2 + c_1c_2 = 0$$

- (iii) if the lines are parallel then

$$l_1 = l_2, m_1 = m_2, n_1 = n_2 \quad \text{OR, equivalently, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$