CHAPTER 5

CONE, CYLINDER, CONICOID

Cone:

A cone is a surface generated by line (the lines are called generators) that passes through a fixed point and touches the given surface.

Note: The fixed point is called vertex of the cone.

Note: The lines are called generator of the cone.

General equation of a cone:

An equation,

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$$

represents a cone with vertex at (u, v, w) if

$$\begin{vmatrix} a & h & g & u \\ h & b & f & v \\ g & f & c & w \\ u & v & w & d \end{vmatrix} = 0.$$

Equation of cone having vertex at origin:

[2019 (Fall), 1999; 2001; 2003(Fall); 2006(Fall); 2008(Spring)–Short]

The equation,

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

represents a cone with vertex at origin.

Equation of cone having three mutually perpendicular generators:

[2000; 2002; 2004(Spring); 2007(Fall); 2008(Spring)–Short][2017 Fall]

An equation,

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$$

have a set of three mutually perpendicular generators when $a + b + c = 0$.

Cylinder:

A cylinder is a locus of lines which remains parallel to a fixed line and intersects a given curve.

The lines are called generator of cylinder.

Equation of cylinder:

The equation of cylinder whose generators intersect the curve $x^2 + y^2 + 2fy + 2gx + c = 0$, $z = 0$ and are parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ is,

$$(nx - lz)^2 + (ny - mz)^2 + 2gn(nx - lz) + 2fx(ny - mz) + cn^2 = 0.$$

Note: If the generators are parallel to z–axis then $l = 0 = m$ and $n = 1$. So, the equation of cylinder is, **[2004 (Fall) – Short]**

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

Equation of right circular cylinder:

The equation of right circular cylinder whose axis is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

and having radius r is

$$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 - [a(x - x_1) + b(y - y_1) + c(z - z_1)]^2 = r^2$$

Note: If the axis is z–axis. Then $a = b = 0$ and $c = 1$. Also, $x_1 = y_1 = z_1 = 0$. So, the equation of right circular cylinder is,

$$x^2 + y^2 = r^2$$

Conicoid:

An equation $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$ represents a coincoid.

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