

# Application of Bootstrap samples in Random Forest

In [124]:

```
import numpy as np
from sklearn.datasets import load_boston
from sklearn.metrics import mean_squared_error
```

- Load the boston house dataset

In [125]:

```
boston = load_boston()
x=boston.data #independent variables
y=boston.target #target variable
```

In [126]:

```
x.shape
```

Out[126]:

```
(506, 13)
```

## Task: 1

**Step 1 Creating samples:** Randomly create 30 samples from the whole boston data points.

1. Creating each sample: Consider any random 303(60% of 506) data points from whole data set and then replicate any 203 points from the sampled points
2. Ex: For better understanding of this procedure lets check this examples, assume we have 10 data points [1,2,3,4,5,6,7,8,9,10], first we take 6 data points randomly consider we have selected [4, 5, 7, 8, 9, 3] now we will replicate 4 points from [4, 5, 7, 8, 9, 3], consider they are [5, 8, 3,7] so our final sample will be [4, 5, 7, 8, 9, 3, 5, 8, 3,7]
3. we create 30 samples like this
4. Note that as a part of the Bagging when you are taking the random samples make sure each of the sample will have different set of columns
5. Ex: assume we have 10 columns for the first sample we will select [3, 4, 5, 9, 1, 2] and for the second sample [7, 9, 1, 4, 5, 6, 2] and so on...
6. Make sure each sample will have atleast 3 features/columns/attributes

**Step 2 Building High Variance Models on each of the sample and finding train MSE value:** Build a DecisionTreeRegressor on each of the sample.

1. Build a regression trees on each of 30 samples.
2. computed the predicted values of each data point(506 data points) in your corpus.
3. predicted house price of  $i^{\text{th}}$  data point  $y_{i\_pred} = \frac{1}{30} \sum_{k=1}^{30} (\text{predicted value of } x^i \text{ with } k^{\text{th}} \text{ model})$ .
4. Now calculate the  $MSE = \frac{1}{506} \sum_{i=1}^{506} (y^i - y_{i\_pred})^2$ .

**Step 3 Calculating the OOB score :**

1. Computed the predicted values of each data point(506 data points) in your corpus.
2. Predicted house price of  $i^{\text{th}}$  data point  $y_{i\_pred} = \frac{1}{k} \sum_{\text{k= model which was buit on samples not included } x^i} (\text{predicted value of } x^i \text{ with } k^{\text{th}} \text{ model})$ .
3. Now calculate the  $OOB \text{ Score} = \frac{1}{506} \sum_{i=1}^{506} (y^i - y_{i\_pred})^2$ .

## Task: 2

**Computing CI of OOB Score and Train MSE**

1. Repeat Task 1 for 35 times, and for each iteration store the Train MSE and OOB score

2. After this we will have 35 Train MSE values and 35 OOB scores
3. using these 35 values (assume like a sample) find the confidence intervals of MSE and OOB Score
4. you need to report CI of MSE and CI of OOB Score
5. Note: Refer the Central\_Limit\_theorem.ipynb to check how to find the confidence interval

### Task: 3

**Given a single query point predict the price of house.**

- Consider  $x_q = [0.18, 20.0, 5.00, 0.0, 0.421, 5.60, 72.2, 7.95, 7.0, 30.0, 19.1, 372.13, 18.60]$  Predict the house price for this point as mentioned in the step 2 of Task 1.

In [127]:

```
import random

row_indices_iob = []
row_indices_oob = []
col_indices = []

for i in range(30):
    iob_indices = np.random.choice(range(506), 303)
    iob_indices = np.hstack([iob_indices, np.random.choice(iob_indices, size=203)])
    col_index = np.array(random.sample(range(13), random.randint(3, 13)))
    row_indices_iob.append(iob_indices)
    col_indices.append(col_index)
    oob_indices = np.setdiff1d(range(506), iob_indices)
    row_indices_oob.append(oob_indices)
print(row_indices_iob[29].shape)
print(row_indices_oob[29].shape)
print(col_indices[29].shape)
```

```
(506,)
(273,)
(12,)
```

In [128]:

```
from sklearn.tree import DecisionTreeRegressor
Y_pred = [0 for i in range(506)]
for i in range(30):
    clf = DecisionTreeRegressor()
    X_train = x[row_indices_iob[i][:, None], col_indices[i]]
    Y_train = y[row_indices_iob[i]]
    clf.fit(X_train, Y_train)
    Y_pred += clf.predict(x[:, col_indices[i]])
Y_pred = Y_pred/30

errors = (y-Y_pred)**2
MSE = errors.sum()/506
print(MSE)
```

```
3.974955773336635
```

In [129]:

```
Y_pred_OOB = [0 for i in range(506)]
Ks = [0 for i in range(506)]

for i in range(30):
```

```

for i in range(50):
    k = 0
    clf = DecisionTreeRegressor()
    X_train = x[row_indices_iob[i][:,None],col_indices[i]]
    Y_train = y[row_indices_iob[i]]
    clf.fit(X_train,Y_train)

    for j in range(506):
        if j not in row_indices_iob[i]:
            Ks[j] += 1
            Y_pred_OOB[k] += clf.predict(x[j,col_indices[i]].reshape(1,-1)).item()
            k += 1

Y_pred_OOB = [x/y for x,y in zip(Y_pred_OOB,Ks)]

errors_OOB = (y-Y_pred_OOB)**2
OOB = errors_OOB.sum()/506
print(OOB)

```

11.69163972555204

## Task 2

In [130]:

```

MSEs = []
OOBs = []
for _ in range(35):
    row_indices_iob = []
    row_indices_oob = []
    col_indices = []

    for i in range(30):
        iob_indices = np.random.choice(range(506),303)
        iob_indices = np.hstack([iob_indices,np.random.choice(iob_indices,size=203)])
        col_index = np.array(random.sample(range(13),random.randint(3,13)))
        row_indices_iob.append(iob_indices)
        col_indices.append(col_index)
        oob_indices = np.setdiff1d(range(506),iob_indices)
        row_indices_oob.append(oob_indices)

    Y_pred = [0 for i in range(506)]
    for i in range(30):
        clf = DecisionTreeRegressor()
        X_train = x[row_indices_iob[i][:,None],col_indices[i]]
        Y_train = y[row_indices_iob[i]]
        clf.fit(X_train,Y_train)
        Y_pred += clf.predict(x[:,col_indices[i]])
    Y_pred = Y_pred/30

    errors = (y-Y_pred)**2
    MSE = errors.sum()/506
    MSEs.append(MSE)

    Y_pred_OOB = [0 for i in range(506)]
    Ks = [0 for i in range(506)]

    for i in range(30):
        k = 0
        clf = DecisionTreeRegressor()
        X_train = x[row_indices_iob[i][:,None],col_indices[i]]
        Y_train = y[row_indices_iob[i]]
        clf.fit(X_train,Y_train)

        for j in range(506):
            if j not in row_indices_iob[i]:
                Ks[j] += 1
                Y_pred_OOB[k] += clf.predict(x[j,col_indices[i]].reshape(1,-1)).item()
                k += 1

    Y_pred_OOB = [x/y for x,y in zip(Y_pred_OOB,Ks)]

    errors_OOB = (y-Y_pred_OOB)**2

```

```

OOB = errors_OOB.sum()/506
OOBs.append(OOB)

print(MSEs)
print('='*120)
print(OOBs)

```

```

[3.8472391580207597, 4.423723355818459, 4.674006739780048, 4.627198883931476, 4.137153248508573, 4.8032
36730456621, 4.435555248665485, 4.500476470394291, 4.215775773973344, 4.393658868006697, 4.404003390559
688, 3.8481509290725167, 3.9115398479907797, 4.360412047485164, 4.480441525906438, 4.16797327736497, 4.
485077160495039, 4.214339719960982, 4.676706371933498, 4.569076640143514, 4.636988400388053, 4.34750647
4740669, 3.707258943931785, 3.932626385705784, 3.9388100516029865, 4.60065953973295, 4.192864610193973,
4.584622508191453, 4.591076764163374, 3.206511235179729, 3.6793776507145735, 5.444778986530027, 4.05767
6130061335, 3.6720522944970986, 4.3562992483320055]

```

```

=====
[12.289036235714564, 14.469099649717203, 15.6610961172924, 15.074464357196256, 13.712702361193076, 13.7
41634684940339, 13.757811911671507, 13.607515571375357, 13.334661527442684, 14.076942749254597, 14.1225
75040214967, 13.322618315083716, 12.915276541617825, 12.87967277513743, 14.18989103157671, 13.187074228
164201, 14.023399493978907, 13.968364644292613, 16.050857645288662, 14.14457151391725, 14.8383013227476
41, 14.23878304829649, 12.391335606728425, 13.407851635984334, 13.155656529344723, 14.672944839369068,
12.941664201928, 13.19612973784292, 13.121051379107554, 12.061279350654242, 12.843027491257255, 15.2106
01393959234, 13.118191379966198, 11.942217340148716, 14.50463799158334]

```

In [131]:

```

#calculating 95% confidence interval
no_of_samples = 200
no_of_bins = 10

from prettytable import PrettyTable
MSE_means = []
table = PrettyTable()
table = PrettyTable(["#samples", "Sample Size", "Sample mean", "Sample std", "Left C.I", "Right C.I"])
for i in range(no_of_samples):
    sample=[MSEs[i] for i in random.choices(range(35),k=10)]
    sample_mean = np.mean(sample)
    MSE_means.append(sample_mean)
    sample_std = np.std(sample)
    sample_size = len(sample)
    # here we are using sample standard deviation instead of population standard deviation
    left_limit = np.round(sample_mean - 2*(sample_std/np.sqrt(sample_size)), 3)
    right_limit = np.round(sample_mean + 2*(sample_std/np.sqrt(sample_size)), 3)
    row = []
    row.append(i+1)
    row.append(sample_size)
    row.append(sample_mean)
    row.append(sample_std)
    row.append(left_limit)
    row.append(right_limit)
    table.add_row(row)

print('===== Calculation Interval Calculations for MSE =====')
print(table)

```

```

===== Calculation Interval Calculations for MSE =====
+-----+-----+-----+-----+-----+-----+
| #samples | Sample Size | Sample mean | Sample std | Left C.I | Right C.I |
+-----+-----+-----+-----+-----+-----+
| 1 | 10 | 4.075129360867921 | 0.5244911977141258 | 3.743 | 4.407 |
| 2 | 10 | 4.187460691531364 | 0.46223527503691647 | 3.895 | 4.48 |
| 3 | 10 | 4.361574390407271 | 0.22383955406118727 | 4.22 | 4.503 |
| 4 | 10 | 4.393131192833627 | 0.38473488931167044 | 4.15 | 4.636 |
| 5 | 10 | 4.440739067359237 | 0.46719209803288103 | 4.145 | 4.736 |
| 6 | 10 | 4.3318684759251935 | 0.4333555910466703 | 4.058 | 4.606 |
| 7 | 10 | 4.411318698917131 | 0.4571887688236746 | 4.122 | 4.7 |
| 8 | 10 | 4.341467418944108 | 0.45830694294967156 | 4.052 | 4.631 |
| 9 | 10 | 4.235089179698016 | 0.2582658943463807 | 4.072 | 4.398 |
| 10 | 10 | 4.381665165354279 | 0.21956577994476403 | 4.243 | 4.521 |
| 11 | 10 | 4.425287091976797 | 0.4550097216228392 | 4.138 | 4.713 |
| 12 | 10 | 4.075441276584313 | 0.5184280702463353 | 3.748 | 4.403 |
| 13 | 10 | 4.355671533404453 | 0.43138851259058686 | 4.083 | 4.629 |
| 14 | 10 | 4.199941270111568 | 0.3504781695306132 | 3.978 | 4.422 |
| 15 | 10 | 4.072262482449671 | 0.2710351262939642 | 3.901 | 4.244 |
| 16 | 10 | 4.242347578369914 | 0.376829737015435 | 4.004 | 4.481 |

```

|    |    |                    |                     |       |       |
|----|----|--------------------|---------------------|-------|-------|
| 16 | 10 | 4.242347078009914  | 0.378823707019430   | 4.004 | 4.401 |
| 17 | 10 | 4.44583742215536   | 0.43941975530650623 | 4.168 | 4.724 |
| 18 | 10 | 4.048012584370863  | 0.41711945554545626 | 3.784 | 4.312 |
| 19 | 10 | 4.195655880725784  | 0.5307218310217404  | 3.86  | 4.531 |
| 20 | 10 | 4.276929184650393  | 0.3409663244586502  | 4.061 | 4.493 |
| 21 | 10 | 4.3336973239273116 | 0.5236106657488898  | 4.003 | 4.665 |
| 22 | 10 | 3.968312716241698  | 0.4389649223547913  | 3.691 | 4.246 |
| 23 | 10 | 4.431146055845599  | 0.460952351147867   | 4.14  | 4.723 |
| 24 | 10 | 4.4033297621072185 | 0.421750716941502   | 4.137 | 4.67  |
| 25 | 10 | 4.332252941786477  | 0.2970174739444613  | 4.144 | 4.52  |
| 26 | 10 | 4.36333229499304   | 0.28386450712942285 | 4.184 | 4.543 |
| 27 | 10 | 4.4410433146176675 | 0.5744357010226209  | 4.078 | 4.804 |
| 28 | 10 | 4.477089687792327  | 0.20486073566890953 | 4.348 | 4.607 |
| 29 | 10 | 4.376916894886185  | 0.5077584071838566  | 4.056 | 4.698 |
| 30 | 10 | 4.4399094054855395 | 0.3121058084517692  | 4.243 | 4.637 |
| 31 | 10 | 4.475020617038686  | 0.41778367959018037 | 4.211 | 4.739 |
| 32 | 10 | 4.273230650005795  | 0.34965781016853714 | 4.052 | 4.494 |
| 33 | 10 | 4.20156832737765   | 0.36818775287931005 | 3.969 | 4.434 |
| 34 | 10 | 4.391853748669609  | 0.24496579445750033 | 4.237 | 4.547 |
| 35 | 10 | 4.2410689867089975 | 0.35334072575310677 | 4.018 | 4.465 |
| 36 | 10 | 4.344262797362012  | 0.35479121054725193 | 4.12  | 4.569 |
| 37 | 10 | 4.3039837390067595 | 0.30365625918414835 | 4.112 | 4.496 |
| 38 | 10 | 4.25354849720157   | 0.34678031091119893 | 4.034 | 4.473 |
| 39 | 10 | 4.271734496378676  | 0.300979262125389   | 4.081 | 4.462 |
| 40 | 10 | 4.302548010773355  | 0.3647224040943424  | 4.072 | 4.533 |
| 41 | 10 | 4.253621734693028  | 0.6183431536772038  | 3.863 | 4.645 |
| 42 | 10 | 4.1768870518246235 | 0.3813756540118753  | 3.936 | 4.418 |
| 43 | 10 | 4.232862291522237  | 0.40659039792049667 | 3.976 | 4.49  |
| 44 | 10 | 4.3675461215352165 | 0.2569465163899393  | 4.205 | 4.53  |
| 45 | 10 | 4.170581166700353  | 0.24491606498478133 | 4.016 | 4.325 |
| 46 | 10 | 4.3741030833807155 | 0.17165042625851135 | 4.266 | 4.483 |
| 47 | 10 | 4.329374667292412  | 0.3556530513069827  | 4.104 | 4.554 |
| 48 | 10 | 4.408237655906279  | 0.2533351346744976  | 4.248 | 4.568 |
| 49 | 10 | 4.08998883827853   | 0.2035939225897997  | 3.961 | 4.219 |
| 50 | 10 | 4.42072858929213   | 0.40570580479114815 | 4.164 | 4.677 |
| 51 | 10 | 4.078454532191374  | 0.22353157715220356 | 3.937 | 4.22  |
| 52 | 10 | 4.209827865399232  | 0.3655156202141566  | 3.979 | 4.441 |
| 53 | 10 | 4.273187587446409  | 0.37155651396888856 | 4.038 | 4.508 |
| 54 | 10 | 4.327856818716116  | 0.3794701708655512  | 4.088 | 4.568 |
| 55 | 10 | 4.302610159961826  | 0.5745975602950831  | 3.939 | 4.666 |
| 56 | 10 | 4.197315031502664  | 0.6186289709474516  | 3.806 | 4.589 |
| 57 | 10 | 4.285515677617549  | 0.4263571806991547  | 4.016 | 4.555 |
| 58 | 10 | 4.158077803796397  | 0.33183976212463157 | 3.948 | 4.368 |
| 59 | 10 | 4.401675924374109  | 0.19714492192023444 | 4.277 | 4.526 |
| 60 | 10 | 4.448518421533545  | 0.5677042478751653  | 4.089 | 4.808 |
| 61 | 10 | 4.419209619251843  | 0.2141813334566503  | 4.284 | 4.555 |
| 62 | 10 | 4.028817446294896  | 0.47155702020501805 | 3.731 | 4.327 |
| 63 | 10 | 4.184401850911213  | 0.27035416442769    | 4.013 | 4.355 |
| 64 | 10 | 4.387907012422085  | 0.41405580835513356 | 4.126 | 4.65  |
| 65 | 10 | 4.19479902304659   | 0.37737861111724547 | 3.956 | 4.433 |
| 66 | 10 | 4.108054510190284  | 0.580990009504106   | 3.741 | 4.476 |
| 67 | 10 | 4.506563548355062  | 0.33753933773505507 | 4.293 | 4.72  |
| 68 | 10 | 4.231154724551995  | 0.3472781547967595  | 4.012 | 4.451 |
| 69 | 10 | 4.260613640459321  | 0.2800812546570991  | 4.083 | 4.438 |
| 70 | 10 | 4.206112959296876  | 0.45605132171264395 | 3.918 | 4.495 |
| 71 | 10 | 4.19883582101866   | 0.5341884661129539  | 3.861 | 4.537 |
| 72 | 10 | 4.379992278007377  | 0.42016064891133664 | 4.114 | 4.646 |
| 73 | 10 | 4.247467984460302  | 0.3523490792416313  | 4.025 | 4.47  |
| 74 | 10 | 4.1964357541257105 | 0.4406428279006781  | 3.918 | 4.475 |
| 75 | 10 | 4.219312461268167  | 0.6137127744799872  | 3.831 | 4.607 |
| 76 | 10 | 4.310368401843408  | 0.29186103772327887 | 4.126 | 4.495 |
| 77 | 10 | 4.527486348990401  | 0.456611672471968   | 4.239 | 4.816 |
| 78 | 10 | 4.438426024002249  | 0.30271247669141765 | 4.247 | 4.63  |
| 79 | 10 | 4.311301259846751  | 0.276712369817227   | 4.136 | 4.486 |
| 80 | 10 | 4.3325350286318125 | 0.3469187497158291  | 4.113 | 4.552 |
| 81 | 10 | 4.3146630203103316 | 0.5594773898066183  | 3.961 | 4.669 |
| 82 | 10 | 4.092188883312789  | 0.549893408394151   | 3.744 | 4.44  |
| 83 | 10 | 4.181833206069126  | 0.3319582543402922  | 3.972 | 4.392 |
| 84 | 10 | 4.312629419571212  | 0.3326305990537819  | 4.102 | 4.523 |
| 85 | 10 | 4.299951561170543  | 0.4212556758099     | 4.034 | 4.566 |
| 86 | 10 | 4.31379735831575   | 0.5174180091822302  | 3.987 | 4.641 |
| 87 | 10 | 4.552385320018493  | 0.36495149030885066 | 4.322 | 4.783 |
| 88 | 10 | 4.0488623529960535 | 0.36457429859474216 | 3.818 | 4.279 |
| 89 | 10 | 4.2384955624733225 | 0.31257423156758624 | 4.041 | 4.436 |
| 90 | 10 | 4.202118133837056  | 0.29245829676305185 | 4.017 | 4.387 |
| 91 | 10 | 4.324608670885089  | 0.3248077816282627  | 4.119 | 4.53  |
| 92 | 10 | 4.315730272247161  | 0.3620360350431151  | 4.087 | 4.545 |
| 93 | 10 | 4.581072052146085  | 0.4881207277856628  | 4.272 | 4.88  |

|     |    |                    |                     |       |       |
|-----|----|--------------------|---------------------|-------|-------|
| 93  | 10 | 4.361072032140003  | 0.4691307271030020  | 4.272 | 4.09  |
| 94  | 10 | 4.028205060197276  | 0.3862150264271091  | 3.784 | 4.272 |
| 95  | 10 | 4.439778829146148  | 0.6443678941570047  | 4.032 | 4.847 |
| 96  | 10 | 4.270498499463452  | 0.2405794756998506  | 4.118 | 4.423 |
| 97  | 10 | 4.13660993945347   | 0.3538992423366045  | 3.913 | 4.36  |
| 98  | 10 | 4.358166175177362  | 0.4201212205268271  | 4.092 | 4.624 |
| 99  | 10 | 4.280883019346846  | 0.5723537614634464  | 3.919 | 4.643 |
| 100 | 10 | 4.357027358128028  | 0.23841926519750037 | 4.206 | 4.508 |
| 101 | 10 | 4.161792040671796  | 0.33816545027539996 | 3.948 | 4.376 |
| 102 | 10 | 4.341686688355322  | 0.3842431996363881  | 4.099 | 4.585 |
| 103 | 10 | 4.394171619864716  | 0.27241970367282586 | 4.222 | 4.566 |
| 104 | 10 | 4.150612504290577  | 0.31321725955640095 | 3.953 | 4.349 |
| 105 | 10 | 4.288253120827452  | 0.2810341122956648  | 4.111 | 4.466 |
| 106 | 10 | 4.433350960995998  | 0.3927972203390502  | 4.185 | 4.682 |
| 107 | 10 | 4.031359434819784  | 0.38736379238047963 | 3.786 | 4.276 |
| 108 | 10 | 4.3773219091915605 | 0.26192393652512047 | 4.212 | 4.543 |
| 109 | 10 | 4.368407957245855  | 0.2765407218648971  | 4.194 | 4.543 |
| 110 | 10 | 4.468368438952299  | 0.4352683801263066  | 4.193 | 4.744 |
| 111 | 10 | 4.4840482210867885 | 0.4285028086196516  | 4.213 | 4.755 |
| 112 | 10 | 4.462902635488367  | 0.4125979037247647  | 4.202 | 4.724 |
| 113 | 10 | 4.29901182903801   | 0.5834451352015428  | 3.93  | 4.668 |
| 114 | 10 | 4.189619242649081  | 0.5101875674379888  | 3.867 | 4.512 |
| 115 | 10 | 4.330409963360255  | 0.4983741674081429  | 4.015 | 4.646 |
| 116 | 10 | 4.468921441091622  | 0.40101100631847775 | 4.215 | 4.723 |
| 117 | 10 | 4.309049949245335  | 0.2978209905674358  | 4.121 | 4.497 |
| 118 | 10 | 4.303757978467325  | 0.2521724427582933  | 4.144 | 4.463 |
| 119 | 10 | 4.3129148158912    | 0.3321509227186139  | 4.103 | 4.523 |
| 120 | 10 | 4.155417870561331  | 0.2909065053479239  | 3.971 | 4.339 |
| 121 | 10 | 4.305506163465236  | 0.3931000241646166  | 4.057 | 4.554 |
| 122 | 10 | 4.376271055624073  | 0.2808896737651267  | 4.199 | 4.554 |
| 123 | 10 | 4.231516967012242  | 0.2760832041423898  | 4.057 | 4.406 |
| 124 | 10 | 4.365834777182277  | 0.2251304249235294  | 4.223 | 4.508 |
| 125 | 10 | 4.393689784726401  | 0.4719268244479052  | 4.095 | 4.692 |
| 126 | 10 | 4.042471895836565  | 0.5202732060512785  | 3.713 | 4.372 |
| 127 | 10 | 4.1078108781132014 | 0.474285258650741   | 3.808 | 4.408 |
| 128 | 10 | 4.298680810451028  | 0.4955202674098534  | 3.985 | 4.612 |
| 129 | 10 | 4.54611800761941   | 0.411929521570711   | 4.286 | 4.807 |
| 130 | 10 | 4.6003056013737575 | 0.43956740734591626 | 4.322 | 4.878 |
| 131 | 10 | 4.351225188889753  | 0.4895838852537171  | 4.042 | 4.661 |
| 132 | 10 | 4.3390741672722415 | 0.2851155524681953  | 4.159 | 4.519 |
| 133 | 10 | 4.195490978997666  | 0.39136548420701844 | 3.948 | 4.443 |
| 134 | 10 | 4.419082660682159  | 0.269630644882823   | 4.249 | 4.59  |
| 135 | 10 | 4.478976468172344  | 0.1983986141123515  | 4.353 | 4.604 |
| 136 | 10 | 4.2328020161637525 | 0.3915102737723284  | 3.985 | 4.48  |
| 137 | 10 | 4.2889143884346925 | 0.27291789638008496 | 4.116 | 4.462 |
| 138 | 10 | 4.274838743635025  | 0.37398454606467857 | 4.038 | 4.511 |
| 139 | 10 | 4.271052761174527  | 0.3094134194324746  | 4.075 | 4.467 |
| 140 | 10 | 4.3955255852425825 | 0.28879488055916486 | 4.213 | 4.578 |
| 141 | 10 | 4.014348288781148  | 0.2925710852112883  | 3.829 | 4.199 |
| 142 | 10 | 4.333260789551552  | 0.46858819307172717 | 4.037 | 4.63  |
| 143 | 10 | 4.283664906264242  | 0.561303654019465   | 3.929 | 4.639 |
| 144 | 10 | 4.15742073488      | 0.35206822627035705 | 3.935 | 4.38  |
| 145 | 10 | 4.198478823941744  | 0.4437774728854969  | 3.918 | 4.479 |
| 146 | 10 | 4.269856846276591  | 0.3489885082082865  | 4.049 | 4.491 |
| 147 | 10 | 4.138473700909417  | 0.370246241027461   | 3.904 | 4.373 |
| 148 | 10 | 4.387359684046432  | 0.2718865342345988  | 4.215 | 4.559 |
| 149 | 10 | 4.39334611672539   | 0.2845574130964868  | 4.213 | 4.573 |
| 150 | 10 | 4.2421998441608295 | 0.5868516711610451  | 3.871 | 4.613 |
| 151 | 10 | 4.348344859672006  | 0.21352618092020811 | 4.213 | 4.483 |
| 152 | 10 | 4.441611599068411  | 0.48261135755371326 | 4.136 | 4.747 |
| 153 | 10 | 4.361399292898897  | 0.47044238606501704 | 4.064 | 4.659 |
| 154 | 10 | 4.174415587668088  | 0.2970264883873234  | 3.987 | 4.362 |
| 155 | 10 | 4.264593570548321  | 0.5417059520531473  | 3.922 | 4.607 |
| 156 | 10 | 4.242456887044944  | 0.508118298459996   | 3.921 | 4.564 |
| 157 | 10 | 4.33297432807001   | 0.31702809726090125 | 4.132 | 4.533 |
| 158 | 10 | 4.565509215975761  | 0.3623221696136326  | 4.336 | 4.795 |
| 159 | 10 | 4.332008713103274  | 0.14472682528746983 | 4.24  | 4.424 |
| 160 | 10 | 4.329738340796612  | 0.3021968262218468  | 4.139 | 4.521 |
| 161 | 10 | 4.369180048543918  | 0.4941442720972559  | 4.057 | 4.682 |
| 162 | 10 | 4.332152553174168  | 0.32359852461588756 | 4.127 | 4.537 |
| 163 | 10 | 4.317283398131355  | 0.5018214477384102  | 4.0   | 4.635 |
| 164 | 10 | 4.453231438153747  | 0.29842650980506874 | 4.264 | 4.642 |
| 165 | 10 | 4.5119042080005425 | 0.3440589396793874  | 4.294 | 4.73  |
| 166 | 10 | 4.617826925913596  | 0.32366153635534123 | 4.413 | 4.823 |
| 167 | 10 | 4.3960151235276195 | 0.4695510317010775  | 4.099 | 4.693 |
| 168 | 10 | 4.493951877102377  | 0.5611717220985689  | 4.139 | 4.849 |
| 169 | 10 | 4.115460303724278  | 0.3668455148633937  | 3.883 | 4.347 |
| 170 | 10 | 4.118555571188851  | 0.25266762278566316 | 3.858 | 4.230 |

|     |    |                    |                     |       |       |
|-----|----|--------------------|---------------------|-------|-------|
| 170 | 10 | 4.1185575188851    | 0.25366763278526216 | 3.958 | 4.279 |
| 171 | 10 | 4.1993462392606915 | 0.31082995170094946 | 4.003 | 4.396 |
| 172 | 10 | 4.063502348934774  | 0.5269927756181045  | 3.73  | 4.397 |
| 173 | 10 | 4.278762932456374  | 0.33142210250194026 | 4.069 | 4.488 |
| 174 | 10 | 4.464535867847365  | 0.4211749434793046  | 4.198 | 4.731 |
| 175 | 10 | 4.198408040852332  | 0.38876953256013036 | 3.953 | 4.444 |
| 176 | 10 | 4.110145981307923  | 0.4059711894929559  | 3.853 | 4.367 |
| 177 | 10 | 4.26657345757534   | 0.39054716799465133 | 4.02  | 4.514 |
| 178 | 10 | 4.428067957632272  | 0.4481902927235779  | 4.145 | 4.712 |
| 179 | 10 | 4.703269275183353  | 0.3798787062800563  | 4.463 | 4.944 |
| 180 | 10 | 4.335640333086867  | 0.257528720885746   | 4.173 | 4.499 |
| 181 | 10 | 4.320783179083196  | 0.424955816752172   | 4.052 | 4.59  |
| 182 | 10 | 4.1620720482410904 | 0.3152883880205154  | 3.963 | 4.361 |
| 183 | 10 | 4.57947900008761   | 0.5054467132847035  | 4.26  | 4.899 |
| 184 | 10 | 4.2211598731367195 | 0.39671852967341725 | 3.97  | 4.472 |
| 185 | 10 | 4.080777831963866  | 0.43512747484823466 | 3.806 | 4.356 |
| 186 | 10 | 4.342902749530561  | 0.3041453247679896  | 4.151 | 4.535 |
| 187 | 10 | 4.251849402190762  | 0.3569200477767188  | 4.026 | 4.478 |
| 188 | 10 | 4.204828224826241  | 0.27162316497631056 | 4.033 | 4.377 |
| 189 | 10 | 4.281516902172294  | 0.3363002761224759  | 4.069 | 4.494 |
| 190 | 10 | 4.518863210874611  | 0.5602834435404571  | 4.165 | 4.873 |
| 191 | 10 | 4.5059389683749185 | 0.18983727165801126 | 4.386 | 4.626 |
| 192 | 10 | 4.343557571879518  | 0.3314390514431859  | 4.134 | 4.553 |
| 193 | 10 | 4.348464434287419  | 0.47307307013158073 | 4.049 | 4.648 |
| 194 | 10 | 4.285470586869936  | 0.3291090695319485  | 4.077 | 4.494 |
| 195 | 10 | 4.278376625730462  | 0.5202283563087551  | 3.949 | 4.607 |
| 196 | 10 | 4.151219238587083  | 0.3325883621346217  | 3.941 | 4.362 |
| 197 | 10 | 4.147584191973595  | 0.4467574833938899  | 3.865 | 4.43  |
| 198 | 10 | 4.141113791827148  | 0.22420372695307902 | 3.999 | 4.283 |
| 199 | 10 | 4.337319583425104  | 0.2765387439886831  | 4.162 | 4.512 |
| 200 | 10 | 4.322703530285968  | 0.2979363155369192  | 4.134 | 4.511 |

In [132]:

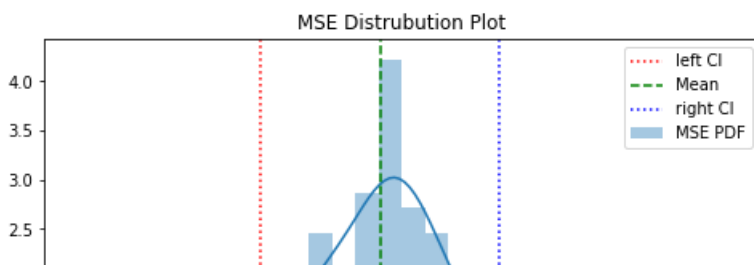
```
mean_of_means = np.mean(MSE_means)
std_dev_of_means = np.mean(sample_std)
mean_Left_CI = np.round(mean_of_means - 2*(std_dev_of_means/np.sqrt(10)), 3)
mean_Right_CI = np.round(mean_of_means + 2*(std_dev_of_means/np.sqrt(10)), 3)

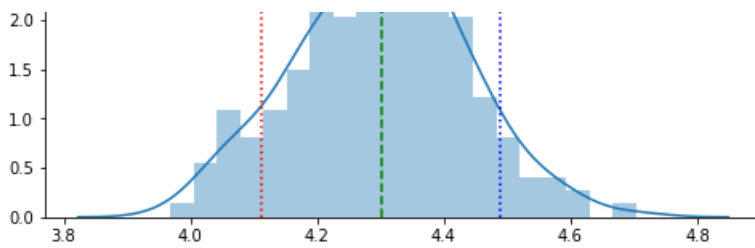
print('Mean of Means: ',mean_of_means)
print('Mean Std-Dev : ',std_dev_of_means)
print('Mean Left CI : ',mean_Left_CI)
print('Mean Right CI: ',mean_Right_CI)
```

```
Mean of Means: 4.301133940773954
Mean Std-Dev : 0.2979363155369192
Mean Left CI : 4.113
Mean Right CI: 4.49
```

In [133]:

```
import seaborn as sns
import matplotlib.pyplot as plt
plt.figure(figsize=(8,5))
sns.distplot(MSE_means, bins=no_of_samples/no_of_bins)
plt.axvline(x=mean_Left_CI, c='red', ls=':')
plt.axvline(x=mean_of_means, c='green', ls='--')
plt.axvline(x=mean_Right_CI, c='blue', ls=':')
plt.legend(['left CI', 'Mean', 'right CI', 'MSE PDF'])
plt.title('MSE Distrubution Plot')
plt.show()
```





In [134]:

```
#calculating 95% confidence interval
from prettytable import PrettyTable
OOB_means = []
table = PrettyTable()
table = PrettyTable(["#samples", "Sample Size", "Sample mean", "Sample std", "Left C.I", "Right C.I"])
for i in range(no_of_samples):
    sample=[OOBs[i] for i in random.choices(range(35),k=10)]
    sample_mean = np.mean(sample)
    OOB_means.append(sample_mean)
    sample_std = np.std(sample)
    sample_size = len(sample)
    # here we are using sample standard deviation instead of population standard deviation
    left_limit = np.round(sample_mean - 2*(sample_std/np.sqrt(sample_size)), 3)
    right_limit = np.round(sample_mean + 2*(sample_std/np.sqrt(sample_size)), 3)
    row = []
    row.append(i+1)
    row.append(sample_size)
    row.append(sample_mean)
    row.append(sample_std)
    row.append(left_limit)
    row.append(right_limit)
    table.add_row(row)
print('==== Calculation Interval Calculations for OOB =====')
print(table)
```

| #samples | Sample Size | Sample mean        | Sample std          | Left C.I | Right C.I |
|----------|-------------|--------------------|---------------------|----------|-----------|
| 1        | 10          | 13.112979626691521 | 0.46627438692820333 | 12.818   | 13.408    |
| 2        | 10          | 13.881885781992633 | 0.515476379021685   | 13.556   | 14.208    |
| 3        | 10          | 13.507101877443228 | 1.1323713960397965  | 12.791   | 14.223    |
| 4        | 10          | 13.307519521149732 | 0.979608799702532   | 12.688   | 13.927    |
| 5        | 10          | 13.974850434183608 | 0.9159545402995406  | 13.396   | 14.554    |
| 6        | 10          | 13.558455922888545 | 1.104545393753663   | 12.86    | 14.257    |
| 7        | 10          | 14.004380945580374 | 1.1102809955882513  | 13.302   | 14.707    |
| 8        | 10          | 13.980834670510115 | 0.9114667784082296  | 13.404   | 14.557    |
| 9        | 10          | 14.219883281506963 | 1.0733858848824889  | 13.541   | 14.899    |
| 10       | 10          | 13.205576100996135 | 0.5326396099100604  | 12.869   | 13.542    |
| 11       | 10          | 13.443557085146988 | 0.7331691095079382  | 12.98    | 13.907    |
| 12       | 10          | 13.962974892726038 | 1.1419242102185279  | 13.241   | 14.685    |
| 13       | 10          | 13.515288373706118 | 0.9590374759838882  | 12.909   | 14.122    |
| 14       | 10          | 13.807725307134735 | 0.641398320560128   | 13.402   | 14.213    |
| 15       | 10          | 13.952820133900364 | 0.6951183256741179  | 13.513   | 14.392    |
| 16       | 10          | 13.675005317561567 | 0.4565835227230701  | 13.386   | 13.964    |
| 17       | 10          | 14.07702517180049  | 1.3296857015037944  | 13.236   | 14.918    |
| 18       | 10          | 13.772202399844364 | 0.7679411581879485  | 13.287   | 14.258    |
| 19       | 10          | 13.472387437823306 | 1.050651591376646   | 12.808   | 14.137    |
| 20       | 10          | 13.712766917981748 | 1.1579503789512582  | 12.98    | 14.445    |
| 21       | 10          | 13.48706771855198  | 1.2240142739050242  | 12.713   | 14.261    |
| 22       | 10          | 13.414716843522465 | 0.580635895181133   | 13.047   | 13.782    |
| 23       | 10          | 13.13362506809934  | 0.44657112587946163 | 12.851   | 13.416    |
| 24       | 10          | 13.804055814488208 | 0.9514774640168396  | 13.202   | 14.406    |
| 25       | 10          | 13.79959537408914  | 0.6579514896188957  | 13.383   | 14.216    |
| 26       | 10          | 13.659560472550734 | 0.880051397776119   | 13.103   | 14.216    |
| 27       | 10          | 13.503395488362225 | 0.8214703676029353  | 12.984   | 14.023    |
| 28       | 10          | 13.614104685898923 | 1.036583557096561   | 12.959   | 14.27     |
| 29       | 10          | 13.543761433985825 | 0.7371830415067259  | 13.078   | 14.01     |
| 30       | 10          | 13.722639284292498 | 1.1836935974849756  | 12.974   | 14.471    |
| 31       | 10          | 13.494188138151681 | 0.9956379886255069  | 12.864   | 14.124    |
| 32       | 10          | 13.343774396477155 | 0.680568622872209   | 12.913   | 13.774    |
| 33       | 10          | 13.768574341379766 | 0.6929589071955642  | 13.33    | 14.207    |
| 34       | 10          | 13.749509449324592 | 1.099418053404586   | 13.054   | 14.445    |



|     |    |                    |                     |        |        |
|-----|----|--------------------|---------------------|--------|--------|
| 35  | 10 | 13.620858223956409 | 0.6344254501869839  | 13.22  | 14.022 |
| 36  | 10 | 13.636067631449638 | 0.5940829584427557  | 13.26  | 14.012 |
| 37  | 10 | 13.802946410911797 | 1.018415045255796   | 13.159 | 14.447 |
| 38  | 10 | 14.093880696438807 | 1.0583697351471932  | 13.425 | 14.763 |
| 39  | 10 | 13.368159929683875 | 0.48418685891152496 | 13.062 | 13.674 |
| 40  | 10 | 13.98418330301843  | 1.2990923499800107  | 13.163 | 14.806 |
| 41  | 10 | 13.859090938210135 | 0.8682710605705088  | 13.31  | 14.408 |
| 42  | 10 | 14.047333014949846 | 0.9792113439517445  | 13.428 | 14.667 |
| 43  | 10 | 14.310344487615572 | 0.5633454459799838  | 13.954 | 14.667 |
| 44  | 10 | 14.095382728993155 | 0.9105114831343747  | 13.52  | 14.671 |
| 45  | 10 | 13.657486291549016 | 0.765707842796435   | 13.173 | 14.142 |
| 46  | 10 | 13.638820311837568 | 0.9685558912555319  | 13.026 | 14.251 |
| 47  | 10 | 14.164808825145826 | 0.7961148414104717  | 13.661 | 14.668 |
| 48  | 10 | 13.547877542272705 | 0.6230601821472106  | 13.154 | 13.942 |
| 49  | 10 | 13.59639492824419  | 0.6673697163313641  | 13.174 | 14.018 |
| 50  | 10 | 13.48816113268623  | 0.65747723006561    | 13.072 | 13.904 |
| 51  | 10 | 12.95976253145148  | 0.6193122930892468  | 12.568 | 13.351 |
| 52  | 10 | 13.427835192455305 | 0.800919900783292   | 12.921 | 13.934 |
| 53  | 10 | 14.261919832419583 | 1.1916965786209812  | 13.508 | 15.016 |
| 54  | 10 | 13.355097823994967 | 0.8261367932877898  | 12.833 | 13.878 |
| 55  | 10 | 14.286006280843434 | 0.5391402149353359  | 13.945 | 14.627 |
| 56  | 10 | 13.745835366053527 | 1.1352451617154034  | 13.028 | 14.464 |
| 57  | 10 | 13.307160907590912 | 0.8669134483817097  | 12.759 | 13.855 |
| 58  | 10 | 13.725726110910449 | 1.0461053607623716  | 13.064 | 14.387 |
| 59  | 10 | 13.935266023487738 | 0.6030438513586582  | 13.554 | 14.317 |
| 60  | 10 | 13.839063076851687 | 1.1265981525689628  | 13.127 | 14.552 |
| 61  | 10 | 13.845847677301697 | 0.9909448151855297  | 13.219 | 14.473 |
| 62  | 10 | 13.576932791097832 | 0.9411519511207838  | 12.982 | 14.172 |
| 63  | 10 | 13.769529116834935 | 0.9129151400612526  | 13.192 | 14.347 |
| 64  | 10 | 13.818860488820928 | 0.5304896342854132  | 13.483 | 14.154 |
| 65  | 10 | 13.41964849215297  | 0.8235264726197892  | 12.899 | 13.94  |
| 66  | 10 | 13.646993814641075 | 0.8962697114078776  | 13.08  | 14.214 |
| 67  | 10 | 13.916407614205506 | 0.8032346178999079  | 13.408 | 14.424 |
| 68  | 10 | 13.91808158642668  | 0.9131097488256447  | 13.341 | 14.496 |
| 69  | 10 | 13.655698013415309 | 0.6482695611526723  | 13.246 | 14.066 |
| 70  | 10 | 14.105879558857007 | 0.606989490288477   | 13.722 | 14.49  |
| 71  | 10 | 13.495812637467271 | 0.8013796876979508  | 12.989 | 14.003 |
| 72  | 10 | 14.487828573370985 | 0.8465275357641551  | 13.952 | 15.023 |
| 73  | 10 | 13.646667772180283 | 1.085986905765037   | 12.96  | 14.334 |
| 74  | 10 | 14.2541089922526   | 0.6804074711203072  | 13.824 | 14.684 |
| 75  | 10 | 13.878412393397088 | 0.8563210665195076  | 13.337 | 14.42  |
| 76  | 10 | 13.866977693501363 | 1.057446841730971   | 13.198 | 14.536 |
| 77  | 10 | 13.772583611576858 | 0.5664411809884432  | 13.414 | 14.131 |
| 78  | 10 | 13.133565420509441 | 0.7735935703865539  | 12.644 | 13.623 |
| 79  | 10 | 13.39333445050461  | 0.5185237380846206  | 13.065 | 13.721 |
| 80  | 10 | 13.976085727019935 | 1.0834332527390573  | 13.291 | 14.661 |
| 81  | 10 | 13.32743742286739  | 0.5995636221407828  | 12.948 | 13.707 |
| 82  | 10 | 13.944789411714657 | 1.1198570139893245  | 13.237 | 14.653 |
| 83  | 10 | 13.339170540412322 | 0.7668644898064816  | 12.854 | 13.824 |
| 84  | 10 | 13.742101789332974 | 0.8548939596838168  | 13.201 | 14.283 |
| 85  | 10 | 13.827845605416105 | 1.0763613420684537  | 13.147 | 14.509 |
| 86  | 10 | 14.115345138808593 | 1.3032665902120286  | 13.291 | 14.94  |
| 87  | 10 | 13.81526816795971  | 0.9428889725287877  | 13.219 | 14.412 |
| 88  | 10 | 13.83524259299151  | 0.819596099472356   | 13.317 | 14.354 |
| 89  | 10 | 13.151452247687047 | 0.7988470429180767  | 12.646 | 13.657 |
| 90  | 10 | 14.115746544708724 | 0.9614224457636767  | 13.508 | 14.724 |
| 91  | 10 | 13.71961300957363  | 0.7821947783998291  | 13.225 | 14.214 |
| 92  | 10 | 14.044757376274429 | 1.3558351546633964  | 13.187 | 14.902 |
| 93  | 10 | 13.21048369633636  | 0.9418877511481196  | 12.615 | 13.806 |
| 94  | 10 | 13.626070360269086 | 0.8332609349752114  | 13.099 | 14.153 |
| 95  | 10 | 13.673213737288387 | 0.7878628136517354  | 13.175 | 14.172 |
| 96  | 10 | 13.68782279792122  | 0.6574075590347267  | 13.272 | 14.104 |
| 97  | 10 | 14.423982764429601 | 1.0890258577350609  | 13.735 | 15.113 |
| 98  | 10 | 13.595820764719894 | 1.0197416528865855  | 12.951 | 14.241 |
| 99  | 10 | 13.538182904333809 | 1.0588696726719207  | 12.868 | 14.208 |
| 100 | 10 | 13.601809068783677 | 1.3537056931919649  | 12.746 | 14.458 |
| 101 | 10 | 13.469614905646385 | 1.009374487958984   | 12.831 | 14.108 |
| 102 | 10 | 13.76111143738217  | 0.9374957093957982  | 13.168 | 14.354 |
| 103 | 10 | 13.501713236244623 | 0.41291761966716983 | 13.241 | 13.763 |
| 104 | 10 | 13.938282016979931 | 0.796168106598396   | 13.435 | 14.442 |
| 105 | 10 | 13.661810334248027 | 1.1683279570190657  | 12.923 | 14.401 |
| 106 | 10 | 13.508391615808582 | 0.5818946290127265  | 13.14  | 13.876 |
| 107 | 10 | 13.64435464630922  | 1.0755605785664732  | 12.964 | 14.325 |
| 108 | 10 | 13.925729531992436 | 1.015681574142479   | 13.283 | 14.568 |
| 109 | 10 | 14.143333565153043 | 1.172523266179568   | 13.402 | 14.885 |
| 110 | 10 | 13.226385311936857 | 1.0201860688616133  | 12.581 | 13.872 |
| 111 | 10 | 13.890070671252236 | 1.0429465531226993  | 13.23  | 14.55  |

|     |    |                    |                    |        |        |
|-----|----|--------------------|--------------------|--------|--------|
| 111 | 10 | 13.061549577846964 | 0.8727179219513936 | 12.51  | 13.614 |
| 112 | 10 | 14.287059248787179 | 1.0434427176373535 | 13.627 | 14.947 |
| 113 | 10 | 14.057538248301825 | 0.9853673365007692 | 13.434 | 14.681 |
| 114 | 10 | 13.345123683094803 | 1.001746560655549  | 12.712 | 13.979 |
| 115 | 10 | 13.74069968843357  | 0.8334409909496293 | 13.214 | 14.268 |
| 116 | 10 | 13.367109764148623 | 0.7322240294246088 | 12.904 | 13.83  |
| 117 | 10 | 13.597609672823726 | 0.6684920138638437 | 13.175 | 14.02  |
| 118 | 10 | 13.515391786594623 | 0.9312124942029567 | 12.926 | 14.104 |
| 119 | 10 | 13.541729396885628 | 0.8264359897062415 | 13.019 | 14.064 |
| 120 | 10 | 13.618711835793814 | 1.0561310161103405 | 12.951 | 14.287 |
| 121 | 10 | 13.16757063177575  | 0.8302576295786066 | 12.642 | 13.693 |
| 122 | 10 | 13.713192308724041 | 1.0421001048804293 | 13.054 | 14.372 |
| 123 | 10 | 13.192769620970108 | 0.690053174993788  | 12.756 | 13.629 |
| 124 | 10 | 13.60822794208841  | 1.0579564755313184 | 12.939 | 14.277 |
| 125 | 10 | 13.937975251544993 | 0.8776559782448835 | 13.383 | 14.493 |
| 126 | 10 | 13.729520176565515 | 0.7493493220301823 | 13.256 | 14.203 |
| 127 | 10 | 13.7592162601813   | 0.7756021675873032 | 13.269 | 14.25  |
| 128 | 10 | 13.903587029714766 | 1.0796017599719818 | 13.221 | 14.586 |
| 129 | 10 | 13.057690971839573 | 1.1900202006374467 | 12.305 | 13.81  |
| 130 | 10 | 13.592099047847512 | 0.5353051120293568 | 13.254 | 13.931 |
| 131 | 10 | 13.255478500538155 | 0.7840378480803222 | 12.76  | 13.751 |
| 132 | 10 | 13.804043186843156 | 0.7629275250731344 | 13.322 | 14.287 |
| 133 | 10 | 13.7176473710799   | 1.1783073108633113 | 12.972 | 14.463 |
| 134 | 10 | 13.680500490467375 | 1.2064464536698298 | 12.917 | 14.444 |
| 135 | 10 | 14.215719061457097 | 0.9702451415869133 | 13.602 | 14.829 |
| 136 | 10 | 13.952289577337897 | 0.7832225314960755 | 13.457 | 14.448 |
| 137 | 10 | 13.410064185743872 | 0.9833737918301747 | 12.788 | 14.032 |
| 138 | 10 | 13.492748338996503 | 0.7715215346349182 | 13.005 | 13.981 |
| 139 | 10 | 13.827101567150047 | 1.0360746646225412 | 13.172 | 14.482 |
| 140 | 10 | 13.526546391957279 | 0.7312344959173991 | 13.064 | 13.989 |
| 141 | 10 | 13.272846963460527 | 0.6576281680996897 | 12.857 | 13.689 |
| 142 | 10 | 13.890857721365071 | 0.9887665561579002 | 13.266 | 14.516 |
| 143 | 10 | 13.681668520510431 | 0.7096912141503816 | 13.233 | 14.131 |
| 144 | 10 | 14.399626451912052 | 0.9665953587378437 | 13.788 | 15.011 |
| 145 | 10 | 13.6838807742352   | 1.0285924676802727 | 13.033 | 14.334 |
| 146 | 10 | 13.825965642552466 | 1.1651301849613727 | 13.089 | 14.563 |
| 147 | 10 | 12.91756679119517  | 0.6585742347659369 | 12.501 | 13.334 |
| 148 | 10 | 13.516493762916264 | 0.9267254098982766 | 12.93  | 14.103 |
| 149 | 10 | 13.728450738110379 | 0.5536696560986004 | 13.378 | 14.079 |
| 150 | 10 | 13.718586535180481 | 0.8492683084008906 | 13.181 | 14.256 |
| 151 | 10 | 13.497650632269995 | 1.0482473939771215 | 12.835 | 14.161 |
| 152 | 10 | 13.282052823406369 | 0.6253098236106515 | 12.887 | 13.678 |
| 153 | 10 | 13.989910292745463 | 1.2836652099959367 | 13.178 | 14.802 |
| 154 | 10 | 13.853935044800801 | 1.0066060681836313 | 13.217 | 14.491 |
| 155 | 10 | 13.958419179478136 | 0.9038718983076373 | 13.387 | 14.53  |
| 156 | 10 | 13.911230804914817 | 0.5318894549184553 | 13.575 | 14.248 |
| 157 | 10 | 14.096531137745222 | 0.8314380212782191 | 13.571 | 14.622 |
| 158 | 10 | 14.122171402860484 | 0.9061486993938356 | 13.549 | 14.695 |
| 159 | 10 | 13.672372763544985 | 0.7559659903653259 | 13.194 | 14.15  |
| 160 | 10 | 13.255476721361049 | 1.0260107833095597 | 12.607 | 13.904 |
| 161 | 10 | 13.897659311402919 | 1.061742029485145  | 13.226 | 14.569 |
| 162 | 10 | 13.630559271595796 | 0.8608193101679733 | 13.086 | 14.175 |
| 163 | 10 | 14.097461105196228 | 0.8448724929799737 | 13.563 | 14.632 |
| 164 | 10 | 13.554404425902561 | 1.1915712442986506 | 12.801 | 14.308 |
| 165 | 10 | 13.469264343210716 | 0.6512109190358273 | 13.057 | 13.881 |
| 166 | 10 | 13.747816807194102 | 0.853632202691883  | 13.208 | 14.288 |
| 167 | 10 | 13.649204957144937 | 0.9632951521640031 | 13.04  | 14.258 |
| 168 | 10 | 13.41832150804969  | 0.3961551465259636 | 13.168 | 13.669 |
| 169 | 10 | 13.315500497365282 | 0.6378083501906289 | 12.912 | 13.719 |
| 170 | 10 | 13.12705849118366  | 1.0624694769115803 | 12.455 | 13.799 |
| 171 | 10 | 13.512042236048776 | 0.8797799275302615 | 12.956 | 14.068 |
| 172 | 10 | 14.136514500863587 | 0.7289156716361341 | 13.676 | 14.598 |
| 173 | 10 | 13.441746209441998 | 0.6243668100643824 | 13.047 | 13.837 |
| 174 | 10 | 13.708724998355606 | 1.2046080552866356 | 12.947 | 14.471 |
| 175 | 10 | 14.284846563056806 | 0.9289716109037777 | 13.697 | 14.872 |
| 176 | 10 | 13.898593671904967 | 0.3878345315314379 | 13.653 | 14.144 |
| 177 | 10 | 13.85916626723161  | 0.8303924589165261 | 13.334 | 14.384 |
| 178 | 10 | 13.673756719764118 | 0.803156833038837  | 13.166 | 14.182 |
| 179 | 10 | 13.411493003150834 | 0.6811428722456672 | 12.981 | 13.842 |
| 180 | 10 | 14.139109849221398 | 0.9329676318477345 | 13.549 | 14.729 |
| 181 | 10 | 13.898507550925027 | 0.8755535835258201 | 13.345 | 14.452 |
| 182 | 10 | 13.92284180675093  | 0.705099032925744  | 13.477 | 14.369 |
| 183 | 10 | 14.86025514782096  | 1.2620209158877553 | 14.062 | 15.658 |
| 184 | 10 | 13.724696039569627 | 0.53862916459991   | 13.384 | 14.065 |
| 185 | 10 | 13.920907544161901 | 1.160472430008931  | 13.187 | 14.655 |
| 186 | 10 | 13.66178728118432  | 0.7737536843510621 | 13.172 | 14.151 |
| 187 | 10 | 13.506365890209404 | 0.5387222063792582 | 13.166 | 13.847 |
| 188 | 10 |                    |                    |        |        |

|     |    |                    |                     |        |        |
|-----|----|--------------------|---------------------|--------|--------|
| 188 | 10 | 13.300303030209404 | 0.3307222003192302  | 13.100 | 13.017 |
| 189 | 10 | 13.575429731386057 | 0.8308120876699266  | 13.05  | 14.101 |
| 190 | 10 | 13.430939646269556 | 0.34702536703234516 | 13.211 | 13.65  |
| 191 | 10 | 13.922142010611832 | 0.8889046471329658  | 13.36  | 14.484 |
| 192 | 10 | 13.552696070338886 | 0.9689962451802915  | 12.94  | 14.166 |
| 193 | 10 | 13.768020677965003 | 0.896589812288967   | 13.201 | 14.335 |
| 194 | 10 | 13.380863979676425 | 0.6507599122779483  | 12.969 | 13.792 |
| 195 | 10 | 14.473816142136096 | 1.0670006916408707  | 13.799 | 15.149 |
| 196 | 10 | 14.14297070671525  | 0.978616426451656   | 13.524 | 14.762 |
| 197 | 10 | 13.94942902146202  | 1.301006807035547   | 13.127 | 14.772 |
| 198 | 10 | 13.963599178341564 | 1.2000927514018709  | 13.205 | 14.723 |
| 199 | 10 | 13.263168788181748 | 0.8976918248714162  | 12.695 | 13.831 |
| 200 | 10 | 13.276618381892561 | 0.6423745739234066  | 12.87  | 13.683 |

In [135]:

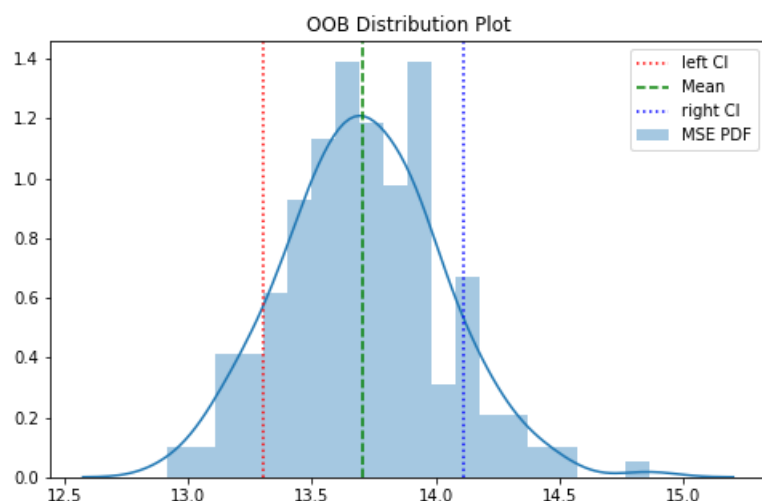
```
mean_of_means = np.mean(OOB_means)
std_dev_of_means = np.mean(sample_std)
mean_Left_CI = np.round(mean_of_means - 2*(std_dev_of_means/np.sqrt(10)), 3)
mean_Right_CI = np.round(mean_of_means + 2*(std_dev_of_means/np.sqrt(10)), 3)

print('Mean of Means: ',mean_of_means)
print('Mean Std-Dev : ',std_dev_of_means)
print('Mean Left CI : ',mean_Left_CI)
print('Mean Right CI: ',mean_Right_CI)
```

```
Mean of Means: 13.708229873730334
Mean Std-Dev : 0.6423745739234066
Mean Left CI : 13.302
Mean Right CI: 14.115
```

In [136]:

```
plt.figure(figsize=(8,5))
sns.distplot(OOB_means, bins=no_of_samples//no_of_bins)
plt.axvline(x=mean_Left_CI, c='red',ls=':')
plt.axvline(x=mean_of_means, c='green',ls='--')
plt.axvline(x=mean_Right_CI, c='blue',ls=':')
plt.legend(['left CI', 'Mean', 'right CI', 'MSE PDF'])
plt.title('OOB Distribution Plot')
plt.show()
```



### Task 3

In [137]:

```
xq= np.array([0.18,20.0,5.00,0.0,0.421,5.60,72.2,7.95,7.0,30.0,19.1,372.13,18.60])
Y_pred = 0
for i in range(30):
    clf = DecisionTreeRegressor()
```

```
    xq = x[row_indices_iob[i][:,None],col_indices[i]]
    X_train = x[row_indices_iob[i][:,None],col_indices[i]]
    Y_train = y[row_indices_iob[i]]
    clf.fit(X_train,Y_train)
    Y_pred += clf.predict(xq[col_indices[i]].reshape(1,-1)).item()
Y_pred = Y_pred/30

print(Y_pred)
```

21.650000000000002

In [ ]: