8E_F_LR_SVM

February 3, 2020

9E and 9F: Finding the Probability P(Y==1|X)

9E: Implementing Decision Function of SVM RBF Kernel

After we train a kernel SVM model, we will be getting support vectors and their corresponsing coefficients α_i

Check the documentation for better understanding of these attributes:

https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html

As a part of this assignment you will be implementing the `decision_function()` of kernel SVM, here decision_function() means based on the value return by `decision_function()` model will classify the data point either as positive or negative

Ex 1: In logistic regression After traning the models with the optimal weights w we get, we will find the value $\frac{1}{1+\exp(-(wx+b))}$, if this value comes out to be < 0.5 we will mark it as negative class, else its positive class

Ex 2: In Linear SVM After traning the models with the optimal weights w we get, we will find the value of $sign(wx + b)$, if this value comes out to be -ve we will mark it as negative class, else its positive class.

Similarly in Kernel SVM After traning the models with the coefficients α_i we get, we will find the value of $sign(\sum_{i=1}^{n}(y_i\alpha_i K(x_i, x_q)) + intercept)$, here $K(x_i, x_q)$ is the RBF kernel. If this value comes out to be -ve we will mark x_q as negative class, else its positive class.

RBF kernel is defined as: $K(x_i, x_q) = exp(-\gamma||x_i - x_q||^2)$

For better understanding check this link: https://scikit-learn.org/stable/modules/svm.html#svm-mathematical-formulation

0.1 Task E

1. Split the data into X_{train}(60), X_{cv}(20), X_{test}(20)
2. Train $SVC(gamma = 0.001, C = 100.)$ on the (X_{train}, y_{train})
3. Get the decision boundry values f_{cv} on the X_{cv} data i.e. f_{cv} = `decision_function`(X_{cv}) you need to implement this decision_function()

```
In [1]: import numpy as np
        import pandas as pd
        from sklearn.datasets import make_classification
        from sklearn.model_selection import train_test_split
        from sklearn.svm import SVC
```

```
In [2]: X, y = make_classification(n_samples=5000, n_features=5, n_redundant=2,
                                   n_classes=2, weights=[0.7], class_sep=0.7, random_state=15)
```

0.1.1 Pseudo code

clf = SVC(gamma=0.001, C=100.) clf.fit(Xtrain, ytrain)

def decision_function(Xcv, ...): #use appropriate parameters for a data point x_q in Xcv: #write code to implement $(\sum_{i=1}^{\text{all the support vectors}}(y_i\alpha_i K(x_i, x_q)) + intercept)$, here the values y_i, α_i, and *intercept* can be obtained from the trained model return # the decision_function output for all the data points in the Xcv

fcv = decision_function(Xcv, ...) # based on your requirement you can pass any other parameters

Note: Make sure the values you get as fcv, should be equal to outputs of clf.decision_function(Xcv)

In [3]:
```
# you can write your code here
data = np.hstack((X, y.reshape(5000,1)))
data.shape
```

Out[3]: (5000, 6)

In [4]:
```
X_train,X_rem,Y_train,Y_rem = train_test_split(X,y,test_size=0.40,random_state=16)
X_cv,X_test,Y_cv,Y_test = train_test_split(X_rem,Y_rem,test_size=0.50,random_state=16)

X_train.shape,X_cv.shape,X_test.shape
```

Out[4]: ((3000, 5), (1000, 5), (1000, 5))

In [5]:
```
model = SVC(gamma = 0.001, C = 100)
model.fit(X_train,Y_train)
```

Out[5]:
```
SVC(C=100, cache_size=200, class_weight=None, coef0=0.0,
  decision_function_shape='ovr', degree=3, gamma=0.001, kernel='rbf',
  max_iter=-1, probability=False, random_state=None, shrinking=True,
  tol=0.001, verbose=False)
```

In [6]:
```
f_cv = model.decision_function(X_cv)
# f_cv
```

In [7]:
```
def calculate_rbf(X_i,X_q):
#     k = np.exp(-model.gamma * np.linalg.norm(X_i-X_q))
    k = np.exp(-model.gamma * np.dot(X_i-X_q, X_i-X_q))
    return k
```

In [8]:
```
from tqdm import tqdm

def custom_decision_function(Xcv): #use appropriate parameters
    ycv = np.zeros(Xcv.shape[0]).reshape((Xcv.shape[0],1))
    for i in tqdm(range(Xcv.shape[0])):
        sum = 0
        for j in range(model.support_vectors_.shape[0]):
            kernal = calculate_rbf(model.support_vectors_[j],Xcv[i])
            sum += kernal*model.dual_coef_[0][j]
        ycv[i] = sum + model.intercept_
    return ycv
```

```
In [9]: f_cv_custom = custom_decision_function(X_cv)
        f_test_custom = custom_decision_function(X_test)
```

```
100%|| 1000/1000 [00:05<00:00, 197.24it/s]
100%|| 1000/1000 [00:06<00:00, 145.36it/s]
```

```
In [10]: (f_cv_custom - f_cv.reshape(1000,1))[:5]
```

```
Out[10]: array([[0.],
                [0.],
                [0.],
                [0.],
                [0.]])
```

9F: Implementing Platt Scaling to find P(Y==1|X)
Check this PDF

0.2 TASK F

4. Apply SGD algorithm with (f_{cv}, y_{cv}) and find the weight W intercept b `Note: here our data is of one dimensional so we will have a one dimensional weight vector i.e W.shape (1,)`

Note1: Don't forget to change the values of y_{cv} as mentioned in the above image. you will calculate y+, y- based on data points in train data

Note2: the Sklearn's SGD algorithm doesn't support the real valued outputs, you need to use the code that was done in the `'Logistic Regression with SGD and L2'` Assignment after modifying loss function, and use same parameters that used in that assignment. if Y[i] is 1, it will be replaced with y+ value else it will replaced with y- value

5. For a given data point from X_{test}, $P(Y = 1|X) = \frac{1}{1+exp(-(W*f_{test}+b))}$ where f_{test} `= decision_function(`X_{test}`)`, W and b will be learned as metioned in the above step

Note: in the above algorithm, the steps 2, 4 might need hyper parameter tuning, To reduce the complexity of the assignment we are excluding the hyerparameter tuning part, but intrested students can try that

If any one wants to try other calibration algorithm istonic regression also please check these tutorials

1. http://fa.bianp.net/blog/tag/scikit-learn.html#fn:1
2. https://drive.google.com/open?id=1MzmA7QaP58RDzocB0RBmRiWfl7Co_VJ7
3. https://drive.google.com/open?id=133odBinMOIVb_rh_GQxxsyMRyW-Zts7a
4. https://stat.fandom.com/wiki/Isotonic_regression#Pool_Adjacent_Violators_Algorithm

In [11]:
```
N_positive = 0
N_negative = 0
for i in range(len(Y_cv)):
    if Y_cv[i] > 0:
        N_positive +=1
    else:
        N_negative +=1
y_positive = (N_positive + 1) / (N_positive + 2)
y_negative = 1/(N_negative + 2)

N_positive,N_negative,y_positive,y_negative
```

Out[11]: (305, 695, 0.996742671009772, 0.0014347202295552368)

In [12]:
```
f_cv_custom[0]
```

Out[12]: array([-1.80554548])

In [13]:
```
import math

def sigmoid(x,w,b):
    a = np.dot(x,w)+b
    return 1/(1 + math.exp(-a))
```

In [14]:
```
class CustomSGDClassifier:
    def __init__(self,alpha = 0.0001,N = len(X_train),b = 0,w = np.zeros_like(X_train
        self.alpha = alpha
        self.N = N
        self.b = b
        self.w = w
        self.eta0 = eta0
        self.ephocs = ephocs
        self.train_loss = []
        self.result = []

    def calculateLoss(self,w,b,X,Y):
        loss = []
        for i in np.arange(0,len(X)):
            sig = sigmoid(X[i],w,b)
            if Y[i] > 0:
                loss.append(-y_positive*np.log(sig) - (1-y_positive)*np.log(1-sig) + s
            else:
                loss.append(-y_negative*np.log(sig) - (1-y_negative)*np.log(1-sig) + s
        return np.mean(loss)

    def fit(self,X,Y):
        initial_loss = self.calculateLoss(w,b,X,Y)
#         print('Initial Loss:',initial_loss)
        for ep in range(self.ephocs):
```

```
                for i in range(N):
                    sig = sigmoid(X[i],self.w,self.b)
                    w_new = (1- (self.alpha*self.eta0)/self.N)*self.w + self.eta0*X[i]*(Y
                    b_new = self.b + self.eta0*(Y[i]-sig)
                    self.w = w_new
                    self.b = b_new
                next_loss = self.calculateLoss(self.w,self.b,X,Y)
                self.train_loss.append(next_loss)
                print('-- Epoch: {}, Avg. Train Loss: {}'.format(ep+1,next_loss))
                if (next_loss < initial_loss) & ((initial_loss-next_loss)<0.001):
                    break
                initial_loss = next_loss

        def getProbability(self,X):
            res = []
            for i in range(len(X)):
                prob = 1/(1+math.exp(-(self.w*X[i]+self.b)))
                res.append(prob)
            return res
```

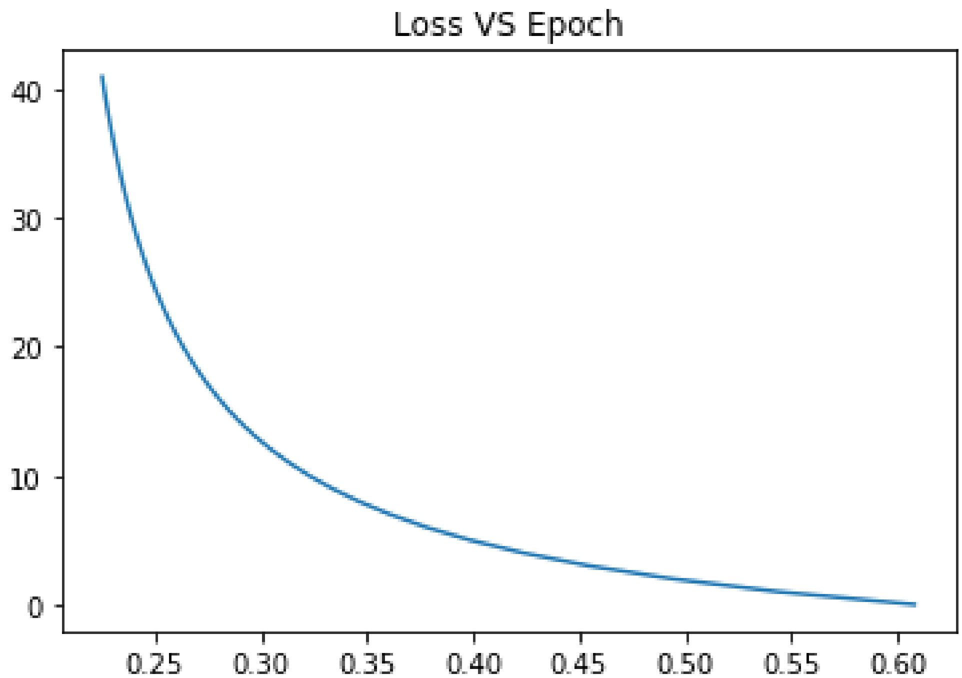
In [15]:
```
w = np.zeros_like(f_cv_custom[0])
b = 0
eta0  = 0.0001
alpha = 0.0001
N = len(f_cv_custom)
# f_test_custom = custom_decision_function(X_test)
ephocs=60
model = CustomSGDClassifier(alpha,N,b,w,eta0,ephocs)
%time model.fit(f_cv_custom,Y_cv)
```

```
-- Epoch: 1, Avg. Train Loss: 0.6079155706248737
-- Epoch: 2, Avg. Train Loss: 0.5432836790075528
-- Epoch: 3, Avg. Train Loss: 0.49358708840014864
-- Epoch: 4, Avg. Train Loss: 0.4547003818858166
-- Epoch: 5, Avg. Train Loss: 0.42371468946471313
-- Epoch: 6, Avg. Train Loss: 0.39859299715826124
-- Epoch: 7, Avg. Train Loss: 0.3778994410869025
-- Epoch: 8, Avg. Train Loss: 0.3606085361894483
-- Epoch: 9, Avg. Train Loss: 0.3459761556072111
-- Epoch: 10, Avg. Train Loss: 0.33345327076465753
-- Epoch: 11, Avg. Train Loss: 0.3226281158110312
-- Epoch: 12, Avg. Train Loss: 0.3131870226728602
-- Epoch: 13, Avg. Train Loss: 0.30488754386278194
-- Epoch: 14, Avg. Train Loss: 0.2975397344274488
-- Epoch: 15, Avg. Train Loss: 0.29099291865545984
-- Epoch: 16, Avg. Train Loss: 0.28512619511616644
-- Epoch: 17, Avg. Train Loss: 0.2798415264456194
-- Epoch: 18, Avg. Train Loss: 0.27505864189224627
```

```
-- Epoch: 19, Avg. Train Loss: 0.2707112289054954
-- Epoch: 20, Avg. Train Loss: 0.2667440535831429
-- Epoch: 21, Avg. Train Loss: 0.2631107589204572
-- Epoch: 22, Avg. Train Loss: 0.2597721635902968
-- Epoch: 23, Avg. Train Loss: 0.2566949345238134
-- Epoch: 24, Avg. Train Loss: 0.2538505416185785
-- Epoch: 25, Avg. Train Loss: 0.2512144275145971
-- Epoch: 26, Avg. Train Loss: 0.248765342861617
-- Epoch: 27, Avg. Train Loss: 0.24648481005728792
-- Epoch: 28, Avg. Train Loss: 0.24435668754887577
-- Epoch: 29, Avg. Train Loss: 0.2423668134719499
-- Epoch: 30, Avg. Train Loss: 0.2405027123436499
-- Epoch: 31, Avg. Train Loss: 0.23875335222033492
-- Epoch: 32, Avg. Train Loss: 0.23710894251026926
-- Epoch: 33, Avg. Train Loss: 0.23556076474355955
-- Epoch: 34, Avg. Train Loss: 0.23410103021730055
-- Epoch: 35, Avg. Train Loss: 0.23272275967932615
-- Epoch: 36, Avg. Train Loss: 0.23141968118067596
-- Epoch: 37, Avg. Train Loss: 0.23018614298225082
-- Epoch: 38, Avg. Train Loss: 0.2290170389950925
-- Epoch: 39, Avg. Train Loss: 0.2279077447035672
-- Epoch: 40, Avg. Train Loss: 0.22685406189458995
-- Epoch: 41, Avg. Train Loss: 0.22585217081505243
-- Epoch: 42, Avg. Train Loss: 0.22489858862016104
Wall time: 1.78 s
```

In [20]:
```python
import matplotlib.pyplot as plt

# Plotting Loss and Epoch
plt.plot(model.train_loss, range(len(model.train_loss)))
plt.title('Loss VS Epoch')
plt.show()
```



In [21]:
```
model.w,model.b
```

Out[21]:
```
(array([1.12246588]), -0.10995353241853036)
```

In [22]:
```
prob_test = model.getProbability(f_test_custom)
prob_test[0:5]
```

Out[22]:
```
[0.015962758698788887,
 0.6256673966097421,
 0.8474897736634276,
 0.8920350885870294,
 0.05398627856798464]
```

In [23]:
```
correct = 0
for i in range(len(Y_test)):
    if (prob_test[i] > 0.5 and Y_test[i]==1) or (prob_test[i] < 0.5 and Y_test[i]==0)
        correct += 1

correct,len(Y_test)
```

Out[23]:
```
(932, 1000)
```