8E_F_LR_SVM

January 28, 2020

9E and 9F: Finding the Probability P(Y==1 | X)

9E: Implementing Decision Function of SVM RBF Kernel

After we train a kernel SVM model, we will be getting support vectors and their corresponsing coefficients α_i

Check the documentation for better understanding of these attributes:

https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html

As a part of this assignment you will be implementing the decision_function() of kernel SVM, here decision_function() means based on the value return by decision_function() model will classify the data point either as positive or negative

Ex 1: In logistic regression After training the models with the optimal weights w we get, we will find the value $\frac{1}{1+\exp(-(wx+b))}$, if this value comes out to be < 0.5 we will mark it as negative class, else its positive class

Ex 2: In Linear SVM After training the models with the optimal weights w we get, we will find the value of sign(wx + b), if this value comes out to be -ve we will mark it as negative class, else its positive class.

Similarly in Kernel SVM After training the models with the coefficients α_i we get, we will find the value of $sign(\sum_{i=1}^{n}(y_i\alpha_iK(x_i,x_q))+intercept)$, here $K(x_i,x_q)$ is the RBF kernel. If this value comes out to be -ve we will mark x_q as negative class, else its positive class.

RBF kernel is defined as: $K(x_i, x_q) = exp(-\gamma ||x_i - x_q||^2)$

For better understanding check this link: https://scikit-learn.org/stable/modules/svm.html#svm-mathematical-formulation

0.1 Task E

- 1. Split the data into $X_{train}(60)$, $X_{cv}(20)$, $X_{test}(20)$
- 2. Train SVC(gamma = 0.001, C = 100.) on the (X_{train}, y_{train})
- 3. Get the decision boundry values f_{cv} on the X_{cv} data i.e. f_{cv} = decision_function(X_{cv}) you need to implement this decision_function()

0.1.1 Pseudo code

```
clf = SVC(gamma=0.001, C=100.) clf.fit(Xtrain, ytrain)
```

def decision_function(Xcv, ...): #use appropriate parameters for a data point x_q in Xcv: #write code to implement $(\sum_{i=1}^{\text{all the support vectors}}(y_i\alpha_iK(x_i,x_q)) + intercept)$, here the values y_i , α_i , and intercept can be obtained from the trained model return # the decision_function output for all the data points in the Xcv

 $fcv = decision_function(Xcv, ...)$ # based on your requirement you can pass any other parameters

Note: Make sure the values you get as fcv, should be equal to outputs of clf.decision_function(Xcv)

```
In [3]: # you can write your code here
        data = np.hstack((X, y.reshape(5000,1)))
        data.shape
Out[3]: (5000, 6)
In [4]: X_train, X_rem, Y_train, Y_rem = train_test_split(X,y,test_size=0.40,random_state=16)
        X_cv,X_test,Y_cv,Y_test = train_test_split(X_rem,Y_rem,test_size=0.50,random_state=16)
        X_train.shape,X_cv.shape,X_test.shape
Out[4]: ((3000, 5), (1000, 5), (1000, 5))
In [5]: model = SVC(gamma = 0.001, C = 100)
        model.fit(X_train,Y_train)
Out[5]: SVC(C=100, cache_size=200, class_weight=None, coef0=0.0,
          decision_function_shape='ovr', degree=3, gamma=0.001, kernel='rbf',
          max_iter=-1, probability=False, random_state=None, shrinking=True,
          tol=0.001, verbose=False)
In [6]: f_cv = model.decision_function(X_cv)
        # f_cv
In [7]: def calculate_rbf(X_i,X_q):
              k = np.exp(-model.gamma * np.linalg.norm(X_i-X_q))
            k = np.exp(-model.gamma * np.dot(X_i-X_q, X_i-X_q))
            return k
In [8]: from tqdm import tqdm
        def custom_decision_function(Xcv): #use appropriate parameters
            ycv = np.zeros(Xcv.shape[0]).reshape((Xcv.shape[0],1))
            for i in tqdm(range(Xcv.shape[0])):
                for j in range(model.support_vectors_.shape[0]):
                    kernal = calculate_rbf(model.support_vectors_[j],Xcv[i])
                    sum += kernal*model.dual_coef_[0][j]
                ycv[i] = sum + model.intercept_
            return ycv
```

```
In [9]: f_cv_custom = custom_decision_function(X_cv)
        f_test_custom = custom_decision_function(X_test)
100%|| 1000/1000 [00:05<00:00, 186.36it/s]
100%|| 1000/1000 [00:04<00:00, 206.61it/s]
In [10]: f_cv_custom - f_cv.reshape(1000,1)
Out[10]: array([[ 0.00000000e+00],
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                [ 0.0000000e+00],
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9F: Implementing Platt Scaling to find P(Y==1 | X)Check this PDF

0.2 TASK F

4. Apply SGD algorithm with (f_{cv}, y_{cv}) and find the weight W intercept b Note: here our data is of one dimensional so we will have a one dimensional weight vector i.e W.shape (1,)

Note1: Don't forget to change the values of y_{cv} as mentioned in the above image. you will calculate y+, y- based on data points in train data

Note2: the Sklearn's SGD algorithm doesn't support the real valued outputs, you need to use the code that was done in the 'Logistic Regression with SGD and L2' Assignment after modifying loss function, and use same parameters that used in that assignment. if Y[i] is 1, it will be replaced with y+ value else it will replaced with y- value

5. For a given data point from X_{test} , $P(Y = 1|X) = \frac{1}{1 + exp(-(W*f_{test} + b))}$ where f_{test} = decision_function(X_{test}), W and b will be learned as metioned in the above step

Note: in the above algorithm, the steps 2, 4 might need hyper parameter tuning, To reduce the complexity of the assignment we are excluding the hyerparameter tuning part, but intrested students can try that

If any one wants to try other calibration algorithm istonic regression also please check these tutorials

- 1. http://fa.bianp.net/blog/tag/scikit-learn.html#fn:1
- 2. https://drive.google.com/open?id=1MzmA7QaP58RDzocB0RBmRiWfl7Co_VJ7
- 3. https://drive.google.com/open?id=133odBinMOIVb_rh_GQxxsyMRyW-Zts7a
- 4. https://stat.fandom.com/wiki/Isotonic_regression#Pool_Adjacent_Violators_Algorithm

```
In [12]: f_cv_custom[0]
Out[12]: array([-1.80554548])
In [13]: import math
         def sigmoid(x,w,b):
             a = np.dot(x,w)+b
             return 1/(1 + math.exp(-a))
In [14]: class CustomSGDClassifier:
             def __init__(self,alpha = 0.0001,N = len(X_train),b = 0,w = np.zeros_like(X_train)
                 self.alpha = alpha
                 self.N = N
                 self.b = b
                 self.w = w
                 self.eta0 = eta0
                 self.ephocs = ephocs
                 self.train_loss = []
                 self.test_loss = []
                 self.result = []
             def calculateLoss(self,w,b,X,Y):
                 loss = []
                 for i in np.arange(0,len(X)):
                     sig = sigmoid(X[i],w,b)
                     if Y[i] > 0:
                         loss.append(-y_positive*np.log(sig) - (1-y_positive)*np.log(1-sig) + ;
                     else:
                         loss.append(-y_negative*np.log(sig) - (1-y_negative)*np.log(1-sig) + =
                 return np.mean(loss)
             def fit(self,X,Y):
                 initial_loss = self.calculateLoss(w,b,X,Y)
         #
                   print('Initial Loss:',initial_loss)
                 for ep in range(self.ephocs):
                     for i in range(N):
                         sig = sigmoid(X[i],self.w,self.b)
                         w_new = (1- (self.alpha*self.eta0)/self.N)*self.w + self.eta0*X[i]*(Y
                         b_new = self.b + self.eta0*(Y[i]-sig)
                         self.w = w_new
                         self.b = b_new
                     next_loss = self.calculateLoss(self.w,self.b,X,Y)
                     self.train_loss.append(next_loss)
                     print('-- Epoch: {}, Avg. Train Loss: {}'.format(ep+1,next_loss))
                     if (next_loss < initial_loss) & ((initial_loss-next_loss)<0.0001):</pre>
                         break
                     initial_loss = next_loss
```

```
def getProbability(self,X):
                 res = []
                 for i in range(len(X)):
                     prob = 1/(1+math.exp(-(self.w*X[i]+self.b)))
                     res.append(prob)
                 return res
In [17]: w = np.zeros_like(f_cv_custom[0])
         b = 0
         eta0 = 0.0001
         alpha = 0.0001
         N = len(f_cv_custom)
         # f_test_custom = custom_decision_function(X_test)
         model = CustomSGDClassifier(alpha, N, b, w, eta0, ephocs=30)
         %time model.fit(f_cv_custom,Y_cv)
-- Epoch: 1, Avg. Train Loss: 0.6079155706248737
-- Epoch: 2, Avg. Train Loss: 0.5432836790075528
-- Epoch: 3, Avg. Train Loss: 0.49358708840014864
-- Epoch: 4, Avg. Train Loss: 0.4547003818858166
-- Epoch: 5, Avg. Train Loss: 0.42371468946471313
-- Epoch: 6, Avg. Train Loss: 0.39859299715826124
-- Epoch: 7, Avg. Train Loss: 0.3778994410869025
-- Epoch: 8, Avg. Train Loss: 0.3606085361894483
-- Epoch: 9, Avg. Train Loss: 0.3459761556072111
-- Epoch: 10, Avg. Train Loss: 0.33345327076465753
-- Epoch: 11, Avg. Train Loss: 0.3226281158110312
-- Epoch: 12, Avg. Train Loss: 0.3131870226728602
-- Epoch: 13, Avg. Train Loss: 0.30488754386278194
-- Epoch: 14, Avg. Train Loss: 0.2975397344274488
-- Epoch: 15, Avg. Train Loss: 0.29099291865545984
-- Epoch: 16, Avg. Train Loss: 0.28512619511616644
-- Epoch: 17, Avg. Train Loss: 0.2798415264456194
-- Epoch: 18, Avg. Train Loss: 0.27505864189224627
-- Epoch: 19, Avg. Train Loss: 0.2707112289054954
-- Epoch: 20, Avg. Train Loss: 0.2667440535831429
-- Epoch: 21, Avg. Train Loss: 0.2631107589204572
-- Epoch: 22, Avg. Train Loss: 0.2597721635902968
-- Epoch: 23, Avg. Train Loss: 0.2566949345238134
-- Epoch: 24, Avg. Train Loss: 0.2538505416185785
-- Epoch: 25, Avg. Train Loss: 0.2512144275145971
-- Epoch: 26, Avg. Train Loss: 0.248765342861617
-- Epoch: 27, Avg. Train Loss: 0.24648481005728792
-- Epoch: 28, Avg. Train Loss: 0.24435668754887577
-- Epoch: 29, Avg. Train Loss: 0.2423668134719499
-- Epoch: 30, Avg. Train Loss: 0.2405027123436499
```

```
In [18]: model.w,model.b
Out[18]: (array([0.98423654]), -0.11499080123800826)
In [23]: prob_test = model.getProbability(f_test_custom)
         prob_test
Out [23]: [0.025769892876980352,
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Wall time: 1.88 s

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