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2020S2307
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MA102: Introduction to Discrete Mathematics

Midsem Remote Exam

1. Let $f(x) = y$

(a) If A is countable, then elements of A can be numbered as natural numbers.

So, $f(x)$ can also be numbered using natural numbers.

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & \cdots \\ f(x_1) & f(x_2) & f(x_3) & f(x_4) & f(x_5) & f(x_6) & \cdots \end{matrix}$$

$\therefore$ If A is countable, so is $f(A)$.

(b) If f is one-one, it means that each $x \in A$ will have a unique image, i.e. $f(x)$ will be unique $\forall x \in A$.

If A is uncountable, element of A cannot be numbered using natural numbers.

$\therefore$ $f(A)$ is uncountable when A is uncountable and f is one-one.

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4. $f: S \to S$ is a bijection

Let $f(x) = x$ be a bijection.

In that case $x - f(x) = 0 \quad \forall x \in S$

$$\therefore \quad R = \prod_{i=1}^{n} \{i - f(i)\}$$

$$= 0 \times 0 \times 0 \times 0 \cdots \times 0$$

$$= 0$$

which is even for any n.

Hence, proved.

another bijection possible is

$$f(x) = \begin{cases} x+1 , & \text{when } x < n \\ 1 , & \text{when } x = n \end{cases}$$

$$\therefore \quad x - f(x) = \begin{cases} x - (x+1) , & \text{when } x < n \\ x - 1 , & \text{when } x = n \end{cases}$$

$$x - f(x) = \begin{cases} -1 , & \text{when } x < n \\ x - 1 , & \text{when } x = n \end{cases}$$

$\because$ n is odd,

So, $R = \underbrace{(1 - f(1)) \cdot (2 - f(2)) \cdots}_{(n-1) \text{ terms}} \times (n - f(n))$

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$\because$ n is odd, $(n-1)$ is even

$\therefore$ $R = (-1)^{n-1} \times (n-1)$

$(-1)^{even} = 1$

$\because$ $(n-1) \rightarrow$ even

$\therefore$ $R = (n-1) \rightarrow$ even.

Hence, proved

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(5) Let R be a relation such that xRy iff $\{(x,y) \mid x-y \in R\}$ on the set of complex numbers

$\therefore$ for every complex number $z = a+ib$
$z - z = 0$, which is real.

$\therefore$ R is reflexive.

Now, let us assume two complex numbers

$$z_1 = a + ib$$
$$z_2 = c + id$$

If $z_1 - z_2 \in R$, then
$a - c \in R$
$b - d = 0$

Now, $z_2 - z_1 = (c-a) + i(d-b)$
$= -(a-c)$

$\because$ $(a-c) \in R$, $\therefore$ $(c-a) \in R$

Hence, R is reflexive

Now, assume three complex numbers
$z_1 = a+ib$, $z_2 = c+id$
and $z_3 = e+if$.

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and let,

$$z_1 - z_2 = (a-c) + i(b-d) \in R$$

$$\therefore (b-d) = 0$$

and, $z_2 - z_3 = (c-e) + i(d-f) \in R$

$$\therefore (d-f) = 0$$

$$z_1 - z_2 = (a-c) + i(b-d)$$
$$z_2 - z_3 = (c-e) + i(d-f)$$

Adding above equations.

$$z_1 - z_3 = (a-e) + i(b-d) + i(d-f)$$

$$\because (b-d) = d-f = 0$$
$$\therefore z_1 - z_3 \in R$$

Hence, R is transitive.

Since, R is reflexive, symmetric and transitive, R is an equivalence relation on set of complex numbers.

Equivalence class of i is $\{z = a+ib \mid a \in R$ and $b = -1\}$

Equivalence class of $\sqrt{2}+1$ is R (set of real numbers.

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(6) $f : A \to B$
f is one-to-one (given)

$S_f : P(A) \to P(B) \mid S_f(X) = f(X)$

$\because$ f is one-one, the image of a subset of A under f in B is unique.

i.e. for every subset X of A, the set that is obtained by imaging the X elements to B under f is unique.

$\therefore$ this set is a unique subset of ~~~~ set B (or ~~~~ a unique element of power set of B).

Hence, S_f is one-one/injective iff f is one-one/injective

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(7) As n is a positive integer, and since $n^3 \geq 100 \;\forall\; n \geq 4$. There fore, we just need to verify if $n^2+n^3 = 100$ for $n = 1, 2, 3, 4$.

for $n = 1$,
$$n^2+n^3 = 2$$

for $n = 2$,
$$n^2+n^3 = 12$$

for $n = 3$,
$$n^2+n^3 = 36$$

for $n = 4$,
$$n^2+n^3 = 80$$

Hence, there is no positive integer n such that $n^2+n^3 = 100$. The method of proof used is Exhaustive Proof.

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(8).

This argument is incorrect. There can be some pet bird other than ~~sparrow~~ ~~sparrow~~ sparrow which likes fruit.

$P(x)$: x is a sparrow
$Q(x)$: x likes fruit

Domain of discourse of x ~~[illegible]~~ is all pet birds, then the argument is

$$\forall x\,(P(x) \rightarrow Q(x))$$
$$\neg P(\text{Sparrow})$$
$$\therefore \neg Q(\text{Sparrow})$$

After applying universal instantiation, it contains the fallacy of denying the hypothesis.