Newton's Second Law of Motion

Aim :- (i) To verify Newton's Second Law of Motion.
(ii) To find acceleration of cart.
(iii) To find the distance covered by cart in given time interval.

Theory :- Newton's Second Law ~~of~~ Motion can be used to derive the equations of motion of a system under following condition.
i) The system must undergo either pure translation or pure rotation.
ii) Motion must occur in a single plane.
iii) Force acting must have either a constant orientation or parallel to direction along which the point of application moves.

Newton's Second Law of motion states that the net force applied on an object is directly proportional to the rate of change of momentum of that object.

$$\vec{F} \propto \frac{d\vec{p}}{dt}$$

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt}$$ (proportionality constant is 1)

$$\vec{F} = m\frac{d\vec{v}}{dt} + v\frac{dm}{dt}$$

$$\vec{F} = m\vec{a} + \vec{v}\frac{dm}{dt} \qquad (\because dv/dt \text{ is acceleration})$$

For a body whose mass is not changing with time, $dm = 0$ and,

$$\vec{F} = m\vec{a}$$

If there are n forces acting on an object, then the vector sum of all the ~~forces~~ forces is used. i.e,

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n = m\vec{a}$$

In the figure there is a cart placed over a table at a distance of 100 cm from one end of the table. The cart is connected to one end of an massless and inextensible string which goes over a massless pulley and a variable hanging mass is attached to its other end. The cart mass can also be changed by putting stones in it. The coefficient of friction between table and cart is 'μ' and can be changed.

When the system is set free, the hanging weight will move downwards with constant acceleration 'a'. As the string is massless and inextensible, the cart will also move forward with the same acceleration. As hanging mass moves downward, tension T is generated in the string and since the cart moves over the table, friction acts between the cart and table.

Applying Newton's Second Law on hanging mass

$$mg - T = ma \quad \text{—(I)}$$

where m is mass of hanging weight.

Applying Newton's Second Law on cart,

$$T - f = Ma$$

$$T - \mu Mg = Ma \quad \text{—(II)} \quad (\because f = \mu Mg)$$

where M is mass of cart.

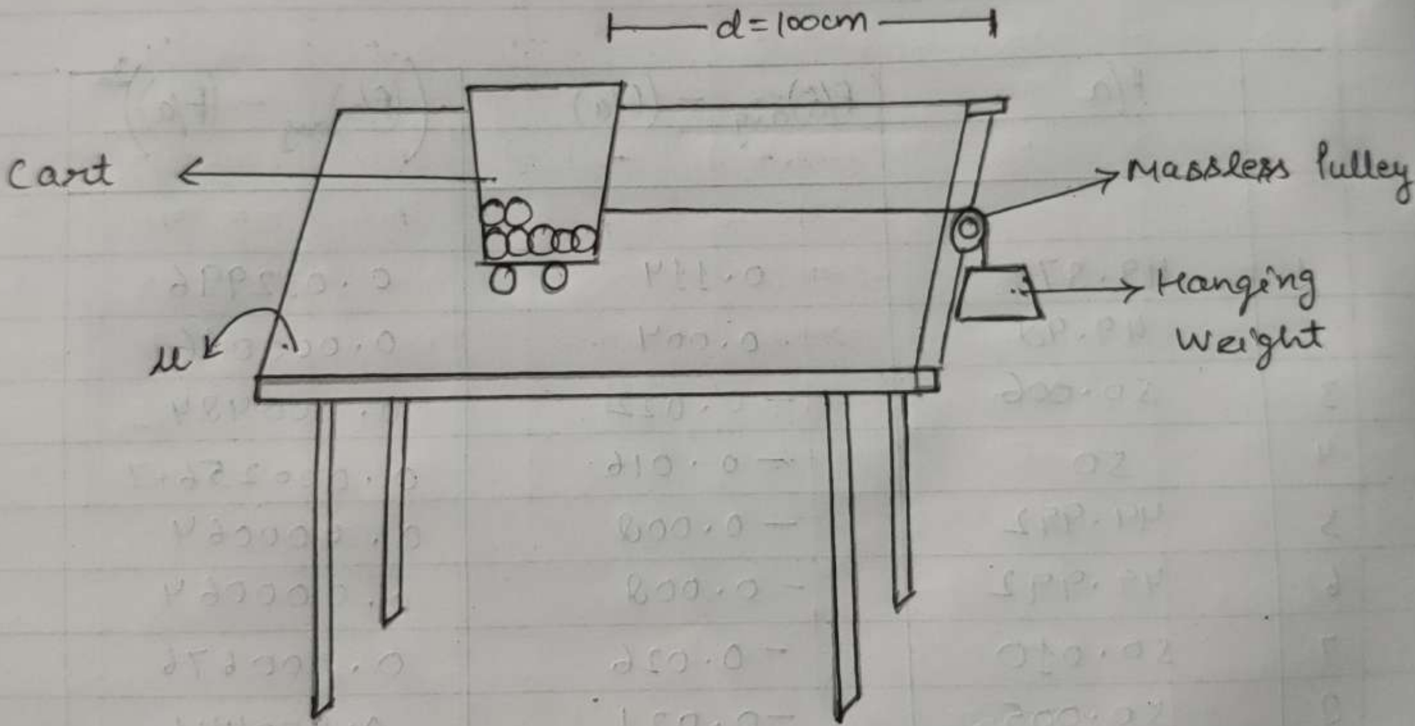
Eliminating T from both (I) and (II),

$$a = \frac{mg - \mu Mg}{m + M}$$

Using second equation of motion, distance travelled by cart is given by

$$s = \frac{1}{2} a t^2$$

where t is time in seconds.



d=100cm
Cart
Massless Pulley
Hanging Weight
μ

Observations and Errors :-

i) Checking $F \propto a$ i.e. F/a constant

a) Keeping hanging mass constant and $\mu = 0.001$ and distance of cart from pulley is 100 cm

S.No	Hanging Mass m (grams)	Cart Mass M (grams)	Time (seconds)	Acceleration a (m/s^2)	Force $F = T - \mu Mg$ (mN)	F/a	Error $M - F/a$
1	1	50	3.29	0.183	9.127	49.87	+0.130
2	4	50	1.67	0.717	35.842	49.98	+0.020
3	11	50	1.07	1.759	87.961	50.006	−0.006
4	15	50	0.94	2.254	112.70	50	0
5	21	50	0.83	2.892	144.58	49.992	+0.008
6	28	50	0.760	3.512	175.57	49.992	+0.008
7	33	50	0.72	3.890	194.54	50.010	−0.010
8	38	50	0.69	4.226	211.32	50.005	−0.005
9	45	50	0.66	4.637	231.84	49.990	+0.002
10	49	50	0.64	4.846	242.25	49.990	+0.010

b) Keeping hanging mass constant and $\mu = 0.003$ and distance of cart from pulley is 100 cm.

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S.No	Hanging Mass m (grams)	Cart Mass M (grams)	Time (seconds)	Acceleration a (m/s^2)	Force $F = T - \mu Mg$ (mN)	F/a	Error M − F/a
1	1	50	3.48	0.163	8.167	50.104	−0.104
2	4	50	1.69	0.699	34.934	49.977	0.023
3	11	50	1.070	1.743	87.157	50.004	−0.004
4	15	50	0.95	2.239	111.945	49.997	0.003
5	21	50	0.833	2.878	143.892	49.996	0.004
6	28	50	0.762	3.499	174.958	50.002	−0.002
7	33	50	0.723	3.879	193.923	49.993	0.007
8	38	50	0.692	4.215	210.760	50.002	−0.002
9	45	50	0.661	4.627	231.315	49.992	0.008
10	49	50	0.641	4.836	241.766	49.995	0.005

(ii) checking $F \propto m$ i.e $\frac{F}{m} = a$

a) Keeping hanging mass constant, $\mu = 0.001$ and distance of cart from pulley 100 cm.

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S.No	Hanging Mass (m) (grams)	Cart Mass M (grams)	Time (seconds)	Acceleration a (m/s^2)	Net force on cart $F = T - \mu Mg$ (mN)	F/M	Error $a - \frac{F}{M}$
1	27	10	0.530	7.149	71.479	7.147	0.002
2	27	20	0.600	5.626	112.502	5.625	0.001
3	27	30	0.660	4.637	139.107	4.636	0.001
4	27	40	0.710	3.943	157.747	3.943	0
5	27	50	0.760	3.430	171.520	3.431	−0.001
6	27	60	0.810	3.035	182.067	3.034	0.001
7	27	70	0.860	2.721	190.447	2.720	0.001
8	27	80	0.900	2.466	197.234	2.465	0.001
9	27	90	0.940	2.254	202.862	2.252	0.002
10	27	100	0.980	2.076	207.568	2.075	0.001

b) Keeping hanging mass constant, $\mu = 0.004$ and distance of cart from pulley 100 cm.

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S.No	Hanging Mass m (grams)	Cart Mass M (grams)	Acceleration (a)	Net force on cart $F = T - \mu Mg$ (mN)	F/M	Time (seconds)	Error $a - \frac{F}{M}$
1	27	10	7.141	71.401	7.1401	0.530	0.0009
2	27	20	5.613	112.265	5.6132	0.600	-0.0002
3	27	30	4.621	138.657	4.6219	0.660	-0.0009
4	27	40	3.926	157.030	3.9257	0.720	0.0003
5	27	50	3.411	170.543	3.4108	0.770	0.0002
6	27	60	3.014	180.870	3.0145	0.820	-0.0005
7	27	70	2.700	188.956	2.6993	0.860	0.0007
8	27	80	2.444	195.476	2.4434	0.910	0.0006
9	27	90	2.231	200.835	2.2315	0.950	-0.0005
10	27	100	2.053	205.249	2.0524	0.990	0.0006

(iii) Now the cart will start from rest and move with a constant acceleration, the hanging mass and cart mass are constant.
The value of $\mu = 0.002$ and the pointer location will be changed.
The time taken by cart to reach the pointer will be noted and we have to calculate the distance between cart and pointer using the relation $s = \frac{1}{2}at^2$.

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$\mu = 0.002$

	Hanging Mass m (grams)	Cart Mass M (grams)	acceleration a (cm/s^2)	Pointer Distance from cart (P) (cm)	Time t (seconds)	Distance $s = \frac{1}{2}at^2$ (cm)	Error P − S
1	34	60	353.8	5	0.180	5.73	−0.73
2	34	60	353.8	14	0.290	14.87	−0.87
3	34	60	353.8	29	0.410	29.73	−0.73
4	34	60	353.8	38	0.460	37.43	0.57
5	34	60	353.8	50	0.530	49.69	0.31
6	34	60	353.8	57	0.570	57.47	−0.47
7	34	60	353.8	70	0.630	70.21	−0.21
8	34	60	353.8	78	0.660	77.05	0.95
9	34	60	353.8	85	0.690	84.22	0.78
10	34	60	353.8	95	0.730	94.27	0.73

Now, calculating the distance using different values that were used in the above table

$\mu = 0.005$

	Hanging Mass m (grams)	Cart Mass M (grams)	Acceleration a (cm/s^2)	Pointer distance from cart P (cm)	Time t (seconds)	Distance $s = \frac{1}{2}at^2$ (cm)	Error P − S
1	24	70	246.6	10	0.280	9.66	0.34
2	24	70	246.6	25	0.450	24.96	0.04
3	24	70	246.6	29	0.480	28.40	0.60
4	24	70	246.6	39	0.560	38.66	0.34
5	24	70	246.6	46	0.610	45.87	0.13
6	24	70	246.6	55	0.670	55.34	−0.34
7	24	70	246.6	73	0.770	73.10	−0.10
8	24	70	246.6	80	0.800	79.91	+0.09
9	24	70	246.6	93	0.870	93.32	−0.32
10	24	70	246.6	100	0.900	99.87	0.13

Errors in last two table are comparatively large because the time was given only upto two decimal places in the simulator.

Errors :- In table 1, the average value of F/a of ten experiments is,

$$(F/a)_{avg} = \frac{49.87 + 49.98 + \cdots + 49.990}{10}$$

$$(F/a)_{avg} = 49.984 \text{ g}$$

	F/a	$(F/a)_{avg} - (F/a)$	$((F/a)_{avg} - (F/a))^2$
1	49.87	0.114	0.012996
2	49.98	0.004	0.000016
3	50.006	−0.022	0.000484
4	50	−0.016	0.000256
5	49.992	−0.008	0.000064
6	49.992	−0.008	0.000064
7	50.010	−0.026	0.000676
8	50.005	−0.021	0.000441
9	49.998	−0.014	0.000196
10	49.990	−0.006	0.000036

$$\sum_{i=1}^{10} \{(F/a)_{avg} - (F/a)\}^2 = 0.015229$$

deviation 'd' is given by,

$$d = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \{(F/a)_{avg} - (F/a)_i\}^2}$$

$$\therefore d = \sqrt{\frac{1}{10} \times 0.015229} = 0.0390 = 3.9 \times 10^{-2} g$$

Hence $\frac{F}{a} = \left(\frac{F}{a}\right)_{avg} \pm d$

$$= 49.984 \pm 3.9 \times 10^{-2} \; g$$

therefore, the error $\Delta(F/a)$ is $d = 3.9 \times 10^{-2}$

percentage error is given by $= \frac{\Delta(F/a)}{(F/a)} \times 100$

$$= \frac{3.9 \times 10^{-2}}{50} \times 100$$

$$= 7.8 \times 10^{-2} \; \%$$

Therefore the percentage error in the calculation of (F/a) is 7.8×10^{-2} %.

Result :- By performing the experiment, it is concluded that the net force applied on a body is directly proportional to its mass times acceleration.

i.e. $F \propto ma$

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Also, the acceleration of a body can be calculated if its mass and the force applied on it is known using the formula

$$F = ma$$

and, the distance which a body will travel (starting from rest) having a constant acceleration a in time t is given by

$$s = \frac{1}{2} a t^2$$