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Date: Dec 21, 2020
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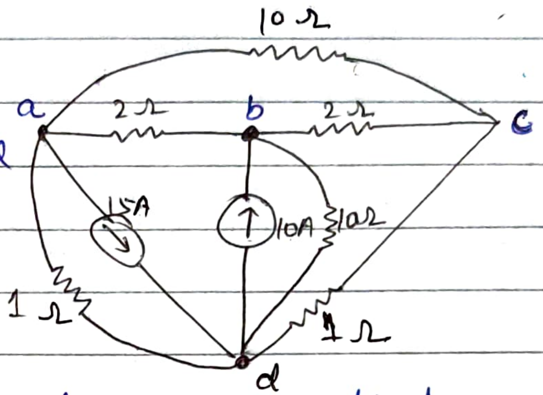
EC100: Assignment 1

2 Questions Using Nodal Analysis

(1.1) Example 3.16 For the network shown in figure
a) Write nodal voltage equations
b) Find current and its direction in 10Ω resistor.



Assuming node d as reference node i.e. potential at node d = 0.
and assuming the potential at node A, node B and node C to be V_a, V_b and V_c respectively.

Since the algebraic sum of all the currents leaving a node is zero,

a) KCL equation at node a

$$\frac{V_a - 0}{1} + 15 + \frac{V_a - V_b}{2} + \frac{V_a - V_c}{10} = 0$$

$$16 V_a - 5 V_b - V_c = -150 \quad \text{(I)}$$

KCL equation at node B,

$$\frac{V_b - V_a}{2} + \frac{V_b - V_c}{2} + \frac{V_b - 0}{10} + (-10) = 0$$

$$-5 V_a + 11 V_b - 5 V_c = 100 \quad \text{(II)}$$

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At node c,

$$\frac{V_c - V_a}{10} + \frac{V_c - V_b}{2} + \frac{V_c - 0}{1} = 0$$

$$-V_a - 5V_b + 16V_c \quad \text{---(iii)}$$

The equations are

$$16V_a - 5V_b - V_c = -150 \quad \text{---(i)}$$
$$-5V_a + 11V_b - 5V_c = 100 \quad \text{---(ii)}$$
$$-V_a - 5V_b + 16V_c = 0 \quad \text{---(iii)}$$

Augmented matrix of given system is

$$\left[\begin{array}{ccc|c} 16 & -5 & -1 & -150 \\ -5 & 11 & -5 & 100 \\ -1 & -5 & 16 & 0 \end{array}\right]$$

$R_1 \rightarrow R_1/16$

$$\left[\begin{array}{ccc|c} 1 & -5/16 & -1/16 & -75/8 \\ -5 & 11 & -5 & 100 \\ -1 & -5 & 16 & 0 \end{array}\right]$$

$R_2 \rightarrow R_2 + 5R_1$

$$\left[\begin{array}{ccc|c} 1 & -5/16 & -1/16 & -75/8 \\ 0 & 151/16 & -85/16 & 425/8 \\ -1 & -5 & 16 & 0 \end{array}\right]$$

$R_3 \rightarrow R_3 + R_1$

$$\left[\begin{array}{ccc|c} 1 & -5/16 & -1/16 & -75/8 \\ 0 & 151/16 & -85/16 & 425/8 \\ 0 & -85/16 & 255/16 & -75/8 \end{array}\right]$$

$R_2 \rightarrow \frac{16}{151} R_2$

$$\left[\begin{array}{ccc|c} 1 & -5/16 & -1/16 & -75/8 \\ 0 & 1 & -85/151 & 850/151 \\ 0 & -85/16 & 255/16 & -75/8 \end{array}\right]$$

$R_3 \rightarrow R_3 + \frac{85}{16} R_2$

$$\left[\begin{array}{ccc|c} 1 & -5/16 & -1/16 & -75/8 \\ 0 & 1 & -85/151 & 850/151 \\ 0 & 0 & \frac{1955}{151} & 3100/151 \end{array}\right]$$

$R_3 \rightarrow \frac{151}{1955} R_3$

$$\left[\begin{array}{ccc|c} 1 & -5/16 & -1/16 & -75/8 \\ 0 & 1 & -85/151 & 850/151 \\ 0 & 0 & 1 & 620/391 \end{array}\right]$$

$R_2 \rightarrow R_2 + \frac{85}{151} R_3$

$$\left[\begin{array}{ccc|c} 1 & -5/16 & -1/16 & -75/8 \\ 0 & 1 & 0 & 150/23 \\ 0 & 0 & 1 & 620/391 \end{array}\right]$$

$R_1 \rightarrow R_1 + \frac{1}{16} R_3$

$$\left[\begin{array}{ccc|c} 1 & -5/16 & 0 & -29015/3128 \\ 0 & 1 & 0 & 150/23 \\ 0 & 0 & 1 & 620/391 \end{array}\right]$$

$R_1 \rightarrow R_1 + \frac{5}{16} R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-2830}{391} \\ 0 & 1 & 0 & 150/23 \\ 0 & 0 & 1 & 620/391 \end{array}\right]$$

This gives $v_a = \frac{-2830}{391}$, $v_b = \frac{150}{23}$ and

$v_c = \frac{620}{391}$

b) Current in $10\,\Omega$ resistor between node a and c is $\frac{V_a - V_c}{10}$ from a to c.

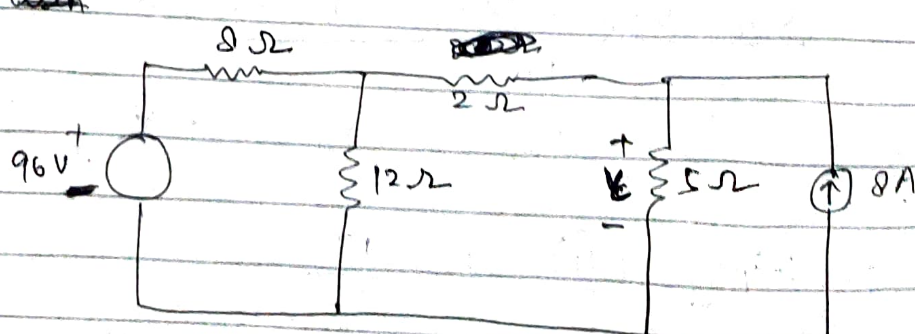
i.e. $\frac{\left(\frac{-2830}{391} - \frac{620}{391}\right)}{10}$ from a to c.

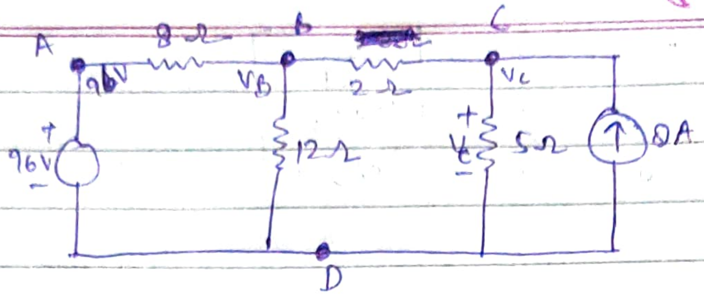
current in $10\,\Omega$ resistor (b/w node a and c) $= \frac{-\cancel{3450}\ 150}{\cancel{391}\ 17 \times 10}$

$= \frac{-15}{17}$ A

Hence, current in $10\,\Omega$ resistor (b/w node a and node c) is $\frac{15}{17}$ A from node c to node a.

(1.1) Example 3.12 Determine the voltage V_c in the circuit.





Let us assume the node D as reference node. Since, potential at reference node is zero, potential at node A, $V_A = 96$ V.
Let the voltage at node B and C be V_B and V_C.
Since, the algebraic sum of all the currents leaving a node is zero,

KCL equation at node B is,

$$\frac{V_B - 96}{8} + \frac{V_B - V_C}{2} + \frac{V_B - 0}{12} = 0$$

$$\frac{3V_B - 288 + 12V_B - 12V_C + 2V_B}{24} = 0$$

$$17V_B - 12V_C = 288 \quad \text{--- (I)}$$

KCL equation at node C is,

$$\frac{V_C - V_B}{2} + \frac{V_C - 0}{5} + (-8) = 0$$

$$\frac{5V_C - 5V_B + 2V_C + (-80)}{10} = 0$$

$$7V_C - 5V_B = 80 \quad \text{--- (II)}$$

Multiply (II) by $\frac{17}{5}$

$$\frac{119}{5}V_C - 17V_B = 272 \quad \text{--- (III)}$$

Adding (I) and (II),

$$V_c\left(\frac{119}{5}-\frac{60}{5}\right) = 560$$

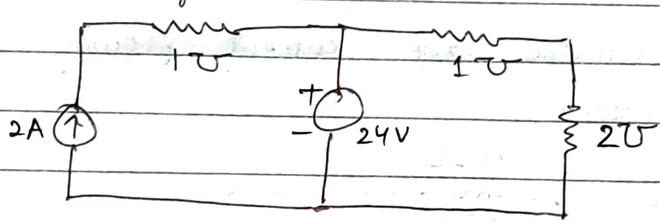
$$V_c = \frac{560\times 5}{59}$$

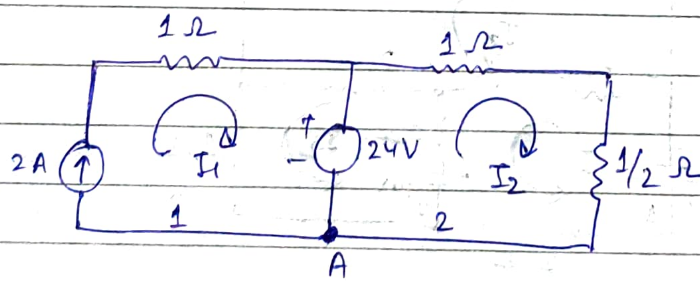
$$V_c = 47.457$$

Hence, the required voltage is 47.457 Volts.

2 Questions Using Mesh Analysis

(1.1) 11. Determine the mesh currents of the network by mesh analysis





Let us assume the currents in mesh 1 and mesh 2 be I_1 and I_2 clockwise.
Since, the current source is supplying current of 2 A in direction of I_1.

$$\therefore I_1 = 2A$$

~~Writing~~

KVL states that the algebraic sum of all the voltages in closed loop is 0.

Applying KVL in mesh 2, starting from A:

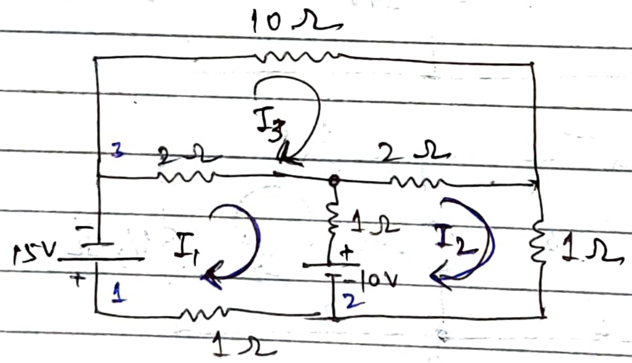
$$24 - 1(I_2) - \frac{1}{2}(I_2) = 0$$

$$\frac{3}{2} I_2 = 24$$

$$I_2 = 16 \text{ A}$$

Hence the currents in mesh 1 and mesh 2 are 2 A and 16 A.

(1.1) Example 3.15 Determine the current flowing through 1 Ω resistor.



Let the currents in mesh 1, 2 and 3 be I_1, I_2 and I_3 in clockwise direction as shown in figure.

According to KVL, voltage drop in a loop is zero.

Applying KVL in mesh 1,

$$(-15) - 2(I_1 - I_3) - (I_1 - I_2) - 10 - 1(I_1) = 0$$

$$-4I_1 + I_2 + 2I_3 = 25 \quad \text{—(1)}$$

Applying KVL in mesh 2,

$$+10 - 1(I_2 - I_1) - 2(I_2 - I_3) - 1(I_2) = 0$$

$$I_1 - 4I_2 + 2I_3 = -10 \quad \text{--- (II)}$$

Applying KVL in mesh 3,

$$-10(I_3) - 2(I_3 - I_2) - 2(I_3 - I_1) = 0$$

$$2I_1 + 2I_2 - 14I_3 = 0$$

$$I_1 + I_2 - 7I_3 = 0 \quad \text{--- (III)}$$

$$I_1 + I_2 - 7I_3 = 0 \quad \text{--- (III)}$$

$$I_1 - 4I_2 + 2I_3 = -10 \quad \text{--- (II)}$$

$$-4I_1 + I_2 + 2I_3 = 25 \quad \text{--- (III)}$$

Augmented matrix for the system of equation is

$$\left[\begin{array}{ccc|c} 1 & 1 & -7 & 0 \\ 1 & -4 & 2 & -10 \\ -4 & 1 & 2 & 25 \end{array}\right]$$

$R_2 \rightarrow R_2 - R_1$
$R_3 \rightarrow R_3 + 4R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -7 & 0 \\ 0 & -5 & 9 & -10 \\ 0 & 5 & -26 & 25 \end{array}\right]$$

$R_3 \rightarrow R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -7 & 0 \\ 0 & -5 & 9 & -10 \\ 0 & 0 & -17 & 15 \end{array}\right]$$

$R_2 \rightarrow -1/5\, R_2$
$R_3 \rightarrow -1/17\, R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & -7 & 0 \\ 0 & 1 & -9/5 & 2 \\ 0 & 0 & 1 & -15/17 \end{array}\right]$$

$$R_2 \rightarrow R_2 + \frac{9}{5} R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -7 & 0 \\ 0 & 1 & 0 & 7/17 \\ 0 & 0 & 1 & -15/17 \end{array}\right]$$

$$R_1 \rightarrow R_1 + 7 R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -105/17 \\ 0 & 1 & 0 & 7/17 \\ 0 & 0 & 1 & -15/17 \end{array}\right]$$

$$R_1 \rightarrow R_1 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -112/17 \\ 0 & 1 & 0 & 7/17 \\ 0 & 0 & 1 & -15/17 \end{array}\right]$$

This gives,

$$I_1 = -112/17 \text{ A}$$

$$I_2 = \frac{7}{17} \text{ A}$$

and, $I_3 = \frac{-15}{17}$ A.

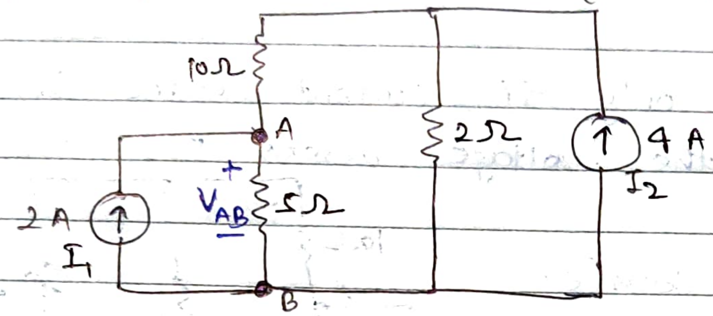
These are loop currents according to figure.

As the $1\,\Omega$ resistor is not specified, only loop currents are found.

Hence $I_1 = \frac{-112}{17}$ A, $I_2 = \frac{7}{17}$ A and $I_3 = \frac{-15}{17}$ A

Two Questions Using Superposition Theorem :-

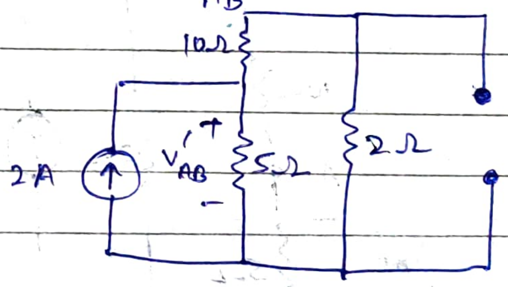
(1·2) Example 10·12 : Apply superposition theorem to network of figure 10.51 to find voltage across AB.



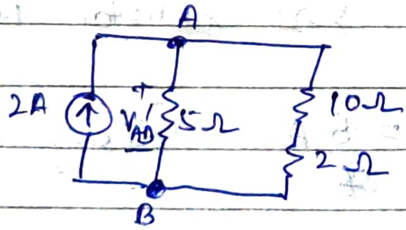
Solution :- The polarity of V_{AB} is shown in fig.

Case - 1 : When only 2A current source is active, let the voltage across AB be V'_{AB}.

Since, only 2A current source is active, 4A current source is open circuited.



The circuit is reduced to



the current through 5Ω resistor i.e. i'_{AB} is

$$i'_{AB} = \frac{2 \times 12}{17} \text{ A}$$ (Using current division)

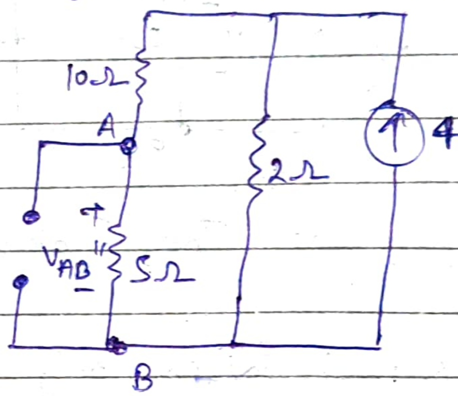
$$i'_{AB} = \frac{24}{17} \text{ A}$$

therefore, V'_{AB} i.e. voltage across 5Ω resistor is

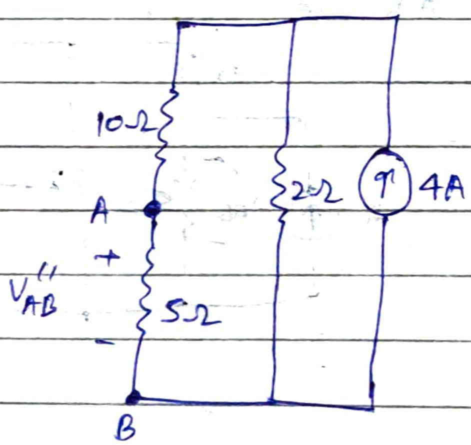
$$V'_{AB} = i'_{AB}(5) = \frac{24 \times 5}{17} = \frac{120}{17} V$$

Case-II:- When only 4A current source is active.
Let V''_{AB} be the voltage across 5Ω resistor.

Since only 4A source is active, open circuit the 2A source.



The circuit is reduced to



the current through 5Ω resistor i.e. i''_{AB} is

$$i''_{AB} = \frac{4 \times 2}{17} = \frac{8}{17} A \quad \text{(Using current division)}$$

therefore voltage V''_{AB} across 5Ω resistor is

$$V''_{AB} = i''_{AB}(5) = \frac{8 \times 5}{17} = \frac{40}{17} V$$

Using superposition theorem, the net voltage across 5Ω resistor V_{AB} is

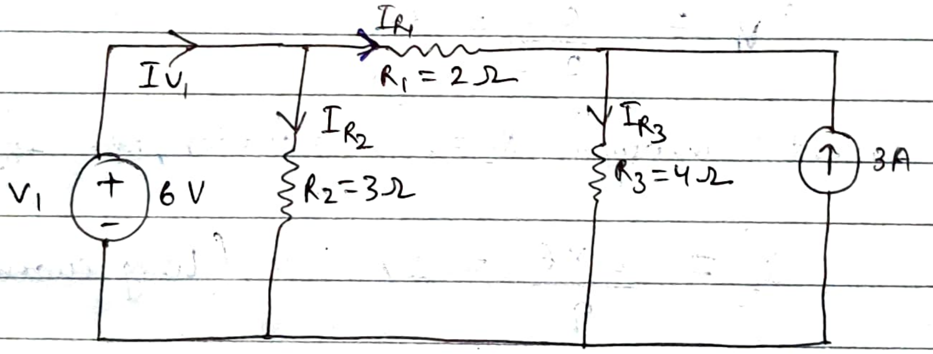
$$V_{AB} = V'_{AB} + V''_{AB}$$

$$V_{AB} = \frac{120}{17} + \frac{40}{17} = \frac{160}{17} V$$

$$V_{AB} = 9.412 V$$

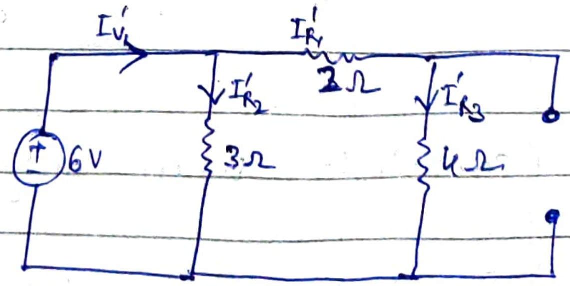
Hence, the voltage across 5Ω resistor is 9.412 V.

(1.2) Example 10.13 . Find the current flowing through every branch of network shown in Fig 10.52,



The current through every element is mentioned in figure.

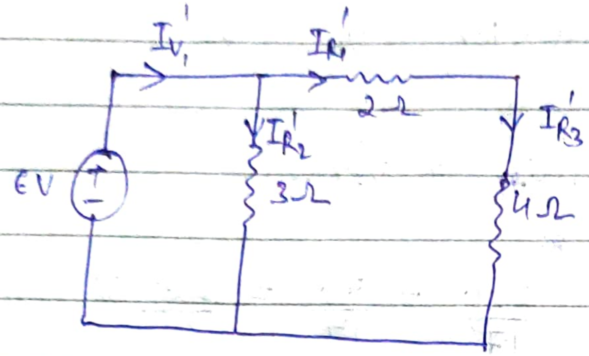
Case 1: When only 6V source is active



As only 6V source is active, the 3A source is ~~spen~~ open - circuited.

The currents through every branch corresponding to figure in question are I'_{V_1}, I'_{R_1}, I'_{R_2} and I'_{R_3}.

The circuit is reduced to



the equivalent resistance for the circuit, R_{eq}

$$R_{eq} = 3 \parallel (2+4) = 2\Omega$$

therefore, the current supplied by 6V source (I'_{V_1})

$$I'_{V_1} = \frac{6V}{2\Omega} = 3A$$

the current through $R_2 = 3\Omega$ resistor, i.e. I'_{R_2}

$$I'_{R_2} = \frac{3 \times 6}{9} = 2A \quad \text{(Using current division)}$$

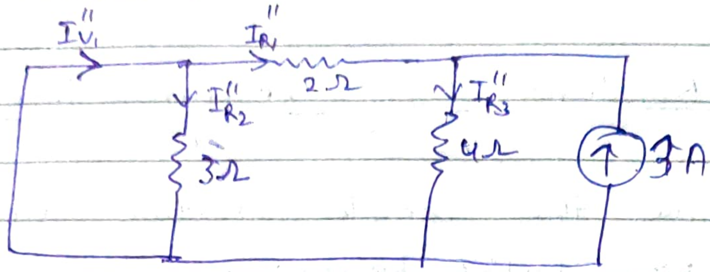
the current through R_1 (I'_{R_1}) and R_3 (I'_{R_3}) is equal (as they are in series),

$$I'_{R_1} = I'_{R_3} = \frac{3 \times 3}{9} = 1A$$

Case II :- when only 3A source is active,

The currents in different branches corresponding to figure in question are I''_{V_1}, I''_{R_1}, I''_{R_2} and I''_{R_3}.

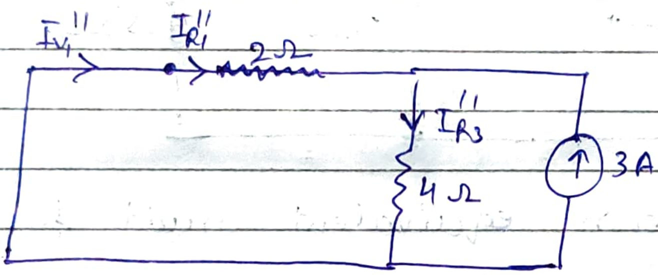
As only 3A source is active, the 6V source is short circuited



As the 3Ω resistance is short circuited

$I''_{R_2} = 0$ A.

The circuit reduces to



the currents I''_{V_1} and I''_{R_1} are equal

$$I''_{V_1} = I''_{R_1} = -\frac{3 \times 4}{6} \quad \text{(Using current division)}$$

$$= -2 \text{ A}$$

the current through 4Ω resistor i.e. I''_{R_3}

$$I''_{R_3} = \frac{3 \times 2}{6} = 1 \text{ A} \quad \text{(Using current division)}$$

Now, using superposition theorem,

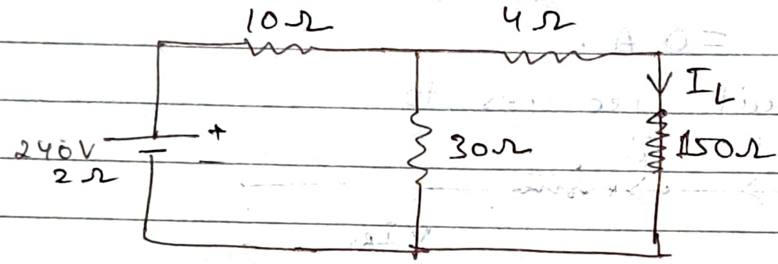
$$I_{V_1} = I'_{V_1} + I''_{V_1} = 3A + (-2A) = 1 A$$
$$I_{R_1} = I'_{R_1} + I''_{R_1} = 1A + (-2A) = -1 A$$

$$I_{R_2} = I'_{R_2} + I''_{R_2} = 2A + 0A = 2A$$

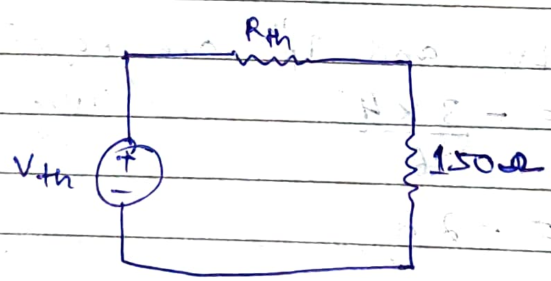
and $I_{R_3} = I'_{R_3} + I''_{R_3} = 1A + 1A = 2A.$

Two Questions Using Thevenin's Theorem

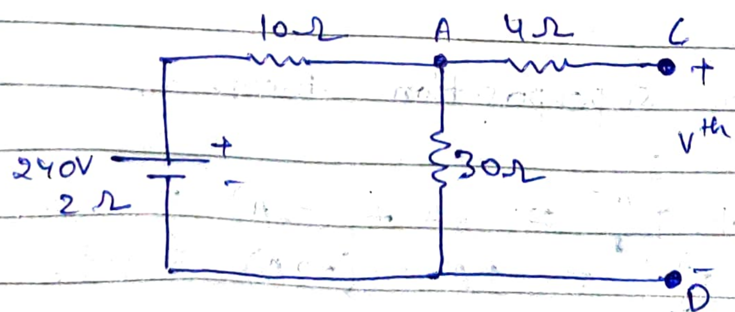
(1.2) 21. Solve the simple electric circuit shown in Fig. 10.137 below for I_L using Thevenin's theorem.



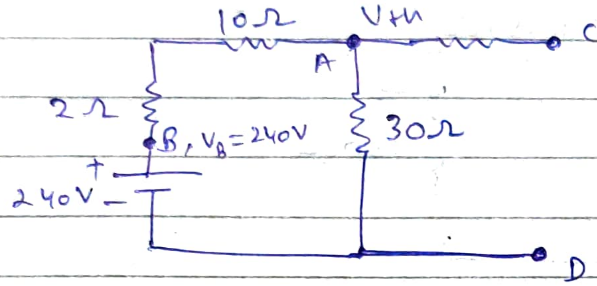
The Thevenin equivalent circuit for the given circuit is



To find V_{th}, we have to open circuit the load first, the voltage across the load is V_{th}.



Since, the 4 Ω resistor is open, 0 current will flow through it. Hence the voltage at A will be V_{th} only (considering D as reference node).



Applying Nodal analysis at node A,

$$\frac{V_{th}-240}{12} + \frac{V_{th}-0}{30} = 0$$

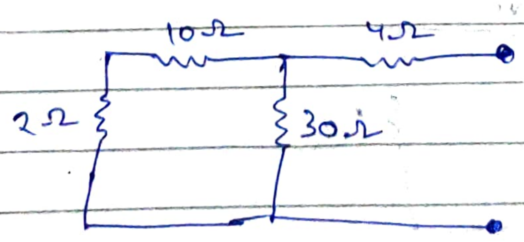
$$\frac{5V_{th}-1200}{60} + \frac{2V_{th}}{60} = 0$$

$$7V_{th} = 1200$$

$$V_{th} = \frac{1200}{7} \text{ V}$$

$$V_{th} = 171.428 \text{ V}$$

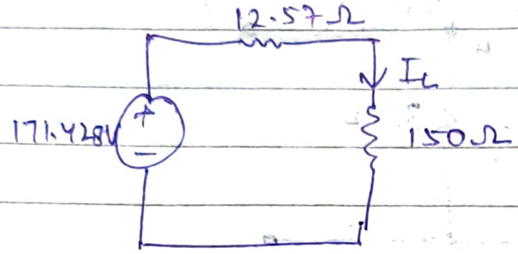
To find R_{th}, open circuit the load resistance and make all ~~sources~~ independent sources inactive. The equivalent resistance is R_{th}.



$$R_{th} = \{(2+10) \parallel 30\} + 4$$

$$R_{th} = \{8.57\} + 4$$
$$= 12.57\ \Omega$$

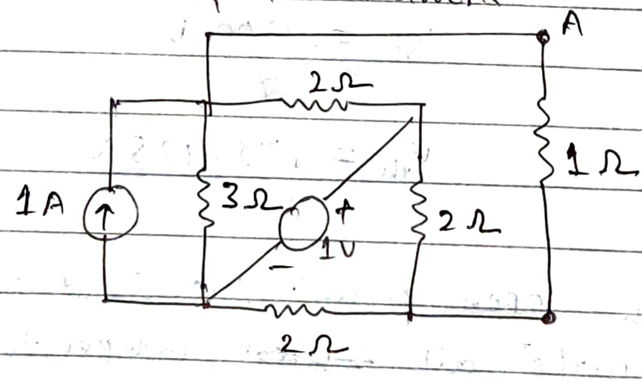
The Thevenin Equivalent Circuit is



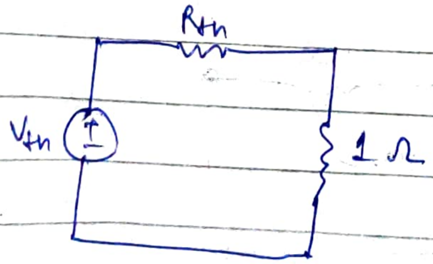
$$I_L = \frac{171.428}{150 + 12.57}$$
$$= 1.0545\ A$$

Hence, the current I_L is 1.0545 A.

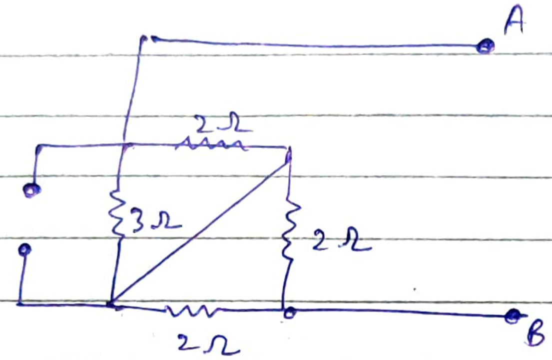
(1.2) Example 10.2 :- Determine the current in 1Ω resistor across AB of the network



The equivalent Thevenin circuit will be

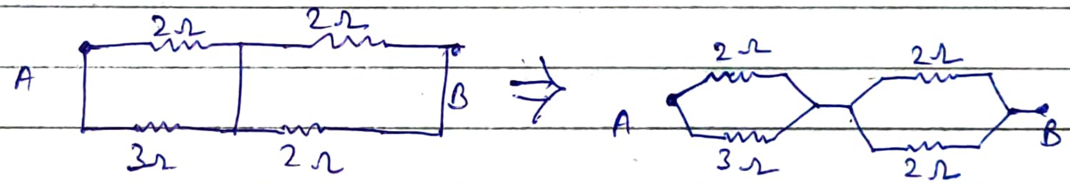


To find R_{th}, open circuit the load resistance and make all the independent sources inactive



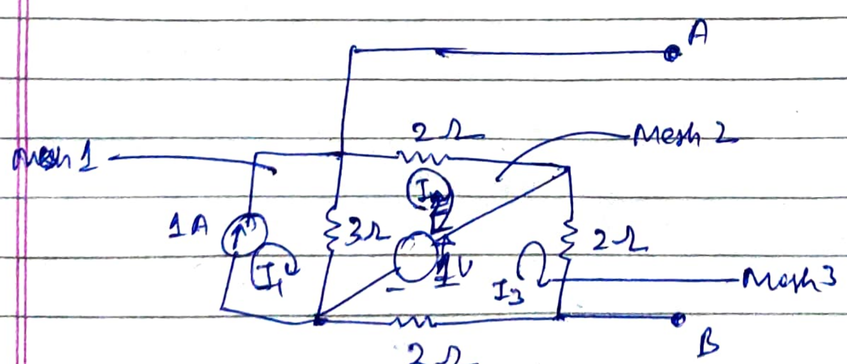
R_{th} will be the equivalent resistance between AB.

The above circuit can be re-drawn as,



$$R_{th} = (2 \| 3) + (2 \| 2)$$
$$= \frac{6}{5} + 1 = 2.2\ \Omega$$

To find V_{th}, open source the load and the voltage across it will be V_{th}.



Assuming currents I_1, I_2 and I_3 in the meshes in clockwise direction as shown in figure.

From figure, it is clear that $I_1 = 1A$.

Applying KVL in mesh 2;

$$-3(I_2 - I_1) - 2I_2 - 1 = 0$$

$$-5I_2 + 3I_1 = 1$$

$$3I_1 - 5I_2 = 1 \quad \text{—(1)}$$

$$I_2 = 2/5 \; A \qquad (\text{as } I_1 = 1A)$$

Applying KVL in mesh 3,

$$+1 - 2I_3 - 2I_3 = 0$$

$$I_3 = 1/4 \; A$$

Now, $V_{th} = V_A - V_B$

$$V_A - 2I_2 - 2I_3 = V_B$$

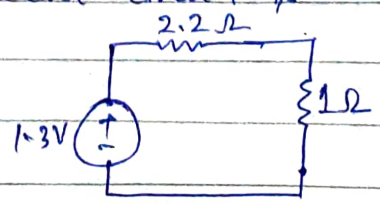
$$V_A - V_B = 2(I_2 + I_3)$$

$$= 2\left(\frac{2}{5} + \frac{1}{4}\right) = 2 \times \frac{13}{20}$$

$$= 1.3V$$

Hence $V_{th} = 1.3 \; V$

Hence, the thevenin equivalent circuit is

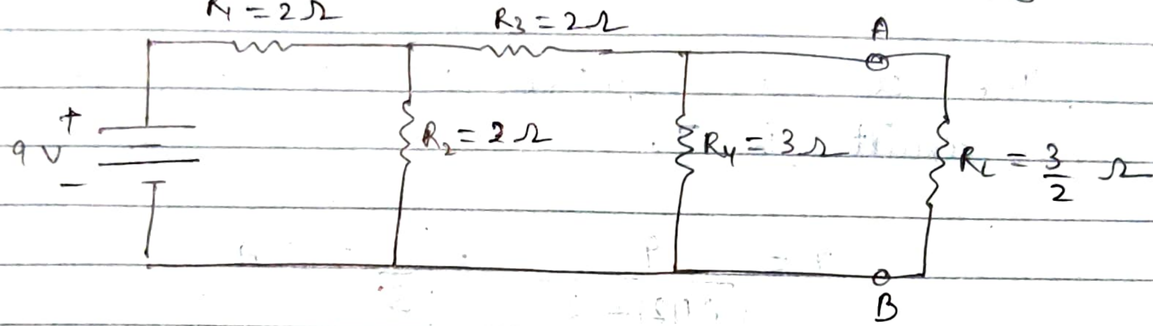


$$i = \frac{1.3}{2.2+1} = \frac{1.3}{3.2} = 0.406 \; A$$

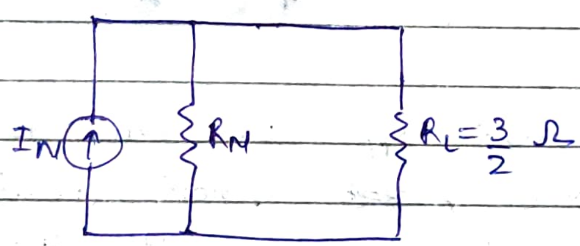
Hence, the current through 1Ω resistor is 0.406A.

Two Questions Using Norton's Theorem

(1.2) Example 10.8 :- Find the current flowing through resistor R_L in the network shown in fig.

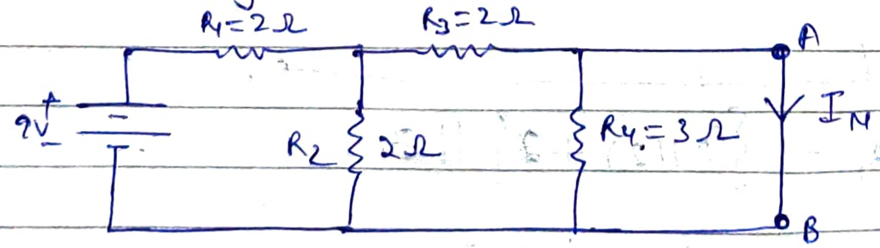


The Norton's Equivalent Circuit for the given network will be

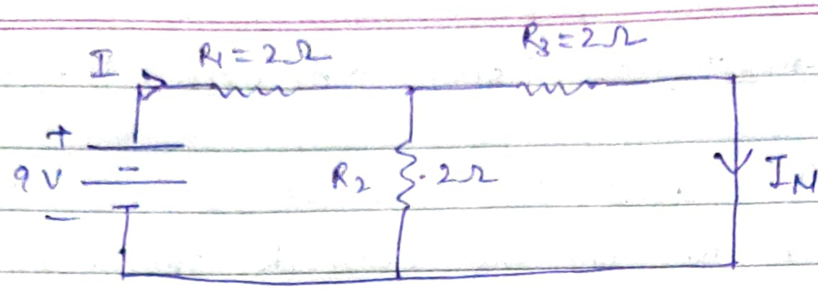


To find I_N, we have to short circuit the load resistance and find the current in that branch. The current is I_N.

Short-circuiting the load resistance



The resistance R_4 is short-circuited and no-current will flow through it. So, we can remove R_4.

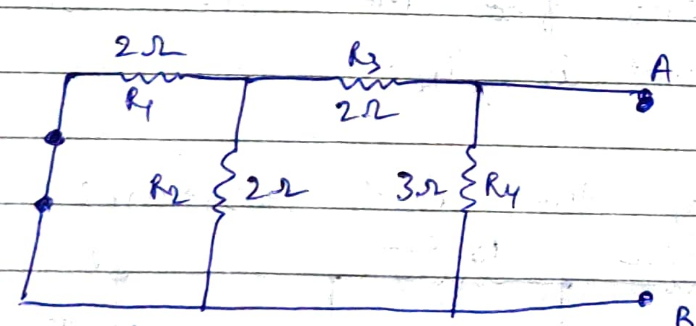


The current supplied by 9 V source to the circuit is I.

$$I = \frac{9}{(2 \| 2) + 2} = \frac{9}{3} = 3\,A$$

$\therefore$ current $I_N = \frac{3 \times 2}{4} = \frac{3}{2}\,A$ (using current division)

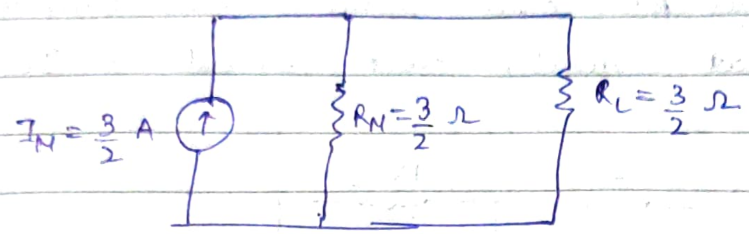
Now, to find R_N, we have to open circuit the load, ~~and~~ short circuit the voltage source and open circuit the current sources. The equivalent resistance between A and B afterwards is R_N.



$$R_N = \{(2 \| 2) + 2\} \| 3$$

$$= \{1 + 2\} \| 3 = \frac{3}{2}\,\Omega$$

So, the Norton's Equivalent circuit is



the current (I_L) through R_L is given by

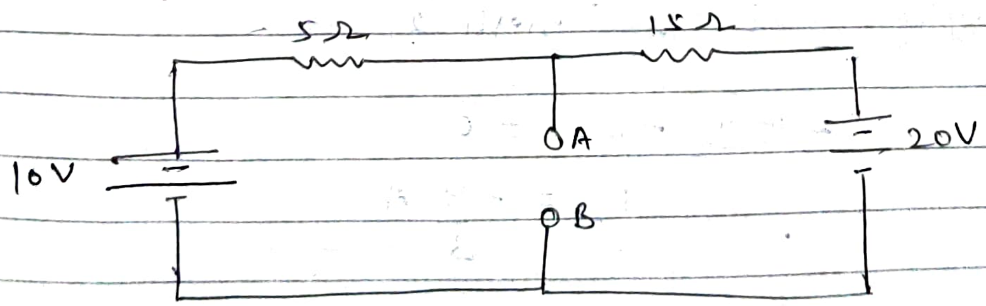
$$I_L = \frac{(I_N)(R_N)}{(R_N + R_L)} \quad \text{(Using current division)}$$

$$I_L = \frac{\frac{3}{2} \times \frac{3}{2}}{\frac{3}{2} + \frac{3}{2}} = \frac{\frac{9}{4}}{3} = \frac{9}{12}$$

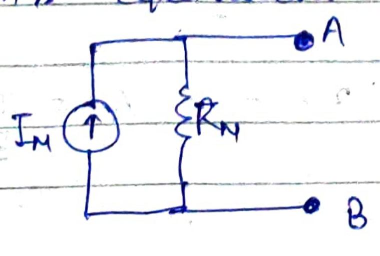
$$I_L = 0.75\ A$$

Hence, the current through R_L is 0.75 A.

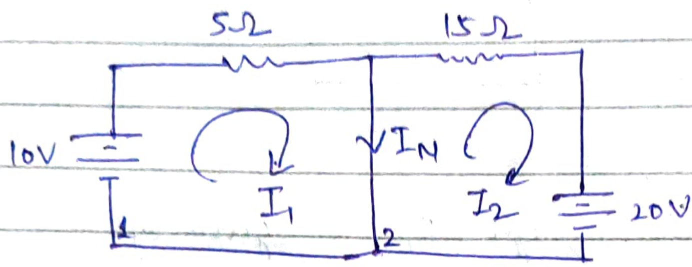
(1.2) Example 10.6 -: Obtain the Norton's equivalent circuit w.r.t the terminals AB for the network shown in figure



The Norton's equivalent for given circuit is



To find I_N, we have to short circuit the load AB and find the current in that branch, that current is I_N only.



Let the current in mesh 1 and mesh 2 be I_1 and I_2 respectively (direction is clockwise.

KVL states that the algebraic sum of all the voltages in a mesh is zero.

Appelying KVL in mesh 1,

$$+10 - 5I_1 = 0$$
$$I_1 = 2\ A$$

Applying KVL in mesh 2,

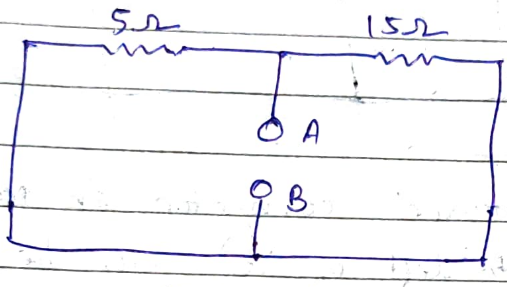
$$-15I_2 - 20 = 0$$
$$I_2 = -\frac{4}{3}\ A$$

From the figure

$$I_N = I_1 - I_2$$

$$= 2\ A - \frac{4}{3}\ A = \frac{10}{3}\ A$$

To find R_N, we should open circuit the load and make all independent sources in active, and calculate net resistance of the circuit. the circuit then will be

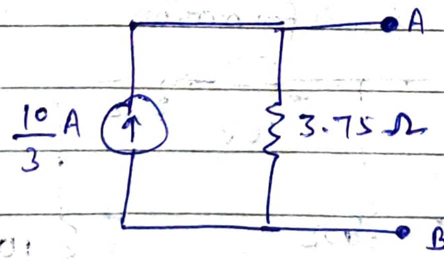


$$R_N = (5 \parallel 15)$$

$$= \frac{15 \times 5}{15 + 5} = \frac{\cancel{75}^{15}}{\cancel{20}_{4}}\ \Omega$$

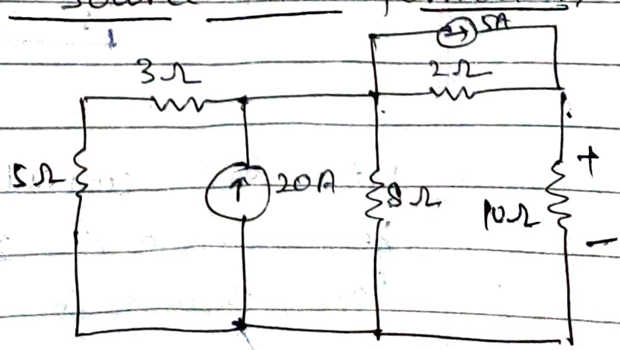
$$R_N = 3.75\ \Omega$$

Hence, the Norton's equivalent circuit for the given circuit is

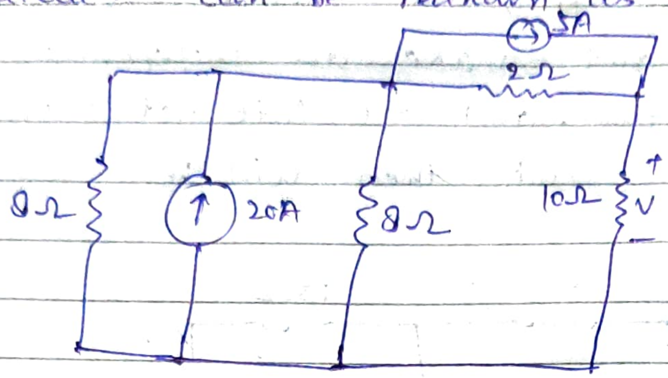


Two Questions Using Source Transformation

1) Ex. 3.13 Determine the voltage v in the circuit, using source transformation technique.



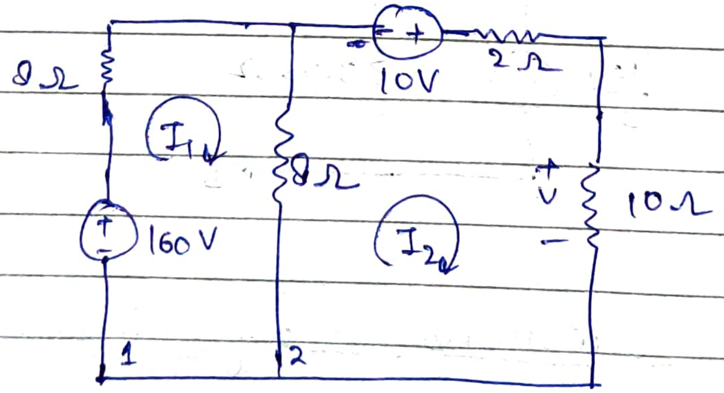
The circuit can be redrawn as



The 8Ω resistance connected in parallel with 20 A source (to left of it) can be converted to a voltage source of voltage $20 \times 8 = 160\,V$ with 8Ω resistance in series with it.

Similarly, the 5A current source in parallel with 2Ω resistance can be transformed to a voltage source of $5 \times 2 = 10\,V$ in series with 2Ω resistance.

The circuit after transformation is shown below.



Let the current in mesh 1 and 2 be I_1 and I_2 in clockwise direction. (as shown in fig.)

According to KVL, the algebraic sum of all the voltages in a loop is zero.

Applying KVL in mesh 1

$$-8I_1 - 8(I_1 - I_2) + 160 = 0$$

$$-16I_1 + 8I_2 = -160$$

$$2I_1 - I_2 = 20 \quad \text{--- (I)}$$

Applying KVL in mesh 2,

$$10 - 2I_2 - 10I_2 - 8(I_2 - I_1) = 0$$

$$8I_1 - 20I_2 = -10$$

$$4I_1 - 10I_2 = -5 \quad \text{--- (II)}$$

Multiplying eq^n (I) by 2

$$4I_1 - 2I_2 = 40 \quad \text{--- (III)}$$

Subtracting (II) from (III),

$$8I_2 = 45$$

$$I_2 = \frac{45}{8} \text{ A}$$

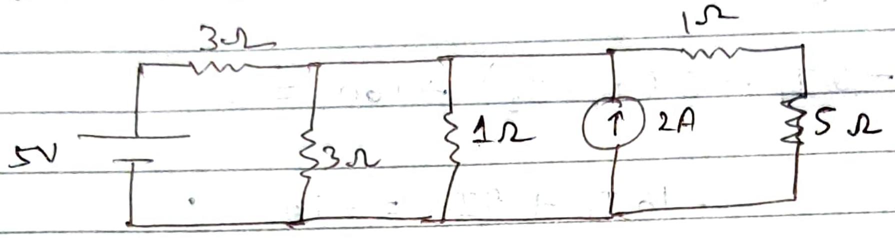
The voltage across $10\,\Omega$ resistor (V) is

$$V = (10)\, I_2$$

$$V = \frac{\cancel{10}^{5} \times 45}{\cancel{8}_{4}} = \frac{225}{4} = 56.25 \text{ V}$$

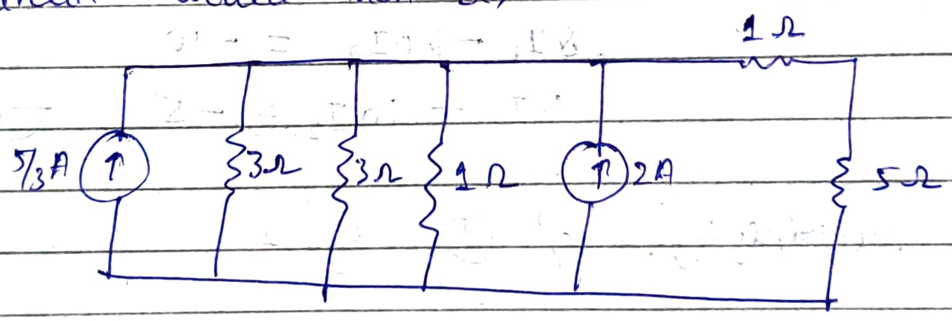
Hence, the required voltage is 56.25 V.

(ii) Example 3.14 Find the current flowing through 5 Ω resistor.



The 3 Ω resistor and 5 V source in series can be transformed into a current source of 5/3 A with 3 Ω resistor in parallel.

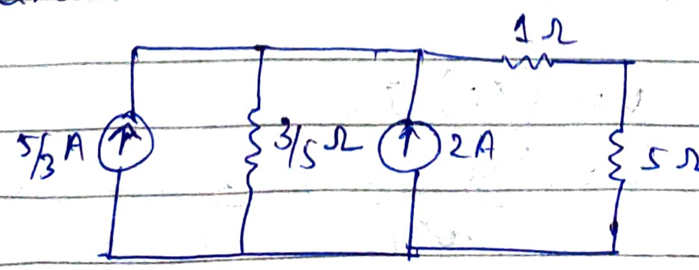
The circuit would then be,



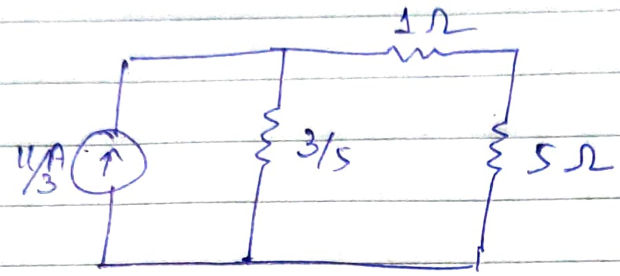
As the resistors 3 Ω, 3 Ω and 1 Ω are in parallel, they can be replaced by r_{eq} resistance.

$$r_{eq} = (3 \| 3 \| 1) = \frac{3}{5}\ \Omega$$

The circuit now reduces to



The two current sources are supplying current in same direction, hence they can be added.



the current i through the 5Ω resistor is

$$i = \frac{\frac{11}{3} \times \left(\frac{3}{5}\right)}{\frac{3}{5} + (1+5)}$$ (using current division)

$$i = \frac{11/5}{33/5} = \frac{1}{3} \text{ A}$$

Hence, the current through 5Ω resistor is $\frac{1}{3}$ A.