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MA 201 : Probability and Statistics

Remote Midsem Exam

1. Given $f_X(x) = 3x^2, \; 0 \le x \le 1$

$\therefore$ the cdf can be computed as :

$$F_X(x) = \int f_X(x)\,dx$$

$$\therefore F_X(x) = \int_0^x 3t^2\,dt$$

$$= t^3 \Big|_0^x$$

$$F_X(x) = x^3 \quad \text{—} (1)$$

(a) Now, $P(X > a) = \frac{26}{27}$

and, $P(X > a) = 1 - F(a)$

$$\therefore 1 - F(a) = \frac{26}{27}$$

$$\therefore F(a) = \frac{1}{27}$$

$$a^3 = \frac{1}{27} \quad \{\text{from } (1)\}$$

$$\boxed{\therefore a = \frac{1}{3}} \text{ Ans}$$

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(b) $P(X > b) = 1 - F(b)$

and, $P(X \leq b) = F(b)$

We need to find b such that

$$1 - F(b) = 7F(b)$$

$$F(b) = \frac{1}{8}$$

$$\therefore b^3 = \frac{1}{8} \quad (\text{from } ①)$$

$$\boxed{\therefore b = \frac{1}{2}}$$ Ans

$\therefore$ 'a' such that $P\{X > a\} = \frac{26}{27}$ is $a = \frac{1}{3}$.

and 'b' such that $P\{X > b\} = 7P\{X \leq b\}$ is $b = \frac{1}{2}$.

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2. P(man aged 60 will live to be 70) $= p = \frac{2}{3}$

Now, probability that out of 10 men aged 60, at least 8 will live to be 70 is equal to

Probability that 8 of 10 such men will live to 70
+ Probability that 9 " 10 " " " " " 70
+ " " 10 " " " " " " " "

To find Probability of X-men living to 70 out of 10 (aged 60), we can choose any X men from these 10 in ${}^{10}C_x$ ways and multiply it with p^x where p is probability that man will live to be 70. And then multiply it with $(1-p)^{10-x}$.

∴ ~~P(at lea~~ i.e. $P(X=x) = {}^{10}C_x \, p^x (1-p)^{10-x}$

∴ $P(X=8) = {}^{10}C_8 \left(\frac{2}{3}\right)^8 \left(\frac{1}{3}\right)^2$

$= \frac{45 \times 256}{3^{10}}$

$P(X=9) = {}^{10}C_9 \left(\frac{2}{3}\right)^9 \left(\frac{1}{3}\right)$

$= \frac{10 \times 512}{3^{10}}$

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$$P(X=10) = {}^{10}C_{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{0}$$

$$= \frac{1 \times 1024}{3^{10}}$$

∴ probability that ~~the~~ at least 8 men out of 10 men aged 60 will live to be 70 is

$$= \frac{11520 + 5120 + 1024}{3^{10}}$$

$$= \frac{17664}{59049}$$

$$\boxed{= 0.2991}$$ Ans

Therefore, probability that atleast 8 out of 10 men aged 60 will live to be 70 is 0.2991

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3(a) Given,

$$\mu = 60$$

and $\sigma = 5$

To find what percentage of students scored more than 60. We can use the Standard Normal Distribution and convert the given distribution(X) to Standard Normal Distribution (Z) as

$$Z = \frac{X-\mu}{\sigma}$$

Now, probability that student scored more than 60 i.e.

$$P(X > 60) = P\left(\frac{X-\mu}{\sigma} > \frac{60-60}{5}\right)$$

$$= P(Z > 0)$$

$$\therefore P(X > 60) = 1 - \Phi(0)$$

$$P(X > 60) = 1 - 0.5$$

$$= 0.5$$

$\left\{ \Phi(0) = \text{normcdf}(0,0,1) = 0.5 \text{ (given)} \right\}$

$\therefore$ 50% of Students scored more than 60.

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3(b)

mean resistance of resistors (μ) = $100\ \Omega$

standard deviation of resistance (σ) = $2\ \Omega$

We need to find $P(98 < X < 102)$
where X is given distribution.

We can convert X to standard normal distribution (Z) as

$$Z = \frac{X-\mu}{\sigma}$$

$$\therefore P(98 < X < 102) = P\left(\frac{98-100}{2} < \frac{X-\mu}{\sigma} < \frac{102-100}{2}\right)$$

$$= P(-1 < Z < 1)$$

$$\therefore P(98 < X < 102) = \Phi(1) - \Phi(-1)$$

Using normcdf(1,0,1) and
normcdf(-1,0,1) in Octave
$\phi(1)$ and $\phi(-1)$ is calculated.

$$\therefore P(98 < X < 102) = 0.8413 - 0.1587$$
$$= 0.6826$$

$\therefore$ 68.26 % of resistors will have resistance between $98\ \Omega$ and $102\ \Omega$.