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MA101: Linear Algebra and Matrices

1. Find an LU factorisation of the matrices, if possible.

a) $A = \begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$

The upper triangular matrix is row echelon form of A.

$$A = \begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ $\quad (l_{21} = 2)$

$$\begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ -1 & 7 & 0 \end{bmatrix}$$

$R_3 \to R_3 - \left(-\frac{1}{3} R_1\right)$ $\quad (l_{31} = -1/3)$

$$\begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 5 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$ $\quad (l_{32} = 1)$

$$U = \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & 5 \end{bmatrix}$$

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To find L, the multiplier l_{21}, l_{31} and l_{32} will go at respective position as L_{ij} and principal diagonal entries will be 1.

$$\therefore \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 1 & 1 \end{bmatrix}$$

Hence,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & 5 \end{bmatrix}$$

b) $$A = \begin{bmatrix} 1 & 3 & -5 & -3 \\ -1 & -5 & 8 & 4 \\ 4 & 2 & -5 & -7 \\ -2 & -4 & 7 & 5 \end{bmatrix}$$

U is row echelon form of A,

$$A = \begin{bmatrix} 1 & 3 & -5 & -3 \\ -1 & -5 & 8 & 4 \\ 4 & 2 & -5 & -7 \\ -2 & -4 & 7 & 5 \end{bmatrix}$$

$R_2 \rightarrow R_2 - (-1)R_1$ $\quad (l_{21} = -1)$
$R_3 \rightarrow R_3 - 4R_1$ $\quad (l_{31} = 4)$
$R_4 \rightarrow R_4 - (-2)R_1$ $\quad (l_{41} = -2)$

$$A = \begin{bmatrix} 1 & 3 & -5 & -3 \\ 0 & -2 & 3 & 1 \\ 0 & -10 & 15 & 5 \\ 0 & 2 & -3 & -1 \end{bmatrix}$$

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$R_3 \rightarrow R_3 - 5R_2$ $(l_{32} = +5)$
$R_4 \rightarrow R_4 - (-R_3)$ $(l_{42} = -1)$

$$A = \begin{bmatrix} 1 & 3 & -5 & -3 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

No pivot columns left. Since we have not used ~~use~~ a multiplier to make A_{43} zero, hence $l_{43} = 0$.
Hence,

$$U = \begin{bmatrix} 1 & 3 & -5 & -3 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

l_{ij} goes directly at i^{th} column and j^{th} row of L and all diagonal entries of L are 1.

$$\therefore \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 4 & 5 & 1 & 0 \\ -2 & -1 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 4 & 5 & 1 & 0 \\ -2 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -5 & -3 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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2. Solve the equation $Ax = b$ by using the LU factorisation given for A.

$$A = \begin{bmatrix} 4 & 3 & -5 \\ -4 & -5 & 7 \\ 8 & 6 & -8 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & -5 \\ 0 & -2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$Ax = b$$
$$LUx = b \qquad (\because A = LU)$$
$$Ly = b \qquad (\because Ux = y)$$

where y is $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

Solving $Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$$

This gives us,

$$y_1 = 2 \quad \text{—(I)}$$
$$-y_1 + y_2 = -4 \quad \text{—(II)}$$
$$2y_1 + y_3 = 6 \quad \text{—(III)}$$

Substituting $y_1 = 2$ in (II), gives

$$y_2 = -2$$

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Substituting $y_1 = 2$ in (III) gives,

$$y_3 = 2$$

Hence, $y = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$

Now, we have to solve $Ux = y$

$$\begin{bmatrix} 4 & 3 & -5 \\ 0 & -2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$$

this gives us three equations

$$4x_1 + 3x_2 - 5x_3 = 2 \quad \text{— (IV)}$$
$$-2x_2 + 2x_3 = -2 \quad \text{— (V)}$$
$$2x_3 = 2 \quad \text{— (VI)}$$

Substituting $x_3 = 1$ in (V) gives,

$$-2x_2 = -4$$
$$x_2 = 2$$

Substituting $x_2 = 2$ and $x_3 = 1$ in (I),

$$4x_1 + 3(2) - 5(1) = 2$$
$$4x_1 = 1$$
$$x_1 = \frac{1}{4}$$

Hence, $x = \begin{bmatrix} 1/4 \\ 2 \\ 1 \end{bmatrix}$

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3. Use the definition of Ax to write the given matrix equation as vector equation.

$$\begin{bmatrix} 5 & 1 & -8 & 4 \\ -2 & -7 & 3 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 1 & -8 & 4 \\ -2 & -7 & 3 & -5 \end{bmatrix}, \quad x = \begin{bmatrix} 5 \\ -1 \\ 3 \\ -2 \end{bmatrix}$$

vector equation for given matrix equation is

$$v_1 x_1 + v_2 x_2 + v_3 x_3 + v_4 x_4 = b$$

where v_i is i_{th} column of matrix A and x_i is i_{th} element of x.

$$\therefore \; 5 \begin{bmatrix} 5 \\ -2 \end{bmatrix} + (-1) \begin{bmatrix} 1 \\ -7 \end{bmatrix} + 3 \begin{bmatrix} -8 \\ 3 \end{bmatrix} + (-2) \begin{bmatrix} 4 \\ -5 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}$$

4. Determine if $Ax = b$ has a solution by checking whether b is a linear combination of a_1, a_2 and a_3, where ith column of A is equal to a_i.

$$a_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \quad a_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \quad a_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \quad b = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$$

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Since, i_{th} column of A is a_i:

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 5 & 0 \\ 2 & 5 & 8 \end{bmatrix}$$

and let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and b is equal to $\begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$.

Augmented matrix for the system $Ax = b$ is

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{array}\right]$$

$R_2 \to R_2 + 2R_1$
$R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 5 & 4 & 3 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

Since the last row gives a equation $0x_1 + 0x_2 + 0x_3 = 2$, which is never possible, hence, $Ax = b$ has no solution.

i.e. b is not a linear combination of vectors a_1, a_2 and a_3.

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5. Give a geometric description of span$\{v_1, v_2\}$ for the vectors

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

The span$\{v_1, v_2\}$ is given by

$c_1 v_1 + c_2 v_2$ (where c_1 and c_2 are scalars)

$$\text{span}\{v_1, v_2\} = c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 \\ c_2 \\ 0 \end{bmatrix}$$

Hence, the span$\{v_1, v_2\}$ is a plane in $\mathbb{R}^3$. (or we can say it is xy plane in xyz space).

6. A steam plant burns two types of coals: anthracite (A) and bituminous (B). For each ton of A burned, the plant produces 27.6 million Btu of heat, 3100 grams of sulfur dioxide and 250 g of particulate matter (solid-particulate pollutants). For each ton of B burned, the plant produces 30.2 million Btu, 6400g of sulfur dioxide and 360 g of particulate matter. Over a certain time period, the steam plant produced 162 million of Btu of heat, 23610 g of sulphur dioxide and 1623 g of particulate matter.

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Determine how many tons of each type of coal the steam plant must have burned.

Let a column matrix with entries as given below,

$$\begin{bmatrix} \text{million Btu of heat} \\ \text{grams of sulfur dioxide} \\ \text{grams of particulate pollutant} \end{bmatrix}$$

Let A be the matrix for anthracite,

$$\therefore \quad A = \begin{bmatrix} 27.6 \\ 3100 \\ 250 \end{bmatrix}$$

Let B be the matrix for bituminous,

$$\therefore \quad B = \begin{bmatrix} 30.2 \\ 6400 \\ 360 \end{bmatrix}$$

Let C denote the matrix for production over ~~the~~ ~~coal~~ the given period.

$$\therefore \quad C = \begin{bmatrix} 162 \\ 23610 \\ 16.23 \end{bmatrix}$$

Let x and y be the tonnes of anthracite and bituminous burned to produce 162 million Btu of heat, 23610 g of sulphur dioxide and 1623 g

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of particulate pollutants:

$\therefore$ vector equation to find x and y is

$$xA + yB = C$$

$$x\begin{bmatrix} 27.6 \\ 3100 \\ 250 \end{bmatrix} + y\begin{bmatrix} 30.2 \\ 6400 \\ 360 \end{bmatrix} = \begin{bmatrix} 162 \\ 23610 \\ 1623 \end{bmatrix}$$

The equations are

$$27.6x + 30.2y = 162 \quad \text{—(I)}$$
$$3100x + 6400y = 23610 \quad \text{—(II)}$$
$$250x + 360y = 1623 \quad \text{—(III)}$$

augmented matrix for eq^n (II) and (III) is

$$\left[\begin{array}{cc|c} 3100 & 6400 & 23610 \\ 250 & 360 & 1623 \end{array}\right]$$

$$R_2 \to R_2 - \frac{250}{3100} R_1$$

$$\left[\begin{array}{cc|c} 3100 & 6400 & 23610 \\ 0 & -156.12 & -281.03 \end{array}\right]$$

equations are reduced to

$$3100x + 6400y = 23610$$
$$-156.12y = -281.03$$

$$y = \frac{281.03}{156.12} = 1.8$$

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Substituting $y = 1.8$ in (II),

$$3100x + 6400(1.8) = 23610$$
$$3100x + 11520 = 23610$$
$$3100x = 12090$$
$$x = \frac{12090}{3100}$$
$$x = 3.9$$

eqⁿ (II) and (III) gives $x = 3.9$ and $y = 1.8$

Putting $x = 3.9$ and $y = 1.8$ in (I)

$$(27.6)(3.9) + (30.2)(1.8) = 162$$
$$107.64 + 54.36 = 162$$
$$RHS = LHS$$

Hence $x = 3.9$ and $y = 1.8$ satisfies all three equations.

Therefore, the steam plant must ~~burn~~ have burned 3.9 ton of anthracite and 1.8 ton of bituminous.

7. Determine if the columns of the matrices span R^4.

$$A = \begin{bmatrix} 7 & 2 & -5 & 8 \\ -5 & -3 & 4 & -9 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$$

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The columns of given matrix will span R^4 if the REF of this matrix has no pivot entries as 0.

$$\begin{bmatrix} 7 & 2 & -5 & 8 \\ -5 & -3 & 4 & -9 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$$

$R_2 \to R_2 + \frac{5}{7} R_1$

$R_3 \to R_3 - \frac{6}{7} R_1$

$$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 58/7 & 16/7 & 1/7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$$

$R_4 \to R_4 + R_1$

$$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 58/7 & 16/7 & 1/7 \\ 0 & 11 & -3 & 23 \end{bmatrix}$$

$R_4 \to R_4 + 7 R_2$

$$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 58/7 & 16/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$R_3 \rightarrow R_3 + \frac{58}{11} R_2$$

$$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 0 & 350/77 & -\frac{1323}{77} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since, the last pivot entry is zero, the columns of given matrix will not span R^4.

b) $\begin{bmatrix} 5 & -7 & -4 & 9 \\ 6 & -8 & -7 & 5 \\ 4 & -4 & -9 & -9 \\ -9 & 11 & 16 & 7 \end{bmatrix}$

columns of matrix will span R^4 if there are no 0 in pivot entries in REF of given matrix.

$$\begin{bmatrix} 5 & -7 & -4 & 9 \\ 6 & -8 & -7 & 5 \\ 4 & -4 & -9 & -9 \\ -9 & 11 & 16 & 7 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - \frac{6}{5} R_1$$

$$R_3 \rightarrow R_3 - \frac{4}{5} R_1$$

$$R_4 \rightarrow R_4 + \frac{9}{5} R_1$$

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$$\begin{bmatrix} 5 & -7 & -4 & 9 \\ 0 & 2/5 & -11/5 & -29/5 \\ 0 & 8/5 & -29/5 & -81/5 \\ 0 & -8/5 & 44/5 & 116/5 \end{bmatrix}$$

$R_3 \to R_3 - 4R_2$
$R_4 \to R_4 + 4R_2$

$$\begin{bmatrix} 5 & -7 & -4 & 9 \\ 0 & 2/5 & -11/5 & -29/5 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

since the pivot entry in 4^{th} row is zero, hence columns of given matrix cannot span R^4.

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8. Describe all solutions of $Ax = 0$ in parametric vector form, where A is

$$\begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Vector x, $[x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T$ be the solution.
Augmented matrix for $Ax = 0$ is

$$\left[\begin{array}{cccccc|c} \boxed{1} & -4 & -2 & 0 & 3 & -5 & 0 \\ 0 & 0 & \boxed{1} & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & \boxed{1} & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

(pivot entries are enclosed in $\boxed{\ }$)
Since, ~~pivot~~ variables having no pivot entry are free variable.
∴ x_2, x_4 and x_6 are free variables
and x_1, x_3 and x_5 are dependent variables.

Let $x_2 = a$, $x_4 = b$ and $x_6 = c$

Row 3 of matrix gives the equation

$$x_5 - 4x_6 = 0$$
$$x_5 = 4x_6$$
$$x_5 = 4c \quad \text{—} (1)$$

Row 2 of matrix gives the equation.

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$$x_3 - x_6 = 0$$
$$x_3 = x_6$$
$$x_3 = c \longrightarrow (ii)$$

Row 1 of matrix gives the equation,

$$x_1 - 4x_2 - 2x_3 + 3x_5 - 5x_6 = 0$$

$$x_1 - 4a - 2c + 3(4c) - 5(c) = 0$$
$$x_1 = 4a - 5c \longrightarrow (iii)$$

$$\therefore x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 4a-5c \\ a \\ c \\ b \\ 4c \\ c \end{bmatrix}$$

$$x = a\begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c\begin{bmatrix} -5 \\ 0 \\ 1 \\ 0 \\ 4 \\ 1 \end{bmatrix}$$

Hence, the solutions of x in parametric form are described above.

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9. Construct a 3x3 nonzero matrix such that the following vector is a solution to $AX = 0$

$$v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Let A be a 3x3 matrix such that

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0.$$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(given)}$$

writing the above matrix equation as vector equation

$$1 \begin{bmatrix} a \\ d \\ g \end{bmatrix} + 1 \begin{bmatrix} b \\ e \\ h \end{bmatrix} + 1 \begin{bmatrix} c \\ f \\ i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a + b + c \\ d + e + f \\ g + h + i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence, any 3x3 matrix whose sum of elements of every row is zero will give

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$A[1 \quad 1 \quad 1]^T = 0$

$\therefore$ eg $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -4 & 4 \\ -2 & 3 & -1 \end{bmatrix}$

10. Given $A = \begin{bmatrix} 4 & -6 \\ -8 & 12 \\ 6 & -9 \end{bmatrix}$, find one non-trivial solution of $Ax = 0$ by inspection.

Let $x = [x_1 \quad x_2]^T$ be the non-trivial solution of $Ax = 0$.

Writing vector equation $Ax = 0$

$$x_1 \begin{bmatrix} 4 \\ -8 \\ 6 \end{bmatrix} + x_2 \begin{bmatrix} -6 \\ 12 \\ -9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By observation,

~~$3x_1 = 2x_2$~~

for $x_1 = 3$ and $x_2 = 2$, the vector equation is satisfied.

$$3 \begin{bmatrix} 4 \\ -8 \\ 6 \end{bmatrix} + 2 \begin{bmatrix} -6 \\ 12 \\ -9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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11. Could a set of three vectors in R^4 span all of R^4? Explain. What about n vectors in R^m if $n < m$.

The ~~three~~ three vectors in R^4 will not span all of R^4 because the row echelon form of matrix formed by the vector will have a zero in pivot entry in row echelon form.
Since in a $m \times m$ matrix, the column vectors span R^m only when there are m non-zero pivot entries.
When we will make a matrix using three vectors in R^4, we will always have at least one pivot entry equals 0. Hence, three vectors in R^4 will not span all of R^4.

Similarly, n vectors in R^m will not span all of R^m if $m > n$.