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PH100: Tutorial #02

1. A 5kg mass moves under the influence of a force $F = (4t^2\hat{i} - 3t\hat{j})$ N, where t is time in seconds. It starts at rest from the origin at $t=0$. Find (a) its velocity (b) its position and (c) $\vec{r} \times \vec{v}$ for any later time.

The force is given by

$$F = 4t^2\hat{i} - 3t\hat{j}$$

$\therefore$ acceleration of the object,

$$a = \frac{F}{m} = \frac{4t^2}{5}\hat{i} - \frac{3t}{5}\hat{j}$$

a) the particle starts from rest at $t=0$, i.e. velocity at $t=0$ is 0 m/s.

$$a = \frac{d\vec{v}}{dt} = \frac{4t^2}{5}\hat{i} - \frac{3t}{5}\hat{j}$$

$$d\vec{v} = \left(\frac{4t^2}{5}\hat{i} - \frac{3t}{5}\hat{j}\right) dt$$

Integrating both sides

$$\int_0^V d\vec{v} = \int_0^t \left(\frac{4t^2}{5}\hat{i} - \frac{3t}{5}\hat{j}\right) dt$$

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$$v\Big|_0^v = \left(\frac{4t^3}{15}\hat{i} - \frac{3t^2}{10}\hat{j}\right)\Bigg|_0^t$$

$$\therefore \quad \vec{v} = \left(\frac{4t^3}{15}\hat{i} - \frac{3t^2}{10}\hat{j}\right) m/s$$

(b) at $t=0$, the particle is at origin,

$$v = \frac{d\vec{r}}{dt} = \frac{4t^3}{15}\hat{i} - \frac{3t^2}{10}\hat{j}$$

$$d\vec{r} = \left(\frac{4t^3}{15}\hat{i} - \frac{3t^2}{10}\hat{j}\right) dt$$

Integrating both sides

$$\int_0^r d\vec{r} = \int_0^t \left(\frac{4t^3}{15}\hat{i} - \frac{3t^2}{10}\hat{j}\right) dt$$

$$\vec{r} = \left(\frac{t^4}{15}\hat{i} - \frac{t^3}{10}\hat{j}\right) m$$

(c) $$\vec{r} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t^4/15 & -t^3/10 & 0 \\ 4t^3/15 & -3t^2/10 & 0 \end{vmatrix}$$

$$\vec{r} \times \vec{v} = \left\{\left(\frac{t^4}{15}\right)\left(-\frac{3t^2}{10}\right) - \left(-\frac{t^3}{10}\right)\left(\frac{4t^3}{15}\right)\right\}\hat{k}$$

$$\vec{r} \times \vec{v} = \left(\frac{-3t^6}{150} + \frac{4t^6}{150}\right)\hat{k}$$

$$\vec{r} \times \vec{v} = \frac{t^6}{150}\hat{k}$$

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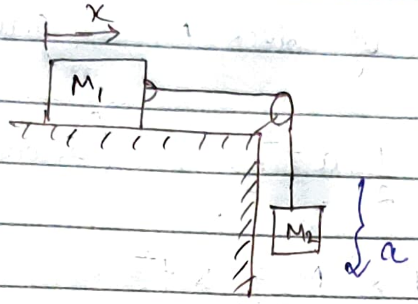
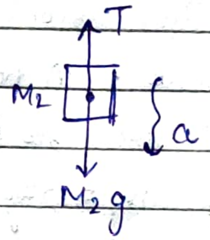
2. Two blocks shown in sketch are connected by a string of negligible mass. If the system is released from rest, find how far block M_1 slides in time t. Neglect friction



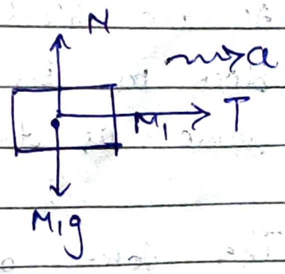
Let us assume the acceleration of block M_2 to be 'a' in downward direction.
The forces acting on block M_2 are its weight and tension due to string.



According to Newton's Second Law,

$$M_2 g - T = M_2 a \quad \text{—} (I)$$

Since, M_1 and M_2 are connected by a string, motion of M_1 is constrained and it will move in forward direction with acceleration 'a'.
Force on block M_1 are its weight, tension due to string and normal due to surface



According to Newton's Second Law,

$$T = M_1 a \quad \text{—} (II)$$

Adding (I) & (II),

$$M_2 g = (M_1 + M_2)\, a$$

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$$a = \frac{M_2 g}{(M_1 + M_2)}$$

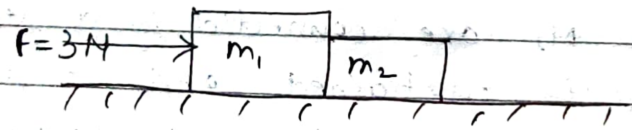
Since, the blocks start from rest, the displacement x of block M_1 is given by

$$x = \frac{1}{2} a t^2 \quad \text{(Second equation of Kinematics)}$$

$$x = \frac{1}{2} \times \frac{M_2 g}{M_1 + M_2} \times t^2$$

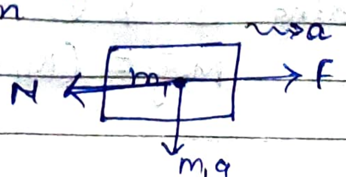
$\therefore$ the block M_1 slides by $\frac{M_2 g t^2}{2(M_1 + M_2)}$ in time t.

3. Two blocks are in contact on a horizontal table. A horizontal force is applied to one of the blocks as shown in the drawing. If $m_1 = 2kg$, $m_2 = 1kg$ and $F = 3N$, find the force of contact between the blocks.



Let us assume the acceleration of m_1 to be 'a' in forward direction.
As the block m_2 is constrained by motion of block m_1 its acceleration will also be same as that of m_1.

The forces on block m_1 are shown in its free body diagram



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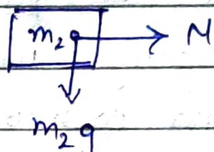
where N is the contact force between the blocks,

By Newton's Second law, we get,

$$F - N = m_1 a \quad \text{—(I)}$$

The forces on block m_2 are shown in its free body diagram:

By Newton's Second law, we get,



$$N = m_2 a \quad \text{—(II)}$$

Multiplying eq (I) by m_2 and eqn (II) by m_1, we get

$$m_2 F - m_2 N = m_1 m_2 a \quad \text{—(III)}$$
$$m_1 N = m_1 m_2 a \quad \text{—(IV)}$$

Subtracting (III) from (IV)

$$(m_1 + m_2) N - m_2 F = 0$$
$$N = \frac{m_2 F}{m_1 + m_2}$$

Putting values $m_1 = 2\,kg$, $m_2 = 1\,kg$ and $F = 3\,N$

$$N = \frac{1 \times 3}{3} = 1\,N$$

Hence, the contact force between the blocks is 1 N.

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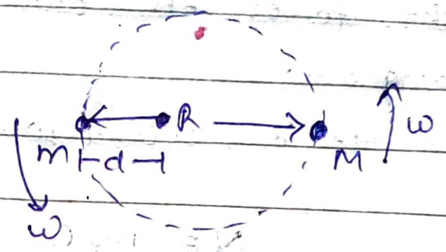
4. Two particles of mass m and M undergo uniform circular motion about each other at a seperation R under the influence of an attractive force F. The angular velocity is ω radians per second. Show that

$$R = \frac{F}{\omega^2}\left(\frac{1}{m} + \frac{1}{M}\right)$$

Since, the particles are undergoing uniform motion around each other, they will actually be revolving around their centre of mass.



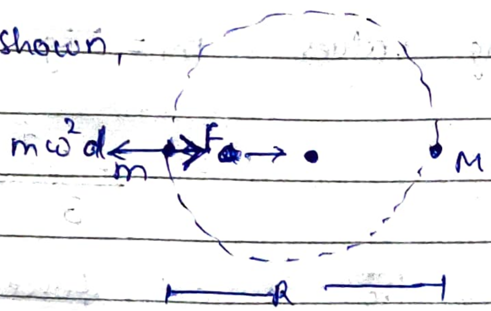
Let the distance of centre of mass from mass m be d.

$$\therefore \quad d = \frac{m(0) + MR}{(m+M)}$$

i.e. $d = \dfrac{MR}{m+M}$

Now, to perform uniform circular, forces along the radial direction (from centre of mass of system to object) should be zero.

Forces on m are shown,



$$F = m\omega^2 d$$

$$F = m\omega^2\left(\frac{MR}{m+M}\right)$$

$$\Rightarrow \quad R = \frac{F}{\omega^2}\frac{(m+M)}{mM}$$

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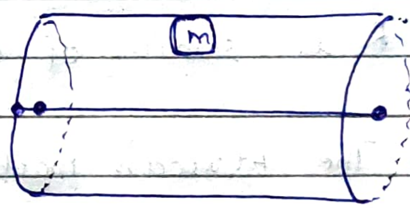
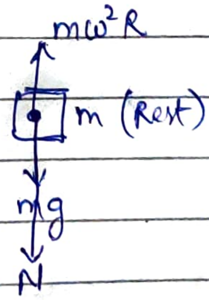
$$R = \frac{F}{\omega^2}\left(\frac{1}{m} + \frac{1}{M}\right)$$

Hence, showed.

5. In a concrete mixer, cement, gravel and water are mixed by tumbling action in a slowly rotating drum. If the drum spins too fast the ingredients stick to the drum wall instead of mixing.
Assume the drum of a mixer has radius R and that it is mounted with its axle horizontal. What is the fastest the drum can rotate without the ingredients sticking to the wall all the time?



The radius of drum is R ft and its axle is horizontal.
Consider a small mass m of cement mixture at top of the cylinder.
The forces acting on this mass will be its weight, Normal contact force due to wall of mixer, and centrifugal force (fbd is drawn)



Observing from frame of reference of mass, its is at rest and hence sum of forces on it must be zero.

$$\therefore \quad mg + N = m\omega^2 R$$

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To protect the mass from sticking to the wall, let the fastest angular velocity be ω.

If the mass is not sticking to the wall, the the normal contact force should be zero.

$\therefore$

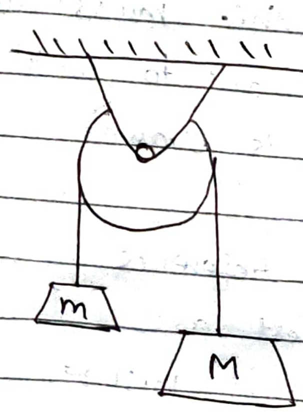
$$mg = m\omega^2 R$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{R}}$$

$$\omega = \sqrt{\frac{32}{R}} \quad \text{(where R is in ft.)}$$

Hence, the fastest angular velocity by which the drum can rotate without the mixture sticking to its wall is $\omega = \sqrt{\frac{32}{R}}$, where R is radius of drum in ft.

6. The Atwood's machine shown in the drawing has a pulley of negligible mass. Find the tension in the rope and acceleration of M



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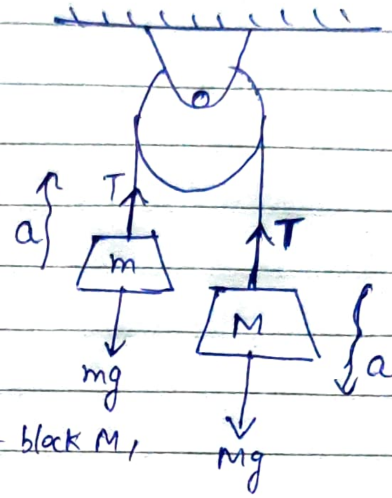
The forces acting on the blocks m and M are shown in figure

Let the acceleration of block M be downwards and equal to 'a'. Therefore acceleration of block m will be a upwards.



Applying Newton's Second Law for block M,

$$Mg - T = Ma \quad \text{— (I)}$$

Applying Newton's Second Law for block m,

$$T - mg = ma \quad \text{— (II)}$$

Adding (I) & (II),

$$(M-m)g = (M+m)a$$

$$a = \frac{(M-m)g}{M+m}$$

Substituting the value of a in eqn (II),

$$T = m\left\{g + \frac{(M-m)g}{M+m}\right\}$$

$$T = \frac{2mMg}{M+m}$$

Hence, the tension in the rope is $\frac{2mMg}{m+M}$ and acceleration of block M is $\frac{(M-m)g}{M+m}$.