MA202

ASSIGNMENT 5

NAME:

ARCHIT AGRAWAL

ROLL NO. :

202051213

SECTION:

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Question 1

Part A

```
clc;
clear;
close all;
a = 1;
trueval = 1./(1 + (a.^2));
h=10.^[-1:-1:-16]
```

```
h =

  Columns 1 through 7

    0.1000    0.0100    0.0010    0.0001    0.0000    0.0000    0.0000

  Columns 8 through 14

    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000

  Columns 15 through 16

    0.0000    0.0000
```

Forward difference formula

```
forward_val = (atan(a+h) - atan(a))./(h);
error_foward = abs(trueval - forward_val);
disp("forward error =");
disp(error_foward);
```

```
forward error =
  Columns 1 through 7
```

```
    0.0242    0.0025    0.0002    0.0000    0.0000    0.0000    0.0000

  Columns 8 through 14

    0.0000    0.0000    0.0000    0.0000    0.0000    0.0004    0.0004

  Columns 15 through 16

    0.0551    0.5000
```

Backward difference formula

```
h1=-h;
backward_val = (atan(a+h1) - atan(a))./(h1);
error_backward = abs(trueval - backward_val);
disp("backword error =");
disp(error_backward);

backword error =
  Columns 1 through 7

    0.0258    0.0025    0.0003    0.0000    0.0000    0.0000    0.0000

  Columns 8 through 14

    0.0000    0.0000    0.0000    0.0000    0.0001    0.0004    0.0004

  Columns 15 through 16

    0.0559    0.5000
```

Central difference formula

```
central_val = (atan(a+h) - atan(a-h)) ./ (2*h);
error_central = abs(trueval - central_val);
disp("central error=");
disp(error_central);

central error=
  Columns 1 through 7

    0.0008    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000

  Columns 8 through 14

    0.0000    0.0000    0.0000    0.0000    0.0000    0.0004    0.0004

  Columns 15 through 16

    0.0004    0.5000
```

Forward improved difference formula

```
forward_val_imp = (4.*atan(a+h) -3.*atan(a)-atan(a+(2.*h))./(2*h);
error_foward_imp = abs(trueval - forward_val_imp);
disp("error_foward=");
disp(error_foward_imp);
```

```
error_foward=
  Columns 1 through 7

    0.0016    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000

  Columns 8 through 14

    0.0000    0.0000    0.0000    0.0000    0.0001    0.0010    0.0004

  Columns 15 through 16

    0.1106    1.0551
```

Backward improved difference formula

```
backward_val_imp = (3.*atan(a)+atan(a-(2.*h))-4*atan(a-h))./(2*h);
error_backard_imp = abs(trueval - backward_val_imp);
disp("error_backward=");
disp(error_backard_imp);
```

```
error_backward=
  Columns 1 through 7

    0.0016    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000

  Columns 8 through 14

    0.0000    0.0000    0.0000    0.0000    0.0002    0.0004    0.0107

  Columns 15 through 16

    0.0559    0.5000
```

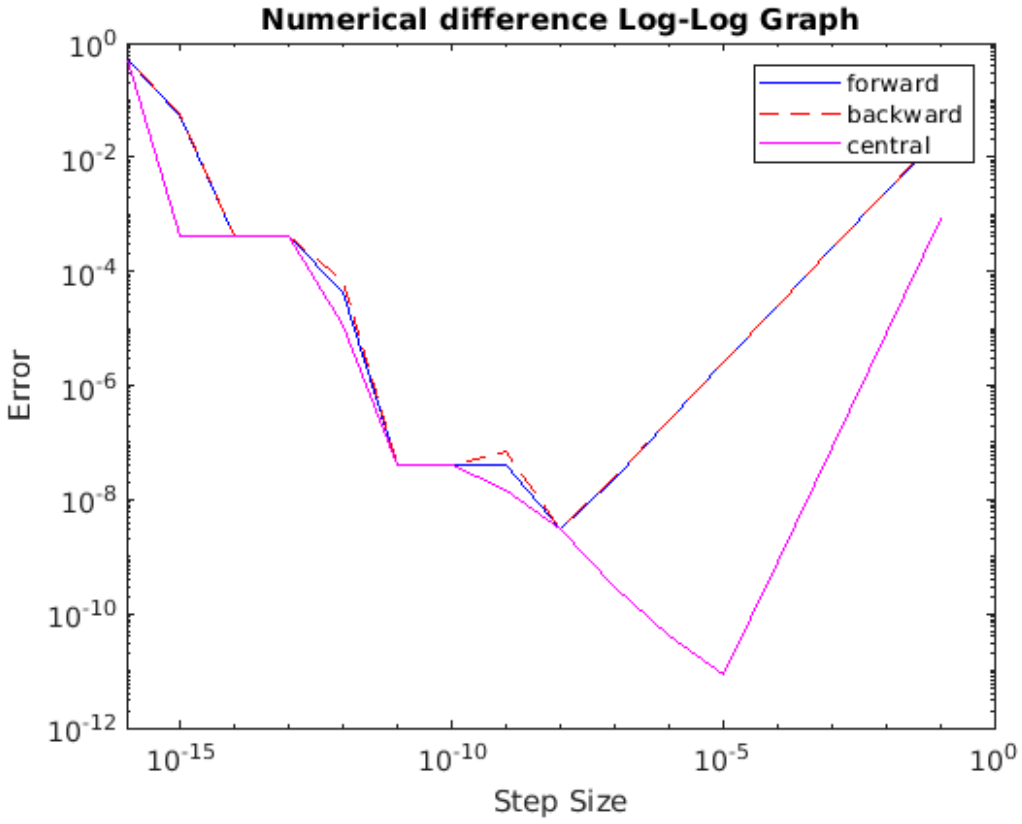
Central improvised difference formula

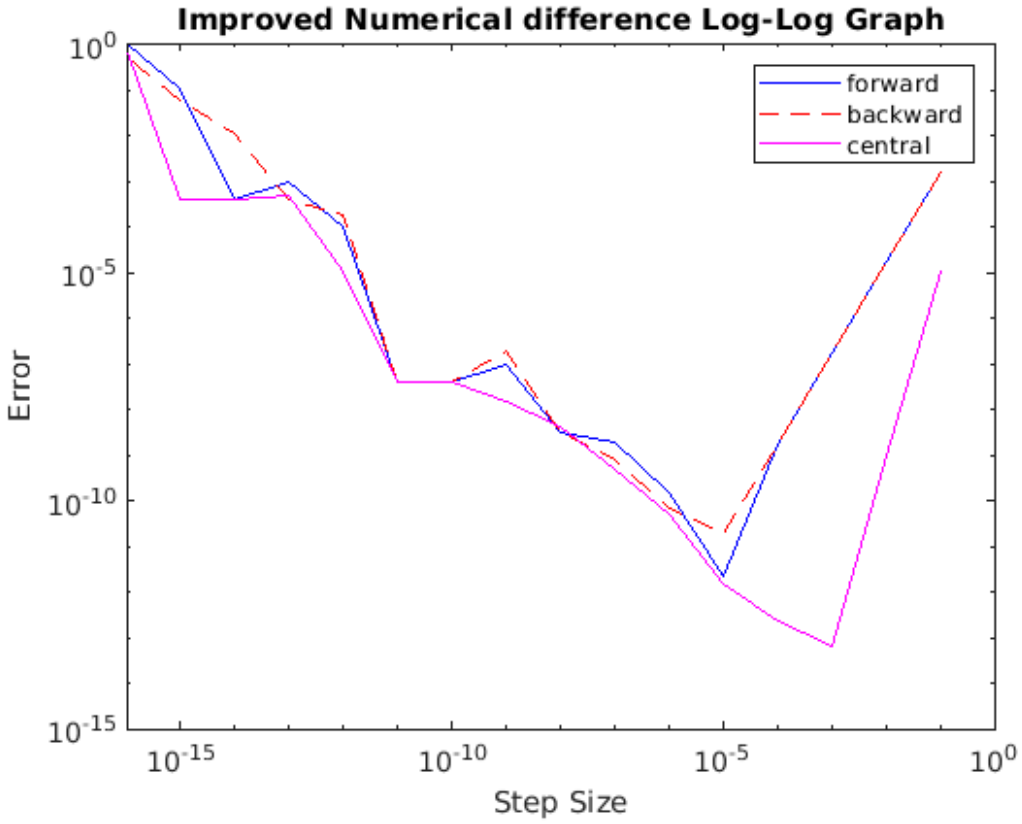
```
central_val_imp=(8*atan(a+h)-8*atan(a-h)-atan(a+(2.*h))+atan(a-
(2.*h)))./(12*h);
error_central_imp = abs(trueval - central_val_imp);
disp("error_central=");
disp(error_central_imp);
```

```
error_central=
  Columns 1 through 7

    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000

  Columns 8 through 14

    0.0000    0.0000    0.0000    0.0000    0.0000    0.0005    0.0004

  Columns 15 through 16

    0.0004    0.6850
```

Plotting graphs

```
figure('Name',"Log-Log Plot");
loglog(h,error_foward,'-b',h,error_backward,'--r',h,error_central,'-
m');
title("Numerical difference Log-Log Graph")
xlabel("Step Size");
ylabel("Error");
legend("forward","backward","central");
figure('Name',"Log-Log Plot Improved");
loglog(h,error_foward_imp,'-b',h,error_backard_imp,'--
r',h,error_central_imp,'-m');
title("Improved Numerical difference Log-Log Graph")
xlabel("Step Size");
ylabel("Error");
legend("forward","backward","central");
```


Numerical difference Log-Log Graph
forward
backward
central
Error
Step Size
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-15}
10^{-10}
10^{-5}
10^{0}


Improved Numerical difference Log-Log Graph
forward
backward
central
Error
Step Size
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-15}
10^{-10}
10^{-5}
10^{0}

Interpretations:

1:There is a trade of between truncation error and round off that when error is minimum then both the errors are minimum and otherwise they are inversely proportional i.e if one decreases other increases. 2:For Forward and Backward difference error is of the order of step size(h) and For Central Difference the error is of the order of square of step size (h^2).

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Question 2:

Q2:Write a MATLAB script to calculate first order as well as second order numerical derivative of 2 _ x + ln(x) at x = 1. Repeat the steps (b), (c), (d), and (e) of Q.1

```
clc;
clear;
close all;
```

Finding Fist order deravative of f(x)=2-x+log(x) at x=1

```
a = 1;
trueval = 1./(1 + (a.^2));
h=10.^[-1:-1:-16]
```

```
h =

  Columns 1 through 7

    0.1000    0.0100    0.0010    0.0001    0.0000    0.0000    0.0000

  Columns 8 through 14

    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000

  Columns 15 through 16

    0.0000    0.0000
```

Forward difference formula

```
forward_val = (calc(a+h) - calc(a))./(h);
error_foward = abs(trueval - forward_val);
disp("error_foward=");
disp(error_foward);
```

Backward difference formula

```
h1=-h;
backward_val = (calc(a+h1) - calc(a))./(h1);
error_backward = abs(trueval - backward_val);
disp("error_backward=");
disp(error_backward);
```

```
error_backward=
  Columns 1 through 7

    0.4464    0.4950    0.4995    0.4999    0.5000    0.5000    0.5000

  Columns 8 through 14

    0.5000    0.5000    0.5000    0.5000    0.5000    0.4989    0.5000

  Columns 15 through 16

    0.3890    0.6102
```

Central difference formula

```
central_val = (calc(a+h) - calc(a-h)) ./ (2*h);
error_central = abs(trueval - central_val);
disp("error_central=");
disp(error_central);
```

```
error_central=
  Columns 1 through 7

    0.4966    0.5000    0.5000    0.5000    0.5000    0.5000    0.5000

  Columns 8 through 14

    0.5000    0.5000    0.5000    0.5000    0.5000    0.4994    0.5000

  Columns 15 through 16

    0.4445    0.0551
```

Plotting garphs

```
figure('Name',"Log-Log Plot");
loglog(h,error_foward,'-b',h,error_backward,'--r',h,error_central,'-
m');
title("Log Log Plot For First Order")
xlabel("Step Size");
ylabel("Error");
legend("forward","backward","central");
```

Finding Second order deravative of f(x)=2-x +log(x) at x=1

```
a = 1;
trueval = -1;
h=10.^[-1:-1:-8];
```

Forward Second order difference formula

```
forward_val = (calc(a+(2.*h))-2*calc(a+h)+calc(a))./(h.*h) ;
error_foward = abs(trueval - forward_val);
disp("error_foward=");
disp(error_foward);
```

```
error_foward=
  Columns 1 through 7

    0.1701    0.0197    0.0020    0.0002    0.0000    0.0001    0.0008

  Column 8

    1.2204
```

Backward Second order difference formula

```
backward_val = (calc(a) - 2.*calc(a-h)+calc(a-(2.*h))).(h.*h);
error_backward = abs(trueval - backward_val);
disp("error_backward=");
disp(error_backward);
```

```
error_backward=
  Columns 1 through 7

    0.2423    0.0204    0.0020    0.0002    0.0000    0.0002    0.0341

  Column 8

    0.1102
```

Central Second order difference formula

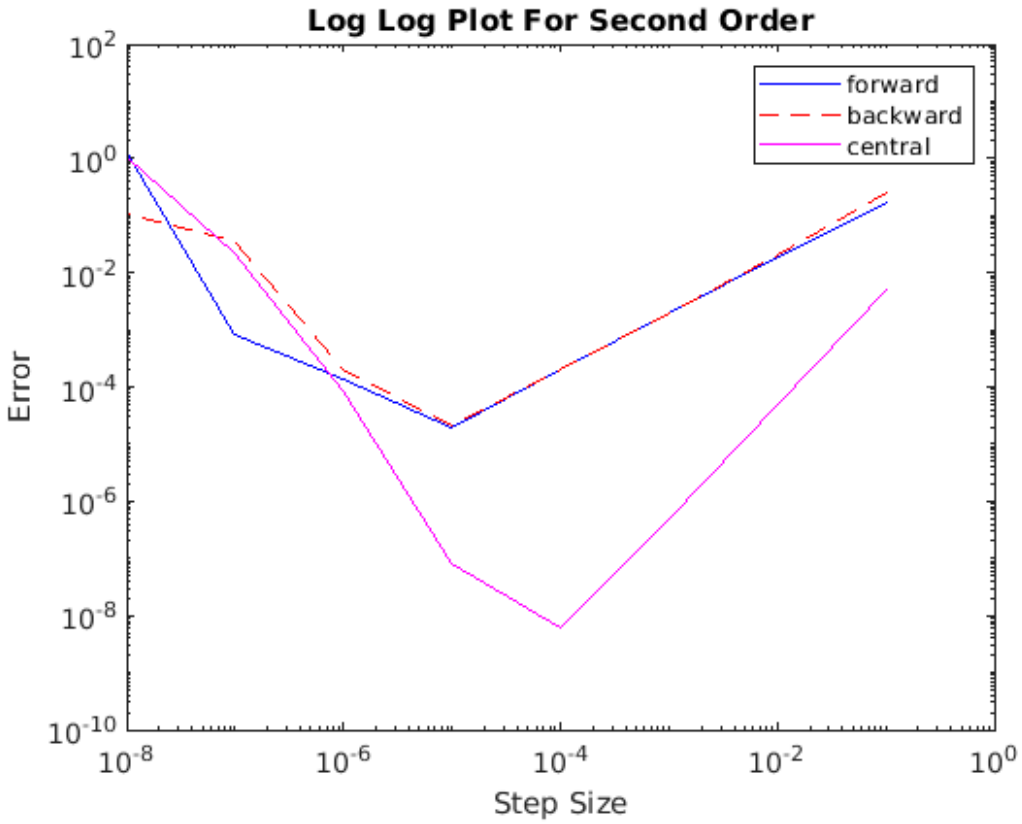
```
central_val = (calc(a+h) + calc(a-h) -2.*calc(a)) ./ (h.*h);
error_central = abs(trueval - central_val);
disp("error_central=");
disp(error_central);
```

```
error_central=
  Columns 1 through 7

    0.0050    0.0001    0.0000    0.0000    0.0000    0.0001    0.0214

  Column 8

    1.0000
```

Plotting garphs

```
figure('Name',"Log-Log Plot");
loglog(h,error_foward,'-b',h,error_backward,'--r',h,error_central,'-
m');
title("Log Log Plot For Second Order")
xlabel("Step Size");
ylabel("Error");
```

```
legend("forward","backward","central");
```



declaring function

```
function fn=calc(a)
    fn=2-a+log(a);
end
```

```
error_foward=
  Columns 1 through 7

    0.5469    0.5050    0.5005    0.5000    0.5000    0.5000    0.5000

  Columns 8 through 14

    0.5000    0.5000    0.5000    0.5000    0.5000    0.5000    0.5000

  Columns 15 through 16

    0.5000    0.5000
```

Interpretations:

1:There is a trade of between truncation error and round off that when error is minimum then both the errors are minimum and otherwise they are inversely proportional i.e if one decreases other increases.

2:For Forward and Backward difference error is of the order of step size(h) and For Central Difference the error is of the order of square of step size (h^2).

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Question 3

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error using central difference method.

Part A

```
clc;
close all;
clear;
% initalising value of x1 and x2
x=[0.5;1];
h=1e-6;
```

Calculating value true value

```
true_val=cos(x(1))*exp(-x(2))-sin(x(1))*((exp(-x(2))));
a(1)=x(1)+h;
a(2)=x(2)+h;
y(1)=x(1)-h;
y(2)=x(2)-h;
```

Finding partial differentiation wrt to x1

```
f1(1)=sin(a(1))*exp(-x(2));
f2(1)=sin(y(1))*exp(-x(2));
```

Finding partial differentiation wrt to x2

```
central(1)=(f1(1)-f2(1))./(2*h);
f1(2)=sin(x(1))*exp(-a(2));
f2(2)=sin(x(1))*exp(-y(2));
central(2)=(f1(2)-f2(2))./(2*h);
central=central(1)+central(2);
```

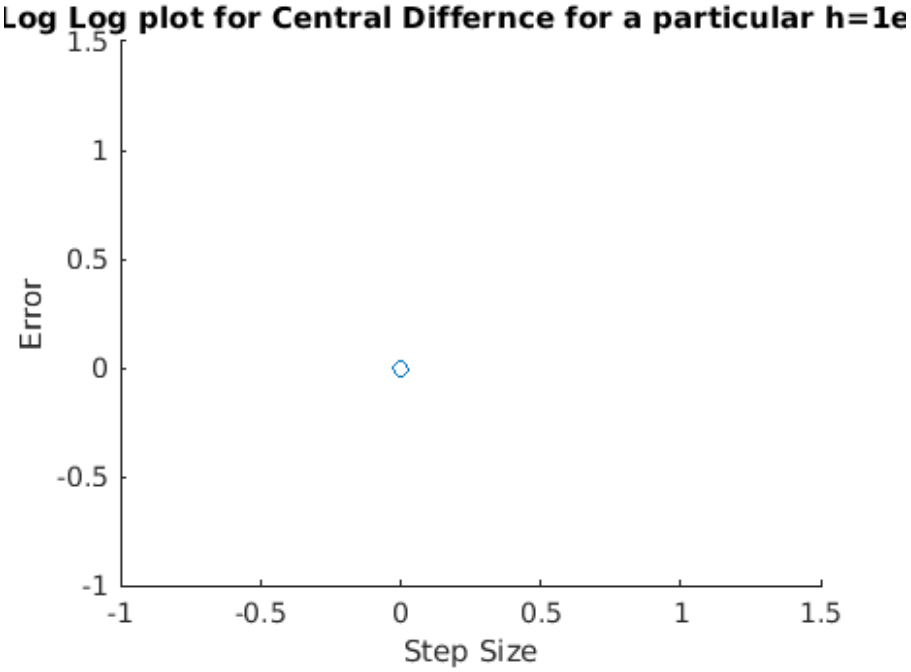
Error calulation

Part B

```
error=abs(central-true_val)
scatter(h,error);
title("Log Log plot for Central Differnce for a particular h=1e-6")
xlabel("Step Size");
ylabel("Error");
```

```
error =

   1.6284e-11
```



Plotting the plot for the range of step size from h=10.^[-1:-1:-16]

initalising value of x1 and x2

```
x1=0.5;
x2=1.0;
h=10.^[-1:-1:-16]
```

```
h =

  Columns 1 through 7
```

```
    0.1000    0.0100    0.0010    0.0001    0.0000    0.0000    0.0000

  Columns 8 through 14

    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000

  Columns 15 through 16

    0.0000    0.0000
```

Calculating value true value

```
true_val=cos(x1)*exp(-x2)-sin(x1)*((exp(-x2)))
a1=x1+h;
a2=x2+h;
y1=x1-h;
y2=x2-h;
f1=sin(a1)*exp(-x2);
f2=sin(y1)*exp(-x2);


true_val =

    0.1465
```

Finding partial differentiation wrt to x1

```
central1=(f1-f2)./(2*h);
ff1=sin(x1)*exp(-a2);
ff2=sin(x1)*exp(-y2);
```

Finding partial differentiation wrt to x2

```
central2=(ff1-ff2)./(2*h);
central=central1+central2
error=abs(central-true_val);
disp(error)


central =

  Columns 1 through 7

    0.1456    0.1465    0.1465    0.1465    0.1465    0.1465    0.1465

  Columns 8 through 14

    0.1465    0.1465    0.1465    0.1465    0.1465    0.1465    0.1471

  Columns 15 through 16
```

```
    0.1388    0.4163

  Columns 1 through 7

    0.0008    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000

  Columns 8 through 14

    0.0000    0.0000    0.0000    0.0000    0.0000    0.0001    0.0006

  Columns 15 through 16

    0.0077    0.2699
```

Plotting log log plot for error v/s step size

```
loglog(h,error);
title("log Log Plot of error Vs step Size");
xlabel("Step Size");
ylabel("Error");
grid on;
```

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