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MA101: Linear Algebra and Matrices
Tutorial #09

1. Find the distance between $u = \begin{bmatrix} 0 \\ -5 \\ 2 \end{bmatrix}$ and $v = \begin{bmatrix} -4 \\ -1 \\ 8 \end{bmatrix}$

Let $A = u = \begin{bmatrix} 0 \\ -5 \\ 2 \end{bmatrix}$ and $B = v = \begin{bmatrix} -4 \\ -1 \\ 8 \end{bmatrix}$.

$$\therefore \text{distance } AB = \sqrt{\{0-(-4)\}^2 + \{-5-(-1)\}^2 + (2-8)^2}$$

$$= \sqrt{16 + 16 + 36}$$

$$= \sqrt{68} \text{ units}$$

$$= 8.246 \text{ units}$$

2. Let $u = \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$, and let W be the set of all vectors ~~of R^3~~ ~~and find an orthogonal basis of W.~~

of R^3 which are orthogonal to u. Show that W is a subspace of R^3 and find an orthogonal basis of W. Given an orthogonal basis of R^3 containing u.

Let a vector $x = [x_1 \ x_2 \ x_3]^T$ be orthogonal to u.

$$\therefore 5x_1 - 6x_2 + 7x_3 = 0$$

$$x_2 = \frac{5x_1 + 7x_3}{6}$$

$$\therefore W = \left\{ \begin{bmatrix} x_1 \\ (5x_1 + 7x_3)/6 \\ x_3 \end{bmatrix} \right\} \text{ where } x_1, x_3 \in R$$

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To prove: W is a subspace

$$W = \begin{bmatrix} x_1 \\ (5x_1+7x_3)/6 \\ x_3 \end{bmatrix}$$

Clearly $[0 \quad 0 \quad 0]^T$ is in W.

Now, let say $a = [a_1 \quad a_2 \quad a_3]^T$ and $b = [b_1 \quad b_2 \quad b_3]^T$ belongs to W.

$$\therefore \quad a = \begin{bmatrix} a_1 \\ (5a_1+7a_3)/6 \\ a_3 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} b_1 \\ (5b_1+7b_3)/6 \\ b_3 \end{bmatrix}$$

$$\therefore \quad a+b = \begin{bmatrix} a_1+b_1 \\ \{5(a_1+b_1)+7(a_3+b_3)\}/6 \\ a_3+b_3 \end{bmatrix}$$

Clearly $a+b \in W$.

Now, let there be a scalar m.

$$\therefore \quad (ma) = \begin{bmatrix} ma_1 \\ m(5a_1+7a_3)/6 \\ ma_3 \end{bmatrix}$$

$$= m \begin{bmatrix} a_1 \\ (5a_1+7a_3)/6 \\ a_3 \end{bmatrix}$$

$$= m(a)$$

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$\because$ W satisfies all three properties of subspace, hence it is a subspace of R^3.

Now, W can be written as

$$W = \left\{ x_1 \begin{bmatrix} 1 \\ 5/6 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 7/6 \\ 1 \end{bmatrix} \right\}$$

Let $P_1 = [1 \;\; 5/6 \;\; 0]^T$ and $P_2 = [0 \;\; 7/6 \;\; 1]^T$

Now, let the orthogonal basis of W be $\{v_1, v_2\}$

Using Gram-Schmidth Process,

$$v_1 = \begin{bmatrix} 1 \\ 5/6 \\ 0 \end{bmatrix} = P_1$$

and $$v_2 = \begin{bmatrix} 0 \\ 7/6 \\ 1 \end{bmatrix} - \frac{P_2 \cdot V_1}{V_1 \cdot V_1} V_1$$

$$V_2 = \begin{bmatrix} 0 \\ 7/6 \\ 1 \end{bmatrix} - \frac{\frac{35}{\cancel{36}}}{\frac{61}{\cancel{36}}} \begin{bmatrix} 1 \\ 5/6 \\ 0 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 0 \\ 7/6 \\ 1 \end{bmatrix} - \begin{bmatrix} 35/61 \\ 175/366 \\ 0 \end{bmatrix} = \begin{bmatrix} -35/61 \\ 252/366 \\ 1 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} -35/61 \\ 42/61 \\ 1 \end{bmatrix}$$

Hence, orthogonal basis for W is $\left\{ \begin{bmatrix} 1 \\ 5/6 \\ 0 \end{bmatrix}, \begin{bmatrix} -35/61 \\ 42/61 \\ 1 \end{bmatrix} \right\}$.

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Now let the orthogonal ~~vector~~ basis of R^3 be $\{u, b_1, b_2\}$.

Now, a vector orthogonal to u is of the form

$$\begin{bmatrix} x_1 \\ (5x_1 + 7x_3)/6 \\ x_3 \end{bmatrix}$$

Put $x_1 = 6$ and $x_3 = 6$

$\therefore$ orthogonal vector b_1 to vector $u = \begin{bmatrix} 6 \\ 12 \\ 6 \end{bmatrix}$

$\therefore$ Now, the cross product of u and b_1 will be orthogonal to u and b_1 and u, b_1, and cross product of u and b_1 will span R^3 also.

$$\therefore \quad b_2 = \vec{u} \times \vec{b_1}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -6 & 7 \\ 6 & 12 & 6 \end{vmatrix}$$

$$= -120\,\hat{i} + 12\,\hat{j} + 96\,\hat{k}$$

$$\therefore \quad b_2 = \begin{bmatrix} -120 \\ 12 \\ 96 \end{bmatrix}$$

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$\therefore$ orthogonal basis of R^3 containing u is

$$\left\{ \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}, \begin{bmatrix} 6 \\ 12 \\ 6 \end{bmatrix}, \begin{bmatrix} -120 \\ 12 \\ 96 \end{bmatrix} \right\}$$

3. Verify the parallelogram law for vectors u and v:

$$||u+v||^2 + ||u-v||^2 = 2||u||^2 + 2||v||^2$$

$$\begin{aligned} ||u+v||^2 &= (u+v)\cdot(u+v) \\ &= u\cdot u + 2u\cdot v + v\cdot v \\ &= ||u||^2 + 2u\cdot v + ||v||^2 \quad \text{---(I)} \end{aligned}$$

$$\begin{aligned} ||u-v||^2 &= (u-v)\cdot(u-v) \\ &= u\cdot u - u\cdot v - u\cdot v + v\cdot v \\ &= ||u||^2 - 2u\cdot v + ||v||^2 \quad \text{---(II)} \end{aligned}$$

Adding (I) & (II),

$$||u+v||^2 + ||u-v||^2 = 2||u||^2 + 2||v||^2$$

Hence, proved.

Q.4 Compute the orthogonal projection of $u = [1 \;\; 7]^T$ on to the line through origin and $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and distance of u to the line.

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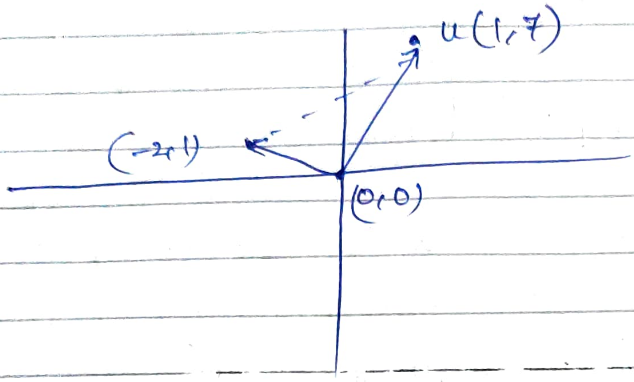
Given $u = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$ and $v = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$



orthogonal projection of u on v = $\left(\frac{u \cdot v}{v \cdot v}\right) v$

$u \cdot v = u^T \cdot v = \begin{bmatrix} 1 & 7 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = 5$

$v \cdot v = v^T \cdot v = \begin{bmatrix} -2 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = 5$

$\therefore$ orthogonal projection of u on v $= \left(\frac{5}{5}\right) v$

$= v$

Since, v is orthogonal projection of u, therefore, perpendicular distance of u from the line is same as $\|u - v\|$

distance $= \left\| \begin{bmatrix} 1 \\ 7 \end{bmatrix} - \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\|$

$= \left\| \begin{matrix} 3 \\ 6 \end{matrix} \right\|$

$= \sqrt{3^2 + 6^2}$

$= \sqrt{45}$ units

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5. Let U be 5×5 orthogonal matrix. Show that columns of U forms an ~~orthogonal~~ orthonormal basis of R^5.

Let $U = [U_1 \;\; U_2 \;\; U_3 \;\; U_4 \;\; U_5]$

as U is orthogonal matrix

$$U^T U = I$$

$$U^T U = \begin{bmatrix} U_1^T U_1 & U_1^T U_2 & \cdots & U_1^T U_5 \\ U_2^T U_1 & \cdots & \cdots & U_2^T U_5 \\ \vdots & & & U_3^T U_5 \\ \vdots & & & U_4^T U_5 \\ U_5^T U_1 & U_5^T U_2 & \cdots & U_5^T U_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore (U_T U)_{ij} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i\neq j \end{cases}$$

$$\therefore U_1^T U_1 = 1$$
$$U_1^T U_2 = 0$$

$$\therefore U_1 \cdot U_1 = U_2 \cdot U_2 = \cdots = U_5 \cdot U_5 = 1$$

Hence, columns of U are orthonormal.

Also, let say $Ux = 0$ for some vector x in R^5.

Premultiplying by U^T.

$$U^T U x = 0$$
$$I x = 0$$
$$\therefore x = 0$$

Hence, only trivial solutions exist and columns of U are linearly independent.

$\therefore$ columns of ~~col~~ U forms an orthonormal basis of R^5.

6. Write $v = \begin{bmatrix} 4 \\ 5 \\ -3 \\ 3 \end{bmatrix}$ as linear combination of u_i,

where

$u_1 = [1 \quad 2 \quad 1 \quad 1]^T$, $u_2 = [-2 \quad 1 \quad -1 \quad 1]^T$

$u_3 = [1 \quad 1 \quad -2 \quad -1]^T$ and $u_4 = [-1 \quad 1 \quad 1 \quad -2]^T$

Let $v = c_1 u_1 + c_2 u_2 + c_3 u_3 + c_4 u_4$

$\therefore \quad c_1 = \dfrac{u_1 \cdot v}{u_1 \cdot u_1} = \dfrac{(4+10-3+3)}{(1+4+1+1)} = 2$

$c_2 = \dfrac{u_2 \cdot v}{u_2 \cdot u_2} = \dfrac{(-8+5+3+3)}{(4+1+1+1)} = \dfrac{3}{7}$

$c_3 = \dfrac{u_3 \cdot v}{u_3 \cdot u_3} = \dfrac{(4+5+6-3)}{(1+1+4+1)} = \dfrac{12}{7}$

$c_4 = \dfrac{u_4 \cdot v}{u_4 \cdot u_4} = \dfrac{(-4+5-3-6)}{1+1+1+4} = \dfrac{-8}{7}$

$\therefore \quad v = 2u_1 + \dfrac{3}{7} u_2 + \dfrac{12}{7} u_3 - \dfrac{8}{7} u_4$

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7. Let $W = \text{span}\{u_1, u_2\}$ and $U = [u_1 \ u_2]$.
Compute UU^T, U^TU, $\text{Proj}_W y$, UU^Ty.
What do you observe

$$y = \begin{bmatrix} 4 \\ 8 \\ 1 \end{bmatrix}, \quad u_1 = \begin{bmatrix} 2/3 \\ 1/3 \\ 2/3 \end{bmatrix} \text{ and } u_2 = \begin{bmatrix} -2/3 \\ 2/3 \\ 1/3 \end{bmatrix}$$

$$U = [u_1 \ u_2] = \begin{bmatrix} 2/3 & -2/3 \\ 1/3 & 2/3 \\ 2/3 & 1/3 \end{bmatrix}$$

$$U^T = \begin{bmatrix} 2/3 & 1/3 & 2/3 \\ -2/3 & 2/3 & 1/3 \end{bmatrix}$$

$$\therefore \quad UU^T = \begin{bmatrix} 2/3 & -2/3 \\ 1/3 & 2/3 \\ 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 2/3 & 1/3 & 2/3 \\ -2/3 & 2/3 & 1/3 \end{bmatrix} = \begin{bmatrix} 8/9 & -2/9 & 2/9 \\ -2/9 & 5/9 & 4/9 \\ 2/9 & 4/9 & 5/9 \end{bmatrix}$$

$$UU^T = \frac{1}{9} \begin{bmatrix} 8 & -2 & 2 \\ -2 & 5 & 4 \\ 2 & 4 & 5 \end{bmatrix}$$

and, $$U^TU = \begin{bmatrix} 2/3 & 1/3 & 2/3 \\ -2/3 & 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 2/3 & -2/3 \\ 1/3 & 2/3 \\ 2/3 & 1/3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Since, $U^TU = I$
Therefore, columns of U i.e. u_1 and u_2 form an orthogonal basis for W.

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$$(\text{proj}_w y) = \left(\frac{y \cdot w}{w \cdot w}\right) w$$

$$= \left(\frac{y \cdot u_1}{u_1 \cdot u_1}\right) u_1 + \left(\frac{y \cdot u_2}{u_2 \cdot u_2}\right) u_2$$

$$= \frac{18/3}{1}\begin{bmatrix} 2/3 \\ 1/3 \\ 2/3 \end{bmatrix} + \frac{9/3}{1}\begin{bmatrix} -2/3 \\ 2/3 \\ 1/3 \end{bmatrix}$$

$$(\text{proj}_w y) = \begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}$$

Now, $UU^T y = \frac{1}{9}\begin{bmatrix} 8 & -2 & 2 \\ -2 & 5 & 4 \\ 2 & 4 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 8 \\ 1 \end{bmatrix}$

$$= \frac{1}{9}\begin{bmatrix} 18 \\ 36 \\ 45 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}$$

Hence, we observed that $\text{proj}_w y = UU^T y$ for all y in R^3.

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Q. Let $A = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 2 & -2 & 9 \\ 4 & -14 & -3 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Find $p, u \in R^3$ such that $x = p + u$, $p \in Row(A)$ and $u \in Null(A)$.

Let r_1, r_2, r_3 and r_4 be rows of A

$\therefore Row(A) = c_1 r_1 + c_2 r_2 + c_3 r_3 + c_4 r_4$

where $c_1, c_2, c_3, c_4 \in R$.

it can be clearly seen that

$r_3 = r_1 + r_2$
and $r_4 = r_3 - 2r_1$

$\therefore Row(A) = c_1 r_1 + c_2 r_2 + c_3(r_1 + r_2) + c_4(r_3 - 2r_1)$

$= r_1(c_1 + c_3 - 2c_4) + r_2(c_2 + c_3 + c_4)$

$= \alpha_1 r_1 + \alpha_2 r_2$

$Null(A) \Rightarrow Ax = 0$

$$\left[\begin{array}{ccc|c} -1 & 6 & 6 & 0 \\ 3 & -8 & 3 & 0 \\ 2 & -2 & 9 & 0 \\ 4 & -14 & -3 & 0 \end{array}\right]$$

$R_2 \to R_2 + 3R_1$
$R_3 \to R_3 + 2R_1$
$R_4 \to R_4 + 4R_1$

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$$\sim \left[\begin{array}{ccc|c} -1 & 6 & 6 & 0 \\ 0 & 10 & 21 & 0 \\ 0 & 10 & 21 & 0 \\ 0 & 10 & 21 & 0 \end{array}\right]$$

$R_3 \to R_3 - R_2$
$R_4 \to R_4 - R_2$

$$\sim \left[\begin{array}{ccc|c} -1 & 6 & 6 & 0 \\ 0 & 10 & 21 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

x_3

Hence, x_3 is free variable.

$$10x_2 + 21x_3 = 0$$
$$x_2 = \frac{-21}{10}x_3$$

and, $$x_1 = 6(x_2 + x_3)$$
$$= \frac{-33}{5}x_3$$

$\therefore$ $\text{Null}(A) = \begin{bmatrix} -33/5 \\ -21/10 \\ 1 \end{bmatrix} x_3$ for $x_3 \in \mathbb{R}$.

$\because$ $p \in \text{Row}(A)$

$p = \alpha_1 \begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} + \alpha_2 \begin{bmatrix} 3 \\ -8 \\ 3 \end{bmatrix}$ for some $\alpha_1, \alpha_2 \in \mathbb{R}$

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and $u \in \text{Nul}\,A$

$$\therefore \quad u = \begin{bmatrix} -33/5 \\ -21/10 \\ 1 \end{bmatrix} x_3 \qquad \text{for some } x_3 \in R$$

Now, $x = p + u$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \alpha_1 \begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} + \alpha_2 \begin{bmatrix} 3 \\ -8 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} -33/5 \\ -21/10 \\ 1 \end{bmatrix}$$

Augmented matrix for the above system of equations is

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & x_2 & \\ \end{array}$$

$$\left[\begin{array}{ccc|c} -1 & 3 & -33/5 & 1 \\ 6 & -8 & -21/10 & 2 \\ 6 & 3 & 1 & 3 \end{array}\right]$$

$$R_2 \rightarrow R_2 + 6R_1$$
$$R_3 \rightarrow R_3 + 6R_1$$

$$\left[\begin{array}{ccc|c} -1 & 3 & -33/5 & 1 \\ 0 & 10 & -417/10 & 8 \\ 0 & 21 & -386/10 & 9 \end{array}\right]$$

$$R_3 \rightarrow R_3 - \frac{21}{10} R_2$$

$$\left[\begin{array}{ccc|c} -1 & 3 & -33/5 & 1 \\ 0 & 10 & -417/10 & 8 \\ 0 & 0 & 4897/100 & -\frac{39}{5} \end{array}\right]$$

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$$\frac{4897}{\cancel{100}20} x_3 = \frac{-39}{8}$$

$$x_3 = \frac{-780}{4897}$$

$$10x_2 - \frac{417}{10} x_3 = 8$$

$$10x_2 = 8 + \frac{417}{10} \times \frac{(-780)}{4897}$$

$$10x_2 = \frac{6650}{4897}$$

$$x_2 = \frac{665}{4897}$$

$$-x_1 + 3x_2 - \frac{33}{5} x_3 = 1$$

$$x_1 = 3x_2 - \frac{33}{5} x_3 - 1$$

$$= \frac{3 \times 665}{4897} - \frac{33}{5}\left(\frac{-780}{4897}\right) - 1$$

$$= \frac{1995}{4897} + \frac{5148}{4897} - \frac{4897}{4897}$$

$$x_1 = \frac{2246}{4897}$$

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$$\therefore \quad p = \frac{2246}{4897}\begin{bmatrix}-1\\6\\6\end{bmatrix} + \frac{665}{4897}\begin{bmatrix}3\\-8\\3\end{bmatrix}$$

$$p = \frac{1}{4897}\begin{bmatrix}-251\\8156\\15471\end{bmatrix}$$

and $$u = \begin{bmatrix}-33/5\\-21/10\\1\end{bmatrix}\left(\frac{-780}{4897}\right)$$

$$u = \begin{bmatrix}5148\\1638\\-780\end{bmatrix}\left(\frac{1}{4897}\right)$$

Hence, the vectors p and u are

$$\frac{1}{4897}\begin{bmatrix}-251\\8156\\15471\end{bmatrix} \text{ and } \frac{1}{4897}\begin{bmatrix}5148\\1638\\-780\end{bmatrix} \text{ respectively.}$$

9. Find an orthonormal basis - $\{u_1, u_2, u_3\}$ for the column space of the matrix

$$A = \begin{bmatrix}-1 & 6 & 6\\3 & -8 & 3\\1 & -2 & 6\\1 & -4 & -3\end{bmatrix}.$$

Let $Q = \{u_1\ u_2\ u_3\}$ with its colums and $R = Q^T A$. Is $A = QR$?

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Row reducing A to its row echelon form reveals that Col A is a 3-D subspace in $\mathbb{R}^4$.

Let $b = \{b_1, b_2, b_3\}$ be the orthogonal basis for the Col A.

Let $x_1 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 6 \\ -8 \\ -2 \\ -4 \end{bmatrix}$ and $x_3 = \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix}$

Using Gram-Schmidt Process,

$$b_1 = x_1 = \begin{bmatrix} -1 & 3 & 1 & 1 \end{bmatrix}^T$$

$$b_2 = x_2 - \left(\frac{x_2 \cdot b_1}{b_1 \cdot b_1}\right) b_1$$

$$x_2 \cdot b_1 = (-6) + (-24) + (-2) + (-4)$$
$$= -36$$

$$b_1 \cdot b_1 = 1 + 9 + 1 + 1 = 12$$

$$\therefore b_2 = \begin{bmatrix} 6 \\ -8 \\ -2 \\ -4 \end{bmatrix} - \frac{(-36)}{12} \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

$$b_2 = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

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and $b_3 = x_3 - \left(\frac{x_3 \cdot b_1}{b_1 \cdot b_1}\right) b_1 - \left(\frac{x_3 \cdot b_2}{b_2 \cdot b_2}\right) b_2$

$x_3 \cdot b_1 = (-6) + 9 + 6 - 3 = 6$
$b_1 \cdot b_1 = 12$
$x_3 \cdot b_2 = 18 + 3 + 6 + 3 = 30$
$b_2 \cdot b_2 = 9 + 1 + 1 + 1 = 12$

$$\therefore \; b_3 = \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix} - \frac{6}{12}\begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix} - \frac{30}{12}\begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$b_3 = \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix} - \begin{bmatrix} -1/2 \\ 3/2 \\ 1/2 \\ 1/2 \end{bmatrix} - \begin{bmatrix} 15/2 \\ 5/2 \\ 5/2 \\ -5/2 \end{bmatrix}$$

$$b_3 = \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix}$$

$\therefore$ orthogonal basis for Col A is $\left\{ \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix} \right\}$

$\therefore$ orthonormal basis for Col A is $\left\{ \begin{bmatrix} -1/\sqrt{12} \\ 3/\sqrt{12} \\ 1/\sqrt{12} \\ 1/\sqrt{12} \end{bmatrix}, \begin{bmatrix} 3/\sqrt{12} \\ 1/\sqrt{12} \\ 1/\sqrt{12} \\ -1/\sqrt{12} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{12} \\ -1/\sqrt{12} \\ 3/\sqrt{12} \\ -1/\sqrt{12} \end{bmatrix} \right\}$

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Now, $Q = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}$

$$Q = \frac{1}{\sqrt{12}} \begin{bmatrix} -1 & 3 & -1 \\ 3 & 1 & -1 \\ 1 & 1 & 3 \\ 1 & -1 & -1 \end{bmatrix}$$

$$R = Q^T A$$

$$R = \frac{1}{\sqrt{12}} \begin{bmatrix} -1 & 3 & 1 & 1 \\ 3 & 1 & 1 & -1 \\ -1 & -1 & 3 & -1 \end{bmatrix} \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$$

$$R = \frac{1}{\sqrt{12}} \begin{bmatrix} 12 & -36 & 6 \\ 0 & 12 & 30 \\ 0 & 0 & 12 \end{bmatrix}$$

$$R = \sqrt{3} \begin{bmatrix} 2 & -6 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix}$$

Now, $$QR = \frac{1}{\sqrt{12}} \begin{bmatrix} -1 & 3 & -1 \\ 3 & 1 & -1 \\ 1 & 1 & 3 \\ 1 & -1 & -1 \end{bmatrix} \sqrt{3} \begin{bmatrix} 2 & -6 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -2 & 12 & 12 \\ 6 & -16 & 6 \\ 2 & -4 & 12 \\ 2 & -8 & -6 \end{bmatrix}$$

$$QR = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$$

$$QR = A$$

Hence, $A = QR$