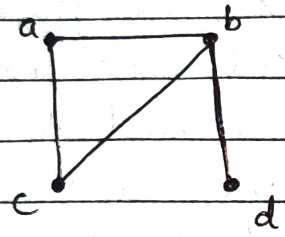
Archit Agrawal
20205207

MA102 : Introduction to Discrete Mathematics

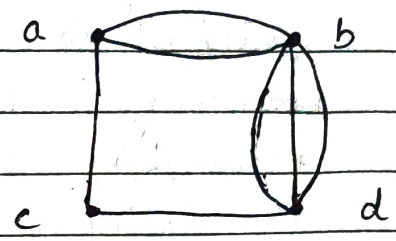
Tutorial 12

1. Determine whether the graphs shown below has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops.

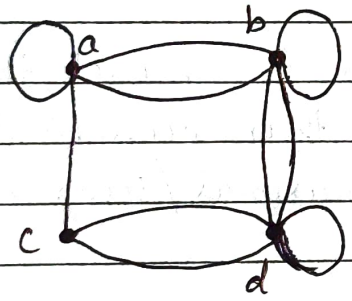
(a)



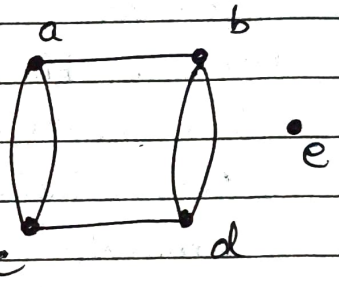
(b)



(c)



(d)



(a) No directed edges, no multiple edges, no loops.

(b) no directed edges, multiple edges, no loops.

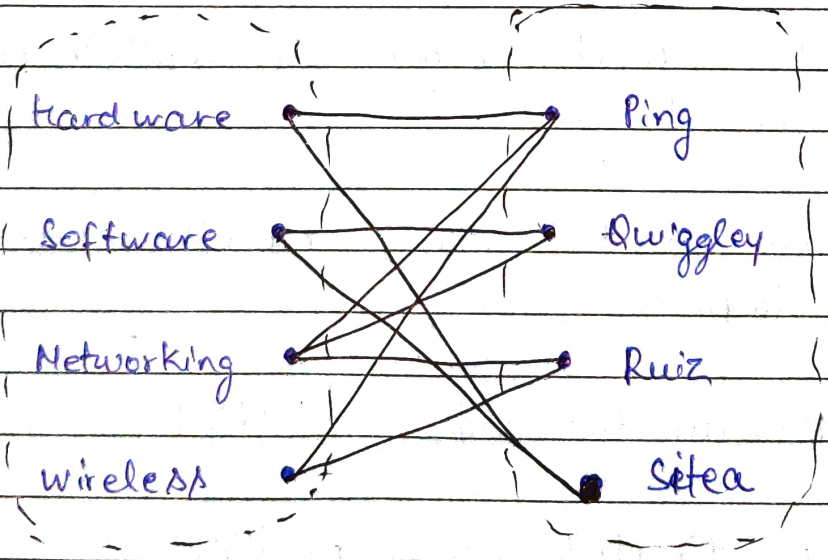
(c) no directed edges, multiple edges, loops.

(d) no directed edges, multiple edges, no loops.

Archit Agrawal
2021052307

2. Suppose that there are four employees in the computer support group of the School of Engineering of a large university. Each employee will be assigned to support one of the four different areas: hardware, software, networking, and wireless. Suppose that Ping is qualified to support hardware, networking and wireless; Quiggley is qualified to support software and networking; Ruiz is qualified to support networking and wireless, and Sitea is qualified to support hardware and software.

Use a bipartite graph to model the four employees and their qualifications. Does there exists maximal/complete matching?



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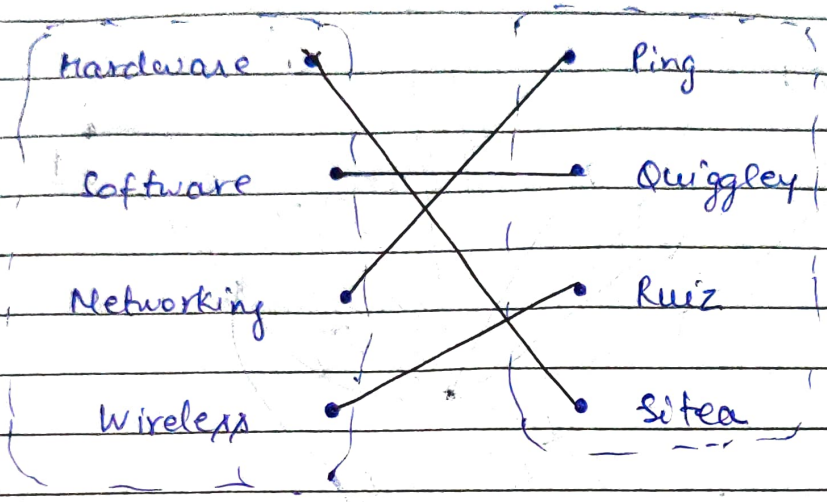
2020052307

Yes, there exists a maximal/complete matching



3. Construct the graph for a set of seven telephone numbers 555-0011, 555-1221, 555-1333, 555-8888, 555-2222, 555-0091, 555-1200 if there were three calls from 555-0011 to 555-8888 and two calls from 555-8888 to 555-0011, two calls from 555-2222 to 555-0091, two calls from 555-1221 to each of the other numbers, and one call from 555-1333 to each of 555-0011, 555-1221 and 555-1200.

vertex 1 → 555-0011
vertex 2 → 555-1221
vertex 3 → 555-1333
vertex 4 → 555-8888
vertex 5 → 555-2222
vertex 6 → 555-0091
vertex 7 → 555-1200

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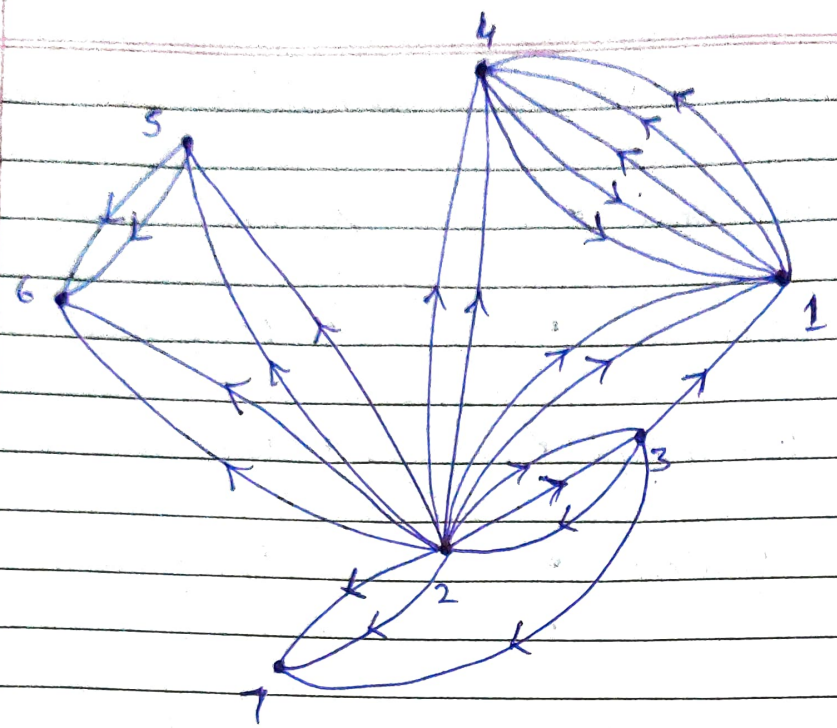
2020052307



The above graph represents the calls.

Q.4 Can a simple graph exist with 15 vertices each of degree 5?

According to hand shaking theorem,

$$2 \times (\text{no. of edges}) = \sum_{v \in V} \deg(v)$$

which implies that the sum of degree of each vertex should be even.

But, $15 \times 5 = 75$, which is odd.

Hence, such a simple graph can not exist

Archit Agrawal
202052307

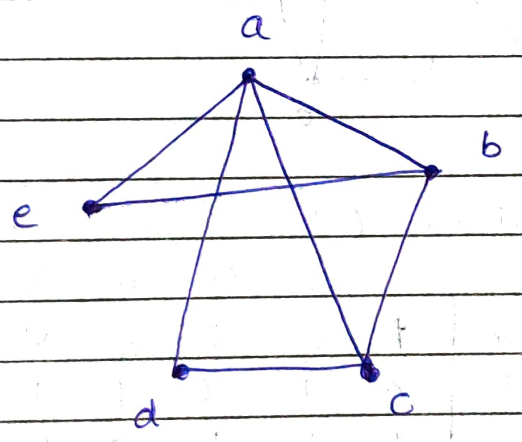
Q.5. How many edges does a graph have if its degree sequence is 4, 3, 3, 2, 2? Draw such a graph. Is it bipartite?

According to handshaking theorem,

$$2 \times (\text{no. of edges}) = \sum_{v \in V} \deg(v)$$

$$\therefore \text{no. of edges} = \frac{4+3+3+2+2}{2}$$

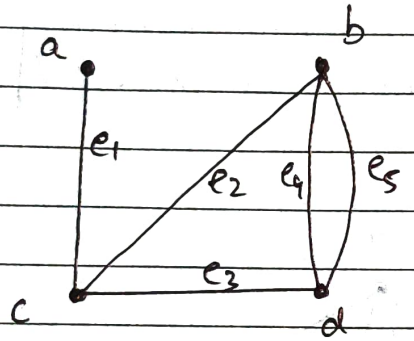
$$\therefore \text{no. of edges} = 7$$



$\deg(a) = 4$
$\deg(b) = 3$
$\deg(c) = 3$
$\deg(d) = \deg(e) = 2$

The graph is not bipartite because the set that will contain vertex 'a' must not contain any other vertex, therefore, vertex 'b', 'c', 'd' and 'e' must be in the other set. Now, since 'b', 'c', 'd', 'e" have edges among them, hence, the graph is not bipartite.

6. Represent following graph using incidence matrix and adjacency matrix.



Incidence matrix E:

$$E = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}_{4\times5}$$

Adjacency matrix M:

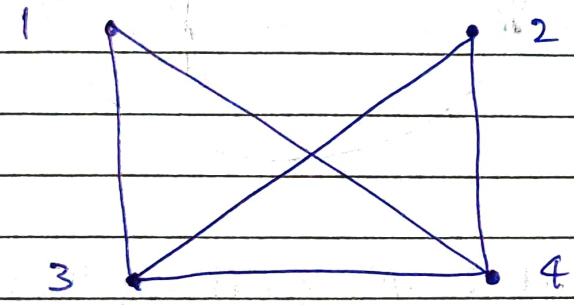
$$M = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix} \end{array}_{4\times4}$$

Archit Agrawal
2020052307

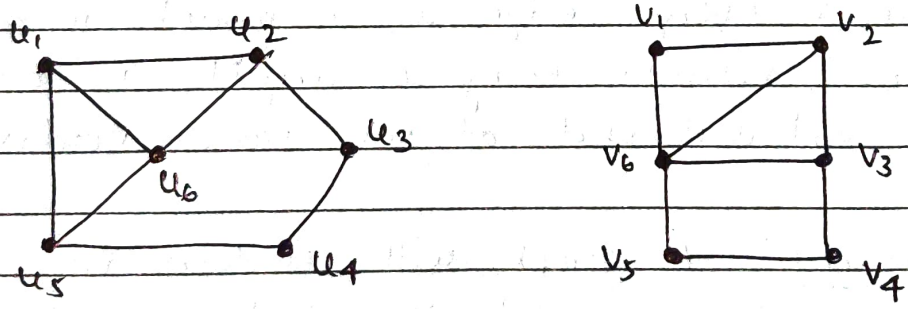
7. Draw an undirected graph represented by following adjacency matrix.

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Let the matrix be w.r.t. ordering of vertices 1, 2, 3 and 4.



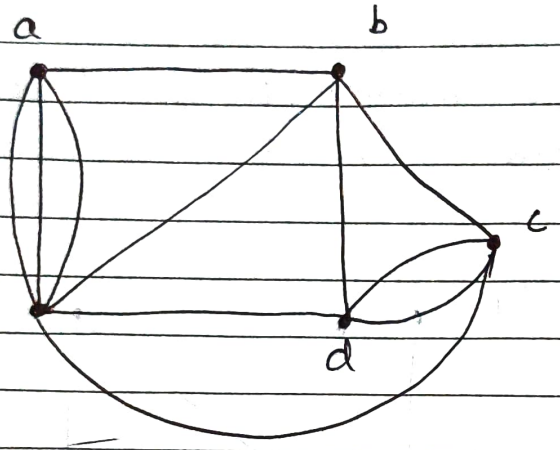
8. Are following graphs isomorphic?



The two graphs are not isomorphic as the graph in the right side has vertex V_6 with degree 4, but there is no vertex in the graph on left side with degree 4.

Archit Agrawal
2020052307

Q.9 Can you draw the following graph without lifting your pen and without tracing an edge twice? If yes, then draw it. Did you stop at a vertex where you started it?



Euler Circuit:- ~~A graph will contain an Euler circuit if all the vertices~~ An Euler circuit is a circuit that uses every edge in a graph with no repeats. Being a circuit, it must start and end at the same vertex.

Theorem: A graph will contain an Euler circuit if all the vertices have even degree.

Archit Agrawal
2020052307

In the given graph,

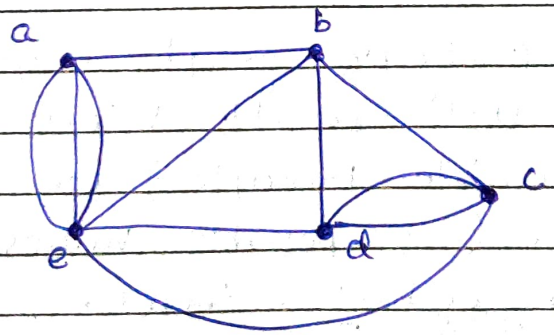
$\deg(a) = 4$
$\deg(b) = 4$
$\deg(c) = 4$
$\deg(d) = 4$
$\deg(e) = 6$

Since, every vertex has even degree, the graph contains an Euler Circuit.

Hence, the graph can be drawn without lifting the pen and without tracing an edge twice.

→ Start at ~~index~~ vertex a and traverse in the given order

a → b → d → e → a → e → c → d → c → b → e → a



Also, we started at vertex 'a' and stopped at vertex 'a' too.

Archit Agrawal
2020S2307

Q:10 Show that in a simple graph with at least two vertices there must be two vertices that have the same degree.

A graph with n vertices ($n \geq 2$), there are n possible values of degrees of each vertex $(0, 1, \cdots, n-1)$

But, in a simple graph, 0 and $(n-1)$ degree value can never be present together. If a vertex has degree 0, it means it is not adjacent to any other vertex and in such case the maximum value of degree can be $(n-2)$.

Similarly, if a vertex has degree $(n-1)$ in a simple graph, it means it is adjacent to each of the vertex and no vertex has a degree 0.

Hence, for a simple graph with n vertices, there are $(n-1)$ possible values of degree. So, by Pigeonhole Principle, we have to arrange $(n-1)$ possible values of degree among n vertices, therefore, there must exist atleast $\left\lceil \frac{n}{(n-1)} \right\rceil$ vertices with same degree. For $n \geq 2$, $\left\lceil \frac{n}{(n-1)} \right\rceil = 2$

Archit Agrawal
2020052307

Q.11 Represent following map of Gujarat using a graph. How many minimum colours are required to paint it so that adjacent districts have different colours?

The graph is attached below.

We start from Kutch and assign it the blue color.

We start from Dwarka. Assign red to Dwarka, blue to Kutch and Porbandar, and green to Jamnagar. Now, Morbi and Rajkot must be of different colours and Morbi cannot be green or blue (possible colour is red) while Rajkot cannot be green or blue (possible colour is Red) but both cannot be Red as they are adjacent. Now, we need to choose a fourth colour (i.e yellow) and colour the map.

Archit Agrawal
2020052307

We see that the map can be coloured using 4 colours.

So, minimum colours required = 4.

A possible colour pattern is shown in the map below.