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PH100: Mechanics and Thermodynamics
Tutorial #08

1. What is the volume of a container that holds exactly 1 mole of an ideal gas at standard temperature and pressure (STP), defined as $T = 0°C$ and $p = 1\ atm = 1.013 \times 10^5\ Pa$?

given, $P = 1.013 \times 10^5\ Pa$
$T = 273\ K$
$n = 1$ moles

Using ideal gas equation, $PV = nRT$

$$\therefore V = \frac{nRT}{P} = \frac{1 \times 8.314 \times 273}{1.013 \times 10^5}$$

$$= 2240.5 \times 10^{-5}\ m^3$$

Since $1\ m^3 = 1000\ L$

$$\therefore V = 2240.5 \times 10^{-2}\ L$$
$$= 22.4\ L$$

Hence, the volume of such a container is 22.4 L.

2. Find the variation of atmospheric pressure with elevation in earth's atmosphere. Assume that at all elevations, $T = 0°C$ and $g = 9.8\ ms^{-2}$.

We know that $P = \rho g h$

where ρ is density and h is height.

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$$\therefore \frac{dP}{dh} = -\rho g$$

negative sign because pressure decreases on going upwards in Earth's atmosphere.

Also from Ideal gas equation we can derive that

$$PM = \rho RT$$

where M is molecular mass.

$$\therefore \rho = \frac{PM}{RT}$$

$$\therefore \frac{dP}{dh} = -\frac{PM g}{RT} \quad \text{—(11)}$$

At sea level i.e. at $h=0$, the pressure is $P_0 = 1\,\text{atm}$.

Integrating both sides of (11) with proper limits

$$\int_{P_0}^{P} \frac{dP}{P} = -\frac{Mg}{RT}\int_0^h dh \qquad \left(\begin{array}{l} g \text{ and } T \text{ is constant} \\ \text{at all elevations} \end{array}\right)$$

$$\ln P \Big|_{P_0}^{P} = \frac{-Mgh}{RT}$$

$$P = P_0 e^{\frac{-Mgh}{RT}}$$

Hence, pressure decreases exponentially with elevation.

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At the summit of Mount Everest

$h = 8848 \text{ m}$

$M = 28.97 \text{ g/mol}$

$$\therefore \frac{Mgh}{RT} = \frac{28.97 \times 10^{-3} \times 9.8 \times 8848}{8.314 \times 273}$$

$$= 1106.7 \times 10^{-3}$$

$$= 1.1$$

$$\therefore p = P_0 e^{-1.1}$$

$$= 1 \text{ atm} \times (0.3328)$$

$$= 0.3328 \text{ atm}.$$

3 (a) What is the average translational kinetic energy of an ideal gas molecule at 27°C. (b) What is the total random translational kinetic energy of molecules in 1 mole of this gas?

(a) Average translational kinetic energy of a molecule is given by $\frac{3}{2} k_b T$ where k_b is Boltzmann constant

Here, $T = 300 K$.

$$\therefore \text{Av. translational KE} = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 \text{ J}$$

$$= 621 \times 10^{-23} \text{ J}$$

$$= 6.21 \times 10^{-21} \text{ J}$$

b) total translational kinetic energy per mole of a gas is given by $\frac{3}{2} RT$.

where R is universal gas constant.

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$\therefore$ total random translational KE $= \frac{3}{2} RT$

$$= \frac{3}{2} \times 8.314 \times 300 \text{ J}$$

$$= 3741.3 \text{ J}$$

Hence, total random translation KE of 1 mole of gas is 3741.3 J.

4. As an ideal gas undergoes an isothermal (constant temperature) expansion at temperature T, its volume changes from V_1 to V_2. How much work does the gas do?

the work done by a gas during a process is given by

$$W = \int P \, dV.$$

from ideal gas equation

$$P = \frac{nRT}{V}$$

$$\therefore W = \int \frac{nRT}{V} dV$$

$\because$ process is isothermal, nRT is constant

$$\therefore W = nRT \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\boxed{W = nRT \ln\left(\frac{V_2}{V_1}\right)}$$

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Also, for an isothermal process

$$P_1 V_1 = P_2 V_2$$

i.e. $$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$\therefore \boxed{W = nRT \ln\left(\frac{P_1}{P_2}\right)}$$

5. You propose to climb several flights of stairs to work off energy you took in by eating a 900 Kcalorie hot fudge sundae. How high must you climb? Assume that your mass is 60 kg.

From first law of Thermodynamics,

$$Q = \Delta U + W$$

Since, temperature remains constant throughout,
$\Delta U = 0$

$$\therefore Q = W$$

given $Q = 900 \text{ kcal} = 900 \times 4184 \text{ J}$
and W = change in gravitational potential energy $= mgh$

$$\therefore h = \frac{Q}{mg} = \frac{900 \times 4184}{60 \times 9.8}$$

$$h = 6404 \text{ metres}$$

Hence, around 6404 metres high one must climb to work off 900 kcal of energy.

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6. A typical dorm room or bedroom contains about 2500 moles of air. Find the change in internal energy of this much air when it is cooled from 35°C to 26°C at constant pressure of 1 atm. Treat air as an ideal gas with $\gamma = 1.4$.

As we know,

$$C_p - C_v = R \quad \text{and} \quad \frac{C_p}{C_v} = \gamma$$

$$C_p - C_v = R$$

dividing by C_v

$$\frac{C_p}{C_v} - 1 = \frac{R}{C_v}$$

$$\gamma - 1 = \frac{R}{C_v}$$

$$C_v = \frac{R}{\gamma - 1}$$

$$\therefore C_v = \frac{8.314}{0.4} = 20.785 \text{ J/mol K}$$

Now, change in internal energy ΔU,

$$\Delta U = n C_v \Delta T$$

$$= 2500 \times 20.785 (26°C - 35°C)$$

$$= -467662.5 \text{ J}$$

$$= -467.66 \text{ kJ}$$

Hence, change in internal energy is equal to -467.66 kJ.

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7. A gasoline truck engine takes in 10000 J of heat and delivers 2000 J of mechanical work per cycle. The heat is obtained by burning gasoline with heat of combustion $L_c = 5 \times 10^4$ J/g

a) What is the thermal efficiency of this engine?
b) How much heat is discarded in each cycle?
c) If the engine goes through 25 cycles per second, what is its power output in watts? In horsepower?
d) How much gasoline is burned in each cycle?
e) How much gasoline is burned per second? Per hour?

a) Thermal efficiency of engine = $\dfrac{\text{Mechanical Work delievered}}{\text{Heat given}} \times 100\%$

$$= \frac{2000}{10000} \times 100\% = 20\%.$$

thermal efficiency of ~~heat~~ the engine is 20%.

b) heat discarded in each cycle = Heat given − Mechanical Work delievered

$$= (10000 - 2000)\text{ J}$$
$$= 8000\text{ J}$$

8000 J of heat is discarded in each cycle.

c) Engine delivers 2000 J of energy per cycle and goes through 25 cycles per second,

∴ Energy delieverd by the engine per second = 25 × 2000 J
= 50000 J

∴ Power output of engine in watts is = 50000 W

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and, power output of engine in horsepower is $= \frac{50000}{746}$

$= 67.02$ hp

d) The energy taken by gasoline truck engine is obtained by burning gasoline.

Energy taken by engine per cycle $= 10000$ J

Let the mass of gasoline burned per cycle be m.

$\therefore \quad m L_c = 10000$

$m \; 50000 \text{ J/g} = 10000$

$\therefore \quad m = 0.2$ g

$\therefore$ 0.2g of gasoline is burned in each cycle.

e) gasoline burned per second = gasoline burned in each cycle X no. of cycles per second

$= 0.2 \text{ g} \times 25$

$= 5$ g

$\therefore$ gasoline burned per hour $= 5 \text{ g} \times 3600$

$= 18000$ g

$= 18$ kg

$\therefore$ 18 kg of gasoline is burned per hour.

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8. A Carnot Engine takes 2000 J of heat from a reservoir at 500K, does some work, and discards some heat to reservoir at 350 K. How much work does it do, how much heat is discarded, and what is its efficiency?

$$\text{efficiency of Carnot engine} = \left(1 - \frac{\text{temp. of sink reservoir}}{\text{temp. of source reservoir}}\right) \times 100\%$$

$$= \left(1 - \frac{350}{500}\right) \times 100\%$$

$$= 30\%$$

$$\text{work done by the heat engine is} = \text{heat taken} \times \text{efficiency}$$

$$= 2000 \times \frac{30}{100}$$

$$= 600 J$$

$$\therefore \text{heat discarded} = (2000 - 600) J$$

$$= 1400 J$$

9. One kilogram of water at $0^\circ C$ is heated to $100^\circ C$. Compute its change in entropy. Assume that the specific heat of water is constant over this temperature range.

Heat required to change the temperature of m kg of water by dT temperature is given by,

$$dQ = m s \, dT$$

s is specific heat capacity of water

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Change in entropy (dS) is given as

$$dS = \frac{dQ}{T}$$

$$\therefore \quad dS = \frac{ms\, dT}{T}$$

$$\therefore \quad \int_0^{\Delta S} dS = ms \int_{273}^{373} \frac{dT}{T}$$

$$\therefore \quad \Delta S = (1\ kg)\left(4190\ \frac{J}{kg \cdot K}\right)\left(\ln T\right)\Big|_{273}^{373}$$

$$\Delta S = 4190 \ln\left(\frac{373}{273}\right) J/K$$

$$\Delta S = 4190 \times 0.3121\ J/K$$

$$= 1307.72\ J/K$$

$$= 1.307 \times 10^3\ J/K$$

Hence, the change in entropy associated with the given change is $1.307 \times 10^3\ J/K$.

10. A gas expands adiabatically and reversibly. What is its change in entropy?

For a reversible adiabatic process, $dQ = 0$

$$\because \quad dS = \frac{dQ}{T}$$

$\therefore$ dS (change in entropy) for reversible adiabatic process is 0.