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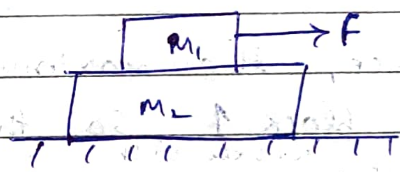
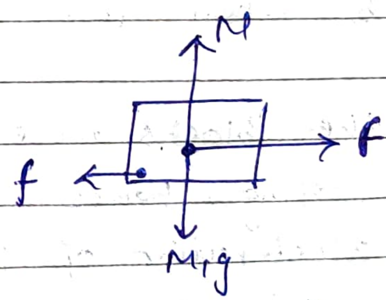
PH100 : Mechanics and Thermodynamics
Tutorial #03

1. A block of mass M_1 rests on a block of mass M_2 which lies on a horizontal ~~table~~ frictionless table. The coefficient of friction between the block is μ. What is the maximum horizontal force which can be applied to the blocks for them to accelerate without slipping on one another if force is applied to
(a) block 1 (b) block 2 ?

(a) when force is applied to block 1,



Since, both blocks are to slip without slipping on each other, acceleration of block will be equal and equal to $a = \frac{F}{M_1+M_2}$.



Applying equation of motion for block 1 in horizontal direction

$$F - f = \frac{M_1 F}{M_1+M_2} \quad \text{—(1)}$$

and in vertical direction,

$$N - M_1 g = 0$$
$$N = M_1 g$$

the friction f will be maximum to find maximum force F,

hence $f = \mu N$

$f = \mu M_1 g$

Putting $f = \mu M_1 g$ in (1),

$$F - \mu M_1 g = \frac{M_1 F}{M_1 + M_2}$$

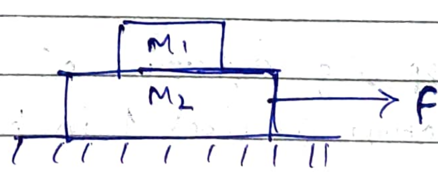
$$\frac{M_2 F}{M_1 + M_2} = \mu M_1 g$$

$$F = \frac{\mu M_1 (M_1 + M_2) g}{M_2}$$

Hence, the maximum force that can be applied on block 1 so that the blocks move without slipping w.r.t each other is $\frac{\mu M_1 (M_1+M_2) g}{M_2}$

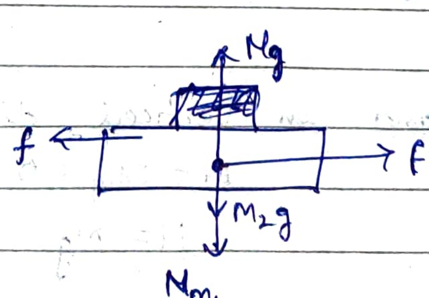
b) when force is applied on block 2.

the blocks move without slipping, hence acceleration of blocks will be equal and given by $a = \frac{F}{M_1 + M_2}$



N_{m_1} is normal on block M_2 due to M_1 and N_g is normal due to ground

$N_{m_1} = M_1 g$



$$N_g = (M_1 + M_2)g$$

Since, we have to find maximum force, friction should act at its maximum value

$$\therefore \quad f = \mu N_{m_1} = \mu M_1 g$$

Applying Newton's second law for block 2 in x-direction,

$$F - f = M_2 a$$

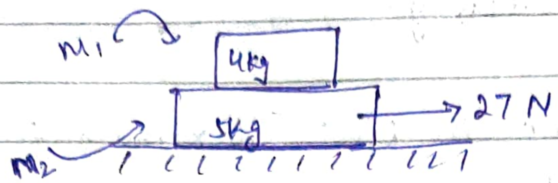
$$F - \mu M_1 g = \frac{M_2 F}{M_1 + M_2}$$

$$\frac{\cancel{M_1} F}{M_1 + M_2} = \mu \cancel{M_1} g$$

$$F = \mu (M_1 + M_2) g$$

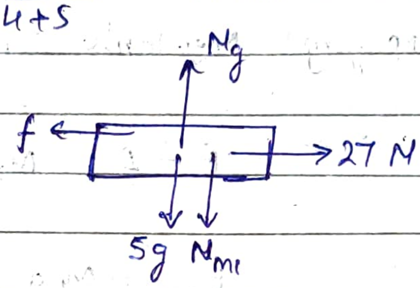
Hence, maximum force that can be applied on block 2 such that both block moves without slipping w.r.t each other is $\mu(M_1 + M_2)g$.

2. A 4-kg block rests on top of a 5-kg block, which rests on a frictionless table. The coefficient of friction between the two blocks is such that the block start to slip when the horizontal force F applied to lower block is 27 N. Suppose that a horizontal force is now applied only to the upper block. What is the maximum value(F) for the blocks to slide ~~relative~~ without slipping relative to each other?



In this situation, the block just starts to slip relative to each other. Hence, acceleration of both blocks is equal to $\frac{27}{4+5} = 3\ m/s^2$.

For 5kg block



Using Newton's Second Law in both directions,

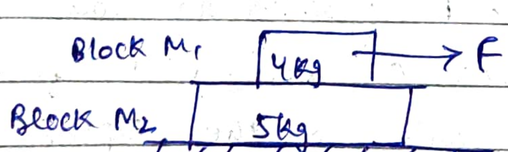
$$Ng = 5g + N_{m_1} \quad \text{—} (1)$$
$$27 - f = 5 \times 3 \quad \text{—} (2)$$
$$f = 12\ N$$

Since the block just starts to slip relative to each other, friction will be maximum

$$f = \mu N_{m_1} = \mu \times M_1 g = 12$$

$$\mu \times 4 \times 10 = 12$$
$$\mu = 0.3$$

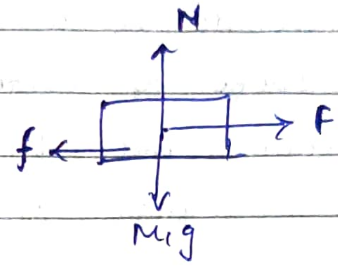
Now, a force F is applied to 4 kg block



The blocks should not slide w.r.t each other, hence acceleration of blocks is

$$a = \frac{F}{4+5} = \frac{F}{9}\ m/s^2$$

<u>For 4kg block</u>



Applying Newton's second law in vertical direction

$$N = M_1 g$$

and in horizontal direction

$$F - f = M_1 a$$

$$F - f = \frac{M_1 F}{9}$$

$$F - f = \frac{4F}{9}$$

$$F = \frac{9f}{5}$$

for maximum force, the friction acting will be maximum

$$\therefore\ f = 0.3 \times 4 \times 10$$
$$= 12\ N$$

$$\therefore\ F = \frac{9}{5} \times 12 = 21.6\ N$$

Hence, maximum value of horizontal force applied to 4kg block such that both blocks slide without slipping w.r.t each other is 21.6 N.

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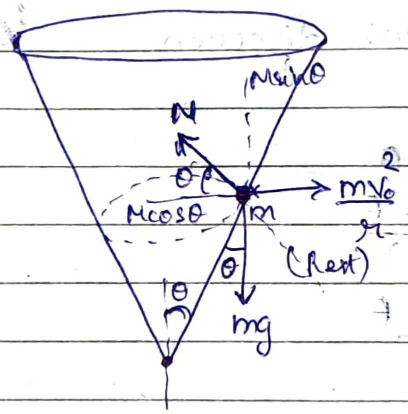
3. A particle of mass m slides without friction on the inside of a cone. The axis of the cone is vertical, and gravity is directed downwards. The apex-half angle of cone is θ as shown. The path of the particle happens to be a circle in horizontal plane. The speed of particle is v_0. Draw a force diagram and find the radius of the circular path in terms of v_0, g and θ.

Let the particle is moving in horizontal circle of radius r.



Suppose the particle is at a position specified in figure, At this time the velocity v_0 is inside the plane of paper (~~X~~) and perpendicular to it.

The forces acting are shown in fig. (from the frame of reference of mass).

Since, we are observing from the frame of reference of mass. Hence, there mass will be at rest in this frame.

∴ Net force on mass will be zero.

$\therefore \quad \Sigma F_x = 0$

$$\frac{mv^2}{r} - N\cos\theta = 0$$

$$N\cos\theta = \frac{mv^2}{r} \quad \text{--- (I)}$$

and, $\Sigma F_y = 0$

$$N\sin\theta - mg = 0$$

$$N\sin\theta = mg \quad \text{--- (II)}$$

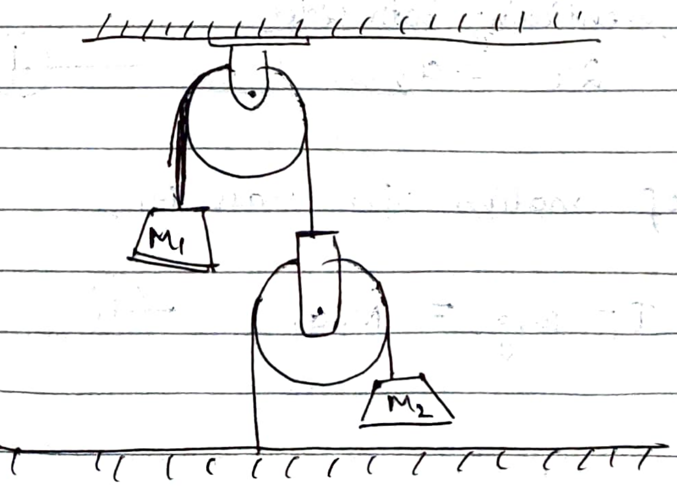
Dividing (II) by (I),

$$\tan\theta = \frac{mgr}{mv_0^2}$$

$$r = \frac{v_0^2 \tan\theta}{g}$$

Hence, radius of the circle in which particle is moving is $\frac{v_0^2 \tan\theta}{g}$.

4. Masses M_1 and M_2 are connected to a system of strings and pulleys as shown. The strings are massless and inextensible, and the pulleys are massless and frictionless. Find the acceleration of M_1.



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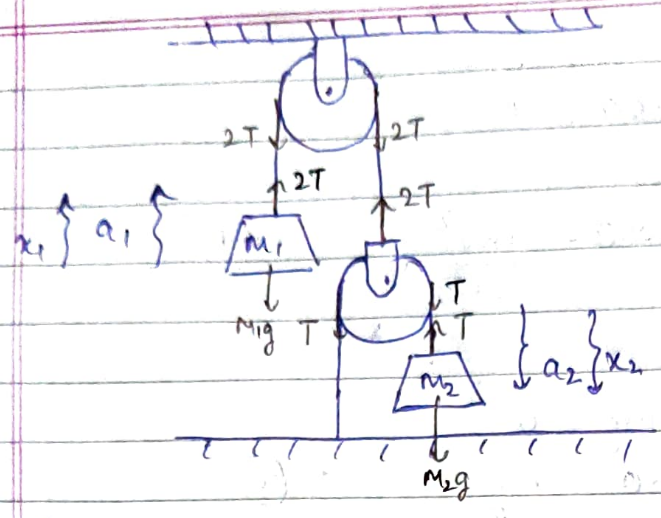


Let us assume that acceleration of M_1 is a_1 upwards and acceleration of M_2 is a_2 downwards.

Let tension in string connected to M_2 is T. The tension in different strings is mentioned.

Since work done by all the internal forces is zero. and only internal force acting here is tension.
∴ Work done by tension is zero.

$$2T x_1 + T(-x_2) = 0$$
$$2Tx_1 - Tx_2 = 0$$
$$2x_1 = x_2$$

differentiating both sides

$$2v_1 = v_2$$

differentiating both sides

$$2a_1 = a_2 \quad \text{— (I)}$$

Equation of motion for mass M_1,

$$2T - M_1 g = M_1 a_1 \quad \text{— (II)}$$

Equation of motion for M_2,

$$M_2 g - T = M_2 a_2 \quad \text{— (III)}$$

Multipyling (III) by 2 and adding it to (II),

$$2M_2 g - M_1 g = M_1 a_1 + 2M_2 a_2$$

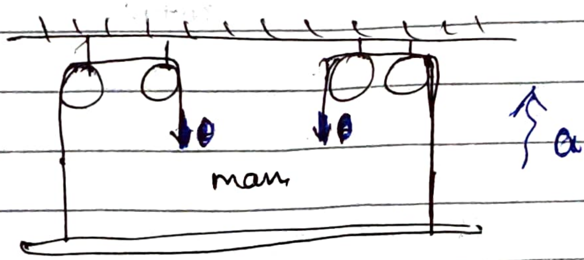
Put $a_2 = 2a_1$ in above equation,

$$(2M_2 - M_1) g = M_1 a_1 + 2M_2 (2a_1)$$
$$(2M_2 - M_1) g = (M_1 + 4M_2) a_1$$

$$\therefore \quad a_1 = \frac{(2M_2 - M_1) g}{(M_1 + 4M_2)}$$

Hence, the acceleration of M_1 is given by $\frac{(2M_2 - M_1) g}{(M_1 + 4M_2)}$, direction is specified in figure.

5. A painter of mass M stands on a platform of mass m and pulls himself up by two ropes which hang over a pulley, as shown. He pulls each rope with force F and accelerates ~~4~~ upward with uniform acceleration a. Find a - neglecting the fact that no one could do this for long.



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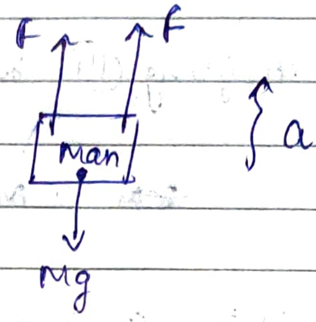
The man is pulling the ropes with force F, hence the ropes are applying a force F on the man in upward direction (Newton's third law)

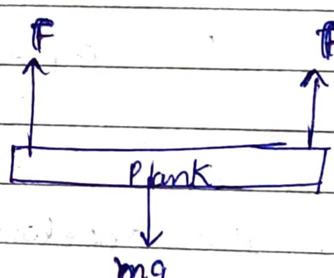
Equation of motion for man,



$$2F - Mg = Ma \quad \text{(I)}$$

The plank is facing two tension forces due to ropes. Since, tension in massless string is same at every point, therefore tension is equal to F.

$$2F - mg = ma \quad \text{(II)}$$



Adding (I) & (II),

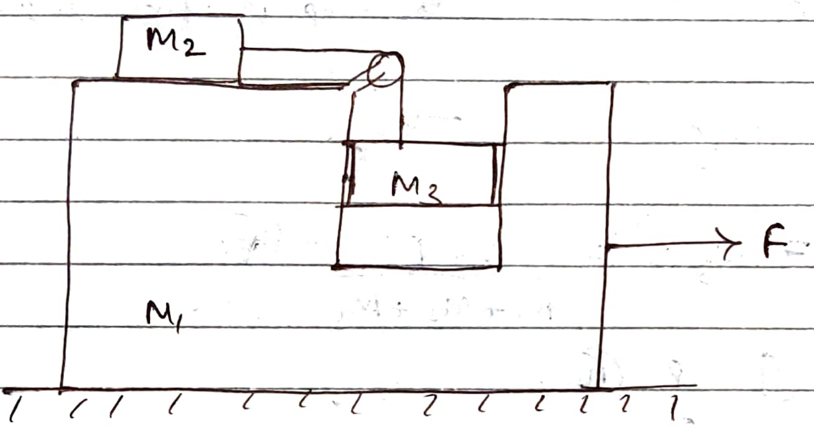
$$4F - (m+M)g = (m+M)a$$

$$\therefore \quad a = \frac{4F}{m+M} - g$$

Hence, acceleration of the plank and man is equal to $\frac{4F}{m+M} - g$

6. a) A "Pedagogical Machine" is illustrated in the sketch below. All surfaces are frictionless. What force F must be applied to M_1 to keep M_3 from rising or falling?

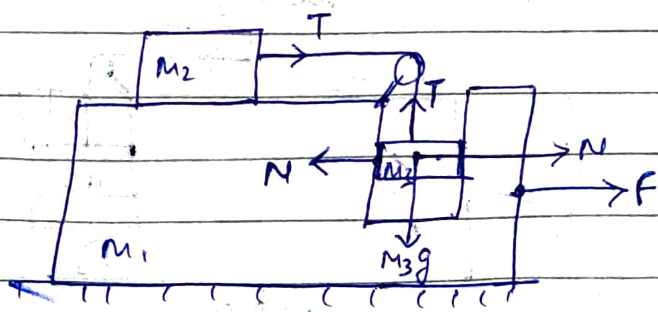
b) Consider the above case where F is zero. Find acceleration of M_1.



a) Since M_3 has to be kept from rising or falling. There will be no relative acceleration between M_1 and M_2. Therefore, the acceleration of the block is same and equal to

$$a = \frac{F}{M_1 + M_2 + M_3}$$

The force diagram of the system is given



For block m_1

$F - N = M_1 a$

$F - N = \frac{M_1 F}{M_1 + M_2 + M_3}$ —— (1) (N is normal on M_1 due to M_3)

For block M_2,

$$T = M_2 a$$

$$T = \frac{M_2 F}{M_1 + M_2 + M_3} \quad \text{--- (II)}$$

For block M_3,

$\Sigma F_y = 0$ (to keep it from rising or falling)

$$T = M_3 g \quad \text{--- (III)}$$

$$N = M_3 a = \frac{M_3 F}{M_1 + M_2 + M_3} \quad \text{--- (IV)}$$

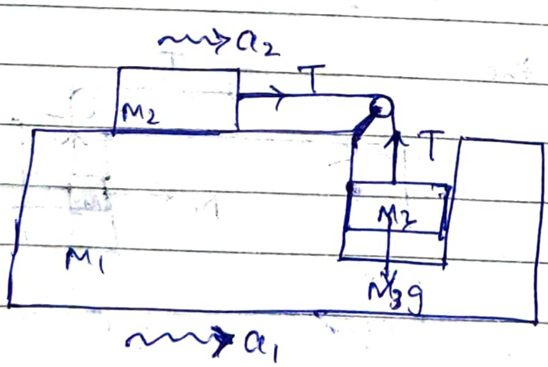
from (II) & (III),

$$\frac{M_2 F}{(M_1 + M_2 + M_3)} = M_3 g$$

$$F = \frac{M_3 (M_1 + M_2 + M_3) g}{M_2}$$

Hence, the force required to keep the block M_3 from rising or falling is $\frac{M_3 (M_1 + M_2 + M_3) g}{M_2}$

b)



Let acceleration of M_1 in horizontal direction be a_1 and let a_2 be the acceleration of M_2 relative to M_1.

$\therefore$ net acceleration of $M_2 = a_1 + a_2$

and, acceleration of M_3 relative to M_1 will be a_2 downwards and relatie to ground it will be a_1 in horizontal direction.

Since, there is no external horizontal force, the net force in horizontal direction must be zero.

$$M_1 a_1 + M_2(a_1 + a_2) + M_3 a_1 = 0 \quad \text{— (I)}$$

The only horizontal force on M_2 is tension;

$$T = M_2(a_1 + a_2) \quad \text{— (II)}$$

Equation of motion for M_3 in vertical direction,

$$M_3 g - T = M_3 a_2 \quad \text{— (III)}$$

Adding (II) & (III),

$$M_3 g = (M_2 + M_3) a_2 + M_2 a_1$$

$$a_2 = \frac{M_3 g - M_2 a_1}{M_2 + M_3}$$

Putting $a_2 = \frac{M_3 g - M_2 a_1}{M_2 + M_3}$ in (I)

$$(M_1 + M_2 + M_3) a_1 + M_2 \left(\frac{M_3 g - M_2 a_1}{M_2 + M_3}\right) = 0$$

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$$a_1\left(M_1+M_2+M_3-\frac{M_2^2}{M_2+M_3}\right) = \frac{-M_2M_3g}{M_2+M_3}$$

$$a_1\left(\frac{M_1M_2+M_1M_3+\cancel{M_2^2}+2M_2M_3+M_3^2-\cancel{M_2^2}}{\cancel{M_2+M_3}}\right) = \frac{-M_2M_3g}{\cancel{M_2+M_3}}$$

$$a_1 = \frac{-M_2M_3g}{M_1M_2+M_1M_3+2M_2M_3+M_3^2}$$

Hence, the acceleration of block 1 is given by $a_1 = \dfrac{-M_2M_3g}{M_1M_2+M_1M_3+2M_2M_3+M_3^2}$