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PH100 : Mechanics & Thermodynamics
Tutorial 7

1. A furnace has walls of temperature 1600°C. What is the wavelength of maximum intensity emitted when a small door is opened?

From Wien's Displacement Law,

$$\lambda_m = \frac{b}{T}$$

$\lambda_m \rightarrow$ maximum intensity wavelength
$T \rightarrow$ temperature in K
$b \rightarrow$ Wien's Constant

$$b = 2.898 \times 10^{-3} \text{ m K}$$

given $T = 1600°C = 1873 \text{ K}$

$$\therefore \lambda_m = \frac{2.898 \times 10^{-3}}{1873}$$

$$\lambda_m = 0.001547 \times 10^{-3} \text{ m}$$
$$= 1.547 \ \mu m$$

Hence, the wavelength corresponding to maximum intensity of radiation emitted is 1.547 µm.

2. Calculate max for blackbody radiation for (a) liquid helium (4.2 K) (b) room temperature (293 K) (c) a steel furnace (2500K) and, (d) a blue star (9000 K)

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Using Wien's displacement law

$$\lambda_m = \frac{b}{T}, \quad \text{where } b = 2.898 \times 10^{-3} \text{ mK}$$

a) $T = 4.2$ K

$$\therefore \lambda_m = \frac{2.898 \times 10^{-3}}{4.2}$$

$$= 0.69 \times 10^{-3} \text{ m}$$

$$= 690 \text{ um}$$

Hence, max intensity for blackbody radiation at $T = 4.2$ K corresponds to wavelength of 690 um.

b) $T = 293$ K

$$\lambda_m = \frac{2.898 \times 10^{-3}}{293}$$

$$\lambda_m = 0.00989 \times 10^{-3} \text{ m}$$

$$= 9.89 \text{ um}$$

Hence, wavelength corresponding to max intensity is 9.89 um.

c) $T = 2500$ K

$$\lambda m = \frac{2.898 \times 10^{-3}}{2500}$$

$$= 0.1159 \times 10^{-5}$$

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$$\lambda_m = 1.16\ \mu m$$

Hence, wavelength corresponding to maximum ~~wavelength~~ intensity is $1.16\ \mu m$.

d) $T = 9000\ K$

$$\lambda_m = \frac{2.898 \times 10^{-3}}{9000}$$

$$= 0.322 \times 10^{-6}$$

$$= 322\ nm$$

Hence, wavelength corresponding to max intensity is 322 nm.

3. Calculate the temperature of a blackbody if the spectral distribution peaks at
(a) gamma rays, $\lambda = 1.5 \times 10^{-14}\ m$
(b) X-rays, $\lambda = 1.5\ nm$
(c) red light, $\lambda = 640\ nm$
(d) broadcast TV waves, $\lambda = 1\ m$
and (e) AM radio waves, $\lambda = 204\ m$

Using wien's displacement law,

$$\lambda_m = \frac{b}{T}$$

$$T = \frac{b}{\lambda_m}$$, where $b = 2.898 \times 10^{-3}\ mK$.

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(a) gamma rays ($\lambda = 1.5 \times 10^{-14}$ m)

$$\therefore \quad T = \frac{2.898 \times 10^{-3}}{1.5 \times 10^{-14}}$$

$$T = 1.932 \times 10^{11} \text{ K}$$

$$T = 1.932 \times 10^{11} \text{ K}$$

Hence, temperature of blackbody is 1.932×10^{11} K.

(b) X-rays ($\lambda = 1.5$ nm)

$$\therefore \quad T = \frac{2.898 \times 10^{-3}}{1.5 \times 10^{-9}}$$

$$= 1.932 \times 10^{6} \text{ K}$$

Hence, temperature of blackbody is 1.932×10^{6} K.

(c) red light ($\lambda = 640$ nm)

$$\therefore \quad T = \frac{2.898 \times 10^{-3}}{640 \times 10^{-9}}$$

$$= 0.00453 \times 10^{6}$$

$$= 4530 \text{ K}$$

Hence, temperature of blackbody is 4530 K.

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d) broadcast TV waves ($\lambda = 1m$)

$$\therefore T = \frac{2.898 \times 10^{-3}}{1}$$

~~= 2.898~~

$$= 2.898 \times 10^{-3} K$$

Hence, temperature of blackbody is 2.898×10^{-3} K

e) AM radio waves ($\lambda = 204 m$)

$$\therefore T = \frac{2.898 \times 10^{-3}}{204}$$

$$= 0.0142 \times 10^{-3}$$

$$= 14.2 \times 10^{-6} K$$

Hence, the temperature of blackbody is 14.2×10^{-6} K.

4. The wavelength of maximum intensity of sun's radiation is observed to be near 500 nm. Assume the sun to be a blackbody and calculate

a) the sun's surface temperature

b) the power per unit area emitted from the sun's surface, and

c) Energy recieved by Earth each day from sun's radiation.

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a) Using Wien's displacement law,

$$T = \frac{b}{\lambda_m}$$

$$\therefore T = \frac{2.898 \times 10^{-3}}{500 \times 10^{-9}}$$

$$= 0.5796 \times 10^{4}$$

$$= 5796 \text{ K}$$

Hence, the Sun's surface temperature is 5796 K.

b) From Stefan-Boltzmann law,

$$E = e\sigma A T^4$$

E is power and e is emissivity, σ is Stefan's constant
For blackbody e = 1.

$\therefore$ power per unit area (I)

$$I = \frac{E}{A} = \sigma T^4$$

$$I = 5.67 \times 10^{-8} \times (5796)^4$$

$$= 6.399 \times 10^{7} \text{ W/m}^2$$

Hence, the power per unit area emitted from Sun's surface is 6.399×10^{7} W/m².

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c) Let d be the distance between Sun and Earth and R be the radius of Earth,

∴ power recieved at Earth due to sun's radiation $= \dfrac{\sigma T^4 (4\pi R^2)}{(4\pi d^2)}$

∴ $\dfrac{R}{d} = \dfrac{696340\,(km)}{1.496 \times 10^8\,(km)} = 4.65 \times 10^{-3}$

∴ Power recieved at Earth $= 5.67 \times 10^{-8} \times (5796)^4 \times (4.65 \times 10^{-3})^2$

$= 1383.62$ W

∴ Energy recieved by Earth in one day

$= 1383.62 \times 24 \times 60 \times 60$
$= 1.195 \times 10^8$ J

Hence, Earth recieves 1.195×10^8 J of energy from sun's radiation each day.

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5. (a) A blackbody's temperature is increased from 900 K to 2300 K. By what factor does the total power radiated per unit area increase?

b) If the original temperature is again 900 K, what final temperature is required to double the power output?

(a) By Stefan's - Boltzmann law, power radiated (E) is given by,

$$E = e\sigma A T^4$$

$\therefore$ power radiated per unit area (I),

$$I = \frac{E}{A} = e\sigma T^4$$

for blackbody $e = 1$,

$$\therefore I = \sigma T^4$$

Now, $T_1 = 900K$ and $T_2 = 2300 K$.

$$\therefore \frac{I_2}{I_1} = \frac{\cancel{\sigma} T_2^4}{\cancel{\sigma} T_1^4}$$

$$\frac{I_2}{I_1} = \frac{(2300)^4}{(900)^4}$$

$$I_2 = \left(\frac{23}{9}\right)^4 I_1$$

$$I_2 = 42.652\, I_1$$

Hence, total power radiated per unit area

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increased by a factor of 42.652 when temperature of blackbody is increased from 900K to 2300 K.

(b) Let power per unit area be I_1 at 900K. and power per unit area be $2I_1$ at T_2 K.

$$\therefore \quad 2I_1 = \sigma T_2^4 \quad \text{—— (I)}$$

$$\text{and,} \quad I_1 = \sigma (900)^4 \quad \text{—— (II)}$$

(I)/(II)

$$2 = \left(\frac{T_2}{900}\right)^4$$

$$\frac{T_2}{900} = 1.1892$$

$$T_2 = 1070.286 \text{ K}$$

Hence, ~~the~~ final temperature of 1070.286 K is required to double the power output.

6. a) At what wavelength will the human body radiate the maximum radiation? (b) Estimate the total power radiated by a person of medium build (assume an area given by a cylinder of 175 cm height and 13 cm radius). (c) Using your answer to (b), compare the energy radiated by a person in one day with energy intake

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of a 2000 - kcal diet.

a) The normal temperature of human body is 310 K.

Using wien's displacement law,

$$\lambda_m = \frac{b}{T}$$

$$= \frac{2.898 \times 10^{-3}}{310}$$

$$= 0.00935 \times 10^{-3}$$

$$= 9.35 \times 10^{-6}\ m$$

$$= 9.35\ \mu m$$

Hence, at a wavelength of 9.35 um human body will radiate the maximum radiation.

b) Using Stefan's - Boltzmann law,

$$E = e \sigma A T^4$$

for black body $e = 1$,
Since, the body is considered to be a cylinder
$A = 2\pi r (h + r)$

where h and r are height and radius of cylinder respectively.

$$\therefore E = \sigma \{2\pi r (r+h)\} T^4$$

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$= 5.67 \times 10^{-8} \{2 \times 3.14 \times 13 \times 10^{-2} (188 \times 10^{-2})\} (310)^4$

$= 5.67 \times 6.28 \times 13 \times 188 \times (31)^4 \times 10^{-8}$

$= 8.037 \times 10^{10} \times 10^{-8}$

$= 803.7 \text{ W}$

Hence, the total power radiated is 803.7 W.

(c) Energy radiated by the person in one-day (u)

$u = Et$

$= 803.7 \times 24 \times 60 \times 60$

$= 6.944 \times 10^{7} \text{ J}$

If person takes 2000 kcal per day,

$\because$ 1 Kcal = 4184 J

$\therefore$ energy taken per day = 2000×4184 J

$= 8.368 \times 10^{6}$ J

Hence, the person radiates more energy in one day than his/her intake in one day.

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7. If we have waves in a one-dimensional box, such that the wave displacement $\psi(x,t) = 0$ for $x=0$ and $x=L$, where L is the length of the box, and

$$\frac{1}{c}\frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi}{\partial x^2} = 0 \quad \text{(wave equation)}$$

Show that the solutions are of the form

$$\psi(x,t) = a(t)\sin\left(\frac{n\pi x}{L}\right) \quad (n = 1,2,3 \ldots)$$

and $a(t)$ satisfies the (harmonic oscillator) equation

$$\frac{d^2 a(t)}{dt^2} + \omega_n^2 a(t) = 0$$

where $\omega_n = \frac{n\pi c}{L}$ is angular frequency $2\pi f$.

The given wave equation

$$\frac{1}{c}\frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi}{\partial x^2} = 0 \quad \text{--- (1)}$$

Let the solution be

$$\psi(x,t) = a(t)\sin(kx)$$

given $\psi(x,t) = 0$ for $x=0$ and $x=L$

for $x = L$

$$\therefore a(t)\sin(kL) = 0$$

$$\sin(kL) = 0$$

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$$KL = n\pi$$

$$\therefore K = \frac{n\pi}{L}$$

Hence, solution is $\psi(x,t) = a(t)\sin\left(\frac{n\pi x}{L}\right)$ — (11)

Substitute (11) in (1)

$$\frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = \frac{1}{c^2}\left(\frac{\partial^2 a(t)}{\partial t^2}\right)\sin\left(\frac{n\pi x}{L}\right)$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2}{\partial x^2}\left[a(t)\sin\left(\frac{n\pi x}{L}\right)\right]$$

$$= -a(t)\left(\frac{n\pi}{L}\right)^2 \sin\left(\frac{n\pi x}{L}\right)$$

(1) now changes to

$$\frac{1}{c^2}\left[\frac{\partial^2 a(t)}{\partial t^2}\right]\sin\left(\frac{n\pi x}{L}\right) + a(t)\left(\frac{n\pi}{L}\right)^2\sin\left(\frac{n\pi x}{L}\right) = 0$$

$$\Rightarrow \frac{\partial^2 a(t)}{\partial t^2} + a(t)\left[\frac{n\pi c}{L}\right]^2 = 0$$

$$\frac{\partial^2 a(t)}{\partial t^2} + \omega_n^2 a(t) = 0$$

Hence, proved.