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MA102 : Introduction to Discrete Mathematics

Tutorial 4

① Let $P(x) : x = x^2$ be a one place predicate with domain of x equal to $\mathbb{Z}$. What are the truth values of the following propositions?

a) $P(0)$
b) $\exists x \, P(x)$
c) $\neg(\forall x \, P(x))$

(a) True, as $0 = (0)^2$

(b) True, as for $x = 1$, $x = x^2$ is true, so there exists at least one $x \in \mathbb{Z}$ such that $x = x^2$.

(c) True.

$\because$ ~~for all $x \geq 2$~~

$\because$ for $x = 2$, $x^2 = 4$

$\therefore x \neq x^2$.

$\therefore \forall x \, P(x)$ is false when x has a domain of $\mathbb{Z}$.

$\therefore \neg \forall x \, P(x)$ is true.

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② Determine whether each of these arguements is correct, is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs?

(a) If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$.

(b) If n is a real number with $n > 3$, then $n^2 > 9$. Suppose that $n^2 \leq 9$. Then $n < 3$

(c) If n is a real number with $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then $n^2 \leq 4$.

(d) If x is a positive real number, then x^2 is a positive number. Therefore, if a^2 is positive, where a is real number, then a is a positive real number.

(e) If $x^2 \neq 0$, where x is a real number, then $x \neq 0$. Let a be a real number with $a^2 \neq 0$, then $a \neq 0$.

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(a) This is an incorrect (Invalid) argument.

Let say $n^2 = 9$, then $n = \pm 3$. Since -3 is less than 1, the argument is invalid.

p : n is a real number greater than 1.
q : $n^2 \geq 1$

This argument has the form of

$$((p \to q) \wedge q) \to p$$

Hence, the logical error in this is fallacy of affirming the conclusion.

(b) This is a valid argument.

Lets say $n^2 \leq 9$
$\therefore -3 \leq n \leq 3$

which is $n \leq 3$

let say the domain of n is $\mathbb{R}$.

p : $n > 3$
q : $n^2 > 9$

Then the argument becomes,

$$((p \to q) \wedge \neg q) \to \neg p$$

Hence, the rule of inference used here is Modus Tollens.

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(c) This is an invalid argument.

Let say $n = -5$ (which is less than 2)
then, $n^2 = 25$
Since 25 is greater than 4, the argument becomes invalid.

Now assume the domain of n to be R.

p: $n > 2$
q: $n^2 > 4$

then the argument becomes

$$((p \rightarrow q) \land \neg p) \rightarrow \neg q$$

Hence, the logical error is fallacy of denying the hypothesis.

(d) This is an invalid argument.
Let say $a^2 = 25$, then $a = \pm 5$
Since, -5 is not a positive real number, the argument is invalid.

Now, domain of discourse of x is R.

p: $x > 0$
q: $x^2 > 0$

then the argument becomes
$$((p \rightarrow q) \land q) \rightarrow p$$

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Hence, the logical error is fallacy of affirming the conclusion.

(e) This is a valid argument

Since, domain of discourse of x is real number.

$p: x^2 \neq 0$
$q: x \neq 0$

then the argument becomes

$$\cancel{\begin{array}{l}\forall x\,(p(x) \to q(x)) \\ \hline \therefore\ p(a) \to q(a)\end{array}}$$

$$\begin{array}{l}\forall x\,(P(x) \to Q(x)) \\ P(a)\text{, where a is in domain} \\ \hline \therefore\ Q(a)\end{array}$$

Hence, the rule of inference used here is Universal Instantiation and Modus ponens.

3. For each of these argument, tell whether the argument is correct or incorrect and explain why.

(a) All students in this class understand logic.
Sachin is a student in this class.
Therefore, Sachin understands logic.

$P(x)$: x is a student in this class.
$Q(x)$: x understands logic

the domain of x is all the students in this

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class.

The given statement can be written as,

$\forall x\,(P(x) \to Q(x))$: All students in this class understand logic

$\therefore$ the argument becomes,

$$\begin{array}{l} \forall x\,(P(x) \to Q(x)) \\ \underline{P(\text{Sachin})} \\ \therefore\ Q(\text{Sachin}) \end{array}$$

Hence, the argument is correct and the rule of inference used is Universal Instantiation and Modus Ponens

(b) Every Computer Science student takes discrete mathematics. Sachin is taking discrete mathematics. Therefore, Sachin is a computer science student.

This ~~is~~ argument is incorrect. Every CS student takes discrete mathematics but not every discrete mathematics student may not be a CS student.

$P(x)$: x is a CS Student
$Q(x)$: x takes Discrete Mathematics

~~x is~~ domain of discourse of x is all the students.

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Now,

$\forall x\,(P(x) \rightarrow Q(x))$ means that "Every CS student takes discrete mathematics"

So, the argeement becomes

$$\begin{array}{l} \forall x\,(P(x) \rightarrow Q(x)) \\ Q(\text{Sachin}) \\ \hline \therefore P(\text{Sachin}) \end{array}$$

So, after applying Universal Instantiation it contains the fallacy of affirming the conclusion.

In simple words, Sachin can take discrete mathematics even if he is not a CS student.

(c) All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.

This argeument is incorrect. There can be some pet bird other than parrot which likes fruit.

$P(x)$: x is a parrot.
$Q(x)$: x likes fruit

domain of discourse of x is all pet birds.

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then the argument becomes

$$\forall x \,(P(x) \rightarrow Q(x))$$
$$\neg P(a)$$
$$\therefore \neg Q(a)$$

After applying the Universal Instantiation, it contains the fallacy of denying the hypothesis.

(d) Everyone who eats an apple every day is healthy. Superman is not healthy. Therefore, Superman does not eat an apple every day.

Let us assume

$P(x)$ = "x eats apple every day"
$Q(x)$ = "x is healthy"

therefore, the argument becomes

P1 : $\forall x \,(P(x) \rightarrow Q(x))$
P2 : $\neg Q(\text{Superman})$
C $\neg P(\text{Superman})$

	Step	Reason
1.	$\forall x (P(x) \rightarrow Q(x))$	Premise 1
2.	$P(\text{Superman}) \rightarrow Q(\text{Superman})$	Universal Instantiation from 1
3.	$\neg Q(\text{Superman})$	Premise 2
4.	$\neg P(\text{Superman})$	Modus Tollens from (2) and (3)

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4. Determine whether $\forall x (P(x) \vee Q(x))$ and $\forall x\, P(x) \vee \forall x\, Q(x)$ are logically equivalent.

Let $P(x)$: x is odd.
and $Q(x)$: x is even.

domain of discourse of x is all positive intergers.

Since, every positive integer is either odd or even. There fore $\forall x (P(x) \vee Q(x))$ is true.

and since, every integer is not odd

$\therefore$ $\forall x\, P(x)$ is false
and similarly $\forall x\, Q(x)$ is false.

$\therefore$ $\forall x\, P(x) \vee \forall x\, Q(x)$ is false.

Hence, we have a counterexample for

$$\forall x (P(x) \vee Q(x)) \equiv \forall x\, P(x) \vee \forall x\, Q(x)$$

Hence, they are not logically equivalent.

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5. Determine whether $\forall x \, (P(x) \rightarrow Q(x))$ and $\forall x \, P(x) \rightarrow \forall x \, (Q(x))$ are logically equivalent.

Assume that
$P(x)$: " x is a multiple of 5"
$Q(x)$: " x is a multiple of 10"

domain of discourse of x is all positive integers.

Now, if $x = 25$, $P(25)$ is true but $Q(25)$ is false.
$\therefore$ $P(25) \rightarrow Q(25)$ is false.
$\therefore$ $\forall x \, (P(x) \rightarrow Q(x))$ is false. — (I)

Now, since every positive integer is not divisible by 5.
$\therefore$ $\forall x \, P(x)$ is false.

$\therefore$ $\forall x \, P(x) \rightarrow \forall x \, (Q(x))$ is true — (II)

from (I) & (II), we have a counterexample and hence we can say that

$\forall x \, (P(x) \rightarrow Q(x))$ and $\forall x \, P(x) \rightarrow \forall x \, Q(x)$ are not logically equivalent.

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6. Show that $\exists x (P(x) \vee Q(x))$ and $\exists x (P(x)) \vee \exists x (Q(x))$ are logically equivalent.

Let $\exists x (P(x) \vee Q(x))$ be true, then there exists a value y such that $P(y) \vee Q(y)$ is true and thus $P(y)$ is true or $Q(y)$ is true. Then $\exists x\, P(x)$ is true or $\exists x\, Q(x)$ is true, which means that

$\exists x\, P(x) \vee \exists x\, Q(x)$ is true.

Let $\exists x (P(x) \vee Q(x))$ be false. This means that for all y, $P(y) \vee Q(y)$ is false. Thus, for all y $P(y)$ is false and $Q(y)$ is false. Then $\exists x\, P(x)$ is false and $\exists x\, Q(x)$ is false.

$\therefore$ $\exists x\, P(x) \vee \exists x\, Q(x)$ is false.

Hence, the two expressions are logically equivalent.

7. $\exists! x\, P(x)$ denotes "there exists a unique x such that $P(x)$ is true". If the domain consists of all integers, what is the truth value of the following statement? $\exists! x (x > 1)$.

The truth value of $\exists! x (x > 1)$ will be false.

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As all the integers greater than 1 will make the proposition ~~$\exists!$~~ $x \geq 1$ true, there are more than one value of x which makes $x > 1$ true. Hence, there is no unique (exactly one) value of x.

Hence, $\exists!\, x\,(x > 1)$ is false in the given domain.

8. Let $S(x)$ be the predicate "x is a student" $F(x)$ be the predicate "x is a faculty member" and $A(x, y)$ be the predicate "x has asked y a question", where the domain consists of all people associated with IIIT V. Use quantifiers to express each of these statements.

(a) Every student has asked Professor Rosen a question.

We can rewrite the statement as,

If a person is student, then the person has asked ~~pro~~ Professor Rosen a question.

∴ the expression will ~~become~~ be

$$\forall x\,(S(x) \rightarrow A(x, \text{Professor Rosen}))$$

domain of discourse of x is every person associated with IIITV.

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(b) Some student has not asked any faculty member a question.

The statement can be rewritten as,

There exists a person who is a student and who has not asked any question to a person who is a teacher.

Therefore, the expression will be,

$$\exists x \left(S(x) \land \neg \exists y \left(F(y) \land A(x, y) \right) \right)$$

(c) There is a faculty member who has asked every other faculty member a question.

The statement can be written as,

There is a person who is a faculty member and who has asked every other person, who also is a faculty member, a question.

Therefore, the expression will be

$$\exists x \left(F(x) \land \left(\forall y \left(F(y) \land (y \neq x) \rightarrow A(x, y) \right) \right) \right)$$

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9. Use logical and mathematical operators, quantifiers and predicates with more than one variable to express these statements.

(a) Some student in this class has visited Manali but not visited Ladakh.

Let $S(x)$ be the predicate: "x is a student in this class"

and $V(x, y)$ be the predicate: "x has visited y"

Then the given statement can be represented as,

$$\exists x \left(S(x) \wedge \left(V(x, \text{Manali}) \wedge \neg V(x, \text{Ladakh}) \right) \right)$$

where ~~x is the~~ domain of x is all people.

(b) The sum of two negative integers is negative

Let the domain consists of all integers.

then the statement can be represented as

$$\forall x \forall y \left((x < 0) \wedge (y < 0) \rightarrow (x + y < 0) \right)$$

10. Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consist of all real numbers. Do they have same meaning?

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(a) $\exists x \, \forall y \, (x + y = y)$

(b) $\forall y \, \exists x \, (x + y = y)$

~~(a) for every rea~~

(a) There exists a real x such that for all real y, $x + y = y$.

(b) For all real y, there exists a real x such that $x + y = y$.

The meaning of both the statements is same in this case.

But in general,

$\exists x \, \forall y \, (P(x, y))$ is not logically equivalent to $\forall y \, \exists x \, (P(x, y))$