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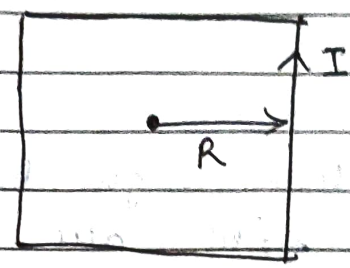
2020S2307

PH110: Waves and Electromagnetics

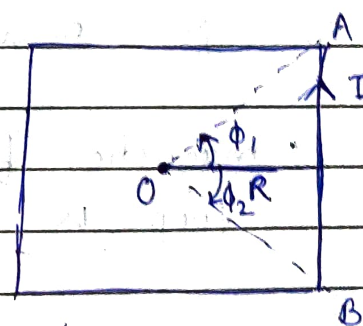
Tutorial 9

Problem 5.8: (a) Find the magnetic field at the center of a square loop, which carries a steady current I. Let R be the distance from centre to side



(b) Find the field at the center of a regular n-sided polygon, carrying a steady current. Again, let R be the distance from centre to any side.

(c) Check that your formula reduces to the field at the center of circular loop, in the limit $n \to \infty$.



(a) Let us calculate the magnetic field at the centre due to the wire AB.

Since, the magnetic field due to a straight wire at a point is given by,

$$B = \frac{\mu_0 I}{4\pi R}\left(\sin\phi_1 + \sin\phi_2\right)$$

In this case, $\phi_1 = 45^\circ$ and $\phi_2 = +45^\circ$

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$\therefore$ magnetic field at centre O due to wire AB, is,

$$B_{OAB} = \frac{\mu_0 I}{4\pi R}\left(\sin 45^\circ + \sin 45^\circ\right)$$

$$B_{OAB} = \frac{\mu_0 I}{4\pi R}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= \frac{\mu_0 I}{2\sqrt{2} R\pi}$$

Using right hand rule, the direction of magnetic field due to wire AB is outside the plane of paper.

By simple observation, the direction and magnitude of magnetic field at centre of loop due to all other sides is same.

$\therefore$ magnetic field at the centre of square loop is,

$$B = 4 \times B_{OAB}$$

$$\boxed{B = \frac{\sqrt{2}\,\mu_0 I}{\pi R}}$$

and the direction of this field is outside the plane of paper.

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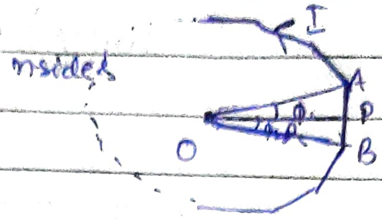
(b) Since it is a n-sided polygon,



$$\phi_1 + \phi_2 = \frac{2\pi}{n}$$

and since OP is bisector of $\angle AOB$,

$$\phi_1 = \phi_2 = \frac{\pi}{n}$$

$\therefore$ magnetic field at centre due to wire AB,

$$B_{OAB} = \frac{\mu_0 I}{4\pi R}\left(\sin\frac{\pi}{n} + \sin\frac{\pi}{n}\right)$$

$$B_{OAB} = \frac{\mu_0 I}{2\pi R}\sin\left(\frac{\pi}{n}\right)$$

$\therefore$ magnetic field at center due to n-sided polygon,

$$\boxed{B_0 = \frac{n\mu_0 I}{2\pi R}\sin\left(\frac{\pi}{n}\right)}$$

the direction of magnetic field is outside the plane of paper.

(c) when $n \to \infty$, $\frac{\pi}{n} \to 0$, i.e. $\frac{\pi}{n}$ is very small.

Since, for very small θ, $\sin\theta \simeq \theta$

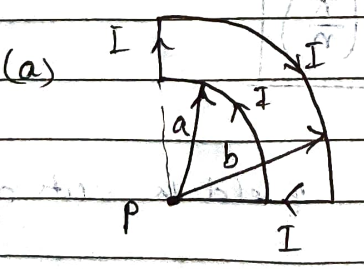
$$\therefore \sin\left(\frac{\pi}{n}\right) \approx \frac{\pi}{n}$$

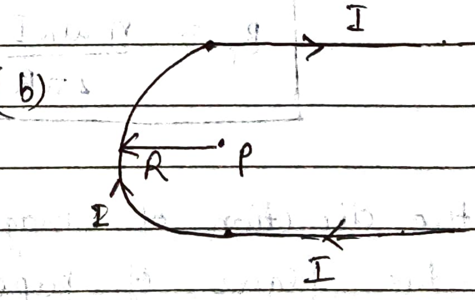
$$\therefore B_0 = \frac{\cancel{n} \mu_0 I}{2 \cancel{\pi} R} \frac{\cancel{\pi}}{\cancel{n}}$$

$$B_0 = \frac{\mu_0 I}{2R}$$

Since, for a n-sided polygon can be assumed to be a circle when $n \to \infty$. and the magnetic field at the center of a circular coil is known to be equal to $\frac{\mu_0 I}{2R}$. Hence, our formula reduces to field at center of circular loop

Problem 5.9. Find the magnetic field at point P for each of the steady ~~state~~ current configurations

(a) 

(b) 

(a) Let us first find the magnetic field at the centre of an arc carrying current I, of radius R

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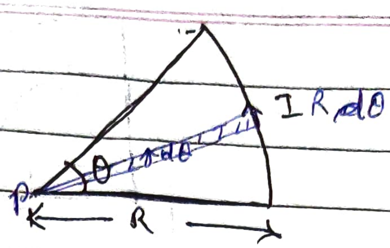
Assume the element $d\theta$, the arc length corresponding to $d\theta$ is $R\,d\theta$.



$\therefore \quad \vec{dl} = R\,d\vec{\theta}\;\hat{\theta}$

$\therefore \quad \vec{dl} \times \vec{R} = dl \cdot R \qquad (\because \phi = 90^\circ)$

$\therefore$ magnetic field at centre due to this $d\theta$ element is,

$$dB = \frac{\mu_0}{4\pi}\,\frac{I\,R\,d\theta}{R^2}$$

$$dB = \frac{\mu_0 I\,d\theta}{4\pi R}$$

$\therefore$ magnetic field at the centre due to arc is,

$$\int dB = \int_0^{\theta} \frac{\mu_0 I\,d\theta}{4\pi R}$$

$$\boxed{B = \frac{\mu_0 I \theta}{4\pi R}}$$

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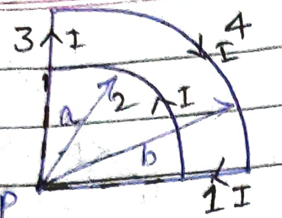

for part 1 and part 3, $\vec{dl}$ and $\vec{r}$ are in same direction, ∴ magnetic field due to these elements is zero.

now, magnetic field due to part 3 at centre P is,

$$B_3 = \frac{\mu_0 I}{4\cancel{\pi} a} \times \frac{\cancel{\pi}}{2}$$ (outside the plane of paper)

$$B_3 = \frac{\mu_0 I}{8 a}$$ (outside the plane of paper)

and, magnetic field due to part 4 at P is,

$$B_4 = \frac{\mu_0 I}{8 b}$$ (inside the plane of paper)

∴ net magnetic field at P,

$$B = \frac{\mu_0 I}{8}\left(\frac{1}{a} - \frac{1}{b}\right)$$ (outside the plane of paper)

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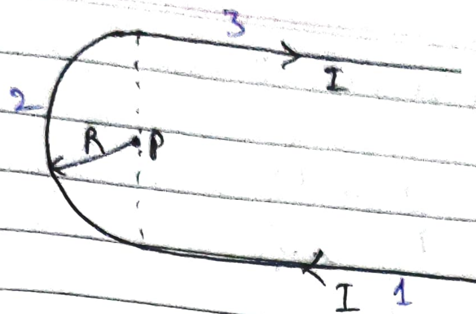
(b)



part 1 is a straight wire, with $\phi_1 = 90^\circ$ and $\phi_2 = 0^\circ$.

∴ magnetic field at P due to part 1,

$$B_1 = \frac{\mu_0 I}{4\pi R}\left(\sin 90^\circ + \sin 0^\circ\right)$$

$$B_1 = \frac{\mu_0 I}{4\pi R} \quad \text{(into the paper)}$$

part 2 is a arc of 180°, therefore, magnetic field at P due to part 2 is,

$$B_2 = \frac{\mu_0 I}{4\pi R} \cdot \pi = \frac{\mu_0 I}{4R} \quad \text{(into the paper)}$$

part 3 is a straight wire, with $\phi_1 = 0^\circ$ and $\phi_2 = 90^\circ$, therefore magnetic field due to part 3 at P is,

$$B_3 = \frac{\mu_0 I}{4\pi R}\left(\sin 0^\circ + \sin 90^\circ\right)$$

$$= \frac{\mu_0 I}{4\pi R} \quad \text{(into the paper)}$$

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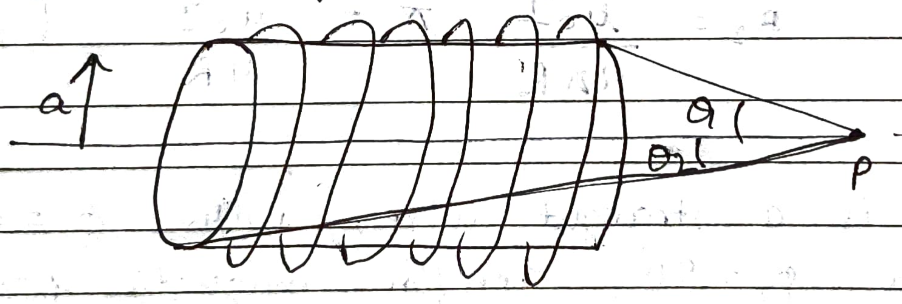
$\therefore$ net magnetic field at P is,

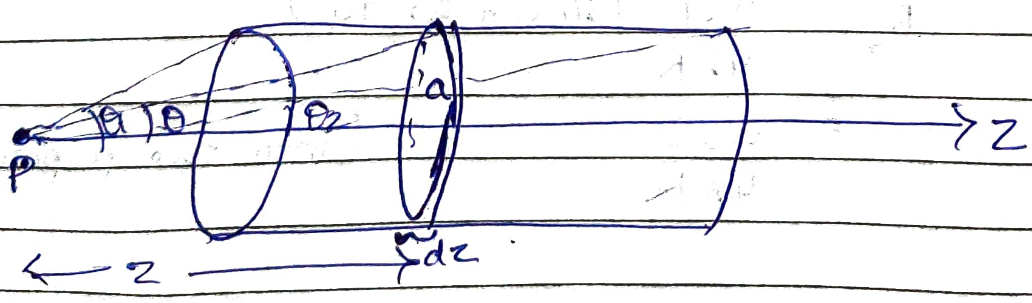
$$B = \frac{\mu_0 I}{4R}\left(\frac{1}{\pi} + \frac{1}{\pi} + 1\right)$$

$$\boxed{B = \frac{\mu_0 I (\pi + 2)}{4\pi R}}$$

Problem 5.11 Find the magnetic field at point P on the axis of a tightly wound solenoid (helical coil) consisting of n turns per unit length wrapped around a cylindrical tube of radius a and carrying current I. Express your answer in terms of θ_1 and θ_2. Consider the turns to be essentially circular. What is the field on the axis of an infinite solenoid?



Let us consider a very small length dz of the solenoid at a distance z from



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number of turns in this length $= n\,dz$

$\because$ magnetic field at the axis of a circular coil at distance x is given by

$$B = \frac{\mu_0 n I R^2}{2}\left(\frac{1}{(R^2+x^2)^{3/2}}\right)$$

$\therefore$ magnetic field at the point P due to the turns of coil present in dz length is

$$dB = \frac{\mu_0 n I \, dz \, a^2}{2\,(a^2+z^2)^{3/2}}$$

$\because$ $\tan\theta = \frac{a}{z}$

$\therefore$ $z = a\cot\theta$

$\therefore$ $dz = -a\,\mathrm{cosec}^2\theta\, d\theta = \frac{-a}{\sin^2\theta}\,d\theta$

and, $\frac{1}{(a^2+z^2)^{3/2}} = \frac{1}{(a^2+a^2\cot^2\theta)^{3/2}}$

$$= \frac{\sin^3\theta}{a^3}$$

$\therefore$ magnetic field due to solenoid at P is,

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$$\int dB = \int_{\theta_1}^{\theta_2} \frac{\mu_0 n I a^2}{2} \left(\frac{-a \, d\theta}{\sin^2\theta} \right) \frac{\sin^3\theta}{a^3}$$

$$B = \frac{+\mu_0 n I}{2} \int_{\theta_1}^{\theta_2} -\sin\theta \, d\theta$$

$$B = \frac{\mu_0 n I}{2} \cos\theta \Big|_{\theta_1}^{\theta_2}$$

$$B = \frac{\mu_0 n I}{2} (\cos\theta_2 - \cos\theta_1)$$

For an infinite solenoid $\theta_1 = \pi$ and $\theta_2 = 0$.

$$\therefore B = \frac{\mu_0 n I}{2} (\cos 0 - \cos \pi)$$

$$B = \frac{\mu_0 n I}{2} (1 - (-1))$$

$$\boxed{B = \mu_0 n I}$$

Problem 5-13 Suppose you have two infinite straight line charges λ, a distance d apart, moving along at a constant speed v. How great would v have to be in order for the magnetic attraction to balance electric repulsion? Work out the actual number. Is this a reasonable sort of speed?

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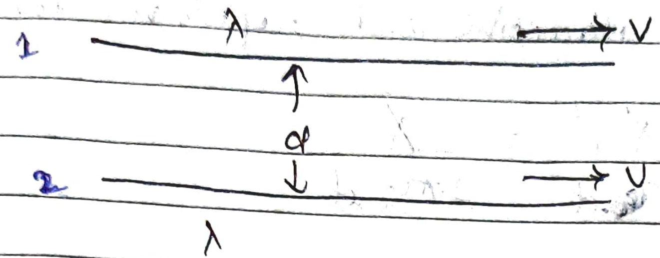

Since, the ~~ch~~ charges are moving, ~~the~~ with speed v, the current in the wire can be assumed to be, charge crossing per unit length per unit time, which is λv.

$$\therefore \quad I_1 = I_2 = \lambda v$$

When two infinite wires have current in them running in same direction, the force between them is attractive and the magnitude of force per unit length is given by

$$F_{mag} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

$$\therefore \quad F_{mag} = \frac{\mu_0 (\lambda v)^2}{2\pi d} = \frac{\mu_0 \lambda^2 v^2}{2\pi d}$$

Now, the electrostatic repulsion force between the wire is given by (force per unit length)

$$F_{ele} = \frac{\lambda_0 \lambda}{2\pi\varepsilon_0 d} = \frac{\lambda^2}{2\pi\varepsilon_0 d}$$

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balancing attractive and repulsive forces,

$$\frac{\mu_0 \lambda^2 v^2}{2\pi d} = \frac{\lambda^2}{2\pi \epsilon_0 d}$$

$$v^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$\because$ speed of light $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$\therefore$ $v = c = 3 \times 10^8$ m/s

Since, particles like electron cannot achieve this speed, this is unreasonable. That is, the forces can never balance each other. Infact, the repulsive (electrostatic) force is dominant in this condition.

Problem 5.15 A thick slab extending from $z = -a$ to $z = +a$ (and infinite in x - and y directions) carries a uniform current $J = J\hat{x}$. Find the magnetic field, as a function of z, both inside and outside the slab.

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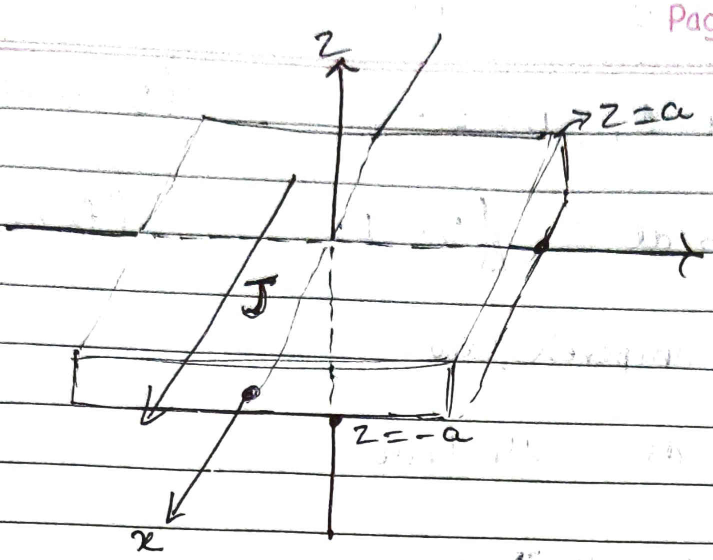
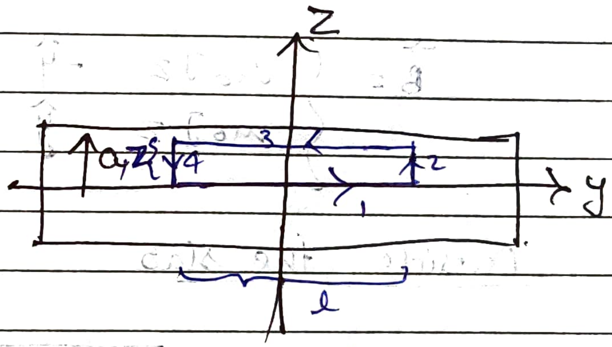


It is clear that direction of magnetic field will be along $-\hat{y}$ for $z > 0$ and $+\hat{y}$ for $z < 0$.

<u>Inside the slab</u>



Consider the amperian loop of length l and width z.

The figure shows the front view (as seen from $+x$-axis)

According to Ampere's Law,

$$\oint B \cdot dl = \mu_0 I_{enc}$$

now, $I_{enc} = J l z$

part 2 and 4 of loop give no contribution in $B \cdot dl$ as B is in $\hat{y}$ (or $-\hat{y}$) and they are in $\hat{z}$ (or $-\hat{z}$) direction $\therefore \cos\theta = \cos 90° = 0$.

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for part 1, $B = 0$. ($B = 0$ at $z = 0$)

$\therefore \oint B \cdot dl = Bl$ (from part (iii) only)

$\therefore$ from Ampere's law

$$\oint B \cdot dl = \mu_0 I_{enc}$$

$$Bl = \mu_0 J l z$$

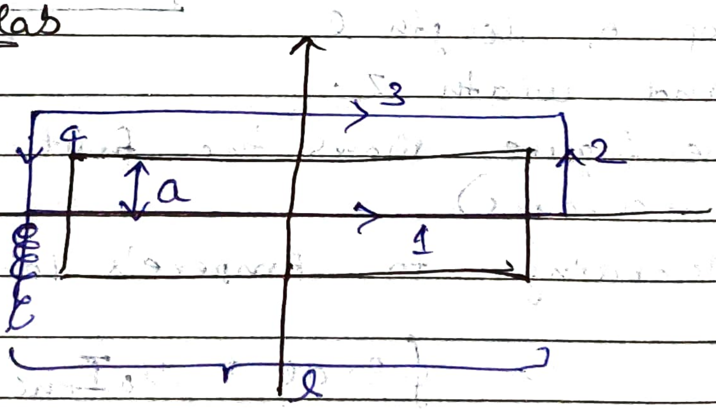
$$B = \mu_0 J z$$

$\therefore$ inside the slab, magnetic field is

$$\vec{B} = \begin{cases} \mu_0 J z \; (-\hat{y}) & \text{for } z > 0 \\ \mu_0 J z \; \hat{y} & \text{, for } z < 0 \end{cases}$$

Outside the slab

Consider the amperian loop shown.



$$\oint B \cdot dl = Bl$$

$$I_{enc} = J l a$$

$\therefore$ Using Ampere's Law;

$$Bl = \mu_0 J l a$$

$$B = \mu_0 J a$$

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$\therefore$ outside the slab, magnetic field is,

$$B = \begin{cases} \mu_0 J a \quad -\hat{y} & , \text{for } z > 0 \\ \mu_0 J a \quad \hat{y} & , \text{for } z < 0 \end{cases}$$

Problem 5-21 Is Ampere's law consistent with the general rule that divergence of curl is always zero? Show that Ampere's law cannot be valid, in general, outside magnetostatics. Is there any such 'defect' in the other three Maxwell equations?

Ampere's law says $\nabla \times B = \mu_0 J$.
Together with the continuity equation this gives $\nabla \cdot (\nabla \times B) = \mu_0 (\nabla \cdot J)$

$$\nabla \cdot (\nabla \times B) = \mu_0 (\nabla \cdot J) = -\mu_0 \frac{\partial \rho}{\partial t}$$

which is inconsistent with divergence of curl is always zero unless ρ is constant (magneto statics).

The other Maxwell equations are fine:
$\nabla \times E = 0 \Rightarrow \nabla \cdot (\nabla \times E) = 0$

and as for ~~the~~ two divergence equations, there is no relevant vanishing second derivative.