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PH100: Mechanics and Thermodynamics

Tutorial #09

1. What do you understand from ultraviolet catastrophe?

The ultraviolet catastrophe, also called Rayleigh-Jeans catastrophe, was the prediction that an ideal blackbody at thermal equilibrium will emit radiation in all frequency ranges, emitting more energy as the frequency increases.

The Rayleigh-Jeans law is

$$B_\lambda(T) = \frac{2ck_BT}{\lambda^4}$$

Electromagnetic spectrum predicted by this formula agrees with experimental results at low frequencies but strongly disagrees at higher frequencies. This inconsistency between observations and the predictions of classical physics is known as Ultraviolet Catastrophe.

By calculating the total amount of radiant energy, it can be shown that a blackbody in this case would release an infinite amount of energy, which is in contradiction with the Law of Conservation of Energy.

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2. A FM radio station of frequency 90.1 MHz puts out a signal of 50000 W. How many photons/s are emitted? How many photons are contained in a beam of electromagnetic radiation of total power 180 W if the source is
(a) an AM radio station of 1100 kHz
(b) 8 nm X-rays
(c) 4 MeV gamma rays?

Number of photons emitted per second $= \dfrac{\text{Power}}{\text{Energy of one photon}}$

$$= \frac{50000}{h\nu}$$

$$= \frac{50000}{6.626 \times 10^{-34} \times 98.1 \times 10^{6}}$$

$$= 0.60769 \times 10^{32}$$

$$= 7.69 \times 10^{30} \text{ photons/s}$$

Hence, 7.69×10^{30} photons/s are emitted.

Now, the number of photons contained in the beam $= \dfrac{\text{total power}}{\text{Energy of photon}}$

a) number of photons in the beam $= \dfrac{180}{6.626 \times 10^{-34} \times 1100 \times 10^{3}}$

$$= 0.024696 \times 10^{31}$$

$$= 2.47 \times 10^{29} \text{ photons}$$

Archit Agrawal
20205 2307

b) total number of photons in the beam $= \dfrac{180\,\lambda}{hc}$

$$= \frac{180 \times 8 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^{8}}$$

$$= 72.44 \times 10^{17}$$
$$= 7.244 \times 10^{18} \text{ photons}$$

c) total number of photons in the beam $= \dfrac{180}{\text{Energy of photon}}$

Energy of photon $= 4 \text{ MeV}$
$= 4 \times 10^{6} \times 1.6 \times 10^{-19}\ J$
$= 6.4 \times 10^{-13}\ J$

$\therefore$ total number of photons $= \dfrac{180}{6.4 \times 10^{-13}}$

$$= 28.125 \times 10^{13} \text{ photons}$$
$$= 2.813 \times 10^{14} \text{ photons}$$

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3. What is the threshold frequency for the photoelectric effect on lithium ($\phi = 2.93 eV$)? What is the stopping potential if the wavelength of incident light is 380 nm?

The threshold frequency (ν_0) is given by,

$$h\nu_0 = \phi$$

$$h = 4.136 \times 10^{-15} \text{ eV s}$$

$$\therefore \nu_0 = \frac{2.93 \text{ eV}}{4.136 \times 10^{-15} \text{ eV s}}$$

$$\nu_0 = 0.708 \times 10^{15} \text{ s}^{-1}$$

$$\nu_0 = 7.08 \times 10^{14} \text{ Hz}$$

The threshold frequency for photoelectric effect on lithium is 7.08×10^{14} Hz.

Now, Energy of incident photon $(E) = \frac{hc}{\lambda} = \frac{4.136 \times 10^{-15} \text{ eVs} \times 3 \times 10^{8} \text{ ms}^{-1}}{380 \times 10^{-9} \text{ m}}$

$$E = 3.265 \text{ eV}$$

$$\therefore \text{Stopping potential } (V_0) = \frac{E - \phi}{e}$$

$$= \frac{3.265 - 2.93 \text{ eV}}{e}$$

$$= 0.335 \text{ V}$$

Hence, the stopping potential is 0.335 V if wavelength of incident light is 380 nm.

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4. What is the maximum wavelength of incident light that can produce photoelectrons from silver ($\phi = 4.64\ eV$)? What will be the maximum kinetic energy of the photoelectrons if the wavelength is halved?

The maximum wavelength (λ_0) that can produce photoelectrons from silver, is given by,

$$\phi = \frac{hc}{\lambda_0}$$

$$\lambda_0 = \frac{hc}{\phi}$$

$$\lambda_0 = \frac{4.136 \times 10^{-15}\ eVs \times 3 \times 10^{8}\ ms^{-1}}{4.64\ eV}$$

$$\lambda_0 = 2.674 \times 10^{-7}\ m$$

$$\lambda_0 = 267.4\ nm$$

Hence, maximum wavelength that can produce photoelectrons from silver is 267.4 nm.

Now, if $\lambda = \frac{\lambda_0}{2} = 133.7\ nm$,

the maximum kinetic energy of photoelectrons (KE_{max}),

$$KE_{max} = \frac{hc}{\lambda} - \phi$$

$$= \frac{4.136 \times 10^{-15}\ eVs \times 3 \times 10^{8}\ ms^{-1}}{133.7 \times 10^{-9}\ m} - 4.64\ eV$$

$$= 9.28 - 4.64 = 4.64\ eV$$

Hence, maximum kinetic energy of photoelectrons is 4.64 eV.

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5. An experimenter finds that no photoelectrons are emitted from tungsten unless the wavelength of light is less than 270 nm. Her experiment will require photoelectrons of maximum kinetic energy 2.0 eV. What frequency of light should be used to illuminate the tungsten?

The threshold wavelength is given and it is equal to 270 nm.

$\therefore$ work function of metal(ϕ) $= \frac{hc}{\lambda} = \frac{4.136 \times 10^{-15} \times 3 \times 10^{8}}{270 \times 10^{-9}\ m}$

$$\phi = 4.595\ eV$$

Now, the maximum kinetic energy of photoelectrons is given by

$$KE_{max} = h\nu - \phi$$

if $KE_{max} = 2\ eV$

$$\therefore \quad 2 = 4.136 \times 10^{-15}\ \nu - 4.595$$

$$\nu = \frac{6.595}{4.136 \times 10^{-15}}$$

$$\nu = 1.595 \times 10^{15}\ Hz$$

Hence, the frequency of light required to generate photoelectrons of maximum kinetic energy 2 eV is 1.595×10^{15} Hz

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6. The human eye is sensitive to a pulse of light containing as few as 100 photons. For orange light of wavelength 610 nm, how much energy is contained in the pulse.

The energy of photon of light with wavelength 610 nm,

$$E = \frac{hc}{\lambda} = \frac{4.136 \times 10^{-15} \times 3 \times 10^{8}}{610 \times 10^{-9}}$$

$$= 2.034 \text{ eV}$$

Since, human eye is sensitive to a pulse of light containing as few as 100 photons,

$\therefore$ Energy of pulse for orange light $= 2.034 \text{ eV} \times 100$
$= 203.4 \text{ eV}$

Hence, 203.4 eV of energy is contained in the pulse.

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7. In a photoelectric experiment it is found that a stopping potential of 1 V is needed to stop all the electrons when incident light of wavelength 260 nm is used and 2.3 V is needed for light of wavelength 207 nm. From these data determine Planck's constant and work function of metal.

The stopping potential(V_0) and wavelength(λ) of incident light are connected by the equation,

$$V_0 = \frac{hc}{e\lambda} - \frac{\phi}{e}$$

if $V_0 = 1\,V$, $\lambda = 260\,nm$,

$$1 = \frac{hc}{e(260\,nm)} - \frac{\phi}{(e)} \longrightarrow (I)$$

and,

$$2.3 = \frac{hc}{e(207\,nm)} - \frac{\phi}{e} \longrightarrow (II)$$

Subtracting (I) from (II),

$$1.3 = \frac{hc}{e(207\,nm)} - \frac{hc}{e(260\,nm)}$$

$$1.3 = \frac{hc}{e \times 10^{-9}} \left\{ \frac{260 - 207}{260 \times 207} \right\}$$

$$h = \frac{1.3 \times 260 \times 207 \times 10^{-9} \times 1.6 \times 10^{-19}}{53 \times 3 \times 10^{8}}$$

$$h = 704.06 \times 10^{-36}\ Js$$

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$h = 7.04 \times 10^{-34}$ Js

or $h = 4.4 \times 10^{-15}$ eVs

if stopping potential is 1 V, KE_{max} of photoelection is 1 eV.

$$\because \quad KE_{max} = \frac{hc}{\lambda} - \phi$$

$$1\ eV = \frac{4.4 \times 10^{-15}\ eV\cancel{s} \times 3 \times 10^{8}\ \cancel{m}\cancel{s^{-1}}}{260 \times 10^{-9}\ \cancel{m}} - \phi$$

$$1\ eV = 0.0507 \times 100 - \phi$$

$$\phi = (5.07 - 1)\ eV$$
$$\phi = 4.07\ eV$$

Hence, the value of Planck's constant is 7.04×10^{-34} Js or 4.4×10^{-15} eVs and work function of metal is equal to 4.07 eV.

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Q. The phase velocity of ocean waves is $\sqrt{g\lambda/2\pi}$, where g is acceleration due to gravity. Find the group velocity of ocean waves.

we have, $V_{ph} = \sqrt{\dfrac{g\lambda}{2\pi}}$

$\because$ $K = \dfrac{2\pi}{\lambda}$

$\therefore$ $V_{ph} = \sqrt{\dfrac{g}{K}}$ —(I)

Now, since $V_{ph} = \dfrac{\omega}{K}$ —(II)

$\dfrac{\omega}{K} = \sqrt{\dfrac{g}{K}}$ (from (I))

$\dfrac{\omega^2}{K^2} = \dfrac{g}{K} \Rightarrow \omega^2 = gK$ —(III)

differentiating both sides of (III) w.r.t. K

$2\omega \dfrac{d\omega}{dK} = g$

$\dfrac{d\omega}{dK} = \dfrac{g}{2\omega}$

$\because$ group velocity $V_{gr} = \dfrac{d\omega}{dK}$

$\therefore$ $V_{gr} = \dfrac{g}{2\omega} = \dfrac{gK}{2\omega K} = \dfrac{\omega^2}{2\omega K}$ (from (III))

$V_{gr} = \dfrac{\omega}{2K} = \dfrac{V_{ph}}{2}$ (from (II))

$$\boxed{\therefore \quad V_{gr} = \frac{V_{ph}}{2} = \sqrt{\frac{g\lambda}{8\pi}}}$$

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9. Find the de-broglie wavelength of a 0.1 mg grain of sand blown by the wind at a speed of 20 m/s.

The de-broglie wavelength is given by,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

∴ de-broglie wavelength of 0.1mg grain of sand is

$$\lambda = \frac{6.626 \times 10^{-34}\ J\ s}{0.1 \times 10^{-6}\ kg \times 20\ m s^{-1}}$$

$$\lambda = 0.3313 \times 10^{-27}\ m$$

$$\lambda = 3.313 \times 10^{-28}\ m$$

Hence, de-Broglie wavelength of 0.1 mg of sand blown by wind at 20 m/s is 3.313×10^{-28} m.

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10. Generating plants in some power systems drop 10% of their load when the AC frequency changes by 0.30 Hz from the standard of 60 Hz. How often must the reading be monitored in order for the automatic operating system to be able to take corrective action? Let the time between measurements be at least half that determined by the bandwidth relation.

The bandwidth is given by

$$\Delta\omega \quad \Delta t = 2\pi$$

$$\Delta(2\pi f)\ \Delta t = 2\pi$$

$$\Delta f\ \Delta t = 1$$

$$\therefore \quad \Delta t = \frac{1}{\Delta f}$$

$$\Delta t = \frac{1}{0.3} = 3.33\ s$$

Since, time between measurements should be at least half of that determined by bandwidth relation.

$\therefore$ time between measurements $= \frac{\Delta t}{2} = 1.67\ s$

Hence, the readings must be monitored every 1.67 s.

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202052307

11. Consider electrons of kinetic energy 0.6 eV and 600 keV. For each electron, find the de Broglie wavelength, particle speed, phase velocity (speed), and group velocity (speed).

For the electron with kinetic energy 0.6 eV, there is no need to use relativistic condition for an electron.

so, de Broglie wavelength

$$\lambda_{dB} = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$$

$$\lambda_{dB} = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 6 \times 1.6 \times 10^{-19}}}$$

$$= \frac{6.626 \times 10^{-34}}{13.225 \times 10^{-25}} = 0.501 \times 10^{-9}$$

$$= 0.5 \text{ nm}$$

$\therefore$ particle speed $v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34}}{9.11 \times 10^{-31} \times 0.5 \times 10^{-9}}$

$$v = 1.455 \times 10^{6} \text{ m/s}$$

phase velocity (v_p)

$$v_p = \frac{\omega}{K} = \frac{2\pi f \lambda}{2\pi} = \lambda f$$

$$v_p = \frac{\cancel{h}}{\cancel{m} v} \times \frac{\cancel{m} c^2}{\cancel{h}} = \frac{c^2}{v} = \frac{9 \times 10^{16}}{1.45 \times 10^{6}}$$

$$v_p = 6.2 \times 10^{10} \text{ m/s}$$

group velocity $v_g = \frac{d\omega}{dk} = \frac{pc^2}{E} = \frac{\cancel{m} v \cancel{c^2}}{\cancel{m} \cancel{c^2}}$

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$V_g = V = 1.45 \times 10^6$ m/s

$\{$or use $V_g \times V_p = c^2\}$

For the electron with KE = 600 keV, there is a need to use relativistic condition, so

$$\lambda_{dB} = \frac{hc}{\sqrt{(KE)^2 + 2(KE)mc^2}}$$

$$= \frac{4.135 \times 3 \times 10^8 \times 10^{-15}}{\sqrt{600 \times 600 \times 10^6 + 2 \times 600 \times 10^3 \times \frac{9.11 \times 10^{-31} \times 9 \times 10^{16}}{1.6 \times 10^{-19}}}}$$

$$= \frac{4.135 \times 3 \times 10^{-7}}{\sqrt{3.6 \times 10^{11} + 2 \times 6 \times 10^5 \times 0.51 \times 10^6}}$$

$$= \frac{4.135 \times 3 \times 10^{-7}}{\sqrt{10^{11}(3.6 + 6.12)}}$$

$$= \frac{12.405 \times 10^{-7}}{10^5 \sqrt{36 + 61.2}}$$

$$= \frac{12.405 \times 10^{-7}}{10^5 \times 9.86}$$

$$\lambda_{dB} = 1.26 \times 10^{-12} \text{ m}$$

$$= 1.26 \text{ pm}$$

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particle speed v,

$$v = \frac{1}{\sqrt{\left(\frac{m^2}{p^2}\right) + \frac{1}{c^2}}} \qquad \left\{ p = \frac{m}{\sqrt{\frac{1}{v^2} - \frac{1}{c^2}}} \right\}$$

$$v = \frac{1}{\sqrt{\frac{\lambda^2 m^2}{h^2} + \frac{1}{c^2}}} \qquad \left(\because p = \frac{h}{\lambda} \right)$$

$$v = \frac{1}{\sqrt{\frac{1.26 \times 1.26 \times 10^{-24} \times (9.11 \times 10^{-31})^2}{(6.626 \times 10^{-34})^2} + \frac{1}{9 \times 10^{16}}}}$$

$$v = \frac{1}{\sqrt{3 \times 10^{-18} + 11.1 \times 10^{-18}}}$$

$$v = 0.268 \times 10^{+9} \text{ m/s}$$

$$v = 2.68 \times 10^{8} \text{ m/s}$$

phase velocity $v_p = \frac{\omega}{K} = \frac{E}{p}$

$$v_p = 3.37 \times 10^{8} \text{ m/s}$$

group velocity, $v_g = \frac{pc^2}{E} = v$

$$v_g = 2.68 \times 10^{8} \text{ m/s}$$

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12. Two waves are travelling simultaneously down a long slinky. They can be represented by

$$\psi_1(x,t) = 0.0030 \sin(6x - 300t)$$
$$\psi_2(x,t) = 0.0030 \sin(7x - 250t)$$

Distances are measured in metres and time in seconds.

(a) Write the expression for resulting wave.

(b) What are the phase and group velocities.

(a) The resulting wave $\psi(x,t)$,

$$\psi(x,t) = \psi_1(x,t) + \psi_2(x,t)$$

$$\psi(x,t) = 0.0030 \{ \sin(6x - 300t) + \sin(7x - 250t) \}$$

Using $\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$

$$\psi(x,t) = 0.0030 \left\{ 2 \sin\left(\frac{13x - 550t}{2}\right) \cos\left(\frac{-x - 50t}{2}\right) \right\}$$

$$\psi(x,t) = 0.0060 \sin(6.5x - 275t) \cos(-0.5x - 25t)$$

(b) phase velocity v_p

$$V_p = \frac{\omega}{K} = \frac{\omega_1 + \omega_2}{K_1 + K_2}$$

$$= \frac{(300 + 250)}{(6 + 7)} \frac{\text{rad/s}}{\text{m}^{-1}}$$

$$V_p = 42.3 \text{ m/s}$$

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2020052307

group velocity V_g

$$V_g = \frac{\Delta \omega}{\Delta K} = \frac{\omega_1 - \omega_2}{K_1 - K_2} = \frac{300 - 250}{7 - 6}$$

$V_g = 50 \text{ m/s}$