MA202

ASSIGNMENT 7

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SECTION:

2

Exercise 1

A) Consider the function $g(x) = \dfrac{1}{x^2 + 0.25}$ where x is a real number. Generate a data random data set $\{x_0, x_1, \ldots, x_n\}$ having $n + 1$ data points in the interval $\{-10,10\}$. At each of these points evaluate the function $y_i = g(x_i)$, and construct the data $\{(x_i, y_i)\}$. Now write a program to find the interpolating polynomial using both the Lagrange method and the Newton divided difference method. Use the above generated data to find the interpolating polynomial using both these methods. Plots the results obtained, along with the original function and the original data. Do this exercise for $n = 3,5,8,10,20$.

B) Do the same exercise but now take the data points $\{x_0, x_1, \ldots, x_n\}$ to be equispaced in the interval.

Comment on the results obtained in both the cases and with both the methods.

```
x = [1,2,3,4,5];
disp(x)
```

```
     1     2     3     4     5
```

```
for i=1:length(x)
A(i) = 1/(x(i)^2 + 0.25)
end
```

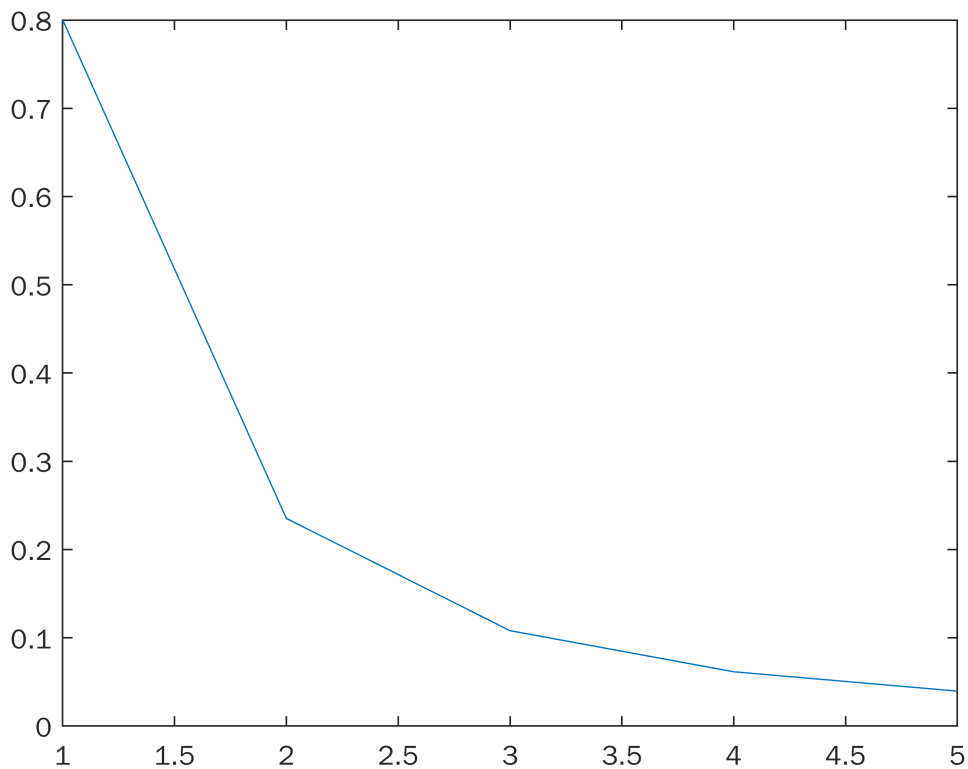
```
A = 1×5
    0.8000    0.2353    0.1081    0.0615    0.0396
A = 1×5
    0.8000    0.2353    0.1081    0.0615    0.0396
A = 1×5
    0.8000    0.2353    0.1081    0.0615    0.0396
A = 1×5
    0.8000    0.2353    0.1081    0.0615    0.0396
A = 1×5
    0.8000    0.2353    0.1081    0.0615    0.0396
```

```

disp(A)
```

```
    0.8000    0.2353    0.1081    0.0615    0.0396
```

```
% plotting (xi , yi).
plot(x, A);
```



```
%Interpolating method (Lagrange's)
sum = 0;
syms a
for i = 1:length(x)
u = 1;
l = 1;
for j = 1:length(x)
if j ~= i
u = u * (a - x(j));
l = l * (x(i) - x(j));
end
end
sum= sum + u / l * A(i);
end
fprintf("Interpolating Polynomial Using Lagrange's ......\n")
```

```
Interpolating Polynomial Using Lagrange's ......
```

```
disp(sum);
```

$$\frac{(a-1)\,(a-2)\,(a-3)\,(a-4)}{606} - \frac{2\,(a-1)\,(a-2)\,(a-3)\,(a-5)}{195} + \frac{(a-1)\,(a-2)\,(a-4)\,(a-5)}{37} - \frac{2\,(a}{}$$

```
%Newton's Divided Difference Implementation and finding polynomial.
ddTable = divDiff(x, A);
disp('divided difference table : ')
```

```
divided difference table :
```

```
disp(ddTable)
```

```
    0.8000   -0.5647    0.2188   -0.0595    0.0125
    0.2353   -0.1272    0.0403   -0.0093         0
    0.1081   -0.0466    0.0123         0         0
    0.0615   -0.0219         0         0         0
    0.0396         0         0         0         0
```

```
syms X
sum = 0;
for i = 1:size(x,2)
sum = sum + prod(X - x(:,1:i))*divDiff(1,i+1);
end
Y = A(1) + sum;
disp('Newton divided difference polynomial');
```

```
Newton divided difference polynomial
```

```
disp(Y)
```

$$2\,X + 3\,(X-1)\,(X-2) + 4\,(X-1)\,(X-2)\,(X-3) + 5\,(X-1)\,(X-2)\,(X-3)\,(X-4) + 6\,(X-1)\,($$

```
function ddTable = divDiff(x, y)
n = length(x) - 1;
ddTable = zeros(n + 1, n + 1);
ddTable(1 : n + 1, 1) = y';
for i = 2 : n + 1
for j = 1 : n - i + 2
ddTable(j, i) = (ddTable(j+1, i-1) - ddTable(j, i-1))/(x(i+j - 1) - x(j));
end
end
end
```

Exercise 2

Do the same exercise as above, now for the function $g(x) = xe^{-x^2}$. Comment on the results obtained in this case.

```
for i=1:length(x)
A(i) = x(i)*(exp((-x(i))^2))
end
```

```
A = 1×5
    2.7183    0.2353    0.1081    0.0615    0.0396
A = 1×5
    2.7183  109.1963    0.1081    0.0615    0.0396
A = 1×5
10^4 ×
    0.0003    0.0109    2.4309    0.0000    0.0000
A = 1×5
10^7 ×
    0.0000    0.0000    0.0024    3.5544    0.0000
A = 1×5
10^11 ×
    0.0000    0.0000    0.0000    0.0004    3.6002
```

```
disp(A)
```

```
   1.0e+11 *

    0.0000    0.0000    0.0000    0.0004    3.6002
```

```
sum = 0;
syms a
for i = 1:length(x)
u = 1;
l = 1;
for j = 1:length(x)
if j ~= i
u = u * (a - x(j));
l = l * (x(i) - x(j));
end
end
sum= sum + u / l * A(i);
end
fprintf("Interpolating Polynomial Using Lagrange's ......\n")
```

```
Interpolating Polynomial Using Lagrange's ......
```

```
disp(sum)
```

$$\frac{5898641353718651\ (a-1)\ (a-2)\ (a-3)\ (a-4)}{393216} - \frac{149084195602433\ (a-1)\ (a-2)\ (a-3)\ (a-5)}{25165824} +$$

```
%Newton's Divided Difference Implementation and finding polynomial.
ddTable = divDiff(x, A);
disp('divided difference table : ')
```

divided difference table :

```
disp(ddTable)
```

```
   1.0e+11 *

    0.0000    0.0000    0.0000    0.0001    0.1500
    0.0000    0.0000    0.0002    0.5999         0
    0.0000    0.0004    1.7998         0         0
    0.0004    3.5999         0         0         0
    3.6002         0         0         0         0
```

```
syms X
sum = 0;
for i = 1:size(x,2)
sum = sum + prod(X - x(:,1:i))*divDiff(1,i+1);
end
Y = A(1) + sum;
disp('Newton divided difference polynomial');
```

Newton divided difference polynomial

```
disp(Y)
```

$2\,X+3\,(X-1)\,(X-2)+4\,(X-1)\,(X-2)\,(X-3)+5\,(X-1)\,(X-2)\,(X-3)\,(X-4)+6\,(X-1)\,($

```
function ddTable = divDiff(x, y)
n = length(x) - 1;
ddTable = zeros(n + 1, n + 1);
ddTable(1 : n + 1, 1) = y';
for i = 2 : n + 1
for j = 1 : n - i + 2
ddTable(j, i) = (ddTable(j+1, i-1) - ddTable(j, i-1))/(x(i+j - 1) - x(j));
end
end
end
```