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MA201: Probability and Statistics

Tutorial 3

1) Shares of Company A cost \$10 per share and give a profit of X%. Independently of A, shares of company B cost \$50 per share and give a profit of Y%. Deciding how to invest \$1000, Mr. X chooses between 3 portfolios:

(a) 100 shares of A
(b) 50 shares of A and 10 shares of B
(c) 20 shares of B.

The distribution of X is given by probabilities:
$P\{X=-3\} = 0.3$, $P(X=0) = 0.2$, $P\{X=3\} = 0.5$.

The distribution of Y is given by probabilities:

$P\{Y=-3\} = 0.4$, $P\{Y=3\} = 0.6$

Compute expectations and variances of the total dollar profit generated by portfolios (a), (b) and (c). What is the least and most risky portfolio.

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Cost of share A = \$10

Profit distribution is X %.

$X = -3$ | $P(X=-3) = 0.3$
$X = 0$ | $P(X=0) = 0.2$
$X = 3$ | $P(X=3) = 0.5$

$\therefore$ profit $= \frac{X}{100} \times \$10$ for shares of A

let distribution of profit in \$ be M,

$\therefore$

M	P(M)
−0.3	0.3
0	0.2
0.3	0.5

$\therefore E(M) = -0.3 \times 0.3 + 0.3 \times 0.5$

$E(M) = 0.06$

and $Var(M) = E(M^2) - (E(M))^2$

$= 0.072 - 0.0036$

$Var(M) = 0.0684$

Now, cost of share B = \$50

profit distribution is Y%

$Y = -3$ | $P(Y=-3) = 0.4$
$Y = 3$ | $P(Y=3) = 0.6$

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$\because$ profit $= \frac{Y}{100} \times \$50$

let distribution of profit in \$ for shares of B be N

N	P(N)
−1.5	0.4
1.5	0.6

$\therefore E(N) = 0.4 \times (-1.5) + 0.6 \times (1.5)$
$= 0.3$

and $Var(N) = E(N^2) - (E(N))^2$
$= 2.16$

(a) 100 shares of A, portfolio I

$\therefore I = 100\, M$

$\therefore E(I) = 100 \cdot E(M)$
$= 100\,(0.06)$

$\boxed{E(I) = 6}$

and, $Var(I) = (100)^2\, Var(M)$
$= 10^4 \times 0.0684$

$\boxed{Var(I) = 684}$

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(b) 50 shares of A and 10 shares of B, portfolio II.

$\therefore$ $II = 50M + 10N$

$\therefore$ $E(II) = 50\,E(M) + 10\,E(N)$

$= 50 \times 0.06 + 10 \times 0.3$

$\boxed{E(II) = 6}$

and, $Var(II) = (50)^2\, Var(M) + (10)^2\, Var(N)$

[$\because$ shares of A and B are independent, hence covariance is 0.]

$\therefore$ $Var(II) = 2500 \times 0.0684 + 100 \times 2.16$

$= 171 + 216$

$\boxed{Var(II) = 387}$

(c) 20 shares of B, portfolio (III)

$\therefore$ $III = 20N$

$\boxed{E(III) = 20 \times 0.3 = 6}$

and, $Var(III) = 400 \times 2.16$

$\boxed{Var(III) = 864}$

As, we can see, the expected returns in all portfolios is same i.e \$6, but the variance in portfolio (b) is least and hence it is least risky. Portfolio (c) with greatest variance is most risky.

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2 (a) A quality control engineer tests the quality of produced computers. Suppose that 5% computers have defects, and defects occur independently of each other.

(i) Find the probability of exactly 3 defective computers in a shipment of 20.

(ii) Find the probability that the engineer has to test at least 5 computers in order to find 2 defective ones.

P(defective computer) = 0.05

(i) Let X be the number of defective computers in ~~20~~ a shipment of 20.

$$\therefore P(X=3) = {}^{20}C_3 \times (0.05)^3 \times (0.95)^{17}$$

$$= 1140 \times 0.00005227$$

$$\boxed{P(X=3) = 0.0596}$$ Ans

(ii) Let Y be the number of computers the engineer has to test to get 2 defective computers.

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$\therefore$ we need to find $P(Y \geq 5)$ and we know that $k = 2$

$$P(Y \geq 5) = 1 - P(Y \leq 4)$$

$$P(Y \leq 4) = \sum_{n=2}^{4} \binom{n-1}{k-1} (0.05)^2 (0.95)^{n-k}$$

$$= (0.05)^2 \sum_{n=2}^{4} (n-1)(0.95)^{n-2}$$

$$= (0.05)^2 \left\{ 1 + 2(0.95) + 3(0.9025) \right\}$$

$$= (0.05)^2 \times 5.6075$$

$$= 0.014$$

$$\therefore P(Y \geq 5) = 1 - 0.014$$

$$\boxed{P(Y \geq 5) = 0.986}$$ Ans

2(b) An internet search engine looks for a certain keyword in a sequence of independent web sites. It is believed that 20% of the sites contain this keyword.

(i) Compute the probability that at least 5 of the first 10 sites contain the given keyword.

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(ii) Compute the probability that the search engine had to visit at least 5 sites in order to find the first occurence of a keyword.

P(site contains the keyword) = 0.2

(i) Let X be the number of sites that contains the keyword in first 10 sites.

∴ We need to find $P(X \geq 5)$

$$P(X \geq 5) = 1 - P(X \leq 4)$$

Now,

$$P(X \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= {}^{10}C_0 (0.2)^0 (0.8)^{10} + {}^{10}C_1 (0.2)^1 (0.8)^9$$

$$+ {}^{10}C_2 (0.2)^2 (0.8)^8 + {}^{10}C_3 (0.2)^3 (0.8)^7$$

$$+ {}^{10}C_4 (0.2)^4 (0.8)^6$$

$$= 0.1074 + 0.2684 + 0.302 + 0.2013 + 0.0881$$

$$= 0.9672$$

$$\therefore P(X \geq 5) = 1 - 0.9672$$

$$\boxed{P(X \geq 5) = 0.0328}$$ Answer

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(ii) Let Y be the number of sites the search engine had to visit to find the first occurence of keyword.

$\therefore$ we need to find $P(Y \geq 5)$.

Now,

$$P(Y \geq 5) = 1 - P(Y \leq 4)$$

$\because$ this is geometric ~~prob~~ distribution.

$$P(y) = (1-p)^{y-1} p \quad (\text{where } p = 0.2)$$

$$\therefore P(Y \leq 4) = P(Y=1) + P(Y=2) + P(Y=3) + P(Y=4)$$

$$= (0.2) + (0.8)(0.2) + (0.8)^2 (0.2) + (0.8)^3 (0.2)$$

$$P(Y \leq 4) = 0.2 + 0.16 + 0.128 + 0.1024$$

$$= 0.5904$$

$$\therefore P(Y \geq 5) = 1 - 0.5904$$

$$\boxed{P(Y \geq 5) = 0.4096}$$ Answer

2(c) After a computer virus entered the system, a computer manager checks the condition of all important files. She knows that each file has probability 0.2 to be damaged by virus, independently of other files.

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(i) Compute the probability that at least 5 of the first 20 files are damaged.

(ii) Compute the probability that the manager has to check at least 6 files in order to find 3 undamaged files.

P (damaged file) = 0.2

(i) Let X be the number of damaged files in the first 20 files.

$\therefore$ We need to find $P(X \geq 5)$

Now,

$$P(X \geq 5) = 1 - P(X < 5)$$

$$P(X<5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= {}^{20}C_0 (0.2)^0 (0.8)^{20} + {}^{20}C_1 (0.2)(0.8)^{19}$$

$$+ {}^{20}C_2 (0.2)^2 (0.8)^{18} + {}^{20}C_3 (0.2)^3 (0.8)^{17}$$

$$+ {}^{20}C_4 (0.2)^4 (0.8)^{16}$$

$$= 0.01153 + 0.05765 + 0.13691$$
$$+ 0.20536 + 0.2182$$

$$P(X<5) = 0.62965$$

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$\therefore P(X \geq 5) = 1 - 0.62965$

$= 0.3704$

$$\boxed{\therefore P(X \geq 5) = 0.3704}$$ Answer

(ii) Let the manager has to check Y files in order to find 3 undamaged files

$\therefore$ We need to find $P(Y \geq 6)$

The manager will ~~need~~ to check at least 3 files to get 3 undamaged files.

$\therefore P(Y \geq 6) = 1 - P(Y=3) - P(Y=4) - P(Y=5)$

$P(Y \geq 6) = 1 - {}^2C_2 \cdot (0.8)^3 - {}^3C_2 (0.2)(0.8)^3 - {}^4C_2 (0.2)^2 (0.8)^3$

$= 1 - 0.512 - 0.3072 - 0.1229$

$$\boxed{P(Y \geq 6) = 0.0579}$$ Answer

3 (a) Every day, a lecture may be cancelled due to inclement weather with probability 0.05. Class cancellations on different days are independent.

(i) There are 15 classes left this semester. Compute the probability that at least 4 of them get cancelled.

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(ii) Compute the probability that the tenth class this semester is the third class that gets cancelled.

P(class gets cancelled) = 0.05

i) Let X be the number of classes that gets cancelled out of the 15 classes left in this semester.

$\therefore$ We need to find $P(X \geq 4)$.

Now,

$$P(X \geq 4) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$$

$$= 1 - {}^{15}C_0 (0.95)^{15} - {}^{15}C_1 (0.05)(0.95)^{14}$$

$$- {}^{15}C_2 (0.05)^2 (0.95)^{13}$$

$$- {}^{15}C_3 (0.05)^3 (0.95)^{12}$$

$$\therefore P(X \geq 4) = 1 - 0.4633 - 0.3657$$

$$- 0.1348 - 0.0307$$

$$\boxed{P(X \geq 4) = 0.0055}$$ Answer

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(ii) If tenth class of semester is the third class that got cancelled, then 2 out of first 9 classes got cancelled.

$\therefore P\left(\begin{array}{l}\text{third class that is cancelled} \\ \text{is } 10^{th} \text{ class of semester}\end{array}\right) = {}^9C_2 (0.05)^3 (0.95)^7$

$$\boxed{= 0.003142}$$ Answer

3(b) A lab network consisting of 20 computers was attacked by a computer virus. The virus enters each computer with probability 0.4, independently of other computers. Find the probability that it entered at least 10 computers.

Let X be the number of computers affected by virus in the lab.

$\therefore$ we need to find $P(X \geq 10)$.

Now, $P(X \geq 10) = 1 - \sum_{n=0}^{9} P(X=n)$ —(1)

$P(X=0) = {}^{20}C_0 (0.6)^{20} = 0.00003656$

$P(X=1) = {}^{20}C_1 (0.4)(0.6)^{19} = 0.0004875$

$P(X=2) = {}^{20}C_2 (0.16)(0.6)^{18} = 0.003087$

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$P(X=3) = {}^{20}C_3 (0.4)^3 (0.6)^{17} = 0.01235$

$P(X=4) = {}^{20}C_4 (0.4)^4 (0.6)^{16} = 0.03499$

$P(X=5) = {}^{20}C_5 (0.4)^5 (0.6)^{15} = 0.07465$

$P(X=6) = {}^{20}C_6 (0.4)^6 (0.6)^{14} = 0.1244$

$P(X=7) = {}^{20}C_7 (0.4)^7 (0.6)^{13} = 0.1659$

$P(X=8) = {}^{20}C_8 (0.4)^8 (0.6)^{12} = 0.1797$

$P(X=9) = {}^{20}C_9 (0.4)^9 (0.6)^{11} = 0.1597$

$$\therefore \sum_{n=0}^{9} P(X=n) = 0.7553$$

$$\therefore P(X \geq 10) = 1 - 0.7553$$

$$\boxed{P(X \geq 10) = 0.2447}$$ Answer

3(c) Five percent of computer parts produced by a certain supplier are defective. What is the probability that a sample of 16 parts contains more than 3 defective ones?

Let X be the no. of defective parts in the sample of 16 parts.

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∴ we need to find $P(X>3)$.

$$P(X>3) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$$

$$= 1 - {}^{16}C_0 (0.95)^{16} - {}^{16}C_1 (0.05)(0.95)^{15} - {}^{16}C_2 (0.05)^2 (0.95)^{14} - {}^{16}C_3 (0.05)^3 (0.95)^{13}$$

$$P(X>3) = 1 - 0.4401 - 0.3706 - 0.1463 - 0.0359$$

$$\boxed{P(X>3) = 0.0071}$$ Answer

4(a) Network breakdowns are unexpected rare events that occur every 3-weeks, on the average. Compute the probability of more than 4 breakdowns during a 21-week period.

∵ 1 breakdown occurs in 3-week period on average,

∴ 7 break down occurs on an average in a 21-week long period.

∴ $\lambda = 7$

∵ for poisson distribution, probability of rare event occuring x times in a given period of time is given by

$$P(X) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

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Let X be the number of network breakdowns that occur in 21- ~~day~~ week period.

∴ we need to find $P(X>4)$.

$$P(X>4) = 1-P(X=0)-P(X=1)-P(X=2)-P(X=3)-P(X=4)$$

$$= 1-e^{-7}\left(1+\frac{7}{1}+\frac{49}{2}+\frac{343}{6}+\frac{2401}{24}\right)$$

$$= 1-e^{-7}\left(\frac{24+168+588+1372+2401}{24}\right)$$

$$= 1-\frac{4553}{24}\times 0.0009119$$

$$= 1-0.173$$

$$\boxed{P(X>4) = 0.827}$$ Answer

4(b) A dangerous computer virus attack a folder consisting of 250 files. Files are affected by the virus independently of one another. Each file is affected with the probability 0.032. What is the probability that more than 7 files are affected by the virus?

Since, number of files is large and probability of computer affected by virus is very small, we can use Poisson

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approximation to binomial distribution here.

$\therefore \quad \lambda = 250 \times 0.032$

$\lambda = 8$

Now, let X be the number of files affected by the virus.

$\therefore$ we need to find $P(X > 7)$.

$$P(X>7) = 1 - \sum_{n=0}^{7} P(X=n) \quad \text{—— (1)}$$

$P(X=0) = e^{-8} = 0.0003355$

$P(X=1) = e^{-8} \cdot 8 = 0.002684$

$P(X=2) = e^{-8} \cdot \frac{64}{2} = 0.01074$

$P(X=3) = e^{-8} \cdot \frac{(8)^3}{6} = 0.02863$

$P(X=4) = e^{-8} \frac{(8)^4}{24} = 0.05726$

$P(X=5) = e^{-8} \frac{(8)^5}{120} = 0.09161$

$P(X=6) = e^{-8} \frac{(8)^6}{720} = 0.1222$

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$$P(X=7) = e^{-8} \frac{8^7}{5040} = 0.1396$$

$$\therefore \sum_{n=0}^{7} P(X=n) = 0.4531$$

$$\therefore P(X>7) = 1 - 0.4531$$

$$\boxed{P(X>7) = 0.5469}$$ Answer

4c) Messages arrive at an electronic message center at random times, with an average of 9 messages per hour?

(i) What is the probability of recieving at least five messages during the next hour?

(ii) What is the probability of recieving exactly five messages during the next hour?

Given $\lambda = 9$

Let X be the number of messages recieved per hour.

(i) We need to find $P(X \geq 5)$.

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$$P(X \geq 5) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3) - P(X=4)$$

$$= 1 - e^{-9}\left\{1 + 9 + \frac{81}{2} + \frac{729}{6} + \frac{6561}{24}\right\}$$

$$= 1 - e^{-9}\left\{\frac{240 + 972 + 2916 + 6561}{24}\right\}$$

$$= 1 - \frac{10689}{24} \times 0.00012341$$

$$= 1 - 0.05496$$

$$\boxed{P(X \geq 5) = 0.945}$$ Answer

(ii) We need to find $P(X=5)$

$$P(X=5) = e^{-9} \times \frac{(9)^5}{120}$$

$$\boxed{P(X=5) = 0.0607}$$ Answer

4 (d) On the average, 1 computer in 800 crashes during a severe thunderstorm. A certain company had 4000 working computers when the area was hit by a severe thunderstorm.

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(i) Compute the probability that less than 10 computers crashed?

(ii) Compute the probability that exactly 10 computers crashed?

$$P(\text{computer crashed}) = \frac{1}{800}$$

$\because$ no. of computers is large and probability of a computer crashing is less, we can use Poisson Approximation to Binomial Distribution.

$$\therefore \lambda = \frac{1}{800} \times 4000$$

$$\lambda = 5$$

Let X be the number of computers crashed in the thunderstorm.

(i) We need to find $P(X<10)$

$$P(X<10) = \sum_{n=0}^{9} P(X=n)$$

$$P(X=0) = e^{-5} = 0.006738$$

$$P(X=1) = e^{-5} \times 5 = 0.03369$$

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$P(X=2) = e^{-5} \times \frac{25}{2} = 0.08423$

$P(X=3) = e^{-5} \times \frac{125}{6} = 0.1404$

$P(X=4) = e^{-5} \times \frac{625}{24} = 0.1755$

$P(X=5) = e^{-5} \times \frac{3125}{120} = 0.1755$

$P(X=6) = e^{-5} \times \frac{15625}{720} = 0.1462$

$P(X=7) = e^{-5} \times \frac{78125}{5040} = 0.1044$

$P(X=8) = e^{-5} \times \frac{390625}{40320} = 0.06528$

$P(X=9) = e^{-5} \times \frac{1953125}{362880} = 0.03627$

$\boxed{\therefore P(X<10) = 0.9682}$ Answer

(ii) $P(X=10) = e^{-5} \times \frac{9765625}{3628800}$

$\boxed{P(X=10) = 0.01813}$ Answer

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4e) The number of computer shutdowns during any month has a Poisson distribution, averaging 0.25 shutdowns per month.

(i) What is the probability of at least 3 computer shutdowns next year?

(ii) During the next year, what is the probability of at least 3 months (out of 12), with exactly one computer shutdown in each?

(i) $\because$ average no. of shutdowns per month $= 0.25$

$\therefore$ average no. of shutdowns per year $= 0.25 \times 12$
$= 3$

Let X be the number of computer shutdowns in next year.

$\therefore$ we need to find $P(X \geq 3)$.

$$P(X \geq 3) = 1 - P(X=0) - P(X=1) - P(X=2)$$

$$= 1 - e^{-3}\left(1 + 3 + \frac{9}{2}\right)$$

$$= 1 - 0.04979 \times 8.5$$
$$= 1 - 0.4232$$

$$\boxed{P(X \geq 3) = 0.5768}$$ Answer

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5(a) In some city, the probability of a thunderstorm on any day is 0.6. During a thunderstorm the number of traffic accidents has Poisson distribution with parameter 10. Otherwise, the number of traffic accidents has Poisson distribution with parameter 4. If there were seven accidents yesterday, what is the probability that there was a thunderstorm?

Let there be X traffic accidents occured yesterday. and let T be the event that there was a thunderstorm yesterday.

$$\therefore P(T) = 0.6$$

$\therefore$ we need to find $P(T \mid X=7)$,

$$P(T \mid X=7) = \frac{P(X=7 \mid T)\, P(T)}{P(X=7 \mid T)\, P(T) + P(X=7 \mid \bar{T})\, P(\bar{T})}$$

$$= \frac{\left(\frac{e^{-10} \cdot 10^7}{7!}\right) \cdot (0.6)}{\left(\frac{e^{-10} \cdot 10^7}{7!}\right) \cdot 0.6 + \left(\frac{e^{-4} \cdot 4^7}{7!}\right) \cdot 0.4}$$

$$= \frac{e^{-10} \cdot 10^7 \cdot (0.6)}{e^{-10} \cdot 10^7 \cdot (0.6) + e^{-4} \cdot 4^7 \cdot (0.4)}$$

$$= \frac{272.3996}{272.3996 + 120.0334}$$

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$$\boxed{P(T \mid X=7) = 0.6941}$$ Answer

5(b) An insurance company divides its customers into 2 groups. Twenty percent of customers are in the high-risk group and 80% are in low risk group. The high risk customers make an average of 1 accident per year while the low-risk customers make an average of 0.1 accidents per year.

Eric had no accidents last year. What is the probability he is a high risk driver.

P(high risk customer) = 0.2
P(low " ") = 0.8

* Poisson distribution for high risk customers has parameter 1.
* Poisson distribution for low-risk customers has parameter 0.1.

Let Eric has X accidents previous year.

∴ We need to find P(high risk driver | X = 0)

$$P(\text{high risk driver} \mid X=0) = \frac{P(X=0 \mid \text{high risk})\, P(\text{high risk})}{P(X=0 \mid \text{high risk})\, P(\text{high risk}) + P(X=0 \mid \text{low risk})\, P(\text{low risk})}$$

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$$P(\text{high risk driver} \mid X=0) = \frac{\left(e^{-1} \cdot \frac{1^0}{0!}\right) \cdot (0.2)}{\left(e^{-1} \cdot \frac{1^0}{0!}\right) \cdot (0.2) + \left(e^{-0.1} \cdot \frac{(0.1)^0}{0!}\right)(0.8)}$$

$$= \frac{e^{-1} \cdot (0.2)}{e^{-1} \cdot (0.2) + e^{-0.1}(0.8)}$$

$$= \frac{0.07358}{0.07358 + 0.7239}$$

$$\boxed{P(\text{high risk driver} \mid X=0) = 0.09227}$$ Answer

5 (c) Eric continues driving. After three years he still has no accidents. Now, what is the conditional probability that he is a high risk driver.

$P(\text{high risk driver}) = 0.2$
$P(\text{low risk driver}) = 0.8$

Poisson distribution for high risk driver for 3 years ~~has~~ will have a parameter
$3 \times (\lambda \text{ for } 1 \text{ year}) = 3.$

Similarly, poisson distribution for low risk driver for 3 years has parameter 0.3.

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Let Eric has X accidents in previous 3 years.

$\therefore$ we need to find P(high risk | X = 0).

$$P(\text{high risk} \mid X=0) = \frac{P(X=0 \mid \text{high risk})\, P(\text{high risk})}{P(X=0 \mid \text{high risk})\, P(\text{high risk}) + P(X=0 \mid \text{low risk}) \cdot P(\text{low risk})}$$

$$= \frac{\left(e^{-3} \cdot \frac{3^0}{0!}\right) \cdot (0.2)}{\left(e^{-3} \cdot \frac{3^0}{0!}\right)(0.2) + \left(e^{-0.3} \cdot \frac{(0.3)^0}{0!}\right)(0.8)}$$

$$= \frac{e^{-3}(0.2)}{e^{-3}(0.2) + e^{-0.3}(0.8)}$$

$$= \frac{0.009957}{0.009957 + 0.59265}$$

$$\boxed{P(\text{high risk} \mid X=0) = 0.016523}$$ Answer

5 (d) This ~~number~~ question is ~~same~~ exactly same with 4(e). It is repeated

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