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PH110: Waves and Electromagnetics

Tutorial 5

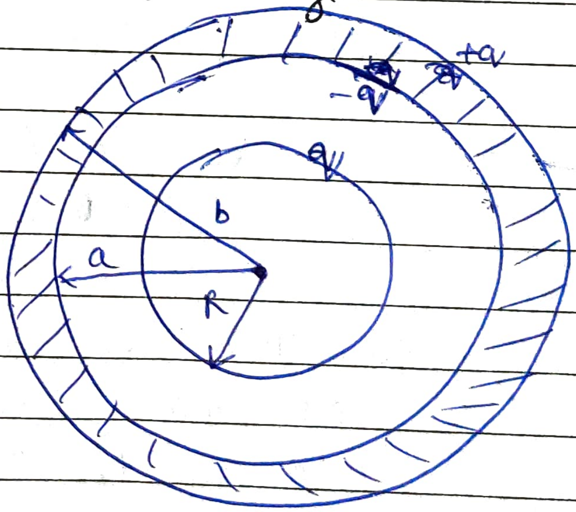
38. A metal sphere of radius R, carrying charge q, is surrounded by thick concentric metal shell (inner radius a, outer radius b). The shell carries no net charge

(a) Find the surface charge density σ at R, a and b.

(b) Find the potential at the center using infinity as the reference point.

(c) Now the outer surface is touched to a grounding wire, which drains off charge and lower its potential to zero. How do your answers to (a) and (b) change?

(a) $\sigma_R = \dfrac{q}{4\pi R^2}$



$\sigma_a = \dfrac{-q}{4\pi a^2}$

$\sigma_b = \dfrac{q}{4\pi b^2}$

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(b) potential at centre will be the sum of potentials ~~due~~ at surface of each shell (principle of superposition).

$$V_0 = \frac{q}{4\pi\epsilon_0 R} - \frac{q}{4\pi\epsilon_0 a} + \frac{q}{4\pi\epsilon_0 b}$$

$$V_0 = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{a} + \frac{1}{b}\right)$$

(c) Since, the surface charge at b drains off on grounding

$\therefore$ $\sigma_b = 0$

but σ_R and σ_a will be same.

and potential at center will be

$$V_0 = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{a}\right)$$

41. Two large metal plates (each of area A) are held a small distance d apart. Suppose we put a charge Q on each plate; what is the electrostatic pressure on the plates

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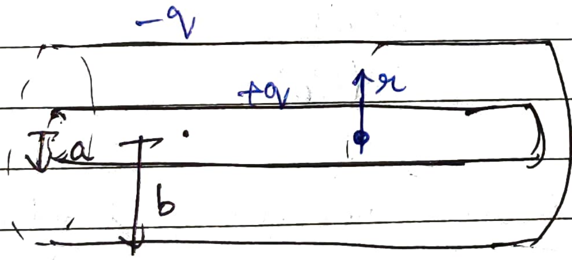
Since, between the plates $E = 0$, but outside the plate $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$.

$\because$ The electrostatic pressure $P = \frac{1}{2}\epsilon_0 E^2$

$$\therefore P = \frac{1}{2}\epsilon_0 \frac{Q^2}{A^2\epsilon_0^2}$$

$$\boxed{P = \frac{Q^2}{2A^2\epsilon_0}}$$

43. Find the capacitance per unit length of two coaxial metal cylindrical tubes of radii a and b



Assume +q charge on inner cylinder and -q on outer cylinder. Assume a length l of the cable,

electric field E in region $a \leq r < b$

$$\int E \cdot (2\pi r l) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{2\pi r l \epsilon_0}$$

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$$\therefore V_b - V_a = -\int_a^b E \cdot dr$$

$$= -\int_a^b \frac{Q}{2\pi\epsilon_0 l r}\, dr$$

$$V_b - V_a = -\frac{Q}{2\pi\epsilon_0 l} \ln\left(\frac{b}{a}\right)$$

Since, inner cylinder is at higher potential.

$$\therefore V = V_a - V_b = \frac{Q}{2\pi\epsilon_0 l} \ln\left(b/a\right)$$

$$\because C = \frac{Q}{V}$$

$$\therefore C = \frac{2\pi\epsilon_0 l}{\ln(b/a)}$$

$\therefore$ capacitance per unit length will be,

$$\boxed{\frac{C}{l} = \frac{2\pi\epsilon_0}{\ln(b/a)}}$$

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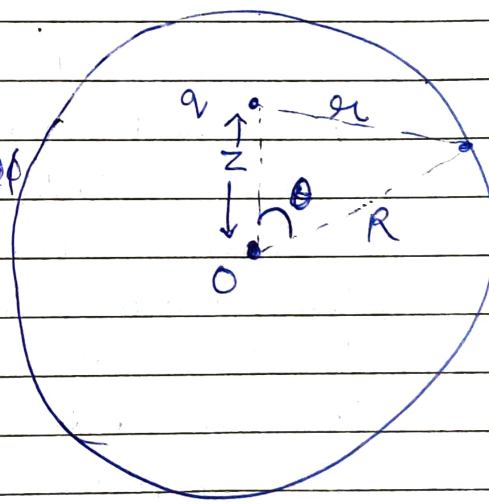
Chapter 3 Questions

1. Find the average potential over a surface (spherical) of radius R due to a point charge q located inside. Show that, in general,

$$V_{ave} = V_{centre} + \frac{Q_{enc}}{4\pi\epsilon_0 R}$$

where V_{center} is the potential at the center due to all the external charges, and Q_{enc} is the total enclosed charge

$$r = \sqrt{R^2 + z^2 - 2Rz\cos\theta}$$



$$V_{ave} = \frac{1}{4\pi R^2} \int \frac{q}{4\pi\epsilon_0 r} R^2 \sin\theta \, d\theta \, d\phi$$

$$V_{ave} = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{2zR} \sqrt{z^2 + R^2 - 2Rz\cos\theta} \Big|_0^\pi$$

$$= \frac{q}{4\pi\epsilon_0 (2zR)} \left[(z+R) - |z-R| \right]$$

$$V_{ave} = \frac{q}{(4\pi\epsilon_0)(2zR)} (2z) \qquad (\text{as } z < R)$$

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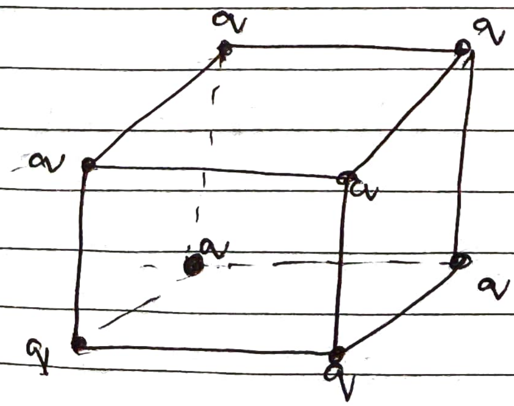
$$\therefore \quad V_{ave} = \frac{q}{4\pi\epsilon_0 R}$$

If there are more than one charges inside the sphere, the average potential due to interior charges is $\frac{Q_{enc}}{4\pi\epsilon_0 R}$, and the average due to exterior charges is V_{center}.

$$\therefore \quad V_{ave} = V_{center} + \frac{Q_{enc}}{4\pi\epsilon_0 R}$$

2. In one sentence, justify Earnshaw's theorem:

A charged particle can not be held in a stable equilibrium by electrostatic forces alone. As an example consider the cubical arrangement of fixed charges. It looks as though a positive charge at the center would be suspended in midair, since it is repelled away from each corner. Where is the leak in this electrostatic "bottle"?



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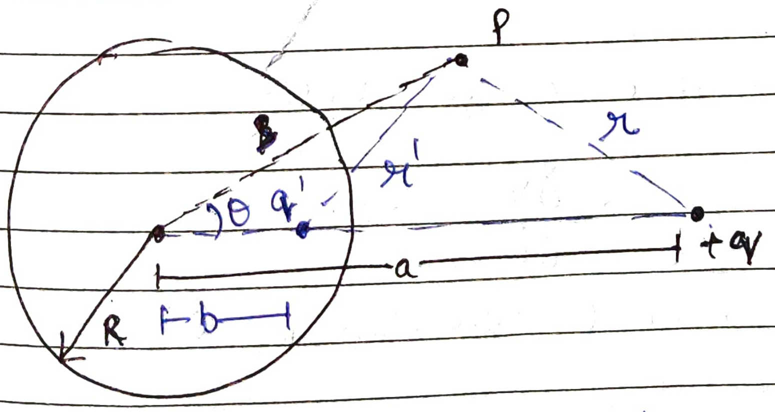
A stable equilibrium is a point of local minimum in potential energy. Here the potential energy is qV. But we know that Laplace's equation allows no local minima for V. What looks like minimum, in the figure, must in fact be a saddle point, and the box "leaks" through the centre of each face.

8 A point charge q is ~~seperated~~ situated at a distance a from the center of a grounded sphere of radius R.

(a) Find the potential outside the sphere.

(b) Find the induced surface charge on the sphere as a function of θ. Integrate this to get the total induced charge.

(c) Calculate the energy of this configuration.



Using the concept of method of images. Let us assume another charge q' at a distance b, right from the center.

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$$q' = -\frac{Rq}{a}$$

$$b = \frac{R^2}{a}$$

$$\therefore \quad V_p = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r} + \frac{q'}{r'}\right)$$

Using cosine rule

$$r = \sqrt{s^2 + a^2 - 2as\cos\theta}$$

$$r' = \sqrt{s^2 + b^2 - 2bs\cos\theta}$$

$$V_p = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{\sqrt{s^2+a^2-2as\cos\theta}} + \frac{q'}{\sqrt{s^2+b^2-2bs\cos\theta}}\right)$$

$$V_p = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{\sqrt{s^2+a^2-2as\cos\theta}} - \frac{R \cdot q}{a\sqrt{s^2+\frac{R^4}{a^2}-2\frac{R^2}{a}s\cos\theta}}\right)$$

$$\boxed{V_p = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{s^2+a^2-2sa\cos\theta}} - \frac{1}{\sqrt{\left(\frac{sa}{R}\right)^2+R^2-2sa\cos\theta}}\right)}$$

Clearly, when $s = R$, $V = 0$.

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(b) Since, $\sigma = -\epsilon_0 \dfrac{\partial V}{\partial n}$

in this case $\dfrac{\partial V}{\partial n} = \dfrac{\partial V}{\partial s}$ at point $s = R$.

$$\frac{4\pi\epsilon_0}{q}\left(\frac{\partial V}{\partial s}\right) = -\frac{1}{2}\left(s^2+a^2-2sa\cos\theta\right)^{-3/2}\left(2s-2a\cos\theta\right)$$
$$+\frac{1}{2}\left(\left(\frac{sa}{R}\right)^2+R^2-2sa\cos\theta\right)^{-3/2}\cdot\left(\frac{a^2}{R^2}2s-2a\cos\theta\right)$$

$$\left(\frac{4\pi\epsilon_0}{q}\right)\left(\left.\frac{\partial V}{\partial s}\right|_{at\, s=R}\right) = \left(R^2+a^2-2Ra\cos\theta\right)^{-3/2}\left\{a\cos\theta - R + \frac{a^2}{R} - a\cos\theta\right\}$$

$$\left(\frac{\partial V}{\partial s}\right)_{at\, s=R} = \frac{q}{4\pi\epsilon_0 R}\left(R^2+a^2-2Ra\cos\theta\right)^{-3/2}\left(a^2-R^2\right)$$

$$\boxed{\therefore\ \sigma(\theta) = \frac{q}{4\pi R}\left(R^2-a^2\right)\left(R^2+a^2-2Ra\cos\theta\right)^{-3/2}}$$

Now, the total charge Q at surface of sphere,

$$Q = \int \sigma\, da$$

$$Q = \frac{q(R^2-a^2)}{4\pi R}\int\left(R^2+a^2-2Ra\cos\theta\right)^{-3/2} R^2 \sin\theta\, d\theta\, d\phi$$

$$Q = \frac{q(R^2-a^2)}{4\pi R}\cdot 2\pi R^2\int_0^{\pi}\left(R^2+a^2-2Ra\cos\theta\right)^{-3/2} d\theta$$

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$$Q = \frac{qR(R^2-a^2)}{2}\left\{\frac{-2\left(R^2+a^2-2Ra\cos\theta\right)^{-1/2}}{-2Ra}\right\}\Bigg|_0^{\pi}$$

$$Q = \frac{q(R^2-a^2)}{2a}\left\{\frac{1}{\sqrt{R^2+a^2+2Ra}} - \frac{1}{\sqrt{R^2+a^2-2Ra}}\right\}$$

$$Q = \frac{q(R^2-a^2)}{2a}\left\{\frac{1}{R+a} - \frac{1}{a-R}\right\}$$

$$Q = \frac{q(R^2-a^2)}{2a(-a^2+R^2)}\left(a-R-R-a\right)$$

$$Q = -\frac{qR}{a} = q'$$

Hence, the induced charge on the surface of sphere will be equal to q'.

(c) $$W = \frac{1}{2}\left\{\frac{1}{4\pi\epsilon_0}\frac{-\frac{Rq}{a}\cdot q}{\left(a-\frac{R^2}{a}\right)}\right\}$$

$$W = \frac{-Rq^2}{8\pi\epsilon_0(a^2-R^2)}$$

the potential energy of the configuration is $\dfrac{-Rq^2}{8\pi\epsilon_0(a^2-R^2)}$