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PH110: Waves and Thermodynamics

Tutorial 12

Q.12 An infinitely long cylinder, of radius R, carries a "frozen in" magnetization parallel to the axis,

$$M = ks\,\hat{z}$$

where k is a constant and s is the distance from the axis; there is no free current anywhere. Find the magnetic field inside and outside the cylinder by two different methods:

(a) As in Sect. 6.2, locate all the bound currents, and calculate the field they produce.

(b) Use Ampere's law (eq 6.20) to find H, and then get B from Eq 6.18.

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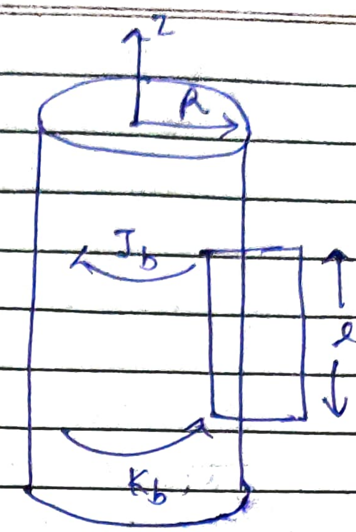

(a) $M = ks\hat{z}$

$\therefore J_b = \nabla \times M$
$= -k\hat{\phi}$

and, $K_b = M \times \hat{n} = kR\hat{\phi}$

B is in z direction.
So, $\boxed{B = 0 \text{ outside.}}$

Using the Amperian loop shown - inner side at radius s:

$$\oint B \cdot dl = Bl$$
$$= \mu_0 I_{enc}$$
$$= \mu_0 \left[\int J_b\, da + K_b l\right]$$
$$= \mu_0 \left[-kl(R-s) + kRl\right]$$
$$= \mu_0 kls$$

$$\therefore \boxed{B = \mu_0 ks\hat{z}} \text{ inside}$$

(b) By symmetry H points in z-direction. That same amperian loop give $\oint H \cdot dl = Hl$
$= \mu_0 I_{fenc} = 0$.

Since, there is no free current here. So $H = 0$, and hence $B = \mu_0 M$. Outside $M = 0$, so $B = 0$;
inside $M = ks\hat{z}$, so $B = \mu_0 ks\hat{z}$.

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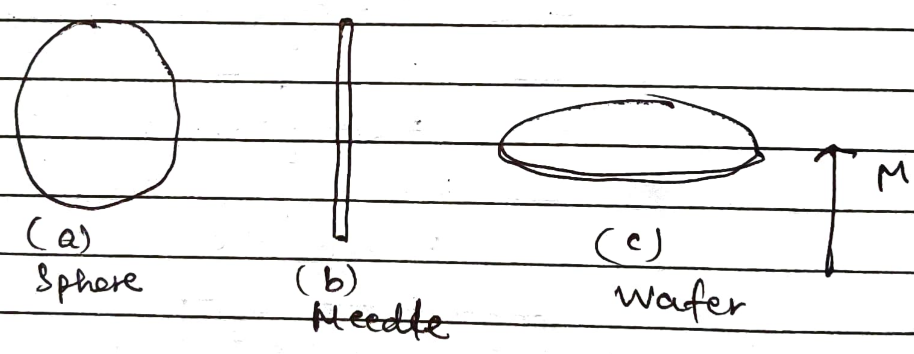
Ques 13. Suppose the field inside a large piece of magnetic material is B_0, so that $H_0 = \frac{1}{\mu_0} B_0 - M$ where M is a "frozen-in" magnetization.

(a) Now, a small spherical cavity is hollowed out of the material. Find the field at the centre of the cavity in terms of B_0 and M. Also find H at the centre of the cavity, in terms of H_0 and M.

(b) Do the same for a long needle-shaped cavity running parallel to M.

(c) Do the same for a thin wafer-shaped cavity perpendicular to M.



Assume that the cavities are small enough so M, B_0 and H_0 are essentially constant. Compare problem 4.16.

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(a) The field of a magnetized sphere is $\frac{2}{3}\mu_0 M$, so

$$B = B_0 - \frac{2}{3}\mu_0 M$$

when the sphere is removed.

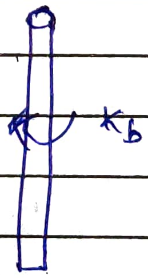
In the cavity, $H = \frac{B}{\mu_0}$, so

$$H = \frac{1}{\mu_0}\left(B_0 - \frac{2}{3}\mu_0 M\right)$$

$$= H_0 + M - \frac{2}{3}M$$

$$\boxed{H = H_0 + \frac{M}{3}}$$

(b) The field inside a long solenoid is $\mu_0 K$. Here $K = M$, so the field of the bound current on the inner surface of the cavity is $\mu_0 M$, pointing down. Therefore,



$$\boxed{B = B_0 - \mu_0 M}$$

and $H = \frac{1}{\mu_0}(B_0 - \mu_0 M)$

$$= \frac{B_0}{\mu_0} - M$$

$$\boxed{\therefore H = H_0}$$

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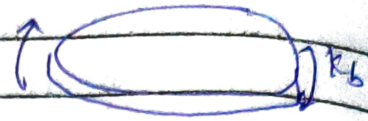
(c) The bound currents are small, and far away from the centre, so



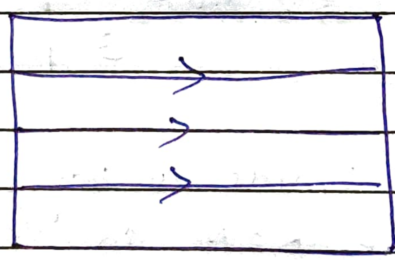
$$\boxed{B = B_0}$$

and $H = \frac{1}{\mu_0} B_0 = H_0 + M$

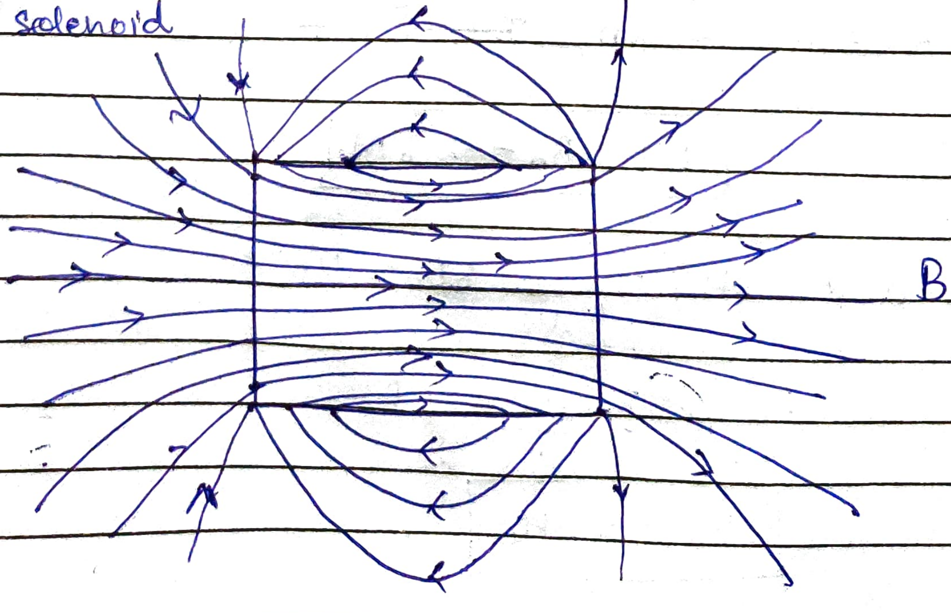
$$\therefore \boxed{H = H_0 + M}$$

Ques 14 For the bar magnet of Prob 6.9, make careful sketches of M, B and H assuming L is about $2a$. Compare Prob 4.17.

M :



B is same as the field of a short solenoid



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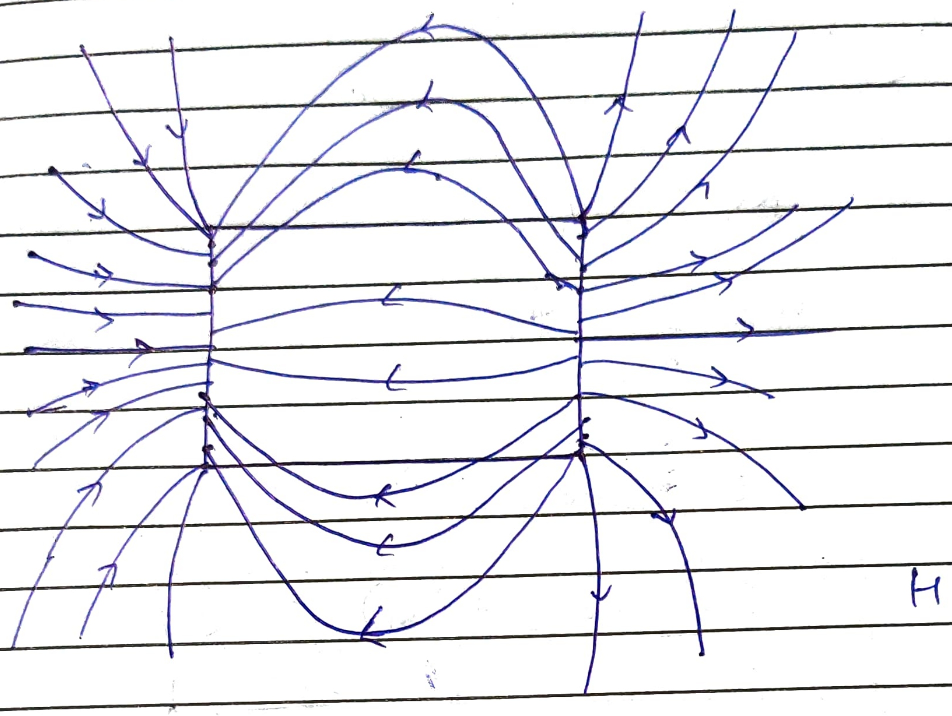
and $H = \frac{B}{\mu_0} - M$



Ques.16 A coaxial cable consists of two very long cylindrical tubes, seperated by linear insulating material of magnetic susceptibility χ_m. A current I flows down the inner conductor and returns along the outer ~~line~~ one : in each case, the current distributes uniformly over the surface. Find the magnetic field in the region between the tubes. As a check, calculate the magnetization and the bound currents, and confirm that they generate the correct field.

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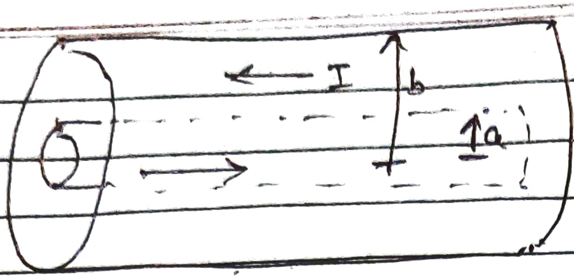

$$\oint H \cdot dl = I_{fenc}$$

So,

$$H = \frac{I}{2\pi s} \hat{\phi}$$

$\because$

$$B = \mu_0 (1 + \chi_m) H$$

$$\boxed{\therefore B = \mu_0 (1 + \chi_m) \frac{I}{2\pi s} \hat{\phi}}$$

and,

$$\boxed{M = \chi_m H = \frac{\chi_m I}{2\pi s} \hat{\phi}}$$

Now, since, $J_b = \nabla \times M$

$$\therefore J_b = \frac{1}{s} \frac{\partial}{\partial s} \left(\cancel{s} \frac{\chi_m I}{2\pi \cancel{s}} \right) \hat{z}$$

$$\boxed{J_b = 0}$$

and, $K_b = M \times \hat{n}$

$$= \begin{cases} \dfrac{\chi_m I}{2\pi a} \hat{z} & \text{at } s = a \\ -\dfrac{\chi_m I}{2\pi b} \hat{z} & \text{at } s = b \end{cases}$$

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∴ the magnetic field in between the ~~region~~ tubes is

$$B = \mu_0 (1 + \chi_m) \frac{I}{2\pi s} \hat{\phi}$$

where s is distance from axis.

Check :-
Total enclosed current, for an amperian loop between the cylinders :

$$I + \frac{\chi_m I}{2\pi a} \cdot 2\pi a = (1 + \chi_m) I$$

$$\therefore \oint B \cdot dl = \mu_0 I_{enc}$$

$$B (2\pi s) = \mu_0 (1 + \chi_m) I$$

$$\therefore B = \mu_0 (1 + \chi_m) \frac{I}{2\pi s} \hat{\phi}$$

Hence, verified.

Ques 17. A current I flows down a long straight wire of radius a. If the wire is made of linear material with susceptibility χ_m, and the current is distributed uniformly, what is the magnetic field at a distance s from the axis? Find all the bound currents. What is the net bound current flowing down the wire?

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From Eq. 6.20 :

$$\oint H \cdot dl = H(2\pi s) = I_{fenc}$$

$$H(2\pi s) = \begin{cases} I\left(s^2/a^2\right) & ; (s<a) \\ I & ; (s>a) \end{cases}$$

$$\therefore H = \begin{cases} \dfrac{Is}{2\pi a^2} & ; s<a \\ \dfrac{I}{2\pi s} & ; s>a \end{cases}$$

$$\boxed{\therefore B = \mu H = \begin{cases} \dfrac{\mu_0(1+\chi_m) Is}{2\pi a^2} & ; (s<a) \\ \dfrac{\mu_0 I}{2\pi s} & ; (s>a) \end{cases}}$$

Calculating bound currents,

$J_b = \chi_m J_f$ and $J_f = \dfrac{I}{\pi a^2}$

$$\therefore \boxed{J_b = \frac{\chi_m I}{\pi a^2} \text{ (direction same as } I)}$$

and $K_b = M \times \hat{n} = \chi_m H \times \hat{n}$

$$\boxed{K_b = \frac{\chi_m I}{2\pi a} \text{ (direction opposite to } I)}$$

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net bound current I_b,

$$I_b = J_b(\pi a^2) + k_b(2\pi a)$$

$$= \chi_m I - \chi_m I$$

$$\boxed{I_b = 0}$$

Ques 21 (a) Show that the energy of a magnetic dipole in a magnetic field B is

$$U = -m \cdot B$$

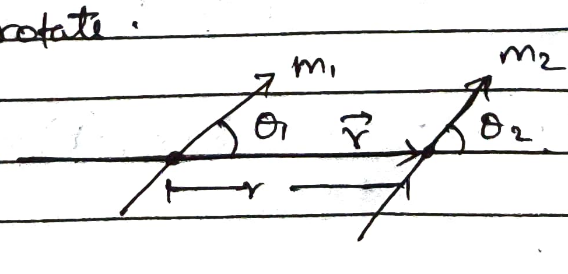
Compare equation 4.6.

(b) Show that the interaction energy of two magnetic dipoles seperated by displacement r is given by

$$U = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[m_1 \cdot m_2 - 3(m_1 \cdot \hat{r})(m_2 \cdot \hat{r}) \right]$$

Compare Eq. 4.7.

(c) Express your answer to (b) in terms of the angles θ_1 and θ_2 in figure, use the result to find the stable configuration two dipoles would adopt if held a fixed distance apart, but left free to rotate.



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(a) Suppose you had a large collection of compass needles, mounted on pins at regular intervals along a straight line. How would they point (assuming earth's magnetic field can be neglected?

(a)
$$U = -\int_{\infty}^{r} F \cdot dl$$

$$= -\int_{\infty}^{r} \nabla (m \cdot B) \cdot dl$$

$$= -m \cdot B(r) - m \cdot B(\infty)$$

$B \to 0$ at $r \to \infty$

$$\boxed{\therefore \quad U = -m \cdot B}$$

(b) $\because$
$$B_1 = \frac{\mu_0}{4\pi r^3} \left[3(m_1 \cdot \hat{r}) - m_1 \right]$$

and we know that

$$U_{12} = -m_2 \cdot B_1$$

$$\therefore U_{12} = \left[m_2 \cdot \left[\frac{\mu_0}{4\pi} \frac{1}{r^3} (3 m_1 \cdot \hat{r} - m_1) \right] \right]$$

$$U_{12} = -\frac{\mu_0}{4\pi} \left[3(m_1 \cdot \hat{r})(m_2 \cdot \hat{r}) - m_1 \cdot m_2 \right]$$

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$$\boxed{U_{12} = \frac{\mu_0}{4\pi r^3}\left[m_1 \cdot m_2 - 3\,(m_1 \cdot \hat{r})(m_2 \cdot \hat{r})\right]}$$

(c) $$U = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[m_1 m_2 \cos(\theta_2 - \theta_1) - 3 m_1 \cos\theta_1 m_2 \cos\theta_2\right]$$

$$= \frac{\mu_0}{4\pi r^3} m_1 m_2 \left[\cos\theta_2 \cos\theta_1 + \sin\theta_2 \sin\theta_1 - 3\cos\theta_1 \cos\theta_2\right]$$

$$\boxed{U = \frac{\mu_0 m_1 m_2}{4\pi r^3}\left[\sin\theta_2 \cdot \sin\theta_1 - 2\cos\theta_2 \cdot \cos\theta_1\right]}$$

Stable position occurs at minimum energy

i.e. $$\frac{\partial U}{\partial \theta_1} = \frac{\partial U}{\partial \theta_2} = 0$$

$$\frac{\partial U}{\partial \theta_1} = \frac{\mu_0 m_1 m_2}{4\pi r^3}\left[\sin\theta_2 \cos\theta_1 + 2\cos\theta_2 \sin\theta_1\right] = 0$$

$$\therefore \quad \sin\theta_2 \cos\theta_1 + 2\cos\theta_2 \sin\theta_1 = 0 \quad \text{—(I)}$$

$$\frac{\partial U}{\partial \theta_2} = \frac{\mu_0 m_1 m_2}{4\pi r^3}\left[\cos\theta_2 \sin\theta_1 + 2\sin\theta_2 \cos\theta_1\right] = 0$$

$$\therefore \quad \sin\theta_1 \cos\theta_2 + 2\sin\theta_2 \cos\theta_1 = 0 \quad \text{—(II)}$$

Rearranging (I) and (II) as,

$$2\sin\theta_1 \cos\theta_2 = -\sin\theta_2 \cos\theta_1 \quad \text{—(III)}$$
$$2\sin\theta_1 \cos\theta_2 = -4\sin\theta_2 \cos\theta_1 \quad \text{—(IV)}$$

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Thus, $\sin\theta_1 \cos\theta_2 = \sin\theta_2 \cos\theta_1 = 0$

$\therefore$ Either $\sin\theta_1 = \sin\theta_2 = 0$

which implies $\underset{①}{\rightarrow \; \rightarrow}$ or $\underset{②}{\rightarrow \; \leftarrow}$

configuration.

or $\cos\theta_1 = \cos\theta_2 = 0$, which implies

$\underset{③}{\uparrow \downarrow}$ or $\underset{④}{\uparrow \uparrow}$ configuration.

Certainly ② and ④ are not stable because for these m_2 is not parallel to B_1, whereas we know m_2 will line up along B_1.

Now computing energy for configuration 1,

$$U_1 = \frac{\mu_0 m_1 m_2}{4\pi r^3}(0 \cdot 0 - 2 \cdot 1 \cdot 1) = \frac{\mu_0 m_1 m_2}{4\pi r^3} \times (-2)$$

and for configuration 3,

$$U_3 = \frac{\mu_0 m_1 m_2}{4\pi r^3}(1 \cdot (-1) - 2 \cdot 0 \cdot 0) = \frac{\mu_0 m_1 m_2}{4\pi r^3}(-1)$$

Since U_1 is the lower energy, hence configuration 1 is more stable configuration.

Hence, they will line up parallel, along the line joining them i.e. $\rightarrow \; \rightarrow$

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(d) They will line up in the same way as concluded in part (c).

$\rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow$