Indian Institute of Information Technology Vadodara

End semester, Autumn/ Winter examination

B.Tech/ M.Tech/ Research student

(Strike off non applicable)

Course Code: CS 203 Course Name: Analysis and Design of Algorithms Date: 6/01/22

Candidate Name: Archit Agrawal Student ID: 202051213

Number of Supplementary booklets:-- 1/2/3

Read the instructions carefully	
1	Listen to the instruction stated by invigilator carefully. It may be in addition to mentioned on answer sheet / question paper.
2	It is mandatory to present your ID card to the invigilator.
3	Answer new question in a new page.
4	Possession of books, notebook, data storage device, scanner, mobile phone is considered as malpractice in examination hall (scientific, non programable calculator are permitted) unless specified by the course instructor.
5	Any type of communication or request for stationery items such as scale, pencil, eraser to other examines during exam will be treated as unfair means.
6	Don't write anything except your roll number on question paper unless specifically instructed.
7	At the end of exam, leave the examination hall quickly and quietly.

Question No.	Marks
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	
13.	
14.	
Total	

Pledge

I shall abide by rules and regulation of Institute. I affirm that I will not take any unauthorize help during exam.

Student's Signature Archit Agrawal

Information Verified

Invigilator's Signature

Archit Agrawal

20205 1213

Question 2

Roll Number → 20205 1213

$S = 16$

$R = 16 \% 5 = 1$

$\therefore R = 1$

$\therefore T = R + 4 = 5$

$\therefore R = 1$ and $T = 5$

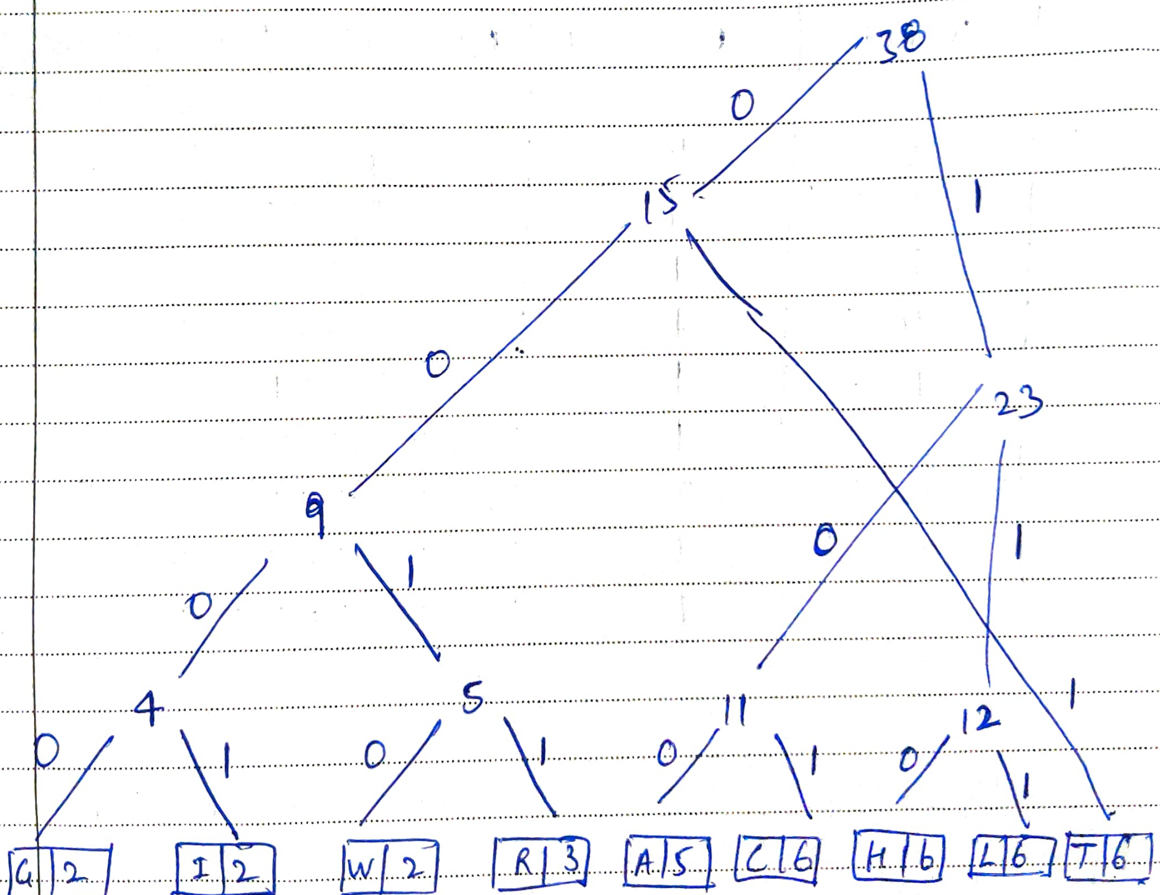
Name: ARCHIT AGRAWAL

Character	Frequency	After adding R and T in column II
A	4	$4+1 = 5$
C	1	$1+5 = 6$
G	1	$1+1 = 2$
H	1	$1+5 = 6$
I	1	$1+1 = 2$
L	1	$1+5 = 6$
R	2	$2+1 = 3$
T	1	$1+5 = 6$
W	1	$1+1 = 2$

The Huffman Tree is drawn on next page

Character	Frequency + R/#
G	2
I	2
W	2
R	3
A	5
C	6
H	6
L	6
T	6

~~C 1~~ ~~G 1~~ ~~H 1~~ ~~I 1~~ ~~L 1~~ ~~T 1~~ ~~W 1~~ ~~R 2~~ A 4



∴ the huffman code for each character is

Character	Huffman code
G	0000
I	0001
W	0010
R	0011
A	100
C	101
H	110
L	111
T	01

adjacency matrix D[0] for given graph is

$$D[0] = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & \infty & 0 & 4 & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \end{array}$$

$$D[1] = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & \infty & 0 & 4 & \infty \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \end{array}$$

$$D[2] = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & \infty & 0 & & \\ & 5 & & 0 & \\ & \infty & & & 0 \end{bmatrix} \end{array}$$

$\boxed{\text{Question 3}}$

adjacency matrix D[0] for given graph

$$D[0] = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \end{array}$$

$$D[1] = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 8 \end{array} & \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \end{array}$$

$$D[2] = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \end{array}$$

$$D[3] = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ \infty & -1 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \end{array}$$

$$D[4] = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 3 & -1 & 4 & -4 \\ \infty & 0 & -4 & 1 & -1 \\ \infty & 4 & 0 & 5 & 3 \\ \infty & -1 & -5 & 0 & -2 \\ \infty & 5 & 1 & 6 & 0 \end{bmatrix} \end{array}$$

$$D[5] = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 & -3 & 2 & -4 \\ \infty & 0 & -4 & 1 & -1 \\ \infty & 4 & 0 & 5 & 3 \\ \infty & -1 & -5 & 0 & -2 \\ \infty & 5 & 1 & 6 & 0 \end{bmatrix} \end{array}$$

Hence, D[5] is the matrix that has all-pair shortest path data.

Question 1

1. $T(n) = O(n \log n)$ {Guess}

∴ ~~we need to prove that~~

$\cancel{T(n) \leq}$

∴ assume hypothesis is true for $m < n$.

∴ $T(m) \leq m \log m$

~~So,~~

$\cancel{T(n) \leq cm}$

Question 1

2. $T(n) = 2T(n/2) + n^2 \log n$

Guess : $n^2 \log n$

We need to prove that
$T(n) <= cn^2 \log n$.

We can assume it is true for values smaller than n.

$T(n) = 2T(n/2) + n^2 \log n$

$<= 2c\frac{n^2}{4}\log\left(\frac{n}{2}\right) + \frac{n^2}{4}\log\left(\frac{n}{2}\right)$

$$<= \frac{cn^2}{2}\log(n) - \frac{cn^2}{2}\log(2) + \frac{n^2}{4}\log\left(\frac{n}{2}\right)$$

$$<= \{ \, cn^2\log n - cn^2\log(2) + \frac{n^2}{4}\log\left(\frac{n}{2}\right)$$

$$<= cn^2\log n$$