Tutorial- 4a Submission

MA 201 Probability and Statistics (2021-22) (3-1-0-4)

B. Tech. II year CSE & IT

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```
clear all
%F = get_latex_string();
```

Continuous Random Variable -

Qus1(a) - The lifetime, in years, of some electronic component is a continuous random variable with the density

$$f(x) = \begin{cases} \frac{k}{x^4} & \text{for} \quad x \geq 1 \\ 0 & \text{for} \quad x < 1 \end{cases}$$

Find k, the cumulative distribution function, and the probability for the lifetime to exceed 2 years.

Hint - Area under the $f_X(x)$ must be 1.

$$\int_{-\infty}^{\infty} f_X(y)dy = \int_{1}^{\infty} \frac{K}{y^4} dy = \frac{K}{3} = 1$$

```
syms K x y
%changed y^4 to y^3
int(K/y^3, y, 1, Inf)
```

ans =

$$\frac{K}{2}$$

Now the CDF is $F_X(x) = \int_1^x \frac{3}{y^4} dy = 1 - \frac{1}{x^3}$

```
F = int(100/y^3, y, 1, x)
```

F =

$$\frac{100}{3} - \frac{100}{3\,x^3}$$

Therefore $F_X^C(x) = \Pr\{X > x\} = \frac{1}{x^3} \Rightarrow F_X^C(2) = \Pr\{X > 2\} = \frac{1}{8}$

```
1 - subs(F,'x', 2)
```

```
ans =
```

$$-\frac{169}{6}$$

which can be written in decimal form as

```
vpa(1 - subs(F,'x', 2))
```

ans = $-28.166666666666666666666666666667$

Qus1(b) - The time, in minutes, it takes to reboot a certain system is a continuous variable with the density

$$f(x) = \begin{cases} C(10-x)^2 & \text{for} \quad 0 < x < 10 \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute C.

(b) Compute the probability that it takes between 1 and 2 minutes to reboot.

Hint - Area under the $f_X(x)$ must be 1.

$$\int_{-\infty}^{\infty} f_X(y)dy = \int_0^{\infty} C(1-y)^2 dy = \frac{1000}{3}C = 1$$

```
syms C x y
%changed constant 10 to 213
int(C*(213-y)^2, y, 0, 10)
```

```
ans =
```

$$\frac{1298170\,C}{3}$$

Now the CDF is $F_X(x) = \int_0^x -\frac{3}{970}(1-y^2)dy = \frac{x}{1000}(x^2 - 30x + 300)$

```
F = int(3/1000*(213-y)^2, y, 0, x)
```

```
F =
```

$$\frac{x\,(x^2 - 639\,x + 136107)}{1000}$$

Now, probability that it takes between 1 and 2 minutes to reboot is

```
subs(F, 'x',2) - subs(F, 'x',1)
```

```
ans =
```

$$\frac{134197}{1000}$$

which can be written in decimal form as

```
vpa(subs(F, 'x',2) - subs(F, 'x',1))
```

ans = 134.197

Qus1(c) - The installation time, in hours, for a certain software module has a probability density function $f(x) = k(1 - x^3)$ for $0 < x < 1$. Find k and compute the probability that it takes less than $1/2$ hour to install this module.

Qus1(d) - Lifetime of a certain hardware is a continuous random variable with density

$$f(x) = \begin{cases} K - x/50 & \text{for} \quad 0 < x < 10 \text{ years} \\ 0 & \text{otherwise} \end{cases}$$

(a) Find K.

(b) What is the probability of a failure within the first 5 years?

(c) What is the expectation of the lifetime?

Hint - Area under the $f_X(x)$ must be 1.

$$\int_{-\infty}^{\infty} f_X(y)dy = \int_{0}^{\infty} \left(K - \frac{y}{50}\right)dy = 10K - 1 = 1$$

```
syms K x y
%changed constant 50 to 13
int(K - y/13, y, 0, 10)
```

ans =

$$10\,K - \frac{50}{13}$$

Now the CDF is $F_X(x) = \int_0^x \left(\frac{1}{5} - \frac{y}{50}\right)dy = \frac{x}{100}(20 - x)$

```
F = int(1/5 - y/13, y, 0, x)
```

F =

$$-\frac{x\,(5\,x - 26)}{130}$$

Now, probability that it takes between 1 and 2 minutes to reboot is $\frac{3}{4}$

```
subs(F, 'x',5)
```

ans =

$$\frac{1}{26}$$

which can be written in decimal form as 0.75

```
vpa(subs(F, 'x',5))
```

ans = $0.038461538461538461538461538461538$

Now Expected value of X is $E(x) = \int_0^{10} y\left(\frac{1}{5} - \frac{y}{50}\right) dy = \frac{10}{3}$

```
E = int(y*(1/5 - y/13), y, 0, 10)
```

E =

$$-\frac{610}{39}$$

Qus1(e) - Two continuous random variables X and Y have the joint density

$f_{X,Y}(x,y) = C(x^2 + y)$ **for** $-1 \leq x \leq 1$**,** $0 \leq y \leq 1$

(a) Compute the constant C**.**

(b) Find the marginal densities of X **and** Y**. Are these two variables independent?**

(c) Compute probabilities $\Pr\{Y < 0.6\}$ **and** $\Pr\{Y < 0.6 | X < 0.5\}$

```
syms C x y
C = 1/integral2(@(x,y) (x.^2+y), -1,1,0,1)
```

C = 0.6000

```
fXx = int(3/5*(x^2+y), y, 0,1)
```

fXx =

$$\frac{3\,x^2}{5} + \frac{3}{10}$$

```
fYy = int(3/5*(x^2+y), x, -1,1)
```

fYy =

$$\frac{6\,y}{5}+\frac{2}{5}$$

```
expand(fXx*fYy) % (b) not independent
```

ans =

$$\frac{9\,y}{25}+\frac{18\,x^2\,y}{25}+\frac{6\,x^2}{25}+\frac{3}{25}$$

```
int(fYy,y,0,0.6)   %(c)
```

ans =

$$\frac{57}{125}$$

```
vpa(integral2(@(x,y) 3/5*(x.^2+y), -1,0.5,0,0.6) / int(fYy,y,0,0.6)) %(c)
```

ans = $0.65131578947382953190308029932\,79$

Qus2(a) - The time it takes a printer to print a job is an Exponential random variable with the expectation of 12 seconds. You send a job to the printer at 10:00 am, and it appears to be third in line. What is the probability that your job will be ready before 10:01 am?

Hint- T is a Gamma Random Varaible $\alpha = 3$, $\frac{1}{\lambda} = 12 seconds$

Probability that the job will not be ready within 60 seconds.

$$\Pr\{T > t\} = \Pr\{X < \alpha\} \Rightarrow \Pr\{T > 60\} = \Pr\{X < 3\}$$

$$\Pr\{T \le t\} = \Pr\{X \ge \alpha\} \Rightarrow \Pr\{T \le 60\} = \Pr\{X \ge 3\}$$

```
poisscdf(2,60/12,'upper')
```

ans = 0.8753

```
gamcdf(60,3,12)
```

ans = 0.8753

Qus2(b) - For some electronic component, the time until failure has Gamma distribution with parameters $\alpha = 2$ and $\lambda = 2(years^{-1})$. Compute the probability that the component fails within the first 6 months.

```
poisscdf(1,2/2,'upper')
```

```
ans = 0.2642
```

```
gamcdf(1/2, 2,1/2)
```

```
ans = 0.2642
```

Qus2(c) - A computer processes tasks in the order they are received. Each task takes an Exponential amount of time with the average of 2 minutes. Compute the probability that a package of 5 tasks is processed in less than 8 minutes.

Hint - Gamma Random Variable $\frac{1}{\lambda} = 2$, $\alpha = 5$

```
gamcdf(8,5,2)
```

```
ans = 0.3712
```

```
poisscdf(4,8/2,'upper')
```

```
ans = 0.3712
```

On the average, it takes 25 seconds to download a file from the internet. If it takes an Exponential amount of time to download one file, then what is the probability that it will take more than 70 seconds to download 3 independent files?

```
gamcdf(70,3,25,'upper')
```

```
ans = 0.4695
```

```
poisscdf(2,70/25)
```

```
ans = 0.4695
```

The time X it takes to reboot a certain system has Gamma distribution with E(X) = 20 min and Std(X) = 10 min.

(a) Compute parameters of this distribution.

(b) What is the probability that it takes less than 15 minutes to reboot this system?

$$\frac{\alpha}{\lambda} = 20 \quad \frac{\alpha}{\lambda^2} = 100 \quad \Rightarrow \lambda = \frac{1}{5}, \quad \alpha = 4.$$

```
gamcdf(15,4,5)
```

```
ans = 0.3528
```

```
poisscdf(3,15/5,'upper')
```

```
ans = 0.3528
```

Qus3(a) - On the average, a computer experiences breakdowns every 5 months. The time until the first breakdown and the times between any two consecutive breakdowns are independent Exponential random variables. After the third breakdown, a computer requires a special maintenance.

(a) Compute the probability that a special maintenance is required within the next 9 months.

(b) Given that a special maintenance was not required during the first 12 months, what is the probability that it will not be required within the next 4 months?

```
gamcdf(9,3,5)
```

```
ans = 0.2694
```

```
poisscdf(2,9/5,'upper')
```

```
ans = 0.2694
```

$$\Pr(T > 16 | T > 12) = \frac{\Pr(T > 16 \cap T > 12)}{\Pr(T > 12)} = \frac{\Pr(T > 16)}{\Pr(T > 12)} = \frac{\Pr(X < 3 | \lambda t = 16/5)}{\Pr(X < 3 | \lambda t = 12/5)}$$

```
gamcdf(16,3,5,'upper')/gamcdf(12,3,5,'upper')
```

```
ans = 0.6668
```

```
poisscdf(2,16/5)/poisscdf(2,12/5)
```

```
ans = 0.6668
```

Tutorial- 4b Submission

MA 201 Probability and Statistics (2021-22) (3-1-0-4)

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Normal Random Variable -

Qus - 1a) Let Z be a Standard Normal random variable. Compute

(a) P(Z < 1.25) (b) P(Z ≤ 1.25) (c) P(Z > 1.25) (d) P(|Z| ≤ 1.25)

```
z1 = 1.25; mu =0;sigma =1;
[1/2*(1+erf(z1/sqrt(2))) normcdf(z1,mu,sigma)] % (a) (b)
```

```
ans = 1×2
    0.8944    0.8944
```

```
[1/2*(1-erf(z1/sqrt(2))) normcdf(z1,mu,sigma,'upper')] % (c)
```

```
ans = 1×2
    0.1056    0.1056
```

```
[1/2*(1+erf(z1/sqrt(2))) - 1/2*(1+erf(-z1/sqrt(2)))  normcdf(z1,mu,sigma) - normcdf(-z1
```

```
ans = 1×2
    0.7887    0.7887
```

(e) P(Z < 6.0) (f) P(Z > 6.0)

```
z1 = 6.0; mu=0; sigma = 1;
[1/2*(1+erf(z1/sqrt(2))) normcdf(z1,mu,sigma)]
```

```
ans = 1×2
    1.0000    1.0000
```

```
[1/2*(1-erf(z1/sqrt(2))) normcdf(z1,mu,sigma,'upper')]
```

```
ans = 1×2
10^-9 ×
    0.9866    0.9866
```

(g) With probability 0.8, variable Z does not exceed what value?

```
y =0.8;  mu=0; sigma = 1;
[sqrt(2)*erfinv(2*(y - 0.5)) norminv(y,mu,sigma)]
```

```
ans = 1×2
    0.8416    0.8416
```

```
z1 = norminv(0.8,mu,sigma);
[1/2*(1+erf(z1/sqrt(2))) normcdf(z1,mu,sigma)]
```

```
ans = 1×2
    0.8000    0.8000
```

Qus -1b)   For a Standard Normal random variable Z, compute

(a) P(Z ≥ 0.99) (b) P(Z ≤ −0.99) (c) P(Z < 0.99)

(d) P(|Z| > 0.99) (e) P(Z < 10.0) (f) P(Z > 10.0)

(g) With probability 0.9, variable Z is less than what?

Repeat it like the previous question.

Qus -1c) For a Normal random variable X with E(X) = −3 and Var(X) = 4, compute

(a) P(X ≤ 2.39) (b) P(X ≥ −2.39) (c) P(|X| ≥ 2.39)

(d) P(|X + 3| ≤ 2.39) (e) P(X < 5) (f) P(|X| < 5)

(g) With probability 0.33, variable X exceeds what value?

```
x = 2.39; mu=-3; sigma = sqrt(4);
z1 = (x-mu)/sigma;
[1/2*(1+erf(z1/sqrt(2))) normcdf(x,mu,sigma)] % (a) (b)
```

```
ans = 1×2
    0.9965    0.9965
```

```
[1/2*(1-erf(z1/sqrt(2))) normcdf(x,mu,sigma,'upper')] % (c)
```

```
ans = 1×2
    0.0035    0.0035
```

P(|X + 3| ≥ 2.39) = P(X ≥ 2.39 -3) + P(-X ≥ 3+ 2.39) = P(X ≥ - 0.61) + P(X ≤ -5.39)

```
c = 3;
%z2 = (x+mu)/sigma;
%[1/2*(1+erf(z1/sqrt(2))) - 1/2*(1+erf(-z2/sqrt(2)))  normcdf(x,mu,sigma) - normcdf(-x,

x = 5; mu=-3; sigma = sqrt(4);
z1 = (x-c-mu)/sigma;
[1/2*(1+erf(z1/sqrt(2))) normcdf(x,mu,sigma)] % (e)
```

```
ans = 1×2
    0.9938    1.0000
```

```
z2 = (x+c+mu)/sigma;
[1/2*(1-erf(z1/sqrt(2))) + 1/2*(1+erf(-z2/sqrt(2)))  normcdf(x-c,mu,sigma,'upper') + nc
```

```
ans = 1×2
    0.0124    0.0124
```

```
y =0.33;
[mu+ sigma*sqrt(2)*erfinv(2*(y - 0.5)) norminv(y,mu,sigma)]
```

```
ans = 1×2
   -3.8798   -3.8798
```

```
z1 = norminv(0.8,mu,sigma);
[1/2*(1+erf(z1/sqrt(2))) normcdf(z1,mu,sigma)] % verify it.
```

```
ans = 1×2
    0.0940    0.8000
```

Qus -4 Suppose that the average household income in some country is 900 coins, and the standard deviation is

200 coins.

(a) Assuming the Normal distribution of incomes, compute the proportion of “the middle class,” whose income is between 600 and 1200 coins.

```
mu =900; sigma = 200;
normcdf(1200,mu,sigma)-normcdf(600,mu,sigma)
```

```
ans = 0.8664
```

(b) The government of the country decides to issue food stamps to the poorest 3% of households. Below what income will families receive food stamps?

```
y =0.03;
[mu+ sigma*sqrt(2)*erfinv(2*(y - 0.5)) norminv(y,mu,sigma)]
```

```
ans = 1×2
  523.8413  523.8413
```

Qus - 5 The lifetime of a certain electronic component is a random variable with the expectation of

5000 hours and a standard deviation of 100 hours. What is the probability that the average

lifetime of 400 components is less than 5012 hours?

Hint - Suppose $M_n = \frac{S_n}{n} = \frac{X_1 + X_2 + X_3 .... X_n}{n}$ with $\mu = E[X_i] = 5000$ and $\sigma = \sqrt{Var[X_i]} = 100$

$E[M_n] = E\left[\frac{S_n}{n}\right] = \frac{n\mu}{n} = \mu$. $Var[M_n] = Var\left[\frac{S_n}{n}\right] = \frac{n \times \sigma^2 \mu}{n^2} = \frac{\sigma^2}{n} \Rightarrow std[Mn] = \frac{\sigma}{\sqrt{n}}$

Since $n = 400$ is large with all X_i with same mean and expectation, we apply CLT.

$\Pr\{M_n < 5012\} = 0.9918$

```
normcdf((5012-5000)/(100/sqrt(400)),0,1)
```

```
ans = 0.9918
```

```
normcdf(5012,5000,100/sqrt(400))
```

```
ans = 0.9918
```

Qus - 10 Seventy independent messages are sent from an electronic transmission center. Messages are

processed sequentially, one after another. Transmission time of each message is Exponential

with parameter λ = 5 $\min^{-1}$. Find the probability that all 70 messages are transmitted in less than 12 minutes. Use the Central Limit Theorem.

```
alpha = 70; lambda = 5;
mu = alpha/lambda; sigma = sqrt(alpha/lambda^2);
normcdf(12,mu,sigma) %central limit theorem
```

```
ans = 0.1160
```

```
gamcdf(12,alpha,1/lambda) %exact
```

```
ans = 0.1118
```

Qus - 13 Upgrading a certain software package requires installation of 68 new files. Files are installed consecutively. The installation time is random, but on the average, it takes 15 sec to install one file, with a variance of 11 $\sec^2$.

(a) What is the probability that the whole package is upgraded in less than 12 minutes?

(b) A new version of the package is released. It requires only N new files to be installed, and it is promised that 95% of the time upgrading takes less than 10 minutes. Given this information, compute N.

```
mu = 68*15; sigma = sqrt(11*68);
normcdf((12*60-mu)/sigma,0,1)
```

```
ans = 2.6907e-28
```

```
normcdf(12*60,mu,sigma) %(a)
```

```
ans = 2.6907e-28
```

Assume $z = \frac{600 - 15N}{\sqrt{11N}}$. Since $\phi^{-1}(0.95) = 1.665$, After solving $\frac{600 - 15N}{\sqrt{11N}} == 1.665$, we get $N = 37.76 \approx 38$ files.

```
y = 0.95;
z = norminv(y,0,1);
syms N
vpa(solve((600 - 15*N)/sqrt(11*N) ==z, N))
```

ans = $37.765002212451730416978960954937$