Archit Agrawal
202052307

PH100: Mechanics & Thermodynamics

Tutorial #10

1. (a) Light Waves as Particles

The photoelectric effect suggests that light of frequency ν can be regarded as consisting of photons of energy $E = h\nu$, where
$h = 6.63 \times 10^{-27}$ erg.s.

(i) Visible light has a wavelength in the range of 400 - 700 nm. What are the energy and frequency of a photon of visible light?

(ii) The microwave in my kitchen operates at roughly 2.5 GHz at a max power of 7.5×10^{9} erg/s. How many photons per second can it emit? What about a low power laser (10^{4} erg/s at 633 nm), or a cell phone (4×10^{6} erg/s at 850 MHz)?

(iii) How many such microwave photons does it take to warm a 200 ml glass of water by 10°C? (The heat capacity of water is roughly 4.18×10^{7} $\frac{erg}{g°K}$).

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(iv) At a given power of an electromagnetic wave, do you expect a classical wave description to work better for radio frequencies, or for X-rays?

(a) (i) For visible light of wavelength $\lambda = 400nm$.

frequency $\nu_1 = \frac{c}{\lambda} = \frac{3 \times 10^8}{400 \times 10^{-9}} = 7.5 \times 10^{14}$ Hz

and energy $E_1 = h\nu$
$= 6.63 \times 10^{-27}$ erg.s $\times 7.5 \times 10^{14}$ Hz
$= 49.725 \times 10^{-13}$ ergs
$= 4.9725 \times 10^{-12}$ ergs

$= 4.9725 \times 10^{-19}$ J
$= 3.104$ eV

(ii) For visible light of wavelength $\lambda = 700nm$,

frequency $\nu_2 = \frac{c}{\lambda} = \frac{3 \times 10^8}{700 \times 10^{-9}} = 4.286 \times 10^{14}$ Hz

and energy $E_2 = h\nu$
$= 6.626 \times 10^{-27}$ erg.s $\times 4.286 \times 10^{14}$ Hz
$= 28.399 \times 10^{-13}$ ergs
$= 2.8399 \times 10^{-19}$ ergs
$= 1.775$ eV

Hence, the frequency of visible light ranges from 4.286×10^{14} Hz to 7.5×10^{14} Hz and its energy ranges from 1.775 eV to 3.104 eV.

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(ii) power output of microwave $= 7.5 \times 10^9$ ergs/sec
$= 750 W$ $(1 erg = 10^{-7} J)$

frequency of microwave radiation $= 2.5 GHz$

$\therefore$ energy of photon $= h\nu$
$= 6.63 \times 10^{-34} Js \times 2.5 \times 10^9 Hz$
$= 1.6575 \times 10^{-24} J$

$\therefore$ number of photons emitted per second $= \dfrac{\text{power output}}{\text{energy of one photon}}$

$= \dfrac{750}{1.6575 \times 10^{-24}}$

$= 4.53 \times 10^{26}$ photons/s

Hence, it emits 4.53×10^{26} photons per second.

Now, power output of low-power laser $= 10^4$ erg/s
$= 10^{-3}$ J/s

$\therefore$ no. of photons emitted per second by it $= \dfrac{10^{-3} \times 633 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^8}$

$= 31.82 \times 10^{14}$ photons/s

$= 3.18 \times 10^{15}$ photons/s

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Similary, for a cell phone,

No. of photons emitted per second $= \dfrac{4 \times 10^{-1}}{6.63 \times 10^{-34} \times 850 \times 10^{6}}$

$= 7.098 \times 10^{23}$ photon/s

(iii) density of water is 1 g/ml.
∴ 200ml water weighs 200g.

Energy required to heat 200 g of water by 10°C is given by

$$Q = m s \Delta T$$
$$= 200g \times 4.18 \frac{J}{g°C} \times 10°C$$
$$= 8360\ J$$

∴ No. of microwave photons required to heat 200 ml of water by 10°C is N,

$$N = \frac{8360\ J}{1.6575 \times 10^{-24}\ J}$$

(Energy of one photon from (ii) part)

$$N = 5043.7 \times 10^{24} \text{ photons}$$
$$= 5.04 \times 10^{27} \text{ photons}$$

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(iv) X-rays are at a much higher frequency than radio waves, which means (according to $E = h\nu$) that an X-ray photon carries much more energy than radio wave photon. Thus an electromagnetic wave of a given power contains many more photons at radio frequencies than at X-ray frequencies. The radiowave is therefore more suitable to be described by a statistical description of photons.

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(b) Matter Particles as Waves

If a wavelength can be associated with every moving particle, then why are we not forcibly made aware of this property in our everyday experience? In answering, calculate the de-Broglie wavelength $\lambda = \frac{h}{p}$ of each of the following particles.

(i) an automobile of mass 2 metric tons (2000 kg) travelling at a speed of 50 mph (22 m/s).

(ii) a marble of mass 10g moving with a speed of 10 cm/s.

(iii) a smoke particle of diameter 10^{-5} cm and a density of, say $0.2\ g/cm^3$ being jostled about by air molecules at room temperature (T = 300K) (assume that the particle has the same translational kinetic energy as the thermal average of the air molecules, $KE = \frac{3}{2} k_B T$, with $k_B = 1.38 \times 10^{-16}$ erg/K).

(iv) an ^{87}Rb atom that has been laser cooled to a temperature of $T = 100\,\mu K$. Again assume $KE_i = \frac{3}{2} k_B T$.

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(b) De-broglie wavelength associated with a body of mass m, moving with velocity v is given by

$$\lambda = \frac{h}{mv}$$

Since, the mass of objects that we come across in our daily lives is much higher, the de-broglie wavelength associated with it is quite small. Hence, the wave nature of matter is not apparent to our daily observations.

(i) given, $m = 2000\,kg$ and $v = 22\,m/s$

$$\therefore \lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}}{2000 \times 22}$$

$$\lambda = 0.1506 \times 10^{-37}\,m$$
$$\lambda = 1.506 \times 10^{-38}\,m$$

Hence, the de-Broglie wavelength associated with the automobile is $1.506 \times 10^{-38}\,m$. We can see it is a very small value.

(ii) given, $m = 10\,g$ and $v = 10\,cm/s = 0.1\,m/s$

$$\therefore \lambda = \frac{6.626 \times 10^{-34}}{10 \times 10^{-3} \times 10^{-1}} = 6.626 \times 10^{-31}\,m$$

Hence, the wavelength associated with the marble is $6.626 \times 10^{-31}\,m$. Again, it is a very small value.

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(iii) diameter of smoke particle = 10^{-5} cm

$\therefore$ volume of smoke particle $= \frac{4}{3}\pi\left(\frac{10^{-5}}{2}\right)^3$ cm^3

$= \frac{4}{3}\pi \times \frac{(10^{-15})}{8}$ cm^3

$= \frac{(10^{-15})\pi}{6}$ cm^3

$\therefore$ mass of smoke particle $= V \times d$

$= \frac{10^{-15}\pi}{\cancel{60}\ 30} \times 0.\cancel{2}\ g/\cancel{cm^3} \times \cancel{cm^3}$

$= \frac{10^{-16}\pi}{3}$ g

$= \frac{10^{-19}\pi}{3}$ kg

Using the assumption given,

KE of particle (smoke particle) $= \frac{3}{2} k_B T$

$= \frac{3}{2} \times 1.38 \times 10^{-23}$ J/K $\times$ 300K

$= 6.21 \times 10^{-21}$ J

$\therefore$ de-broglie wavelength $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m(KE)}}$

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$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times \frac{10^{-19}}{3} \times \pi \times 6.21 \times 10^{-21}}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{3.605 \times 10^{-20}}$$

$$\lambda = 1.838 \times 10^{-14} \text{ m}$$

$$\lambda = 18.38 \text{ fm}$$

Hence, the de-broglie wavelength corresponding to smoke particle is 18.38 fm.

(iv) mass of an ^{87}Rb atom $\approx$ 87 × mass of proton

$$= 87 \times 1.66 \times 10^{-27} \text{ kg}$$

$$= 1.444 \times 10^{-25} \text{ kg}$$

Using the assumption given,

$$\text{KE of } ^{87}Rb \text{ atom} = \frac{3}{2} \times k_B \times T$$

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \text{ J/K} \times 100 \times 10^{-6} \text{ K}$$

$$= 2.07 \times 10^{-27} \text{ J}$$

$\therefore$ de-broglie wavelength $\lambda = \frac{h}{\sqrt{2m(KE)}}$

$$= \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 1.444 \times 10^{-25} \times 2.07 \times 10^{-27}}}$$

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$$\lambda = \frac{6.626 \times 10^{-34}}{2.445 \times 10^{-26}}$$

$$\lambda = 2.71 \times 10^{-8} \text{ m}$$

$$\lambda = 27.1 \text{ nm}$$

Hence, the de-broglie wavelength corresponding to ^{87}Rb atom is 27.1 nm. This wavelength is ~~sm~~ large enough to observe the wave nature, but, we don't often see this phenomena in our daily lifes.

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2. Electrons are accelerated in television tubes through potential difference of 10 KV. Find the highest frequency of the electromagnetic waves emitted when these electrons strike the screen of the tube?

Since, the electrons are accelerated through a potential difference of 10 KV,

therefore, the kinetic energy of electrons = 10 keV

$\therefore$ KE of electrons in joules = $10 \times 10^{3} \times 1.6 \times 10^{-19}$

$\because$ energy $E = h\nu$, where h is Planck's constant and ν is frequency of radiation.

$$\therefore \quad \nu = \frac{10 \times 10^{3} \times 1.6 \times 10^{-19}}{6.626 \times 10^{-34}}$$

$$\nu = 2.4147 \times 10^{18} \text{ Hz}$$

Hence, the highest frequency of EM wave is 2.4147×10^{18} Hz.

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3. The distance between adjacent atomic planes in calcite ($CaCO_3$) is 0.3 nm. Find the smallest angle of Bragg scattering for 0.03 nm X-rays.

According to Bragg's law,

$$2d \sin\theta = n\lambda$$

where d is interatomic spacing, λ is wavelength and n is order of reflection.

For minimum value of θ, n should be 1.

Given, $d = 0.3\,nm$ and $\lambda = 0.03\,nm$

$$\therefore \quad 2 \times (\cancel{0.3\,nm}) \sin\theta = 1 \times \cancel{0.03\,nm}$$

$$\sin\theta = \frac{1}{20}$$

$$\theta = \sin^{-1}(0.05)$$

$$\theta = 2.866^\circ$$

Hence, the minimum angle of Bragg scattering for given X-rays is 2.866°.

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4. Obtain an expression for the energy levels (in MeV) of a neutron confined to a one-dimensional box 1.00×10^{-14} m wide. What is the neutron's minimum energy? (The diameter of an atomic nucleus is of this order of magnitude).

Since, $$E_n = \frac{n^2 h^2}{8mL^2}$$

and mass of neutron $m = 1.67 \times 10^{-27}$ kg and given $L = 10^{-14}$ m

$$\therefore \quad E_n = \frac{n^2 (6.63 \times 10^{-34})^2}{8 \times (1.67 \times 10^{-27}) \times (10^{-14})^2}$$

$$E_n = 3.29 \times 10^{-13}\, n^2 \text{ J}$$
$$E_n = 2.05 \times 10^{6}\, n^2 \text{ eV}$$
$$E_n = (2.05 \text{ MeV})\, n^2$$

For minimum energy of neutron, $n = 1$,

therefore, minimum energy of neutron is 2.05 MeV.

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5. A proton in one-dimensional box has an energy of 400 keV in its first excited state. How wide is the box?

Since, $E_n = \dfrac{n^2 h^2}{8 m L^2} \Rightarrow L = \dfrac{nh}{\sqrt{8mE}}$

given that the proton is in first excited state, therefore $n = 2$.

mass of proton, $m = 1.67 \times 10^{-27}$ kg

and energy $E = 400$ keV

$\therefore$ width of the box $L = \dfrac{2 \times 6.63 \times 10^{-34}}{\sqrt{8 \times 1.67 \times 10^{-27} \times 4 \times 10^5 \times 1.6 \times 10^{-19}}}$

$$L = \frac{2 \times 6.63 \times 10^{-34}}{29.241 \times 10^{-21}}$$

$$L = 0.4535 \times 10^{-13} \text{ m}$$

$$L = 45.35 \text{ fm}$$

Hence, the width of the box is 45.35 fm.

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6. The position and momentum of a 1 keV electron are simultaneously determined. If its position is located to within 0.100 nm, what is the percentage of uncertainty in its momentum?

The kinetic energy of electron is 1 keV.

$$\therefore \frac{1}{2} \times 9.1 \times 10^{-31} \times v^2 = 10^3 \times 1.6 \times 10^{-19} \text{ J}$$

$$v^2 = \frac{2 \times 10^3 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}$$

$$v^2 = 3.5164 \times 10^{14}$$

$$v = 1.88 \times 10^7 \text{ m/sec}$$

$\therefore$ momentum of electron $= mv$

$$= 9.1 \times 10^{-31} \times 1.88 \times 10^7$$

$$p = 1.711 \times 10^{-23} \text{ kg m/s}$$

Given, Δx for electron $= 0.1$ nm.

Using uncertainty principle $\Delta x . \Delta p \geq \frac{\hbar}{2}$

$$\therefore \Delta p = \frac{0.5275 \times 10^{-34}}{0.1 \times 10^{-9}}$$

$$\Delta p = 5.275 \times 10^{-25} \text{ kg m/sec}$$

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$\therefore$ percentage of uncertainty in momentum is;

$$= \frac{\Delta p}{p} \times 100$$

$$= \frac{5.275 \times 10^{-25}}{1.711 \times 10^{-23}} \times 100\%$$

$$= 3.083\%$$

Hence, the ~~unce~~ percentage uncertainty in momentum of electron is 3.083%.

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7. Which of the following wave functions cannot be solutions of Schrödinger's equation for all values of x?

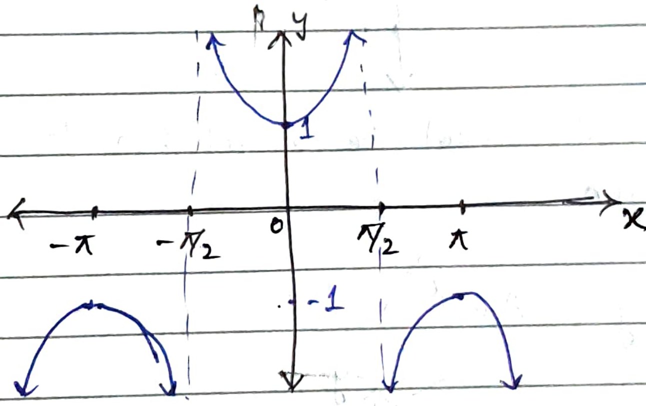
(a) $y = A \sec x$

(b) $y = A \tan x$

(c) $y = A e^{x^2}$

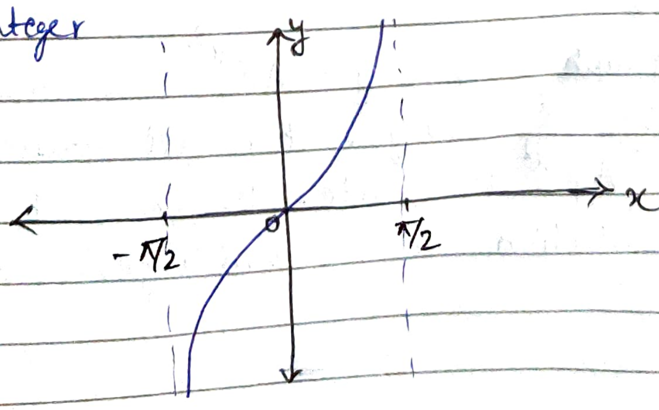
(d) $y = A e^{-x^2}$

(a) Since, $A \sec x$ goes to infinity when $\cos x = 0$
i.e when $x = (2n+1)\frac{\pi}{2}$, where n is an integer.



Hence, it cannot be a solution of the Schrödinger's equation.

(b) Since, $A \tan x$ goes to infinity when $\cos x = 0$
i.e. when $x = (2n+1)\frac{\pi}{2}$, where n is an integer



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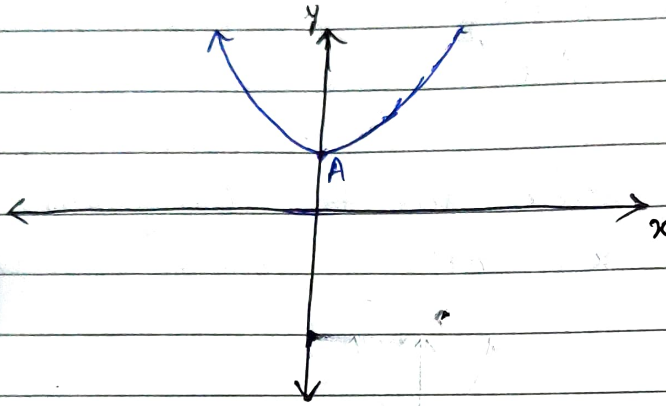
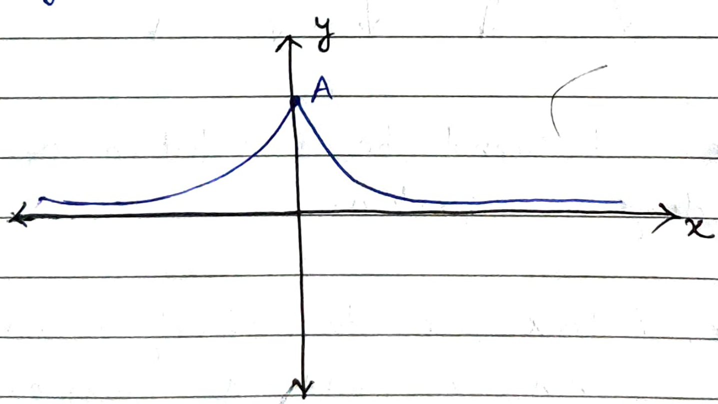
Hence, It cannot be a solution of the Schrodinger's equation.

(c) Since, Ae^{x^2} approaches infinity when x approaches infity.



Hence, it cannot be a solution to Schrödinger's equation.

(d) $y = Ae^{-x^2}$



The graph of $y = Ae^{-x^2}$ is never reaching infinity. For all values of x, Ae^{-x^2} is finite. Hence, it can be a solution of Schrödinger's Equation.