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Tutorial 1- Probability & Statistics

Ques 1

(a) Probability that at least two of them have birthday's on same day = 1 − Probability that all have birthday on different day

$$\cancel{= 1 - {}^{365}C_{30}}$$

No. of ways that all 30 students have birth day's on different days = ${}^{365}C_{30} \times 30!$

$\therefore$ P(atleast two with birthday on same day) = $1 - \left(\frac{{}^{365}C_{30} \times 30!}{(365)^{30}} \right)$

Ans

(b) no. of ways to end up in different nests $= {}^{36}C_3 \times 3!$

$\therefore$ P(ending up in different nests) $= \frac{{}^{36}C_3 \times 3!}{(36)^3}$ Ans

(c) The expression is similar to Maxwell Boltzmann Statistics but the above problems are different as there are no preselected entities.

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→ **Ques 2**

(a) Probability that it will be found in at least two of first 4 searches is equal to (1 - probability that it is found in none - probability that it is found in only one)

∴ P(found in at least two of first 4 searches)

$$= 1 - \left(\frac{{}^5C_0 \times {}^4C_4}{{}^9C_4}\right) - \left(\frac{{}^5C_1 \times {}^4C_3}{{}^9C_4}\right)$$

$$= 1 - \left(\frac{1 \times 1}{126}\right) - \left(\frac{5 \times 4}{126}\right)$$

$$= \frac{105}{126} = \frac{5}{6}$$ Ans

(b) 10 are defective i.e. the ~~batch~~ 12 widgets must be chosen from remaining 134 widgets. for the batch to be accepted.

$$P(\text{batch is accepted}) = \frac{{}^{134}C_{12}}{{}^{144}C_{12}}$$ Ans

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(c) Let us assume that $(k+1)^{th}$ card is ace.
i.e. ace doesn't appear in first k cards.

P(ace doesn't appear in first k cards) $= \frac{^{48}C_k}{^{52}C_k}$

P(that $(k+1)^{th}$ card is ace after drawing k cards) $= \frac{^{4}C_1}{^{52-k}C_1}$

$\therefore$ P(k cards are dealt before first ace appears)

$$= \frac{^{48}C_k}{^{52}C_k} \times \frac{^{4}C_1}{^{52-k}C_1}$$

$$= \frac{\frac{48!}{\cancel{k!}\,(48-k)!}}{\frac{52!}{\cancel{k!}\,(52-k)!}} \times \frac{4}{(52-k)}$$

$$\boxed{= \frac{4(51-k)(50-k)(49-k)}{52\times 51\times 50\times 49}}$$ Ans

⇒ Ques 3.

(i) without replacement

P(first chip is defective) $= 2/6 = 1/3$

Now, 5 chips are left and one is defective.

P(second chip is defective) $= \frac{1}{5}$

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P(that both chips are defective) {without replacement} $= \frac{1}{3} \times \frac{1}{5}$

$$\boxed{= \frac{1}{15}}$$ Ans

(ii) With Replacement

P(first chip is defective) $= \frac{2}{6} = \frac{1}{3}$

$\because$ chip is replaced, therefore there are still 6 chips and 2 are defective.

P (second chip is defective) $= 2/6 = 1/3$

$\therefore$ P(both chips are defective) {with replacement} $= \frac{1}{3} \times \frac{1}{3} \boxed{= \frac{1}{9}}$ Ans

⇒ Ques 4

undergrads = 10
masters = 30
doctoral = 20

P(that of the last 8 students, 2 were undergrads, 5 were masters, and 1 was doctoral)

$$= \binom{8}{2 \cdot 5 \cdot 1} \left(\frac{10}{60}\right)^2 \left(\frac{30}{60}\right)^5 \left(\frac{20}{60}\right)^1$$

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$$= {}^8C_2 \cdot {}^6C_5 \cdot {}^1C_1 \left(\frac{1}{6}\right)^2 \left(\frac{1}{2}\right)^5 \left(\frac{1}{3}\right)$$

$$= \cancel{28}^{\,7} \times \cancel{6} \times 1 \times \frac{1}{\cancel{36}\,6} \times \frac{1}{\cancel{32}\,8} \times \frac{1}{3}$$

$$= \frac{7}{144} \boxed{= 0.0486} \text{ Ans}$$

→ Ques 5(a)

~~Reliabilities~~

~~(a)~~ ~~A~~

Probabilities of proper functioning

$A = 0.9$

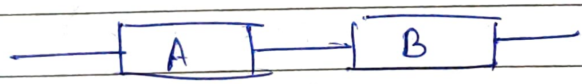
$B = 0.8$

$C = 0.7$

$D = 0.6$

$E = 0.5$

(a)

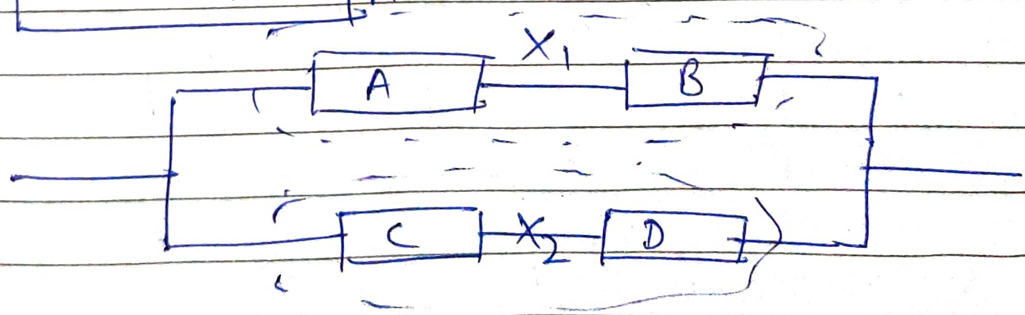


reliability $= P(A) \times P(B)$

$= 0.9 \times 0.8$

$\boxed{= 0.72}$ Ans

(b)



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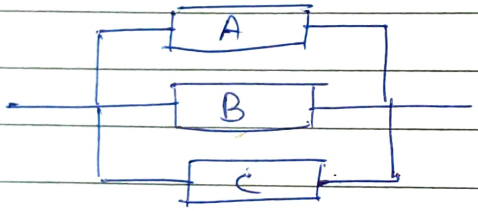
$$R(X_1) = P(A) \times P(B)$$
$$= 0.72$$

$$R(X_2) = P(C) \times P(D)$$
$$= 0.7 \times 0.6$$
$$= 0.42$$

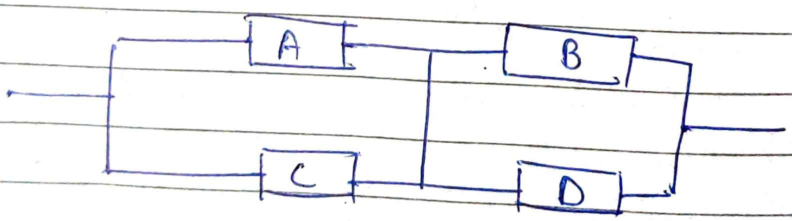
$$P(X_1 \cup X_2) = 1 - P(X_1')P(X_2')$$
$$= 1 - (1-0.72)(1-0.42)$$
$$= 1 - (0.28)(0.58)$$
$$\boxed{= 0.8376} \text{ Ans}$$

(c)



$$P(A \cup B \cup C) = 1 - P(A')P(B')P(C')$$
$$= 1 - (0.1)(0.2)(0.3)$$
$$= 1 - 0.006$$
$$\boxed{= 0.994} \text{ Ans}$$

(d)



$$R = P(A \cup C) \cap P(B \cup D)$$
$$= [1 - P(A')P(C')] \times [1 - P(B')P(D')]$$

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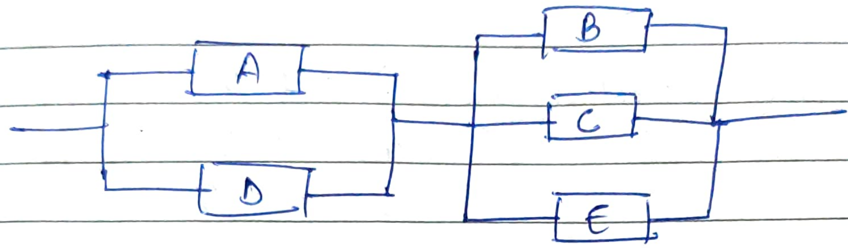
$$R = [1-(0.1)(0.3)] \times [1-(0.2)(0.4)]$$

$$= 0.97 \times 0.92$$

$$\boxed{= 0.8924} \text{ Ans}$$

(e)



$$R = P(A \cup D) \cap P(B \cup C \cup E)$$

$$= [1 - P(A') P(D')] \times [1 - P(B').P(C').P(E')]$$

$$= [1-(0.1)(0.4)] \times [1-(0.2)(0.3)(0.5)]$$

$$= 0.96 \times 0.97$$

$$\boxed{= 0.9312} \text{ Ans}$$

⇒ Ques 5(b)

$P(MB) = 0.4$

$P(HD) = 0.3$

$P(MB \cap HD) = 0.15$

$$P(MB \cup HD) = P(MB) + P(HD) - P(MB \cap HD)$$

$$P(MB \cup HD) = 0.4 + 0.3 - 0.15$$

$$= 0.55$$

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$$P(\text{functioning}) = 1 - 0.55$$
$$= 0.45$$

⇒ Ques 6

Let the tests be x, y, z

$P(\text{error detected by } x) = 0.2$

$P(\quad " \quad " \quad " \quad y) = 0.3$

$P(\quad " \quad " \quad " \quad z) = 0.5$

$$P(\text{atleast one will detect}) = 1 - P(x') P(y') P(z')$$
$$= 1 - (0.8)(0.7)(0.5)$$
$$= 1 - 0.28$$
$$P(\text{atleast one}) \boxed{= 0.72} \text{ Ans}$$

⇒ Ques 7

Box 1	Box 2
b_1 blue balls	b_2 blue balls
r_1 red balls	r_2 red balls

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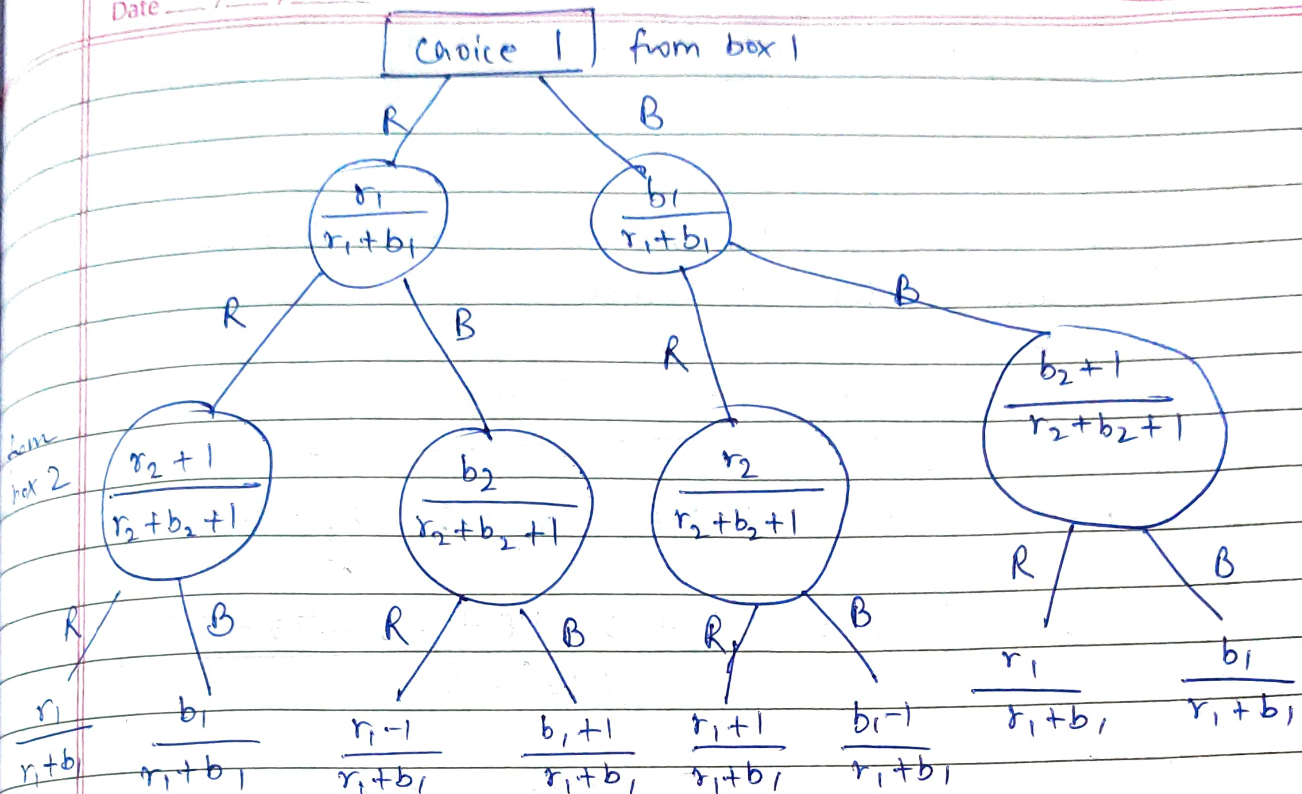

$$P(\text{blue}) = \left(\frac{r_1}{r_1+b_1}\right)\left(\frac{r_2+1}{r_2+b_2+1}\right)\left(\frac{b_1}{r_1+b_1}\right)$$

$$+$$

$$\left(\frac{r_1}{r_1+b_1}\right)\left(\frac{b_2}{r_2+b_2+1}\right)\left(\frac{b_1+1}{r_1+b_1}\right)$$

$$+$$

$$\left(\frac{b_1}{r_1+b_1}\right)\left(\frac{r_2}{r_2+b_2+1}\right)\left(\frac{b_1-1}{r_1+b_1}\right)$$

$$+$$

$$\left(\frac{b_1}{r_1+b_1}\right)\left(\frac{b_2+1}{r_2+b_2+1}\right)\left(\frac{b_1}{r_1+b_1}\right)$$

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$$= \frac{r_1b_1 + r_1b_2 + b_1r_1r_2 + r_1b_1b_2 + b_1^2r_2 + b_1^2b_2 - b_1r_2 + b_1^2}{(r_1+b_1)^2\,(r_2+b_2+1)}$$

If $r_1b_2 > b_1r_2$, then probability of getting blue is higher

$$\boxed{P(\text{red ball}) = 1 - P(\text{blue ball})}$$

Now, $r_1 = 4$, $b_1 = 6$, $r_2 = 2$, $b_2 = 3$

$$P(\text{blue ball}) = \frac{24 + 12 + 48 + 72 + 72 + 168 - 12 + 36}{(10)^2 \cdot 6}$$

$$P(\text{blue ball}) = \frac{360}{600} = 0.6$$

$\therefore P(\text{red ball}) = 0.4$ Ans

⇒ Ques 8.

Box b_1 — 1 W + 1 B

" b_2 — 1 W + 2 B

" b_3 — 1 W + 3 B

" b_4 — 1 W + 4 B

" b_5 — 1 W + 5 B

" b_6 — 1 W + 6 B

$$P(\text{white}) = \sum P(\text{box}_i)\,P(\text{white}/\text{box}_i)$$

$$= \sum \left(\frac{1}{6}\right) P(\text{white}/\text{box}_i)$$

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$P(\text{white}) = \frac{1}{6}\left[\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right]$

$= \frac{1}{\cancel{6}\,2} \times \frac{\cancel{669}\,223}{420}$

$\boxed{P(\text{white}) = 0.2655 \text{ Ans}}$

$P(\text{Box 1}/\text{white}) = \frac{P(\text{white}/\text{box 1}) \times P(\text{box 1})}{P(\text{white})}$

$= \frac{\frac{1}{2} \times \frac{1}{6}}{0.2655}$

$\therefore P(\text{box 1}/\text{white}) = 0.3138$ Ans

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