Indian Institute of Information Technology Vadodara

End semester, Autumn/ Winter examination

B.Tech/ M.Tech/ Research student

(Strike off non applicable)

Course Code: MA201 Course Name: Probability & Statistics Date: 4/01/2022

Candidate Name: Archit Agrawal Student ID: 202051213

Number of Supplementary booklets:-- 1/2/3

Read the instructions carefully	
1	Listen to the instruction stated by invigilator carefully. It may be in addition to mentioned on answer sheet / question paper.
2	It is mandatory to present your ID card to the invigilator.
3	Answer new question in a new page.
4	Possession of books, notebook, data storage device, scanner, mobile phone is considered as malpractice in examination hall (scientific, non programable calculator are permitted) unless specified by the course instructor.
5	Any type of communication or request for stationery items such as scale, pencil, eraser to other examines during exam will be treated as unfair means.
6	Don't write anything except your roll number on question paper unless specifically instructed.
7	At the end of exam, leave the examination hall quickly and quietly.

Question No.	Marks
1.	
2.	
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14.	
Total	

TALK CHEATING

Pledge

I shall abide by rules and regulation of Institute. I affirm that I will not take any unauthorize help during exam.

Student's Signature Archit Agrawal

Information Verified

Invigilator's Signature

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1. Roll Number is odd, hence $x = 95\%$.

(a) for distribution X,

$$n = 5$$

$$\bar{x} = \frac{12+8+6+29+57}{5} = 22.4$$

$$S_x = 2.0975$$

Since, no. of samples is small, we will use student T-distribution

$\therefore$ a 95% confidence ~~level~~ interval for population mean of sample X is

$$\left[22.4 - t_{0.025} \times \frac{2.0975}{\sqrt{5}},\ 22.4 + t_{0.025} \times \frac{2.0975}{\sqrt{5}}\right]$$

Since, we need to calculate only one parameter, degree of freedom will be $n-1 = 4$.

$$t_{0.025} = 2.776.$$

$$\therefore \text{margin} = 2.776 \times \frac{2.0975}{\sqrt{5}}$$

$$= 2.6040$$

$\therefore$ 95% confidence interval for population mean of sample X is

$$\boxed{[19.796,\ 25.004]}$$

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$\therefore$ for distribution Y,

$n = 5$
$\bar{Y} = 19.8$
$S_y = 2.0975$

$\therefore$ margin $= 2.776 \times \frac{2.0975}{\sqrt{5}}$

$= 2.6040$

$\therefore$ 95% confidence interval for population mean of sample Y is

$[19.8 - 2.6040, \; 19.8 + 2.6040]$

$$\boxed{[17.196, \; 22.404]}$$

(b) 95% confidence interval for difference of population mean of X and Y is :

$$\bar{X} - \bar{Y} \pm t_{\alpha/2} \, S_p \sqrt{\frac{1}{n} + \frac{1}{m}}$$

and $S_p^2 = \frac{(n-1) S_x^2 + (m-1) S_y^2}{n+m-2}$

$S_p^2 = \frac{4 \times (2.0975)^2 + 4 \times (2.0975)^2}{8}$

$S_p = 2.0975$

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and ~~$t_{\alpha/2}$~~ $t_{0.025} = 2.306$ as degree of freedom is now 8.

$\therefore$ 95% confidence Interval for difference of population mean of X and Y is

$$22.4 - 19.8 \pm 2.306 \times 2.0975 \sqrt{\frac{1}{5} + \frac{1}{5}}$$

$$\boxed{[-0.4593, 5.6593]}$$

(c) The hardware up-gradation was successful with 95% ~~of~~ confidence. This can be seen from the fact that the confidence interval of difference of population mean has a larger set of positive values.

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3.

$Pr(X=0) = 1/5$

$Pr(X=1) = 3/5$

$Pr(X=2) = 1/5$

(a) $X=1$ occurs 2 times, and $X=2$ occurs 3 times,
∴ remaining 1 time, $X=0$ occurs.

This can be done by selecting two out of 6 trials and multiplying it with (probability of $X=1)^2$, then selecting 3 out of remaining four and multiplying it with $(Pr(X=2))^3$, and then multiply with $Pr(X=0)$, for the remaining trial.

∴ required probability:

$$\boxed{{}^6C_2 \left(\frac{3}{5}\right)^2 \times {}^4C_3 \left(\frac{1}{5}\right)^3 \times \left(\frac{1}{5}\right)}$$

(b) outcome $X=1$ has occured second time in 6^{th} trial, that is, it has occured only once in first 5 trials and sixth trial gives $X=1$ as outcome.

∴ Pr(outcome $X=1$ occured 1 time in first 5 trials)

$$= {}^5C_1 \times \frac{3}{5} \times \left(\frac{2}{5}\right)^4$$

Pr(si

∴ req

(c)

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$Pr(\text{sixth outcome is } X=1) = \frac{3}{5}$

$\therefore$ required probability is

$$\boxed{= {}^5C_1 \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^4}$$

(c)

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4. $P_{xy}(x,y)$

		x = 0	x = 1
y	0	0.24	0.26
	1	0.12	0.38

∴ distribution of X,

X	P(x)
0	0.36
1	0.64

$$\therefore \quad E(X) = \sum_{i=0}^{1} x(P(x))$$

$$= 0 \times 0.36 + 1 \times 0.64$$

$$\boxed{E(X) = 0.64}$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= [0 \times 0.36 + 1 \times 0.64] - 0.64 \times 0.64$$

$$= 0.64 - 0.4096$$

$$\boxed{Var(X) = 0.2304}$$

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now, distribution of Y

Y	P(Y)
0	0.50
1	0.50

$\therefore E(Y) = 0 \times 0.5 + 1 \times 0.5$
$= 0.5$

$Var(Y) = (0 \times 0 \times 0.5 + 1 \times 1 \times 0.5) - (0.5 \times 0.5)$
$= 0.25$

$\therefore$ $\boxed{\begin{array}{l} E(Y) = 0.5 \\ Var(Y) = 0.25 \end{array}}$

We know that,

$Cov(X, Y) = E(XY) - E(X) \cdot E(Y)$

$\therefore$ distribution of XY is

XY	P(XY)
0	0.62
1	0.38

$\therefore E(XY) = 0 \times 0.62 + 1 \times 0.38$
$= 0.38$

$\therefore Cov(X, Y) = 0.38 - 0.64 \times 0.5$
$= 0.06$

$\boxed{\therefore Cov(X, Y) = 0.06}$

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2 $f(x) = \lambda^2 x e^{-\lambda x}, \quad x > 0$

(a) log-likelihood of $f(x)$;

$$\ln f(x) = \sum_{i=1}^{n} \ln\left(\lambda^2 x e^{-\lambda x}\right)$$

$$\ln f(x) = 2n \ln\lambda + n \ln x - \lambda \sum_{i=1}^{n} x_i$$

$$\frac{\partial \ln f(x)}{\partial \lambda} = \frac{2n}{\lambda} - \sum_{i=1}^{n} x_i = 0$$

$$\Rightarrow \quad \frac{2n}{\lambda} = \sum_{i=1}^{n} x_i$$

$$\frac{1}{\lambda} = \frac{\bar{X}}{2}$$

$$\boxed{\hat{\lambda} = \frac{2}{\bar{X}}}$$

(b) The distribution is exponential.

$\because$ for exponential distribution, mean, $\mu_1 = \frac{1}{\lambda}$

$$\therefore \quad \mu_1 = \frac{1}{\lambda} = m_1$$

where m_1 can be computed as

$$m_1 = \int_0^\infty x \lambda^2 x e^{-\lambda x} \, dx$$

$$m_1 = \lambda^2 \int_0^\infty x^2 e^{-\lambda x} \, dx$$

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$$m_1 = \lambda^2 \left[x^2 \frac{e^{-\lambda x}}{-\lambda} \Big|_0^\infty - \int_0^\infty \frac{e^{-\lambda x}}{-\lambda} . 2x \, dx \right]$$

$$m_1 = \lambda^2 \left[\lim_{x \to \infty} \frac{x^2 e^{-\lambda x}}{-\lambda} - \int_0^\infty \frac{e^{-\lambda x}}{-\lambda} . 2x dx \right]$$

$$m_1 = \lambda \left[\int_0^\infty e^{-\lambda x} . 2x \, dx - \lim_{x \to \infty} \frac{2x}{e^{\lambda x} . (+\lambda)} \right]$$

$$m_1 = \lambda \left[2x \frac{e^{-\lambda x}}{-\lambda} \Big|_0^\infty - \int_0^\infty \frac{e^{-\lambda x}}{-\lambda} . 2 \, dx - \lim_{x \to \infty} \frac{2}{e^{\lambda x} (\lambda^2)} \right]$$

$$m_1 = \left[2 \int_0^\infty e^{-\lambda x} dx - 2x e^{-\lambda x} \Big|_0^\infty \right]$$

$$m_1 = \left[2 \frac{e^{-\lambda x}}{-\lambda} \Big|_0^\infty - \cancel{\lim_{x \to \infty}} \overset{0}{\frac{2x}{e^{+\lambda x}}} \right]$$

$$m_1 = \frac{-2}{\lambda} \left[0 - 1 \right]$$

$$\boxed{m_1 = \frac{2}{\lambda}}$$

$\therefore$ we have $m_1 = \frac{2}{\lambda}$

(c)