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PH110 : Remote Exam Midsem

① $\vec{F} = yz\,\hat{x} + zx\,\hat{y} + xy\,\hat{z}$

$$\nabla \cdot F = \frac{\partial}{\partial x}(yz) + \frac{\partial}{\partial y}(zx) + \frac{\partial}{\partial z}(xy)$$

$$\boxed{\nabla \cdot F = 0}$$

$$\nabla \times F = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & zx & xy \end{vmatrix}$$

$$\nabla \times F = \left(\frac{\partial(xy)}{\partial y} - \frac{\partial(zx)}{\partial z}\right)\hat{x} - \left(\frac{\partial(xy)}{\partial x} - \frac{\partial(yz)}{\partial z}\right)\hat{y} + \left(\frac{\partial(zx)}{\partial x} - \frac{\partial(yz)}{\partial y}\right)\hat{z}$$

$$\nabla \times F = 0\,\hat{x} + 0\,\hat{y} + 0\,\hat{z}$$

$$\boxed{\nabla \times F = 0}$$

Hence, F can be expressed as gradient of a scalar function and curl of a vector potential.

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Let $\boxed{U = xyz}$ be the scalar potential.

$$\nabla U = yz\,\hat{x} + xz\,\hat{y} + xy\,\hat{z}$$

$\therefore$ U is the required scalar potential such that

$$F = \nabla U.$$

Now, let the vector potential be

$$A = f(\hat{x}) + g\,\hat{y} + h\,\hat{z}$$

$$\therefore \quad F = \nabla \times A_2 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f & g & h \end{vmatrix}$$

$$\frac{\partial h}{\partial y} - \frac{\partial g}{\partial z} = yz \quad \text{---(1)}$$

$$\frac{\partial f}{\partial z} - \frac{\partial h}{\partial x} = xz \quad \text{---(II)}$$

$$\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} = xy. \quad \text{---(III)}$$

Let us set $h = 0$

$\therefore$ from (1) we get

$$\frac{\partial g}{\partial z} = -yz$$

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$\therefore \quad g = -\frac{1}{2} y z^2 + g_1(x,y)$

from (II) we get

$$\frac{\partial f}{\partial z} = +xz$$

$$f = \frac{1}{2} x z^2 + f_1(x,y)$$

Substituting f and g in (III), we get

$$\frac{\partial g_1}{\partial x} - \frac{\partial f_1}{\partial y} = xy$$

Let us choose $g = 0$

$$f_1(x,y) = -\frac{1}{2} x y^2 + f_2(x)$$

But $-\frac{\partial f}{\partial x}$ does not appear in $\nabla \times A$, so

we can set $f_2 = 0$

$\therefore$ we have

$$f = \frac{xz^2}{2} - \frac{xy^2}{2}$$

and

$$g = -\frac{1}{2} y z^2$$

and $h = 0$

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∴ the required vector potential is

$$\boxed{A = \frac{x}{2}(z^2 - y^2)\,\hat{x} - \frac{yz^2}{2}\,\hat{y} + 0\,\hat{z}}$$

(2) Let us assume f be a compactly supported function.

$$= \int_{-\infty}^{\infty} f(z)\left(\frac{d\theta}{dz}\right) dz$$

$$= f(z)\,\theta(z)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty}\left(\frac{df}{dz}\right)\theta(z)\,dz$$

$$= f(\infty)\,(\theta(\infty)) - f(-\infty)\,\theta(-\infty) - \int_{-\infty}^{\infty}\frac{df}{dz}\,\theta(z)\,dz$$

$$= f(\infty) - \left[\int_{-\infty}^{0}\theta(z)\,\frac{df}{dz}\,dz + \int_{0}^{\infty}\left(\frac{df}{dz}\right)\theta(z)\,dz\right]$$

$\because$ $\theta(z) = 0$ if $z \leq 0$
and $\theta(z) = 1$ if $z > 0$

the above equation becomes

$$= f(\infty) - \left[\int_{0}^{\infty}\left(\frac{df}{dz}\right) dz\right]$$

$$= f(\infty) - f(\infty) + f(0)$$

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$$\therefore \int_{-\infty}^{\infty} f(z)\left(\frac{d\theta}{dz}\right) dz = f(0)$$

$$\int_{-\infty}^{\infty} f(z)\left(\frac{d\theta}{dz}\right) dz = \int_{-\infty}^{\infty} f(z)\,\delta(z)\,dz \quad (1)$$

$\because$ when $\int_{-\infty}^{\infty} g(x)\,D_1(x)\,dx = \int_{-\infty}^{\infty} g(x)\,D_2(x)\,dx$

then, $D_1(x) = D_2(x)$

$\therefore$ from (1) we have

$$\frac{d\,\theta(z)}{dz} = \delta(z)$$

$$\therefore \frac{d\,\theta(z)}{dz} = \delta(z)$$

Hence, proved.

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(2)(a) given $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \left[1 + \frac{r}{\lambda}\right] e^{-r/\lambda} \hat{r}$

therefore, electric field due to a charge q at distance r will be

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \left[1 + \frac{r}{\lambda}\right] e^{-r/\lambda} \hat{r}$$

calculating $\nabla \times E$,

$$\nabla \times V = \frac{1}{r\sin\theta} \left[\frac{\partial(\sin\theta V_\phi)}{\partial\theta} - \frac{\partial V_\theta}{\partial\phi}\right] \hat{r}$$

$$+ \frac{1}{r} \left[\frac{1}{\sin\theta} \frac{\partial V_r}{\partial\phi} - \frac{\partial}{\partial r}(r V_\phi)\right] \hat{\theta}$$

$$+ \frac{1}{r} \left(\frac{\partial}{\partial r}(r V_\theta) - \frac{\partial V_r}{\partial\theta}\right) \hat{\phi}$$

$E_\phi = 0$ and $E_\theta = 0$

$$\frac{\partial E_r}{\partial\phi} = \frac{\partial E_r}{\partial\theta} = 0$$

$$\therefore \nabla \times E = \frac{1}{r} \left[-\frac{\partial(V_r)}{\partial\theta}\right] \hat{\phi}$$

$$\nabla \times E = 0$$

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Since, the curl of E comes out to be zero. This means that this field can be represented using scalar potential.

(b) The Gauss's law does not changes because the electric field is curl-less.

$\nabla \cdot E = \frac{\rho}{\epsilon_0}$ is still true

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3.(a) The point charge $4q$ at $(4,0,0)$ is at $r=4$ and $\theta=\pi$ and $\phi=0$,
so,

$$\vec{E} = \frac{p}{4\pi\epsilon_0(4)^3}\hat{\theta} = \frac{p}{4\pi\epsilon_0(64)}(-\hat{z})$$

$$\therefore \vec{F} = 4q\vec{E}$$

$$\vec{F} = \frac{-4pq}{4\pi\epsilon_0(64)}(\hat{z})$$

$$\vec{F} = \frac{-pq}{64\pi\epsilon_0}(\hat{z})$$

$\therefore$ Force on $4q$ charge at $(4,0,0)$ is $\frac{-pq}{64\pi\epsilon_0}(\hat{z})$

Now, (b) here $r=4$, and $\theta=0$

$$\therefore \vec{E} = \frac{p}{4\pi\epsilon_0(64)}(2\hat{r}) = \frac{2p}{4\pi\epsilon_0(64)}\hat{z}$$

$$\therefore \vec{F} = 4q\vec{E}$$

$$= \frac{8pq}{4\pi\epsilon_0(64)}\hat{z}$$

$\therefore$ force on $4q$ charge at $(0,0,4)$ is $\frac{pq}{32\pi\epsilon_0}\hat{z}$.

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(c) $W = q\left[V_{(0,0,4)} - V_{(4,0,0)}\right]$

$$W = \frac{qP}{4\pi\epsilon_0(4)^2}\left(\cos 0 - \cos \pi/2\right)$$

$$W = \frac{pq}{4\pi\epsilon_0(16)}$$

$$\boxed{W = \frac{pq}{64\pi\epsilon_0}}$$

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4. (a) The impact of electric field is that the atoms are polarized in a dielectric when it is kept in electric field. The gauss's law and boundary conditions changes because an electric field is induced inside the dielectric due to polarization.

(b) Atomic polarizability is defined as the dipole moment induced in the atom in response to an application of electric field.

Electric susceptibility is a dimensionless constant that indicates the degree of polarization of a dielectric material in response to an applied electric field. The greater the electrical susceptibility, the greater the ability of material to polarize.