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1. $P = \begin{bmatrix} I & 0 & 0 \\ A & I & 0 \\ B & C & I \end{bmatrix}$

for finding P^{-1} we can block partitioned this matrix

$$\left[\begin{array}{cc|c} I & 0 & 0 \\ A & I & 0 \\ \hline B & C & I \end{array}\right] \rightarrow \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$$

If $|D'| \neq 0$ & $|A' - B' D'^{-1} C'| \neq 0$

then its inverse

$$P^{-1} = \begin{bmatrix} (A' - B'D'^{-1}C')^{-1} & -A'^{-1}B'(D' - C'A'^{-1}B')^{-1} \\ -D'^{-1}C'(A' - B'D'^{-1}C')^{-1} & (CD' - C'A'^{-1}B')^{-1} \end{bmatrix}$$

$A' = \begin{bmatrix} I & 0 \\ A & I \end{bmatrix}$ $\quad B' = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad C' = \begin{bmatrix} B & C \end{bmatrix}$

$D' = [I]$

$(A'^{-1}) = \begin{bmatrix} I & 0 \\ -A & I \end{bmatrix}$ $\quad (D'^{-1}) = [1/I]$

$a_{11} = A' - B'D'^{-1}C = A - \begin{bmatrix} 0 \\ 0 \end{bmatrix} D^{-1} C = A - \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

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$$a_{11} = A' = \begin{bmatrix} I & 0 \\ A & I \end{bmatrix}$$

$$a_{21} = -D'^{-1}C' \left(A' - B'D'^{-1}C'\right)^{-1}$$

$$= -B'^{-1}C' \left(A' - 0\right)$$

$$= -[I]\begin{bmatrix} B & C \end{bmatrix}\begin{bmatrix} I & 0 \\ A & I \end{bmatrix}$$

$$= -\begin{bmatrix} B + CA & C \end{bmatrix}$$

$$a_{21} = \begin{bmatrix} -B - CA & -C \end{bmatrix}$$

$$a_{22} = \left(D' - C'A'^{-1}B'\right) \qquad \text{If } B = 0$$

$$\left(B' - 0\right)$$

$$= D' = [I]$$

$$a_{12} = -A'^{-1}B'D'$$

$$= -\begin{bmatrix} I & 0 \\ -A & I \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix}[I] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$D^{-1} = \begin{bmatrix} I & 0 & 0 \\ A & I & 0 \\ -B-CA & C & I \end{bmatrix}$$

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(2) $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \end{bmatrix}$

This matrix augmented with 4x4 identity matrix

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 4 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$
$R_4 \to R_4 - 3R_3$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & -3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 & -3 & 1 \end{array}\right]$$

$R_2 \to -\frac{1}{2} R_2$

$R_4 \to -\frac{1}{2} R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 3/2 & -1/2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 3/2 & -1/2 \end{array}\right]$$

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$R_1 \to R_1 - 2R_2$
$R_3 \to R_3 - 2R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 3/2 & -1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 3/2 & -1/2 \end{array}\right]$$

Hence, inverse of A is

$$A^{-1} = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 3/2 & -1/2 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 3/2 & -1/2 \end{bmatrix}$$

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3.

$$4x_1 - 6x_2 - 14x_3 + 10x_4 + 4x_5 = -4$$

$$-x_1 + 2x_2 + 4x_3 - 3x_4 - x_5 = 2$$

$$2x_1 + 0x_2 - 4x_3 + 2x_4 + x_5 = 3$$

$$x_1 - 5x_2 - 7x_3 + 6x_4 + 2x_5 = -7$$

the coefficient matrix for the system is

$$\begin{bmatrix} 4 & -6 & -14 & 10 & 4 \\ -1 & 2 & 4 & -3 & -1 \\ 2 & 0 & -4 & 2 & 1 \\ 1 & -5 & -7 & 6 & 2 \end{bmatrix}$$

the augmented matrix system is

$$\left[\begin{array}{ccccc|c} 4 & -6 & -14 & 10 & 4 & -4 \\ -1 & 2 & 4 & -3 & -1 & 2 \\ 2 & 0 & -4 & 2 & 1 & 3 \\ 1 & -5 & -7 & 6 & 2 & -7 \end{array}\right]$$

$R_2 \leftrightarrow R_1$

$$\left[\begin{array}{ccccc|c} -1 & 2 & 4 & -3 & -1 & 2 \\ 4 & -6 & -14 & 10 & 4 & -4 \\ 2 & 0 & -4 & 2 & 1 & 3 \\ 1 & -5 & -7 & 6 & 2 & -7 \end{array}\right]$$

$R_2 \rightarrow R_2 + 4R_1$

$R_3 \rightarrow R_3 + 2R_1$

$R_4 \rightarrow R_4 + R_1$

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$$\left[\begin{array}{ccccc|c} -1 & 2 & 4 & -3 & -1 & 2 \\ 0 & 2 & 2 & -2 & 0 & 4 \\ 0 & 4 & 4 & -4 & -1 & 7 \\ 0 & -3 & -3 & -3 & 1 & -5 \end{array}\right]$$

$R_3 \to R_3 - 2R_2$
$R_4 \to R_4 + \frac{3}{2} R_2$

$$\left[\begin{array}{ccccc|c} -1 & 2 & 4 & -3 & -1 & 2 \\ 0 & 2 & 2 & -2 & 0 & 4 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -6 & 1 & 1 \end{array}\right]$$

$R_3 \leftrightarrow R_4$

$$\left[\begin{array}{ccccc|c} -1 & 2 & 4 & -3 & -1 & 2 \\ 0 & 2 & 2 & -2 & 0 & 4 \\ 0 & 0 & 0 & -6 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{array}\right]$$

four pivot entries, hence x_3 is free variable

system of equation is reduced to

$$-x_1 + 2x_2 + 4x_3 - 3x_4 - x_5 = 2 \quad \text{(I)}$$
$$2x_2 + 2x_3 - 2x_4 = 4 \quad \text{(II)}$$
$$-6x_4 + x_5 = 1 \quad \text{(III)}$$
$$-x_5 = -1 \quad \text{(IV)}$$

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Let $\boxed{x_3 = a} \in R$

$\boxed{x_5 = 1}$ {from (IV)}

$\therefore$ $-6x_4 + 1 = 1$ {from (III)}

$\therefore$ $\boxed{x_4 = 0}$

$2 \; (x_2 + x_3) = 4$

$x_2 + x_3 = 2$

$x_2 = 2 - x_3$

$\boxed{x_2 = 2 - a}$

$-x_1 + 2x_2 + 4x_3 - \cancel{3x_4}^{0} - x_5 = 2$

$-x_1 = 2 - 2x_2 - 4x_3 + x_5$

$x_1 = 2x_2 + 4x_3 - x_5 - 2$

$x_1 = 2(2-a) + 4(a) - 1 - 2$

$x_1 = 4 - 2a + 4a - 3$

$\boxed{x_1 = 2a + 1}$

$\therefore$ $x = \begin{bmatrix} 2a+1 \\ 2-a \\ a \\ 0 \\ 1 \end{bmatrix}$

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4. $A \sim \begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$

To find U, reduce it to row echelon,

$R_2 \to R_2 - 2R_1 \quad \because l_{21} = 2$

$$\sim \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ -1 & 7 & 0 \end{bmatrix}$$

$R_3 \to R_3 - \left(-\frac{1}{3} R_1\right) \quad \because l_{31} = -\frac{1}{3}$

$$\begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 5 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2 \quad \because l_{32} = 1$

$$\begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & 5 \end{bmatrix}$$

$\therefore U = \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & 5 \end{bmatrix}$

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and L is a matrix with l_{ij} as entry in i^{th} row and j^{th} column and diagonal entries as 1.

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 1 & 1 \end{bmatrix}$$

$$\therefore A = LU$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & 5 \end{bmatrix}$$

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c. $A = \begin{bmatrix} 601 & 501 & 201 & 301 \\ 602 & 502 & 202 & 302 \\ 603 & 503 & 203 & 303 \\ 604 & 504 & 204 & 304 \end{bmatrix}$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 \to R_2$

$A = \begin{bmatrix} 601 & 501 & 201 & 301 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 604 & 504 & 204 & 304 \end{bmatrix}$

$\because$ R_2 and R_3 are equal

$\det(A) = 0$

Rank- Nullity Theorem

For a $m \times n$ matrix, the rank-nullity theorem states that

$$\text{rank}(A) + \dim(\text{Nul } A) = n$$

Reducing A to row echelon form

$R_2 \to R_2 \rightarrow R_1$
$R_3 \to R_3 \rightarrow R_2$
$R_4 \to R_4 \rightarrow R_3$

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$$A \sim \begin{bmatrix} 601 & 501 & 201 & 301 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\sim$

$R_3 \to R_3 - R_2$
$R_4 \to R_4 \to R_2$

$$\sim \begin{bmatrix} 601 & 501 & 201 & 301 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 - \frac{1}{601} R_1$

$$\begin{bmatrix} 601 & 501 & 201 & 301 \\ 0 & \frac{100}{601} & \frac{400}{601} & \frac{300}{601} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since A has two pivot columns
rank (A) = 2

and since it has 4 columns
dim (Nul A) + rank (A) = 4
dim(Nul A) = 4 - rank(A)
= 4 - 2
= 2

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6. $Z = 3X_1 + 2X_2 + 3X_3$

$2X_1 + X_2 + X_3 + X_4 = 2$

$3X_1 + 4X_2 + 2X_3 - X_5 + X_6 = 4$

I

		C_j	3	2	3	0	0	-M	min Ratio
X_B	C_B	P	X_1	$X_2 \downarrow$	X_3	X_4	X_5	X_6	
X_4	0	2	2	1	1	1	0	0	2
X_6	-M	4	3	4	2	0	-1	1	1
		$Z_j - C_j$	-3M-3	-4M-2	-2M-3	0	M	0	

II

X_4	0	1	1.25	0	0.5	1	0.25	-0.25	2
X_2	2	1	0.75	1	0.5	0	-0.25	0.25	2
		$Z_j - C_j$	-1.5	0	-2	0	-0.5	M	

III

X_3	3	2.5	0	1	2	0.5	-0.5
X_2	2	-0.5	1	0	-1	0.5	0.5
		3.5	0	0	4	0.5	0.5

Hence, $X_1 = 0$
$X_2 = 0$
$X_3 = 2$

$Z_{max} = 0 + 0 + 3 \times 2$
$= 6$