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Remote Examination 2021

PH100: Mechanics & Thermodynamics

Answer 1(a) Newton's laws of motion are not valid in non-relativistic domain and microscopic domain. In such domain, velocity of particles is comparable to speed of light.

1(b) This statement is true because in elastic collision both momentum and kinetic energy remains conserved. Hence we only need to observe the scattering angle with respect to centre of frame reference.

1(c) Since, for conservative forces $\vec{\nabla} \times \vec{F} = 0$ by Stoke's law. As $\vec{\nabla} \times \vec{F}$ for spring force is 0, it is a conservative force while $\vec{\nabla} \times \vec{F}$ for frictional force is non-zero. Hence it is not a conservative force.

1(d) The effect of damping is necessary to understand forced damped harmonic oscillations because the starting chaotic motion of mass in driven oscillation can be described ~~be~~ with help of damped oscillations.

1(e)

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(e) $F = 2xy^3\,\hat{i} + 3x^2y^2\,\hat{j}$

$F = F_x\,\hat{i} + F_y\,\hat{j}$

$$\frac{\partial F(\hat{i})}{\partial y} = 2 \cdot 3y^2 \cdot x = 6xy^2$$

$$\frac{\partial F(\hat{j})}{\partial x} = 3\,(2x)\,y^2 = 6xy^2$$

$$\therefore \quad \frac{\partial F(\hat{i})}{\partial y} = \frac{\partial F(\hat{j})}{\partial x}$$

Force F is conservative.

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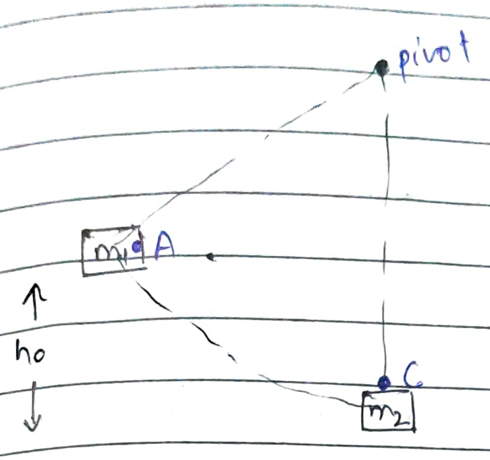
Solution 2(a)



Let the point where collision occurs be C.
and A be the point where m_1 is present initially.
Before the collision, no energy loss occurs

$$\therefore \quad E_A = E_C$$
$$\cancel{m_1} g h_0 = \frac{1}{2} \cancel{m_1} v^2$$
$$v^2 = 2gh_0 \Rightarrow v = \sqrt{2gh_0}$$

where v is velocity of m_1 at C before the collision.

Let the final speed of bob and mass m_2 be v'. Since the bob moves backward and mass moves forward; applying conservation of momentum; gives

$$m_1 v = -m_1 v' + m_2 v'$$
$$\frac{m_1 v}{m_2 - m_1} = v'$$

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$$v' = \frac{m_1 \sqrt{2gh_0}}{m_2 - m_1}$$

Since, the collision is elastic,

$$\frac{1}{2} m_1 v^2 = \frac{1}{2} m_1 v'^2 + \frac{1}{2} m_2 v'^2$$

$$\frac{1}{2} m_1 v^2 = \frac{1 v'^2}{2} (m_1 + m_2)$$

$$\cancel{\frac{1}{2}} \cancel{m_1} \times \cancel{2gh_0} = \cancel{\frac{1}{2}} \frac{m_1^{\cancel{2}} \times \cancel{(2gh_0)}}{(m_2 - m_1)^2} (m_1 + m_2)$$

$$(m_2 - m_1)^2 = m_1 (m_1 + m_2)$$

$$\cancel{m_1^2} + m_2^2 - 2m_1 m_2 = \cancel{m_1^2} + m_1 m_2$$

$$m_2^2 = 3 m_1 m_2$$
$$m_2 = 3 m_1$$

Hence, the mass m_2 is equal to $3m_1$

Solution 2(b)

Now, the bob and block sticks together and move with same velocity (say v_f).

Applying conservation of momentum,

$$\cancel{\frac{1}{2} m_1 v^2 = \frac{1}{2} (m_1 + m_2) v_f^2}$$

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$m_1 v = (m_1 + m_2) v_f$

$$\frac{m_1 \sqrt{2gh_0}}{m_1 + m_2} = v_f$$

$$\therefore \quad v_f = \frac{m_1 \sqrt{2gh_0}}{(m_1 + m_2)}$$

The initial kinetic energy of the system,

$$(KE)_i = \frac{1}{2} m_1 v^2 + (0)$$

$$= \frac{1}{2} m_1 \times 2gh_0 = \frac{\cancel{2} m_1 g h_0}{\cancel{2}}$$

The final kinetic energy of the system K_f

$$K_f = \frac{1}{2}(m_1 + m_2) v_f^2$$

$$= \frac{1}{2} \cancel{(m_1 + m_2)} \frac{m_1^2 \times 2gh_0}{(m_1 + m_2)^{\cancel{2}}}$$

$$= \frac{\cancel{2} m_1^2 g h_0}{\cancel{2} (m_1 + m_2)}$$

$\therefore$ change in KE $= -(K_f - K_i)$

$$= -\left\{ \frac{m_1^2 g h_0}{m_1 + m_2} - m_1 g h_0 \right\}$$

$$= \frac{m_1 m_2 g h_0}{(m_1 + m_2)}$$

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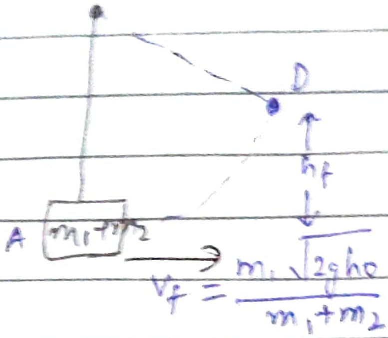
2 (c)



The total mechanical energy of the system is conserved. Let the height be h_f above lowest point of bob's swing when they both first come to rest after the collision.

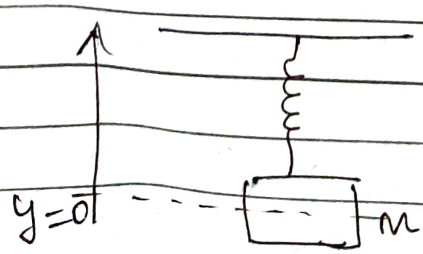
$$\therefore (0) + \frac{1}{2}(m_1+m_2)\left\{\frac{m_1\sqrt{2gh_0}}{m_1+m_2}\right\}^2 = (m_1+m_2)\,g\,h_f + (0)$$

$$\frac{1}{\cancel{2}}\,\frac{m_1^2 \times \cancel{2}\cancel{g}h_0}{m_1+m_2} = (m_1+m_2)\,\cancel{g}\,h_f$$

$$\therefore \quad h_f = \frac{h_0 m_1^2}{(m_1+m_2)^2}$$

Hence, $h_f = \dfrac{h_0 m_1^2}{(m_1+m_2)^2}$

Solution (3) (A) (i)



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Since a piece of mass falls off and leaves αM mass of it, a new equilibrium position is reached

At eq^{ll}m position,

$$Ky = (\alpha M) g$$

$$y = \frac{\alpha M g}{K}$$

Hence, new eq^{ll}m position is $y = \frac{\alpha M g}{K}$

(i) the y position of mass will be,

$$y = \frac{\alpha M g}{K} + \frac{\alpha M g}{K} \cos(\omega t)$$

(ii) Velocity of object

$$v = \frac{dy}{dt} = -\frac{\alpha M g \omega}{K} \sin(\omega t)$$

$$\therefore \; KE(t) = \frac{1}{2}(\alpha M) v^2 = \frac{1}{2}(\alpha M) \frac{\alpha M g \omega}{K} \sin(\omega t)$$

$$= \frac{1}{2} \frac{\alpha^2 M^2 g^2 \omega^2}{K^2} \sin^2(\omega t)$$

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and potential energy (U)

$$U = \frac{1}{2} k \left(\frac{\alpha Mg}{k} - y \right)^2$$

$$= \frac{1}{2} k \left\{ \frac{\alpha Mg}{k} - \frac{\alpha Mg}{k} - \frac{\alpha Mg}{k} \cos(\omega t) \right\}^2$$

$$= \frac{1}{2} k \frac{\alpha^2 M^2 g^2}{k^2} \cos^2(\omega t)$$