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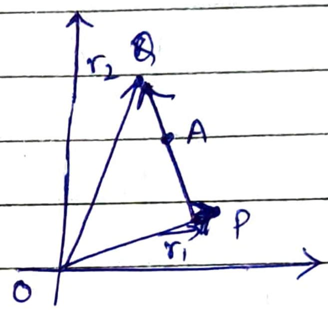
PH100 : Tutorial #01

1. Consider two points located at r_1 and r_2, seperated by distance $r = |r_1 - r_2|$. Find a vector A from the origin to a point on the line between r_1 and r_2 at distance xr from the point at r_1, where x is some number.



Consider $\vec{r_1}$ as $\vec{OP}$ and $\vec{r_2}$ as $\vec{OQ}$.

$\therefore \vec{PQ} = \vec{r_2} - \vec{r_1}$

Now, unit vector $\hat{PQ}$ in the direction of $\vec{PQ}$ is

$$\hat{PQ} = \frac{(\vec{r_2} - \vec{r_1})}{r} \qquad (\text{where } r = |r_1 - r_2|)$$

Let the point A on PQ is at distance xr from $P(\vec{r_1})$

$$\therefore \vec{PA} = xr\,\hat{PQ} = x|r_1 - r_2|\frac{(\vec{r_2} - \vec{r_1})}{|r_1 - r_2|}$$

$$= x(\vec{r_2} - \vec{r_1})$$

In ΔOPQ, applying triangle law,

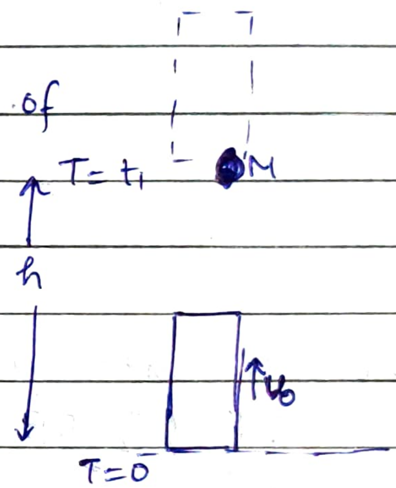
$$\vec{OA} = \vec{OP} + \vec{PQ}$$

$$\vec{OA} = \vec{r_1} + x(\vec{r_2} - \vec{r_1})$$

Hence, the required $\vec{A}$ is $\vec{r_1} + x(\vec{r_2} - \vec{r_1})$.

2. At $t=0$, an elevator departs from the ground with uniform speed. At time T_1 a boy drops a marble through the floor. The marble falls with uniform acceleration $g = 9.8\ m/s^2$ and hits the ground T_2 seconds later. Find the height of elevator at time T_1.

Let us suppose the height of elevator at time T_1 be h.



Since elevator was going up with uniform speed (say v_0) from $T=0$ to $T=T_1$.

$$h = v_0 T_1$$

$$v_0 = \frac{h}{T_1} \quad \text{—— (1)}$$

At T_1, the marble M is dropped ~~with~~ from the floor of elevator.
Hence, initial velocity of marble is v_0 upwards.
Assuming upward direction to be positive j axis.

initial velocity of marble $= v_0 \hat{j}$

acceleration of marble $= -g(\hat{j})$

~~Appl~~ At T_2 the marble ~~ground~~ is at ground.
$\therefore$ displacement of marble in time $(T_2 - T_1) = -h\hat{j}$

Applying second kinematic equation, we get,

$$-h = v_0 (T_2 - T_1) + \frac{1}{2}(-g)(T_2 - T_1)^2$$

$$-h = \frac{h}{T_1}(T_2 - T_1) + \frac{1}{2}(-g)(T_2 - T_1)^2$$

$$h\left[1 + \frac{T_2 - T_1}{T_1}\right] = \frac{g}{2}(T_2 - T_1)^2$$

$$h\frac{T_2}{T_1} = \frac{g}{2}(T_2 - T_1)^2$$

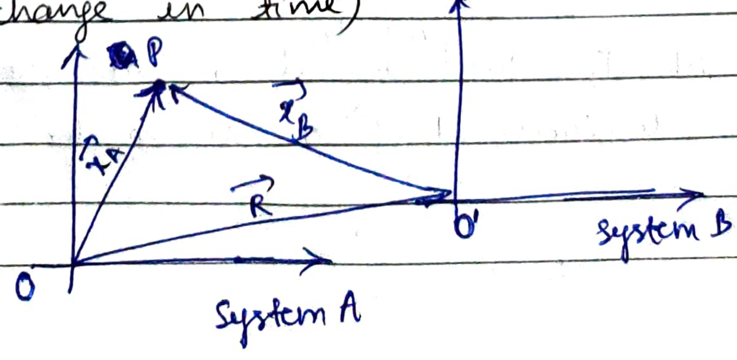
$$h = \frac{gT_1}{2T_2}(T_2 - T_1)^2$$

The height of elevator at $t = T_1$ is

$$\frac{gT_1}{2T_2}(T_2 - T_1)^2$$

3 By relative velocity we mean velocity with respect to a specified coordinate system. (The term velocity, alone, is understood to be relative to the observer's coordinate system).

a) A point is observed to have velocity u_A relative to co-ordinate system A. What is its velocity relative to coordinate system B, which is displaced from system A by distance R?
(R can change in time)



Let position of P w.r.t system A is x_A and position of P w.r.t B be x_B.

from $\Delta OO'P$, using triangular law of vector addition,

$$\vec{x_A} = \vec{R} + \vec{x_B}$$

$$\vec{x_B} = \vec{x_A} - \vec{R}$$

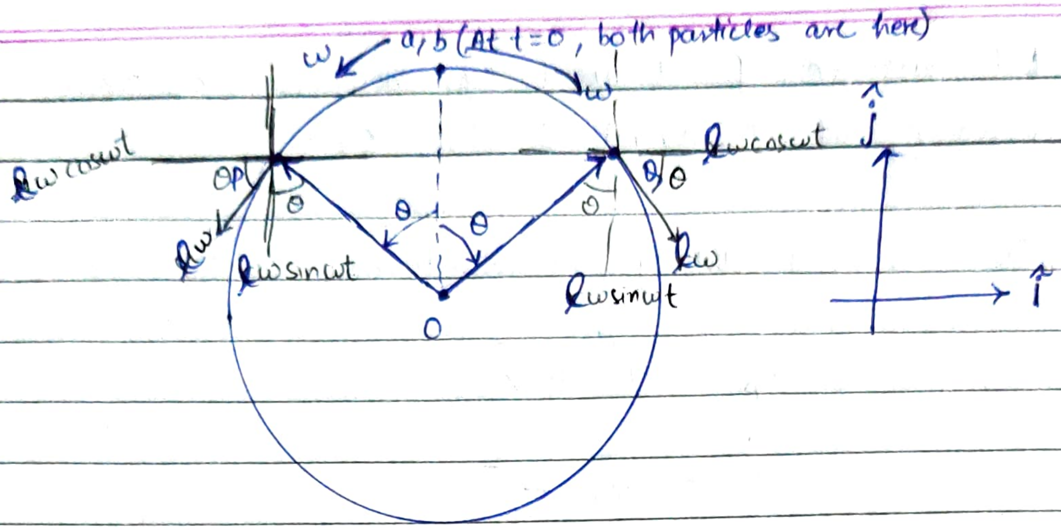
differentiating both sides w.r.t time

$$\frac{d(\vec{x_B})}{dt} = \frac{d(\vec{x_A})}{dt} - \frac{d(\vec{R})}{dt}$$

$$\vec{v_B} = \vec{v_A} - \frac{d\vec{R}}{dt}$$

Hence, velocity of point P relative to coordinate system B is $\left(\vec{v_A} - \frac{d\vec{R}}{dt}\right)$, where $\vec{v_A}$ is velocity of point relative to system A.

b) Particles a and b move in opposite directions around a circle with angular speed ω, At $t=0$ they are both at point $r = l\hat{j}$, where l is the radius of circle. Find velocity of a relative to b.



~~At~~ $t=0$, both particles are at $l\hat{j}$ and particle a has angular velocity ω in counter-clockwise direction, while particle b has angular velocity ω in clockwise direction.

At time t, particle A is at P and particle b is at Q. Both the particle have covered equal angular displacement but in opposite direction.

velocity of particle a w.r.t O, at time t

$$\vec{V}_{ao} = -l\omega\cos\omega t\,\hat{i} - l\omega\sin\omega t\,\hat{j}$$

velocity of particle b w.r.t. O, at time t,

$$\vec{V}_{bo} = l\omega\cos\omega t\,\hat{i} - l\omega\sin t\,\hat{j}$$

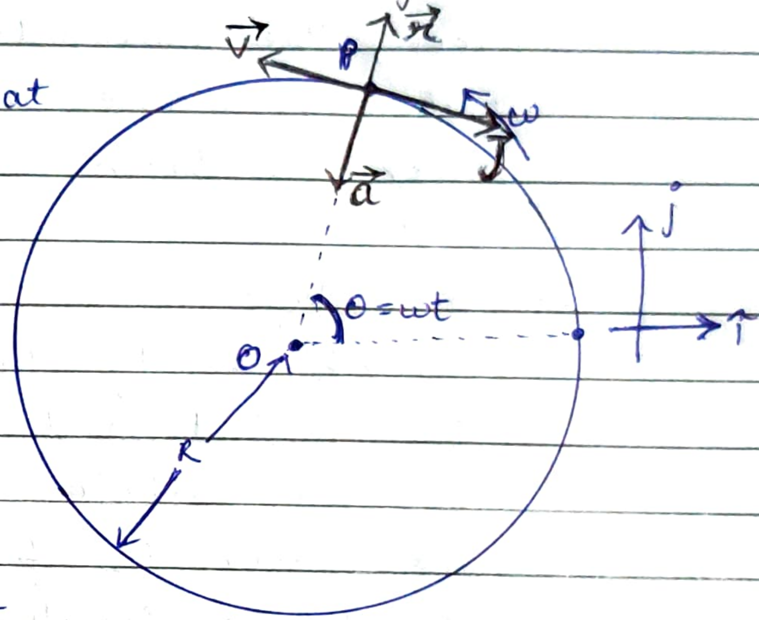
$$\vec{V}_{ab} = \vec{V}_{ao} - \vec{V}_{bo}$$

$$= \left(-l\omega\cos\omega t\,\hat{i} - l\omega\sin\omega t\,\hat{j}\right) - \left(l\omega\cos\omega t\,\hat{i} - l\omega\sin\omega t\,\hat{j}\right)$$

$$= -2l\omega\cos\omega t\,\hat{i}$$

Hence, velocity of a relative to b is $-2l\omega\cos\omega t\,\hat{i}$

4. The rate of change of acceleration is sometimes known as jerk. Find the direction and magnitude of jerk for a particle moving in a circle of radius R at angular velocity ω. Draw a vector diagram showing the instantaneous position, velocity, acceleration and jerk.



The particle is at (R, 0) at time t = 0, considering O as origin. The particle starts moving in a circle of radius R at t = 0 with angular velocity ω in counter clockwise direction.

Let us assume at time t, the particle is at P, such that θ = ωt.

The position vector of particle P at time t is given by.

$$\vec{r} = R\cos\omega t\ \hat{i} + R\sin\omega t\ \hat{j}$$

velocity vector

$$\vec{v} = \frac{d\vec{r}}{dt} = -R\omega\sin\omega t\ \hat{i} + R\omega\cos\omega t\ \hat{j}$$

acceleration vector

$$\vec{a} = \frac{d\vec{v}}{dt} = -R\omega^2\cos\omega t\ \hat{i} - R\omega^2\sin\omega t\ \hat{j} = -\omega\vec{r}$$

jerk vector

$$\vec{J} = \frac{d\vec{a}}{dt} = R\omega^3\sin\omega t\ \hat{i} - R\omega^3\cos\omega t\ \hat{j} = -\omega^2\vec{v}$$

$$|\vec{j}| = \sqrt{(R\omega^3 \sin\omega t)^2 + -(R\omega^3 \cos\omega t)^2}$$

$$|\vec{j}| = \sqrt{R^2\omega^6 (\sin^2\omega t + \cos^2\omega t)}$$

$$|\vec{j}| = R\omega^3$$

Hence, the magnitude of jerk vector is $R\omega^3$ and the direction is opposite to velocity vector as shown in figure.

5. A particle moves in a plane with constant radial velocity 4 m/s. The angular velocity is constant and has magnitude 2 rad/s. When the particle is 3m from origin, find the magnitude of (a) the velocity and (b) the acceleration.

The position vector is given by

$$\vec{r} = r\hat{r}$$

where $\hat{r}$ is unit vector in radial direction.

$$\vec{V} = \frac{d(\vec{r})}{dt} = \frac{d}{dt}(r\hat{r})$$

$$= \left(\frac{dr}{dt}\right)\hat{r} + r\frac{d(\hat{r})}{dt}$$

$$= \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

where $\hat{\theta}$ is a unit vector perpendicular to $\hat{r}$.

radial velocity = $\dot{r}$ = 4 m/s
angular velocity = $\dot{\theta}$ = 2 rad/s

$$|v| = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$$

at $r = 3$

$$|v| = \sqrt{(4)^2 + (3\times 2)^2} = \sqrt{16+36}$$
$$= \sqrt{52} \text{ m/s}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}\right)$$

$$= \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} - r(\dot{\theta})^2\hat{r}$$

As angular acceleration ($\ddot{\theta}$) and radial acceleration ($\ddot{r}$) are zero.

$$\vec{a} = 2\dot{r}\dot{\theta}\hat{\theta} - r(\dot{\theta})^2\hat{r}$$

at instant when $r = 3$

$$\vec{a} = (2\times 4\times 2)\hat{\theta} - 3(2)^2\hat{r}$$
$$= 16\hat{\theta} - 12\hat{r}$$

$$|\vec{a}| = \sqrt{(16)^2 + (12)^2} = \sqrt{400}$$
$$= 20 \text{ m/s}^2$$

Hence, the (a) magnitude of velocity is $\sqrt{52}$ m/s and (b) magnitude of acceleration is 20 m/s^2.

6. A particle moves outward along a spiral. Its trajectory is given by $r = A\theta$, where A is a constant. $A = \frac{1}{\pi}$ m/rad. θ increases in time according to $\theta = \frac{kt^2}{2}$, where k is a constant.

a) Sketch the motion, and indicate the approximate velocity and acceleration at few points.

b) show that when the radial acceleration is zero. At what angles do the radial and tangential accelerations have equal magnitude?

Position vector is given by
$$\vec{r} = r\hat{r}$$

Velocity vector is
$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

and acceleration vector,

$$\vec{a} = \frac{d\vec{v}}{dt} = \left\{\ddot{r} - r(\dot{\theta})^2\right\}\hat{r} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)$$

Since, $\theta = \frac{kt^2}{2}$

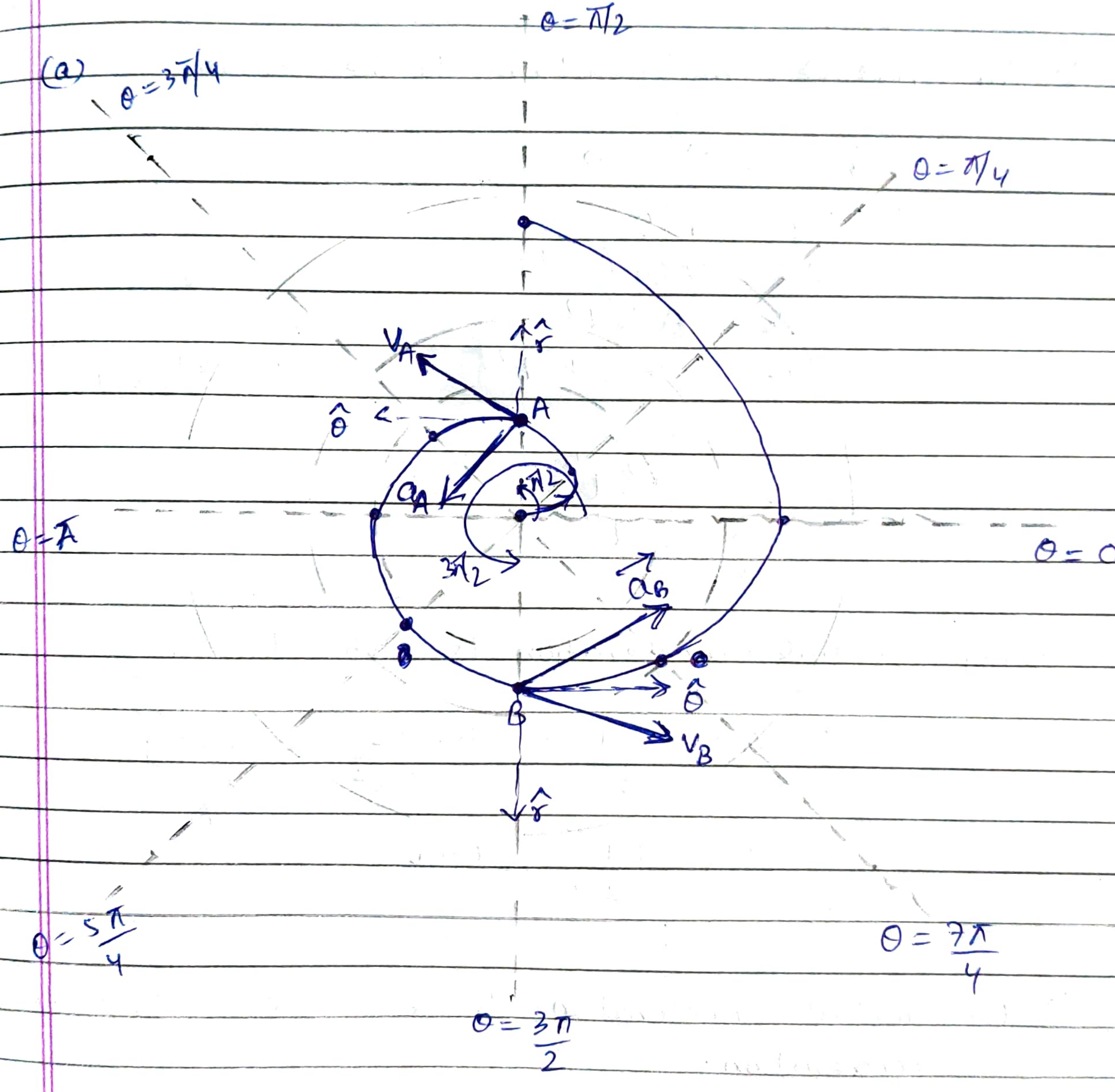
$\therefore$ $\dot{\theta} = kt$

and, $\ddot{\theta} = k$

As,

$$r = A\theta$$
$$\dot{r} = A\dot{\theta} = Akt$$
$$\ddot{r} = A\ddot{\theta} = AK$$

(a)



at point A , $\theta = \pi/2$ $\therefore$ $\vec{r} = A\theta\,\hat{r} = \frac{1}{\pi} \times \frac{\pi}{2} = \frac{1}{2}\hat{r}$

$$\vec{v} = Akt\,\hat{r} + (A\theta)(kt)\,\hat{\theta}$$

$$\vec{V}_{\theta=\pi/2} = AK\left\{\sqrt{\frac{\pi}{K}}\,\hat{r} + \frac{\pi}{2}\sqrt{\frac{\pi}{K}}\,\hat{\theta}\right\}$$

$$\vec{a}_{\theta=\pi/2} = AK\left\{\left(1-2\left(\frac{\pi}{2}\right)^2\right)\hat{r} + 5\left(\frac{\pi}{2}\right)\hat{\theta}\right\}$$

at point B,

$$r = A\theta = A\left(\frac{3\pi}{2}\right)$$

$$\vec{v}_{\theta=\frac{3\pi}{2}} = AK\left(\sqrt{\frac{3\pi}{K}}\,\hat{r} + \frac{3\pi}{2}\sqrt{\frac{3\pi}{K}}\,\hat{\theta}\right)$$

$$\vec{a}_{\theta=\frac{3\pi}{2}} = AK\left(\left(1-\frac{9\pi^2}{2}\right)\hat{r} + \left(\frac{15\pi}{2}\right)\hat{\theta}\right)$$

b) $\vec{a} = \left\{\ddot{r} - r(\dot{\theta})^2\right\}\hat{r} + \left\{2\dot{r}\dot{\theta} + r\ddot{\theta}\right\}$

⇒ for radial acceleration to be zero

$$\ddot{r} - r(\dot{\theta})^2 = 0$$
$$\ddot{r} = r(\dot{\theta})^2$$
$$AK = (A\theta)(Kt)^2$$
$$K = \theta K^2 t^2$$
$$\cancel{K} = \left(\frac{\cancel{K}t^2}{2}\right) K^2 t^2 \qquad \left(\theta = Kt^2/2\right)$$
$$2 = K^2 t^4$$
$$\frac{2}{K^2} = t^4$$

when radial acceleration is zero $t^2 = \sqrt{\frac{2}{K^2}}$

$$\therefore \theta = \frac{Kt^2}{2} = \frac{K}{2}\sqrt{\frac{2}{K^2}} = \sqrt{\frac{2\cancel{K^2}}{4\cancel{K^2}}} \quad \frac{1}{2}$$
$$\theta = \sqrt{\frac{1}{2}} \text{ radian}$$

⇒ For radial acceleration to be equal to tangential acceleration.

$$\ddot{r} - r(\dot{\theta})^2 = 2\dot{r}\dot{\theta} + r\ddot{\theta}$$
$$AK - (A\theta)(Kt)^2 = 2(AKt)(Kt) + (A\theta)K$$
$$AK - AK^2\theta t^2 = 2AK^2t^2 + AK\theta$$
$$1 - K\theta t^2 = 2Kt^2 + \theta$$
$$1 - K\theta\left(\frac{2\theta}{K}\right) = 2K\left(\frac{2\theta}{K}\right) + \theta \qquad \left\{\because \theta = \frac{Kt^2}{2}\right\}$$

$$1 - 2\theta^2 = 5\theta$$

$$2\theta^2 + 5\theta - 1 = 0$$

$$\theta = \frac{-5 \pm \sqrt{25 - 4(2)(-1)}}{4}$$

$$\theta = \frac{-5 \pm \sqrt{33}}{4}$$

Neglecting negative value, we get

$$\theta = \frac{\sqrt{33} - 5}{4}$$

$$t^2 = \frac{2}{K}\left(\frac{\sqrt{33} - 5}{4}\right)$$

i.e $t = \sqrt{\frac{2}{K}\left(\frac{\sqrt{33} - 5}{4}\right)}$

when radial acceleration is equal to tangential acceleration $\theta = \left(\frac{\sqrt{33}-5}{4}\right)$ radian and time is equal to $\sqrt{\frac{2}{K}\left(\frac{\sqrt{33} - 5}{4}\right)}$ seconds.