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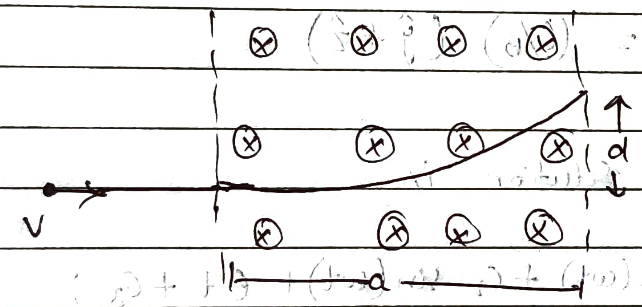
PH110: Waves and Electromagnetics

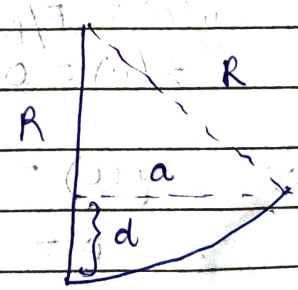
Tutorial 8

Q1. A particle of charge q enters a region of uniform magnetic field B (pointing into the page). The field deflects the particle a distance d above the original line of flight, as shown. Is the charge positive or negative? In terms of a, d, B and q, find the momentum of the particle.



Since $v \times B$ points upwards, and that is also the direction of force, q must be positive.

To find R in terms of a and d use the pythagoras theorem,



$$(R-d)^2 + a^2 = R^2$$

$$R = \frac{a^2+d^2}{2d}$$

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∴ Using the cyclotron formula

$$\boxed{p = qBR = qB\frac{(a^2+d^2)}{2d}}$$

Q.2 Find and sketch the trajectory of the particle in Ex 5.2, if it starts at origin with velocity

(a) $V(0) = (E/B)\,\hat{y}$

(b) $V(0) = (E/2B)\,\hat{y}$

(c) $V(0) = (E/B)\,(\hat{y}+\hat{z})$

The general solution is

$$y(t) = C_1 \cos(\omega t) + C_2 \sin(\omega t) + \frac{E}{B}t + C_3 ;$$

$$z(t) = C_2 \cos(\omega t) - C_1 \sin(\omega t) + C_4 ,$$

(a) $y(0) = z(0) = 0$

$\dot{y}(0) = E/B$

$\dot{z}(0) = 0$

∵ $y(0) = 0$

∴ $0 = C_1 + C_3$ —(1)

and ∵ $\dot{y}(0) = E/B$

$$\frac{E}{B} = -C_1\omega \sin(0) + C_2\omega\cos(0) + \frac{E}{B}$$

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$$\frac{E}{B} = C_2 + \frac{E}{B}$$

$$\therefore \boxed{C_2 = 0}$$

$\because$ $z(0) = 0$

$$0 = C_2 + C_4$$

$$\because C_2 = 0 \;, \therefore \boxed{C_4 = 0}$$

$\because$ $\dot{z}(0) = 0$

$$\therefore \quad 0 = -C_2 \omega \sin(0) + C_1 \omega \cos(0)$$

$$0 = -C_1 \omega.$$

$$\therefore C_1 = 0$$

Put $C_1 = 0$ in eqn ①, gives $C_3 = 0$.

$$\therefore \quad y(t) = \frac{E}{B} t$$

$$z(t) = 0$$

The magnetic force in this case cancels the electric force, hence there is no net force and the particle moves in a straight line at constant speed.

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(b) Since the particle starts at origin.

$\because$ using $y(0)=0$ and $z(0)=0$, we get

$$C_1 = -C_3 \quad \text{and} \quad C_4 = -C_2$$

$\because$ $\dot{z}(0) = 0$

$\therefore$ $C_1 = 0$

and hence $C_3 = 0$

$\because$ $\dot{y}(0) = \frac{E}{2B}$

$\therefore$ $C_2\omega + \frac{E}{B} = \frac{E}{2B}$

$$C_2 = \frac{-E}{2\omega B}$$

$$\therefore C_4 = \frac{E}{2\omega B}$$

$$\therefore y(t) = \frac{-E}{2\omega B}\sin(\omega t) + \frac{Et}{B}$$

$$z(t) = \frac{-E}{2\omega B}\cos(\omega t) + \frac{E}{2\omega B}$$

$$\therefore y(t) = \frac{E}{2\omega B}\left[2\omega t - \sin(\omega t)\right]$$

$$z(t) = \frac{E}{2\omega B}\left[1 - \cos(\omega t)\right]$$

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Let $\beta = \frac{E}{2\omega B}$

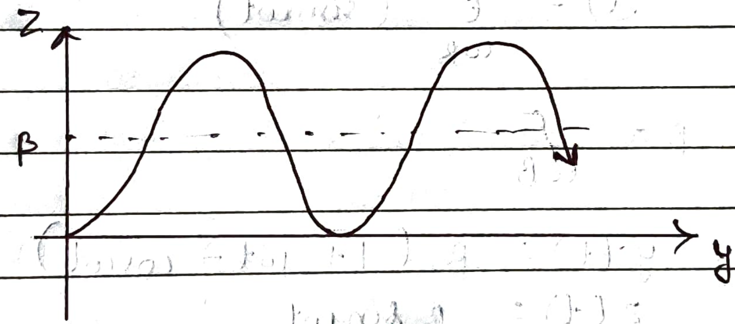
$\therefore$ $y(t) = \beta[2\omega t - \sin(\omega t)]$
$z(t) = \beta[1 - \cos\omega t]$

$y - 2\beta\omega t = -\beta\sin(\omega t)$

and, $z - \beta = -\beta\cos(\omega t)$

$(y - 2\beta\omega t)^2 + (z - \beta)^2 = \beta^2$.

This is a circle of radius β whose center moves to the right at constant speed.



(c) $\dot{z}(0) = \dot{y}(0) = \frac{E}{B}$

$-C_1\omega = \frac{E}{B}$

$\therefore$ $C_1 = -C_3 = \frac{-E}{\omega B}$

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and, $C_2\omega + \frac{E}{B} = \frac{E}{B}$

$$C_2 = C_4 = 0$$

$\therefore$ $y(t) = \frac{-E}{\omega B}\cos(\omega t) + \frac{E}{B}(t) + \frac{E}{\omega B}$

$$z(t) = \frac{E}{\omega B}\sin(\omega t)$$

or $y(t) = \frac{E}{\omega B}\left[1 + \omega t - \cos(\omega t)\right]$

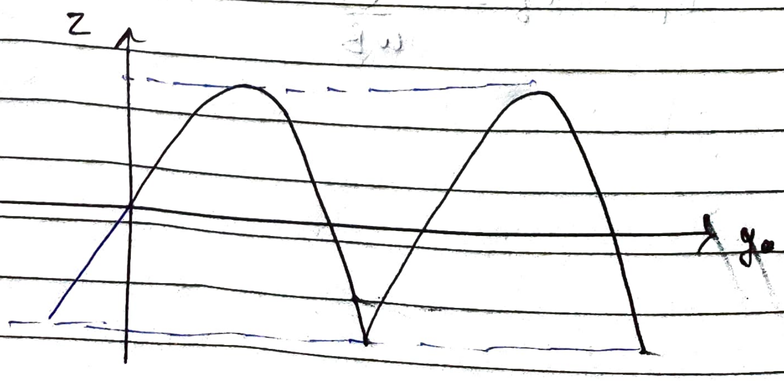
$$z(t) = \frac{E}{\omega B}(\sin \omega t)$$

Let $\beta = \frac{E}{\omega B}$

$$y(t) = \beta(1 + \omega t - \cos \omega t)$$
$$z(t) = \beta \sin \omega t$$

& $\left[y - \beta(1+\omega t)\right]^2 + z^2 = \beta^2$

This is a circle of radius β whose centre is at $y_0 = \beta(1+\omega t)$ and $z_0 = 0$.



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5.3 In 1897, JJ Thompson 'discovered' the electron by measuring the charge-to-mass ratio of "cathode rays" (stream of electrons with charge q and mass m) as follows:

(a) First he passed the beam through uniform crossed electric field and magnetic field E and B (mutually perpendicular and both of them perperdicular to the beam), and adjusted the electric field until he got zero deflection. What, then, was the speed of particles?

(b) Then he turned off the electric field, and measured the radius of curvature R, of the beam, as deflected by the magnetic field alone. In terms of E, B and R; what is the charge to mass ratio (q/m) of particles?

(a) Let the speed of particles be v.
electric force on the particles $= qE$.
magnetic " " " " $= q\,v \times B$

$$\therefore \quad qE = q\,v \times B$$
$$\therefore \quad E = vB$$
$$\boxed{\therefore \quad v = \frac{E}{B}}$$

Hence, the speed of particles is $\frac{E}{B}$.

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(b) Now, the centrifugal force will balance the magnetic force, to keep it in circular motion.

$$\therefore \quad m\left(\frac{E}{B}\right)^2 \cdot \frac{1}{R} = q\frac{E}{B} \cdot B$$

$$\boxed{\frac{q}{m} = \frac{E}{B^2 R}}$$

Hence, charge-to-mass ratio is $\frac{E}{B^2 R}$

5.4. Suppose that magnetic field in some region has the form

$$B = kz\hat{x}$$

where k is a constant. Find the force on a square loop (side a), lying in the yz plane and centered at origin, if it carries a current I, flowing counter clockwise, when you look down the x-axis.

Suppose I flows counterclockwise, the force on the left side cancels the force on the right side; the force on the top is $IaB = Iak\left(\frac{a}{2}\right) = \frac{Ika^2}{2}$ (pointing upward), and the force on the

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bottom is $IaB = -\frac{Ika^2}{2}$ (pointing upward).

So, the net force is $\frac{Ika^2}{2}\hat{z}$.

5.5 A current I flows down a wire of radius a.

(a) If it is uniformly distributed over the surface, what is the surface current density K,

(b) If it is distributed in such a way that the volume current density is ~~merely~~ inversely proportional to the distance from the axis, what is $J(s)$?

(a) As the length perpendicular to flow is circumference

$$\therefore \quad K = \frac{I}{2\pi a}$$

(b) $J = \frac{\alpha}{s} \Rightarrow I = \int J \, da$

$$I = \alpha \int \frac{1}{s} s \, ds \, d\phi$$

$$I = 2\pi\alpha \int ds = 2\pi\alpha a$$

$$\therefore \quad \alpha = \frac{I}{2\pi a}$$

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$$\therefore \boxed{J = \frac{1}{2\pi a s}}$$

Problem 5.6. (a) A phonograph record carries a uniform density of "static electricity" σ. If it rotates at angular velocity ω, what is the surface current density K at a distance r from center?

(b) A uniformly charged sphere of radius R and total charge Q, is centered at the origin and spinning at a constant angular velocity about the z-axis. Find current density J at any point (r, θ, ϕ) within the sphere.

(a) Surface current $\vec{K}$ is defined by

$$\vec{K} = \sigma \vec{v}$$

where $\vec{v}$ is ~~the~~ related to angular velocity ω by

$$\vec{v} = \omega s \hat{\phi}$$

where s is the axial distance.

$$\therefore \boxed{\vec{K} = \sigma \omega s \hat{\phi}}$$

(b) Let the sphere spin about z-axis, then the axial distance from z-axis is $r \sin\theta$ and current density $\vec{J}$ is defined by

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$$\vec{J} = \rho \vec{v}$$

where charge density $\rho = \frac{3Q}{4\pi R^3}$ and

$$\vec{v} = \omega r \sin\theta \, \hat{\phi}$$

$$\therefore \boxed{\vec{J} = \frac{3Q}{4\pi R^3} \omega r \sin\theta \, \hat{\phi}}$$

5.7 For a configuration of charges and currents confined within a volume V, show that

$$\int_V J \, d\tau = \frac{dp}{dt}$$

where p is total dipole moment

$$\because \frac{dp}{dt} = \frac{d}{dt} \int_V \rho \, r \, d\tau$$

$$= \int \left(\frac{\partial \rho}{\partial t}\right) r \, d\tau$$

$$= -\int (\nabla \cdot J) \, r \, d\tau \quad \text{(continuity equation)}.$$

using product rule 5

$$\nabla \cdot (x J) = x (\nabla \cdot J) + J \cdot (\nabla x)$$

but, $\nabla x = \hat{x}$, so

$$\nabla \cdot (x J) = x (\nabla \cdot J) + J_x$$

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Thus $\int_V (\nabla \cdot J)\, x \, d\tau = \int_V \nabla \cdot (xJ)\, d\tau - \int_V J_x \, d\tau$

The first term is $\int_S xJ \cdot da$ (by divergence theorem)

and since J is entirely inside V, it is zero on the surface S.

$$\therefore \int_V (\nabla \cdot J)\, x \, d\tau = -\int_V J_x \, d\tau$$

or, combining this with the y and z components

$$\int_V (\nabla \cdot J)\, r \, dr = -\int_V J \, d\tau.$$

or, referring back to the first line,

$$\frac{dP}{dt} = \int_V J \, d\tau$$

Hence, proved.