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MA102 : Introduction to Discrete Mathematics

Tutorial 7

1. Let R be the relation $\{(a,b) \mid a \neq b\}$ on Z. What is the reflexive closure of R?

(a,a) must be present in R $\forall a \in Z$ for reflexive closure.

So, reflexive closure $R' = R \cup \{(a,a) \mid a \in Z\}$

$\therefore R' = \{(a,b) \mid a \neq b \text{ or } a = b\}$

$R' = \{(a,b) \mid a,b \in \mathbb{Z}\}$

$\therefore R' = \mathbb{Z} \times \mathbb{Z}$

2. Use Warshall's algorithm to find the transitive closure of the relation R $= \{(1,2), (2,1), (2,3), (3,4), (4,1)\}$ on $\{1,2,3,4\}$.

The matrix representation of R,

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

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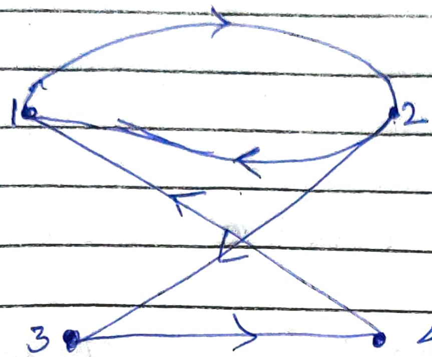
Digraph Representation



Using Warshall's theorem to find transitive closure

W_0 is the zero-one matrix that doesn't allow any interior vertex in the path.

$$\therefore W_0 = M_R$$

$$W_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$$W_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

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$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

W_4 is the transitive closure of relation R.

3. Let $A = \{2, 3, 4, \dots, 100\}$ with partial order of divisibility.

(a) How many maximal elements does $(A, |)$ have?

(b) Give a subset of A that is linear order under divisibility and is as large as possible?

(a) Maximal elements of $(A, |)$ are all numbers from 51 to 100 that are not ~~integer~~ primes.

$\therefore \{51, 52, \dots, 100\} - \{53, 59, 61, 67, 71, 73, 79, 83, 89, 97\}$

$\therefore$ no. of maximal elements $= 50 - 10$
$= 40$

(b) largest linear order set under divisibility would be the set containing powers of 2.

$\therefore \{2, 4, 8, 16, 32, 64\}$

There are other possible sets like

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$\{3, 9, 27, 81\}$ or $\{4, 16, 64\}$ but exponents of 2 is the largest set.

4. A person's blood type is determined by the presence (T) or absence (F) of antigens A, B and Rh, as shown in the table below.

A	B	Rh	Type
F	F	F	O^-
F	F	T	O^+
F	T	F	B^-
F	T	T	B^+
T	F	F	A^-
T	F	T	A^+
T	T	F	AB^-
T	T	T	AB^+

A person with blood type X can donate blood to a person with blood type Y, if and iff only if all the antigens present in X are present in Y. Let P be the set of eight possible blood type, and let R be the relation on P such XRY iff a person with blood type X can donate blood to a person with blood type Y. Answer the following questions:

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(a) Can a person with A^+ blood type donate to one with A^-?

Since A^+ has Rh factor which is not present in A^-. Hence, a person with blood group A^+ can not donate to a person with A^-.

(b) What types of blood can a person with A^+ blood type recieve?

A^+ bloody types person can takes blood from people with blood type O^-, O^+, A^- and A^+.

(c) Draw a directed graph for R?

$R = \{(O^-, O^-), (O^-, O^+), (O^-, A^-), (O^-, A^+), (O^-, B^-),$

$(O^-, B^+), (O^-, AB^-), (O^-, AB^+), (O^+, O^+),$

$(O^+, A^+), (O^+, B^+), (O^+, AB^+), (A^-, A^-),$

$(A^-, A^+), (A^-, AB^-), (A^-, AB^+), (A^+, A^+),$

$(A^+, AB^+), (B^-, B^-), (B^-, B^+), (B^-, AB^-),$

$(B^-, AB^+), (B^+, B^+), (B^+, AB^+), (AB^-, AB^-),$

$(AB^-, AB^+), (AB^+, AB^+)\}$

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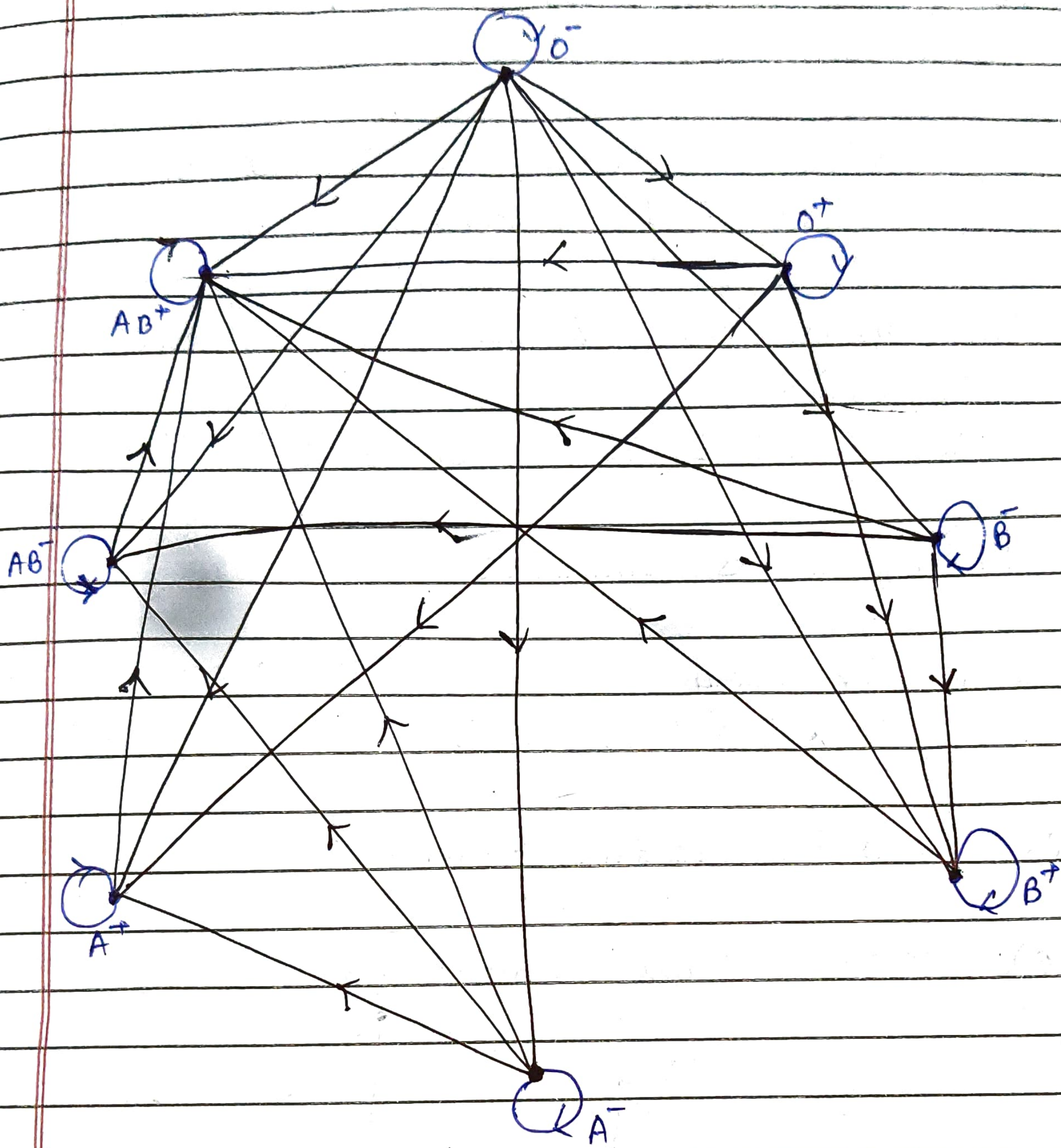

(d) Show that R is a partial order.

Since, a person with blood group X can donate to a person with same blood group
$\therefore$ $(X,X) \in R \ \forall X \in P$. So, R is reflexive.

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If XRY, it means that Y has all antigens that X have.

If YRZ, it means that Z has all antigens that Y have.

It implies that Z has all antigens that X have.

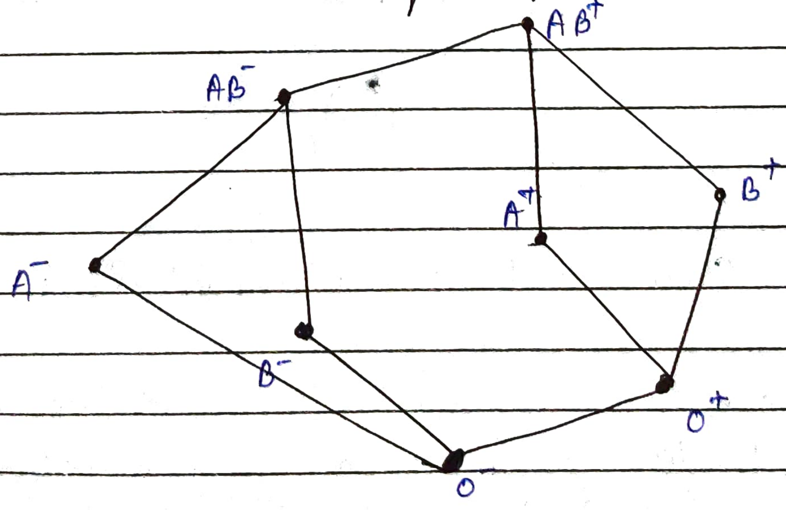
$\therefore$ XRZ iff XRY and YRZ.

Hence, R is transitive.

Now, for every $(X, Y) \in R$, $(Y, X) \in R$ iff $X = Y$.
So, R is anti-symmetric.

$\therefore$ R is a partial order.

(e) Make a Hasse diagram for R.



(f) What are the minimal (universal donor) and maximal (universal acceptor) elements of P?

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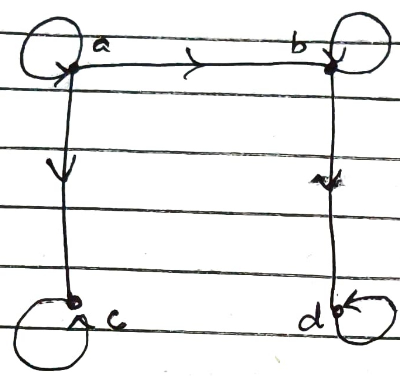
Minimal element (or universal Donor) : O^-

Maximal element (or universal acceptor) : AB^+

5. Determine whether the relation with the directed graph shown is a partial order.

(a)



As there is loop at every vertex, R is reflexive.

Since, $(a,b) \in R$ and $(b,d) \in R$ but $(a,d) \notin R$
Hence, R is not transitive.

R is ~~not~~ anti-symmetric as there is only single path between two distinct vertices.

Since, R is reflexive and anti-symmetric, but not transitive, hence R is not a partial order.

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(b)

as there are loops on both vertices, R is reflexive.

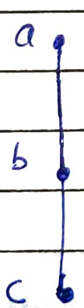
Since, there are only two vertices, R is transitive.

as aRb but b$\not R$a, R is anti-symmetric.

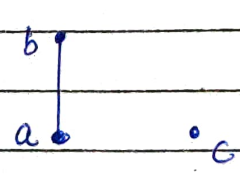
Hence, R is a partial order.

6. Display all the partial orders on a set with three elements with the help of Hasse diagram. How many of them are lattices?

Hasse diagram for this case



permutations = 3! = 6.



permutations = 3! = 6

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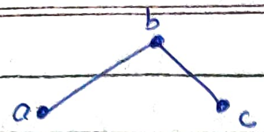
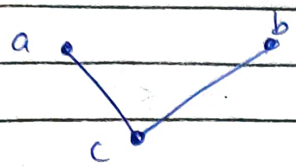
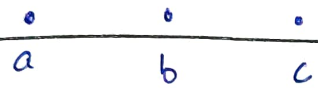


permutations $= 3!/2! = 3$



permutations $= 3!/2! = 3$



permutations $= 1$

$\therefore$ total possibilities $= 6+6+3+3+1$
$= 19$

7. Let R be a partial order on a finite set S. Describe how to use the matrix representation. M_R to find the least and greatest element of A if they exist. Greatest element:
$y \in (S, \preceq)$ is greates if $x \preceq y \ \forall x \in S$.

Least Element: $z \in (S, \preceq)$ is least if $z \preceq x \ \forall x \in S$

Least Element: It will be the element corresponding to which the row in M_R will have all entries as 1's.

Greatest Element: It will be the element corresponding to which the column in M_R will have all entries as 1's.

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8. Give an example of an infinite lattice with neither a least element nor a greatest element.

We can consider the lattice $(Z, \leq)$

least element : $-\infty$

greatest element : ∞

9. Give an example of an infinite lattice with a least element and a greatest element.

We can consider the lattice: $([20,30], \leq)$

least element : 20
greatest element : 30