MA202

ASSIGNMENT 9

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SECTION:

2

Exercise 1

Write a program to numerically evaluate the integral of the function $f(x) = \frac{\sin(\lambda x)}{x}$, from $x = -10$ to $x = 10$, for the values of $\lambda = 0.01, 0.1, 1, 10, 100$. Repeat the same exercise as above for $g(x) = \exp(\sin(\lambda x))$, for which the integration limits are $x = 0$ to $x = 5\pi$. Note that your answers should be accurate upto 5th significant digit. Also mention the estimated error in the evaluation. Do clearly mention number of sampling done in each case.

Exercise 2

Write a program to numerically integrate the multi-dimensional integral $\int_{-L}^{L} dx_1 \int_{-L}^{L} dx_2 \cdots \int_{-L}^{L} dx_n \exp[-\frac{1}{2}(x_1^2 + x_2^2 + \cdots + x_n^2)]$. Take $n = 8, 9, 10$ and $L = 10$ to evaluate the integral. Your answer should be accurate upto 5th significant digit. Please mention the number of sampling needed as also the variance estimate in each case.

Exercise 3

Use the above written program to evaluate this integral $\int_{-L}^{L} dx_1 \int_{-L}^{L} dx_2 \cdots \int_{-L}^{L} dx_n \exp[-\frac{1}{2} x^T \cdot A \cdot x]$, where the matrix $x = (x_1, x_2, \cdots, x_n)$, and the matrix A is such that, $A_{ij} = 1$ when $|i - j| = 1$, else $A_{ij} = 0$. Take $n = 8, 9, 10$ and $L = 10$ to evaluate the integral. Your answer should be accurate upto 5th significant digit. Please mention the number of sampling needed as also the variance estimate in each case.

Exercise 1(a)

Code

The values of c is changed repeatedly.

```
%%Monte Carlo Height implementation
%lower Limit
a = -10;
%upper limit
b = 10;
%total tries
N = 1000;
%c values
c = 0.01
%c = 0.1
%c = 1
%c = 10
%c = 100
%defining the function
f = @(x) sin(c*x)./(x);

for i = 1: N
    x_val(i) = rand(1)*(b-a)+a;
    y_val(i) = (b-a)*f(x_val(i));
end

mean = sum(y_val)/N
MATLAB_method = integral(f,a,b)
error = MATLAB_method - mean
```

Output

```
c = 0.0100

mean = 0.1999

MATLAB_method = 0.1999

error = -2.5707e-06
```

```
c = 0.1000

mean = 1.8935

MATLAB_method = 1.8922

error = -0.0014
```

```
c = 1

mean = 3.3101

MATLAB_method = 3.3167

error = 0.0066
```

```
c = 10

mean = 2.0285

MATLAB_method = 3.1245

error = 1.0960
```

```
c = 100

mean = 1.5000

MATLAB_method = 3.1405

error = 1.6404
```

Exercise 1(b)

Code

```
%%Monte Carlo Height implementation
%lower Limit
a = 0;
%upper limit
b = 5*pi;
%total tries
N = 1000;
%c values
%c = 0.01
%c = 0.1
%c = 1
%c = 10
%c = 100
%defining the function
f = @(x) exp(sin(c*x));

for i = 1: N
    x_val(i) = rand(1)*(b-a)+a;
   y_val(i) = (b-a)*f(x_val(i));
```

```
end

mean = sum(y_val)/N
MATLAB_method = integral(f,a,b)
error = MATLAB_method - mean
```

Output

```
c = 0.0100

mean = 16.9914

MATLAB_method = 17.0060

error = 0.0146
```

```
c = 0.1000

mean = 30.5015

MATLAB_method = 31.0438

error = 0.5422
```

```
c = 1

mean = 22.9619

MATLAB_method = 22.1186

error = -0.8433
```

```
c = 10

mean = 19.9866

MATLAB_method = 19.8873

error = -0.0993
```

```
c = 100

mean = 19.1877

MATLAB_method = 19.8873

error = 0.6996
```

Exercise 2

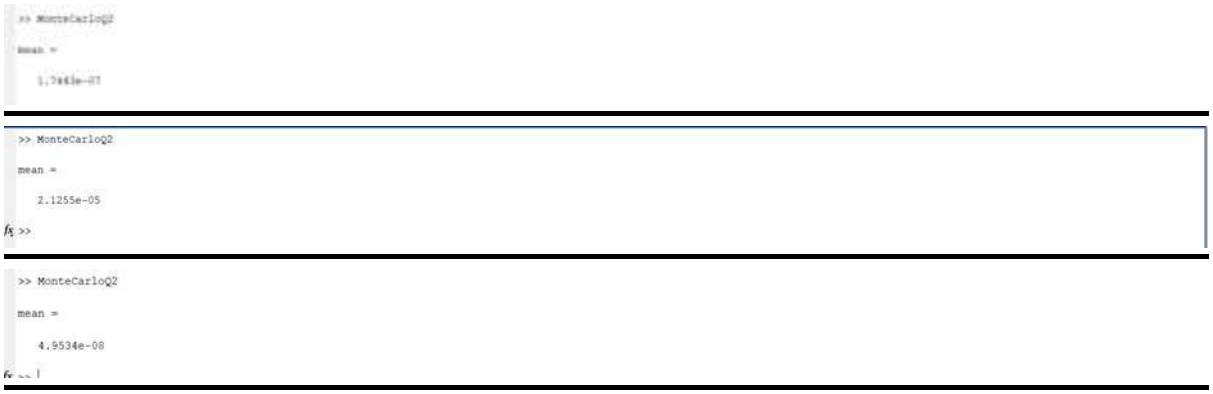
Code

```
%%Monte Carlo Height implementation
%lower Limit of x1
ax1 = -10;
%upper limit of x1
bx1 = 10;
%lower Limit of x2
ax2 = -10;
%upper limit of x2
bx2 = 10;
%lower Limit of x3
ax3 = -10;
%upper limit of x3
bx3 = 10;
%lower Limit of x4
ax4 = -10;
%upper limit of x4
bx4 = 10;
%lower Limit of x5
ax5 = -10;
```

```
%upper limit of x5
bx5 = 10;
%lower Limit of x6
ax6 = -10;
%upper limit of x6
bx6 = 10;
%lower Limit of x7
ax7 = -10;
%upper limit of x7
bx7 = 10;
%lower Limit of x8
ax8 = -10;
%upper limit of x8
bx8 = 10;
%lower Limit of x9
ax9 = -10;
%upper limit of x9
bx9 = 10;
%lower Limit of x9
ax10 = -10;
%upper limit of x9
bx10 = 10;









%total tries
N = 1000;
%defining the function
f = @(x1,x2,x3,x4,x5,x6,x7,x8,x9,x10) exp(-0.5*(x1.^2 + x2.^2+x3.^2+x4.^2
+x5.^2 +x6.^2 +x7.^2 +x8.^2+x9.^2+x10.^2));
for i = 1: N
    x1_val(i) = rand(1)*(bx1-ax1)+ax1;
    x2_val(i) = rand(1)*(bx2-ax2)+ax2;
    x3_val(i) = rand(1)*(bx3-ax3)+ax3;
    x4_val(i) = rand(1)*(bx4-ax4)+ax4;
    x5_val(i) = rand(1)*(bx5-ax5)+ax5;
    x6_val(i) = rand(1)*(bx6-ax6)+ax6;
    x7_val(i) = rand(1)*(bx7-ax7)+ax7;
    x8_val(i) = rand(1)*(bx8-ax8)+ax8;
    x9_val(i) = rand(1)*(bx9-ax9)+ax9;
    x10_val(i) = rand(1)*(bx10-ax10)+ax10;
    plane(i)=
f(x1_val(i),x2_val(i),x3_val(i),x4_val(i),x5_val(i),x6_val(i),x7_val(i),x8_val
(i),x9_val(i),x10_val(i));
end
```

```
mean = (20^10)*sum(plane)/N;


mean
```

Output



Exercise 3

Code

```
clc;
clear all;
lower_limit = -10;
upper_limit = 10;
% For n = 8


for i = 1:8
    for j = 1:8
        if abs(i-j) == 1
            A(i,j) = 1;
        else
            A(i,j) = 0;
        end
    end
end
sum  = 0;
for i = 1 : 10000
    x(:,i) = lower_limit + (upper_limit - lower_limit)*rand(1,8);
    sum = sum + exp((-0.5*transpose(x(:,i))*A*x(:,i)));
end
sum = sum/10000;
sum = sum*(upper_limit - lower_limit).^8


clc;
```

```
clear all;
lower_limit = -10;
upper_limit = 10;
% For n = 9


for i = 1:9
    for j = 1:9
        if abs(i-j) == 1
            A(i,j) = 1;
        else
            A(i,j) = 0;
        end
    end
end
sum  = 1;
for i = 1 : 10000
    x(:,i) = lower_limit + (upper_limit - lower_limit)*rand(1,9);
    sum = sum + exp((-0.5*transpose(x(:,i))*A*x(:,i)));
end
sum = sum/10000;
sum = sum+(upper_limit - lower_limit).^9


clc;
clear all;
lower_limit = -10;
upper_limit = 10;
% For n = 10


for i = 1:10
    for j = 1:10
        if abs(i-j) == 1
            A(i,j) = 1;
        else
            A(i,j) = 0;
        end
    end
end
sum  = 0;
for i = 1 : 10000
    x(:,i) = lower_limit + (upper_limit - lower_limit)*rand(1,10);
    sum = sum + exp((-0.5*transpose(x(:,i))*A*x(:,i)));
end
sum = sum/10000;
sum = sum*(upper_limit - lower_limit).^10
```

Output

```
sum = 1.3718e+185

sum = 1.3678e+175

sum = 4.5073e+236


sum = 1.0239e+177

sum = 2.6489e+176

sum = 3.7590e+199
```