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PH110: Waves and Electrodynamics

Tutorial 11

Ques 34. Show that the magnetic field of a dipole can be written in the co-ordinate ~~form~~ free-form:

$$B_{dip}(r) = \frac{\mu_0}{4\pi r^3}\left[3(m\cdot\hat{r})\hat{r} - m\right]$$

The dipole magnetic field is

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

Now, if the dipole is oriented along z-axis, we can write it in the form

$$\vec{m} = m\hat{z} = m(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})$$

$$m\cos\theta = (\vec{m}\cdot\hat{r})\hat{r}$$

$$m\sin\theta\,\hat{\theta} = m\cos\theta\,\hat{r} - \vec{m} = (\vec{m}\cdot\hat{r})\hat{r} - \vec{m}$$

$$B = \frac{\mu_0}{4\pi r^3}\left[2(\vec{m}\cdot\hat{r})\hat{r} + (\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]$$

$$\boxed{B = \frac{\mu_0}{4\pi r^3}\left[3(m\cdot\hat{r})\hat{r} - m\right]}$$

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Ques 35. A circular loop of wire, with radius R, lies in the xy plane (centered at origin) and carries a current I running counterclockwise as viewed from +z-axis.

(a) What is its magnetic dipole moment?
(b) What is the approximate magnetic field at points far from the origin?
(c) Show that, for points on the z-axis, your answer is consistent with the exact field when z >> R.

(a) $m = Ia = I(\pi R^2)\,\hat{z}$

(b) Far from the origin,

$$\boxed{B \simeq \frac{\mu_0}{4\pi} \cdot \frac{I\pi R^2}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)}$$

(c) field along z-axis

$$B(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}$$

Since, z >> R

$$\therefore \quad B(z) = \frac{\mu_0 I}{2} \frac{R^2}{z^3}\,\hat{z}$$

This agrees with the dipole formula, with r = z and θ = 0. The field is upwards along z-axis.

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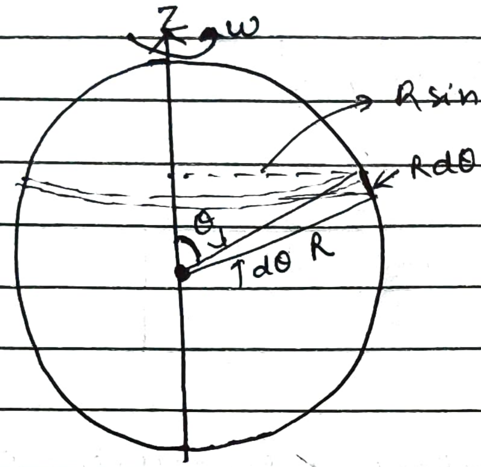
Ques 37(a) A phonograph record of radius R, carrying a uniform surface charge σ, is rotating at constant angular velocity ω. Find its magnetic dipole moment.

(b) Find the magnetic dipole moment of the spinning spherical shell in Ex 5.11. Show that for points $r > R$, the potential is that of a perfect dipole



(a) For a ring, $m = I\pi r^2$

Here, $I = \sigma V dr = \sigma \omega r dr$

So, $m = \int_0^R \pi r^2 \sigma \omega r dr$

$$\boxed{m = \frac{\pi \sigma \omega R^4}{4}}$$

(b) Total charge on shaded ring, $dq = \sigma (2\pi R \sin\theta) R d\theta$

Time for revolution, $dt = \frac{2\pi}{\omega}$.

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∴ the current in the ring is $I = \frac{dq}{dt} = \sigma \omega R^2 \sin\theta \, d\theta$

Area of ring $= \pi (R \sin\theta)^2$

So, $dm = (\sigma \omega R^2 \sin\theta \, d\theta) \pi R^2 \sin^2\theta$

$$m = \sigma \omega \pi R^4 \int_0^\pi \sin^3\theta \, d\theta$$

$$m = \left(\frac{4}{3}\right) \sigma \omega \pi R^4$$

$$\boxed{\therefore \; m = \frac{4\pi}{3} \sigma \omega R^4 \, \hat{z}}$$

The dipole term in the multipole expansion of A is

$$A_{dip} = \frac{\mu_0}{4\pi} \frac{4\pi}{3} \sigma \omega R^4 \frac{\sin\theta}{r^2} \hat{\phi}$$

$$A_{dip} = \frac{\mu_0 \sigma \omega R^4}{3} \frac{\sin\theta}{r^2} \hat{\phi}$$

which is also the exact potential. Evidently, a spinning sphere produces a perfect dipole field, with no higher multipole conditions.

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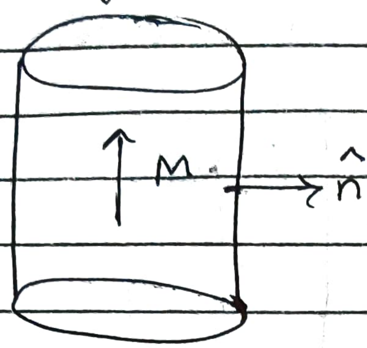
Chapter 6

Ques 7. An infinitely long circular cylinder carries a uniform magnetisation M parallel to its axis. Find the magnetic field (due to M) inside and outside the cylinder.



$$J_b = \nabla \times M = 0$$

The magnetisation of a cylinder is uniform, so there is no volume bound current, but there's surface bound current.

$$K_b = M \times \hat{n} = M\hat{\phi}$$

The field is that of a surface current $K_b = M\hat{\phi}$ but that is just a solenoid, so outside field is zero and inside field is,

$$B = \mu_0 K_b = \mu_0 M.$$

Moreover, it points in the direction of M.

$$B = \begin{cases} 0 & , \quad x > R \\ \mu_0 M & , \quad x \leq R \end{cases}$$

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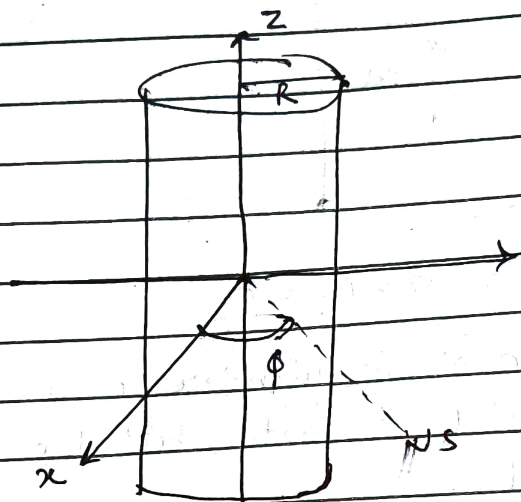
Ques 8. A long circular cylinder of radius R carries a magnetization $M = ks^2\hat{\phi}$, where k is a constant, s is the distance from the axis and $\hat{\phi}$ is the usual azimuthal unit vector in the fig. Find the magnetic field due to M, for points inside and outside the cylinder.



$$J_b = \nabla \times M = \frac{1}{s}\frac{\partial}{\partial s}(3ks^2)\hat{z} = 3ks\,\hat{z}$$

$$K_b = M \times \hat{n} = ks^2(\hat{\phi} \times \hat{s}) = -kR^2\hat{z}$$

Field Outside:-

Using a circular Amperian loop,

$$B \cdot 2\pi s = \mu_0 I_{enc}$$

$$= 2\pi K_b + \int_0^R \int_0^{2\pi} J_b\, s\, ds\, d\phi$$

$$= (2\pi R)(-kR^2) + 3k \int_0^R \int_0^{2\pi} s^2\, ds$$

$$= 0$$

$$\therefore \boxed{B_{out} = 0}$$

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Field Inside:

We draw an Amperian loop ($s < R$)

$$B \cdot 2\pi s = \mu_0 I_{enc}$$

$$= \mu_0 \int_S J_b \, dS$$

$$= \mu_0 3k \int_0^s \int_0^{2\pi} s^2 \, ds \, d\phi$$

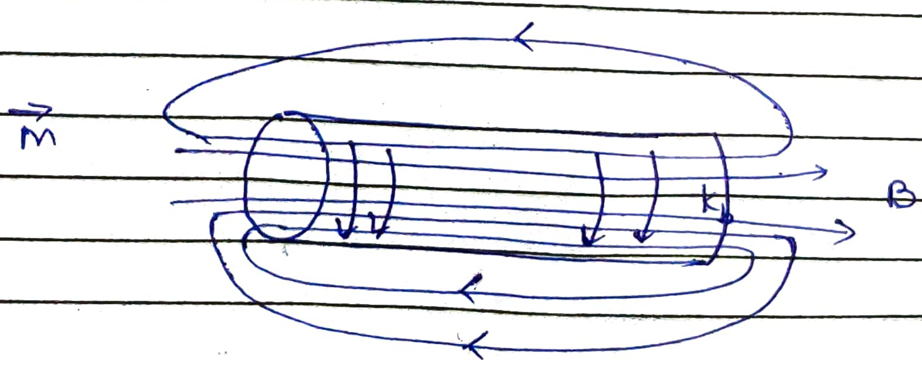
$$= \mu_0 2\pi k s^3$$

$$\therefore \boxed{B_{in} = \mu_0 k s^2 \hat{\phi} = \mu_0 M}$$

$$\therefore B = \begin{cases} \mu_0 k s^2 \hat{\phi} = \mu_0 M & , s < R \\ 0 & , s > R \end{cases}$$

Ques 9. A short circular cylinder of radius a and length L carries a 'frozen-in' uniform magnetization M parallel to its axis. Find the bound current and sketch the magnetic field of the cylinder. (Make three sketches: one for $L \gg a$, one for $L \ll a$ and $L \approx a$). Compare this bar magnet with the bar electret of Prob 4.11.

(i) $L \gg a$



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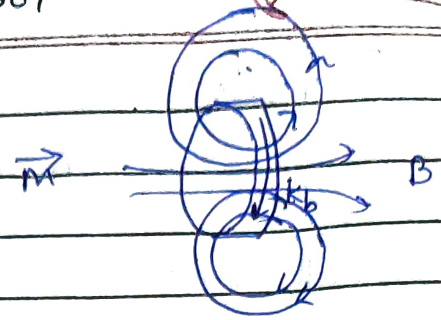
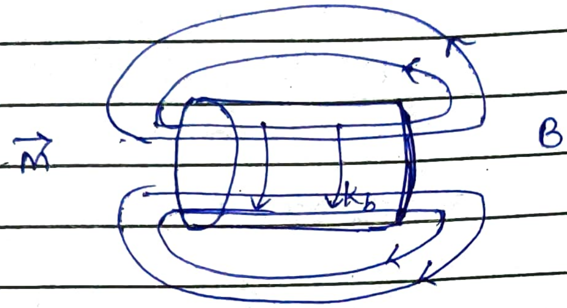
(ii) $L << a$



(iii) $L \approx a$



Bound current is given by

$$k_b = \vec{M} \times \hat{n} = M\hat{\phi}$$

and thus the field is one of a solenoid of length L, radius a and surface current M.

The magnetic field lines are sketched above. Outside, they are very similar to bar electret. But inside, they are completely different since the electric field has a discontinuity at the top and bottom, and the magnetic field does not have a discontinuity.