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EndSem Remote Examination

MA101

1. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix}$

$\det A = 1 + (-1)(3) + 1(1)$
$= -1$

Hence, A^{-1}, inverse exists

adjoining A with identity matrix

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right)$$

$R_2 \to R_2 - R_1$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -1 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right)$$

$R_2 \to -R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & -1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right)$$

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$R_3 \rightarrow R_3 - R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right)$$

$R_2 \rightarrow R_2 - 2R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & -3 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right)$$

$R_1 \rightarrow R_1 - R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & 3 & -3 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right)$$

$R_1 \rightarrow R_1 - R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 1 \\ 0 & 1 & 0 & 3 & -3 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right)$$

$\therefore$ By Gauss elimination

$$A^{-1} = \begin{bmatrix} -1 & 2 & 1 \\ 3 & -3 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

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2. $x + 2y + 4z = 1$
$3x + 8y + 14z = 2$
$2x + 6y + 13z = 3$

$\therefore$ ~~decomp~~ let A be coefficient matrix

$$\therefore A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$, $l_{21} = 3$
$R_3 \to R_3 - 2R_1$, $l_{31} = 2$

$$\sim \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 2 & 5 \end{bmatrix}$$

$R_3 \to R_3 - R_2$, $l_{32} = 1$

$$U = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

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$\therefore \quad A = LU$

Now $\quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Let, $\quad Ax = b$

$LUx = b$

$Ly = b \qquad [\because y = Ux$

Solve $\quad Ly = b$,

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore \quad y_1 = 1$

$3y_1 + y_2 = 2 \Rightarrow y_2 = -1$

$2y_1 + y_2 + y_3 = 3 \Rightarrow y_3 = 2$

$\therefore \quad y = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

Now, since $\quad Ux = y$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$\therefore \quad 3x_3 = 2 \Rightarrow x_3 = 2/3$

$2x_2 + 2x_3 = -1 \Rightarrow x_2 = -7/6$

$x_1 + 2x_2 + 4x_3 = 1 \Rightarrow x_1 = 2/3$

$\therefore \quad x = \begin{bmatrix} 2/3 & -7/6 & 2/3 \end{bmatrix}^T$

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⇒ If all elements at diagonal of L are non-zero, the system has a unique solution.

⇒ If any of the diagonal elements of L is zero, the system is inconsisten.

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3.

Let $u = [u_1 \quad u_2 \quad u_3]^T$ and $v = [v_1 \quad v_2 \quad v_3]^T$, an inner product defined on R^3 as

$$\langle u, v \rangle = u_1 v_1 + 4u_2 v_2 + 9u_3 v_3$$

<u>Property 1</u> → $\langle u, v \rangle = \langle v, u \rangle$

$$\langle v, u \rangle = v_1 u_1 + 4v_2 u_2 + 9v_3 u_3$$
$$= u_1 v_1 + 4u_2 v_2 + 9u_3 v_3$$
$$= \langle u, v \rangle$$

<u>Property-2</u> → $\langle u+v, w \rangle = \langle u, w \rangle + \langle v, w \rangle$

$$\langle u+v, w \rangle = (u_1 + v_1) w_1 + 4(u_2 + v_2) w_2 + 9(u_3 + v_3) w_3$$
$$= u_1 w_1 + v_1 w_1 + 4u_2 w_2 + 4v_2 w_2 + 9u_3 w_3 + 9v_3 w_3$$
$$= (u_1 w_1 + 4u_2 w_2 + 9u_3 w_3) + (v_1 w_1 + 4v_2 w_2 + 9v_3 w_3)$$
$$= \langle u, w \rangle + \langle v, w \rangle$$

<u>Property-3</u> $\langle cu, v \rangle = c(\langle u, v \rangle)$

$$\langle cu, v \rangle = cu_1 v_1 + 4(cu_2) v_2 + 9(cu_3) v_3$$
$$= c(u_1 v_1 + 4u_2 v_2 + 9u_3 v_3)$$
$$= c(\langle u, v \rangle)$$

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Property 4 : $<u, u> \geq 0$ and
$<u, u> = 0$ iff $u = 0$

$<u, u> = u_1 u_1 + 4 u_2 u_2 + 9 u_3 u_3$
$= u_1^2 + 4 u_2^2 + 9 u_3^2$

Hence $<u, v>$ is always positive.
and $<u, v> = 0$ if $u_1 = u_2 = u_3 = 0$.
i.e. $u = 0$.

Now, $i = 7 \bmod 2 = 1$

$\therefore$ $u = [1 \quad 0 \quad 1]^T$

$\therefore$ $<u, u> = (1)^2 + 4(0)^2 + 9(1)^2$
$= \underline{10}$ Ans

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4. Given $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$

$$\det(A - \lambda I) = \begin{vmatrix} 5-\lambda & -6 & -6 \\ -1 & 4-\lambda & 2 \\ 3 & -6 & -\lambda-4 \end{vmatrix}$$

$$= (5-\lambda)\{(\lambda-4)(\lambda+4)+12\} + 6(\lambda+4-6) - 6\{6-3(4-\lambda)\}$$

$$= -\lambda^3 + 5\lambda^2 - 8\lambda + 4$$

$\therefore$ $f(\lambda) = -\lambda^3 + 5\lambda^2 - 8\lambda + 4$

$$= -(\lambda-2)^2(\lambda-1)$$

characteristic polynomial $-(\lambda-2)^2(\lambda-1)$

$f(x) = (x-2)^2(x-1)$
$g(x) = (x-2)(x-1)$

Using Cayley Hamilton Theorem

$$g(A) = (A-2I)(I-A)$$

$$= \begin{bmatrix} 3 & -6 & -6 \\ -1 & 2 & 2 \\ 3 & -6 & -6 \end{bmatrix} \begin{bmatrix} -4 & 6 & 6 \\ 1 & -3 & -2 \\ -3 & 6 & 5 \end{bmatrix}$$

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$$g(A) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\because$ $g(A) = 0$

$\therefore$ minimal polynomial is

$$g(x) : (x-2)(1-x)$$

$\because$ algebraic multiplicity is not equal to geometric ~~diagonizable~~ multiplicity, hence A is not diagonizable.

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5. $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$\therefore A^T A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$

$\therefore \det(A^T A - \lambda I) = \begin{vmatrix} 5-\lambda & 4 \\ 4 & 5-\lambda \end{vmatrix}$

to find eigenvalues of $A^T A$, put

$\det(A^T A - \lambda I) = 0$

$(5-\lambda)^2 - 16 = 0$

$\lambda^2 - 10\lambda + 9 = 0$

$\lambda = 1, 9$

$\therefore$ eigen vector for $\lambda = 1$ is v_1 is null space of $(A-I)$

$\therefore (A - I)x = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$R_2 \to R_2 - R_1$

$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$\therefore x_2$ is free

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and $x_1 = -x_2$

$\therefore \quad v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

and eigenvector for $\lambda = 9$ is v_2 is null space of $A - 9I$.

$(A - 9I)x = 0$

$$\left[\begin{array}{cc|c} -4 & 4 & 0 \\ 4 & -4 & 0 \end{array}\right]$$

$R_2 \rightarrow R_2 + R_1$

$$\left[\begin{array}{cc|c} -4 & 4 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\therefore x_2$ is free variable.

and, $-4x_1 + 4x_2 = 0$
$x_1 = x_2$

$\therefore \quad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Normalizing v_1 and v_2 as u_1 and u_2 respectively

$\therefore \quad u_1 = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$ and $u_2 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

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$\therefore$ V is $[u_1 \quad u_2]$

$$V = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix}$$

where D is diagonal matrix having non-zero single value of A.

$$\therefore \quad D = \begin{bmatrix} \sqrt{9} & 0 \\ 0 & \sqrt{1} \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$

Σ is a 2×2 matrix, therefore

$$D = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$

U can be found using formula

$$u_i = \frac{1}{\sigma_i} A \cdot V_i$$

$\cancel{u_1 = \frac{A_1 V_1}{3}}$ ($\because$ v is an eigenvecto[r]

$\cancel{u_1 =}$

$$\therefore \quad U = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

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6. Power method is an iterative technique used to determine the dominant eigenvalue of a matrix - that is eigenvalue with largest magnitude.

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7. $v = \begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix}$

$\left(\because \text{ Student ID is } 202052307 \quad \therefore i = 7 \right)$

$T : R^3 \to R^3$ such that $T\begin{pmatrix} 1 \\ 1 \\ 7 \end{pmatrix} = 0$

Let $T : R^3 \to R^3$:

$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_1 x_1 + a_2 x_2 + a_3 x_3 \\ b_1 x_1 + b_2 x_2 + b_3 x_3 \\ c_1 x_1 + c_2 x_2 + c_3 x_3 \end{bmatrix}$$

$\because \quad T\begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\therefore \quad a_1 + a_2 + 7a_3 = 0$

$b_1 + b_2 + 7b_3 = 0$

$c_1 + c_2 + 7c_3 = 0$

If $a_1 = a_2 = 7$, then $a_3 = -2$

and let $b_1 = 3$, $b_2 = 4$, then $b_3 = -1$

and let $c_1 = 4$, $c_2 = 3$, then $c_3 = -1$

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$$\therefore \quad T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7x_1 + 7x_2 - 2x_3 \\ 3x_1 + 4x_2 - x_3 \\ 4x_1 + 3x_2 - x_3 \end{bmatrix}$$

Hence, the transformation T is defined on $R^3 \to R^3$ such that

$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7x_1 + 7x_2 - 2x_3 \\ 3x_1 + 4x_2 - x_3 \\ 4x_1 + 3x_2 - x_3 \end{bmatrix}$$

so that $T\begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$