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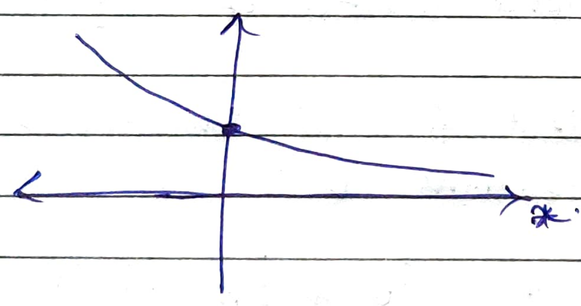
DATE: 22/3/21

End Semester Remote Examination 2021
PH100 : Mechanics & Thermodynamics

1(a) The condition for a well behaved function are.

- It must be continuous and single valued everywhere
- The partial derivatives $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$ must be continuous and single-valued everywhere.
- ψ must be normalizable. that is, ψ ~~must~~ must tend to zero as x tends to ∞, y tends to ~~zero~~ ∞ and z tends to ∞, so that $\int_{-\infty}^{\infty} |\psi|^2 d\tau$ is finite all over the space.

Yes, Ae^{-x} is not a well-behaved function as it ~~is~~ approaches infinite when $x \to -\infty$.



1(b) Given a dispersion relation $\omega(k)$, the phase velocity v_p is defined as the rate at which single plane wave moves within a wave packet.

$$v_p = \frac{\omega}{k}$$

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The group velocity V_g is defined as the rate at which the envelope or the "patter" of a wavepacket move.

$$v_g = \frac{\partial \omega}{\partial K}$$

The phase velocity can be greater than speed of light because the single monochromatic wave or constant oscillation does not carry energy.

1(c) Time - dependent form of Schrödinger's equation for a particle in three dimension

$$i\hbar\left(\frac{\partial \psi}{\partial t}\right) = -\frac{\hbar^2}{2m}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right) + U\psi$$

where U is some function of x, y, z and t.

Time independent form of Schrödinger's Equation in three dimensions is

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{2m}{\hbar^2}(E-U)\psi = 0$$

Ψ is product of a time dependent function $e^{-(E/\hbar)t}$ and a position dependent function ψ. As it happens the time variation of all wave functions of particles acted on by forces independent of time.

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1(d) For the solid, there is a restriction for the position of each atoms and so we can't set Δx as infinity. It means the momentum is finite and so there should be an energy even if the temperature is 0 K. But there is no restriction for the position of ideal gas, so it can have zero energy when temperature is 0 K.

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(e) We know that pressure $P = \rho g h$

where ρ is density and h is height.

$$\therefore \frac{dP}{dh} = -\rho g$$

negative sign because pressure decreases on going upwards in Earth's atmosphere.

Also from ideal gas equation we can derive that

$$PM = \rho RT$$

where M is molecular mass,

$$\therefore \rho = \frac{PM}{RT}$$

$$\therefore \frac{dP}{dh} = -\frac{PMg}{RT}$$

$$\int_{P_i}^{P_f} \frac{dP}{P} = \int_{h_1}^{h_2} \frac{Mg}{RT} \, dh$$

$$\ln \frac{P_f}{P_i} = -\frac{Mg}{RT}(h_2 - h_1)$$

$$\therefore P_f = P_i \, e^{-\frac{Mg}{RT}(h_2 - h_1)}$$

At sea level i.e. at $h = 0$, the pressure is $P_0 = 1\,atm$.

$$\therefore P = P_0 \, e^{-\left(\frac{Mgh}{RT}\right)}$$

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2 (10) (a) Lets figure out which of AB and AC is isothermal and which is adiabatic step.
Along the isothermal step we know,

$$P_A V_A = P_{isothermal} V_{isotherme}$$

But since the pressure and volume is known for point A, and the volumes of both point B and C are known to be same:

$$\therefore \quad P_{isothermal} = \frac{P_A V_A}{V_B} = \frac{(50\,kPa)(25\,L)}{(5\,L)} = 250\,kPa$$

Along the adiabatic step,

$$P_A V_A^{\gamma} = P_{adiabatic} V_{adiabatic}^{\gamma}$$

which can be solved for the pressure resulting from adiabatic step.

$$P_{adiabatic} = P_a \left(\frac{V_a}{V_{adiabatic}}\right)^{\gamma} = (50\,kPa)\left(\frac{25\,L}{5\,L}\right)^{4/3} = 427.5\,kPa$$

Since, we can see from the diagram that point C has a higher pressure than point B, we know AC must be adiabatic and AB must be isothermal.

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$\therefore$ Work done from A to B,

$$W_{A\to B} = -P_A V_A \ln\left(\frac{V_A}{V_B}\right)$$

$$= -(50\,kPa)(25L)(10^{-3}m^3/L)\ln\left(\frac{25L}{5L}\right) = -2012J$$

The -ve sign indicates work is done on the gas

The work done in $B\to C$ is zero as it is isochoric process.

The step that goes from $C\to A$ is adiabatic

$$W_{C\to A} = \frac{P_C V_C - P_A V_A}{\gamma - 1}$$

$$= \frac{(427.5KPA)(5L)(10^{-3}) - (50KPa)(25L)(10^{-3}m^3/L)}{\frac{4}{3}-1}$$

$$= 2662\,J$$

So, total work is

$$W_{ABC} = -2012\,J + 0J + 2662\,J = 650J$$

$\because$ ~~the~~ system returns to its original state $\Delta U = 0$, $\therefore$ heat absorbed in full cycle is 650J.

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$$\because \quad T = \frac{pV}{nR}$$

$$\therefore \quad T_A = \frac{(50\,kPa)(25L)(10^{-3}\,m^3/L)}{(1\,mo)\,(8.314)} \approx 150K$$

$$T_C = \frac{(427.5\,kPa)(5L)(10^{-3}\,m^3/L)}{(1\,mol)\,(8.314)} \approx 257K.$$

$\therefore$ for point A

$(P, V, T) : (50\,kPa, 25L, 150\,K)$

for point B

$(P, V, T) : (250\,kPa, 5L, 150\,K)$

for point C

$(P, V, T) : (427.5\,kPa, 5L, 257\,K)$

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2(i)(b)

mass of pan = 55 g

specific heat of aluminium = $0.92 \frac{J}{g^\circ C}$

mass of water = 2000 g

temp. of water and pan = $5^\circ C$

(i) power of heater = 2300 watt

∴ energy given by it in 5 minutes, Q

$$Q = 2300 \times 300$$
$$= 690000 \text{ J}$$

This heat increases the temperature of pan and water to $T^\circ C$

$$690000 = 55 \times 0.92 \times (T-5) + 2000 \times 4.2 \times (T-5)$$

$$690000 = (50.6 + 8400)(T-5)$$

$$(T-5) = 81.65^\circ C$$
$$T = 86.65^\circ C$$

The final temperature is $86.65^\circ C$

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(ii) total ~~ma~~ mass of rain water = 0.5 g/s × 300

= 150 g

Temperature of rain water = 15°C

total heat given by heater = 690000 J

Let the final temperature be T > 15°C.

∴ heat given by heater = ~~change in temp.~~ heat absorbed by pan, water and rain water

$$690000 = 55 \times 0.92 \times (T-5) + 2000 \times 4.2(T-5) + 150 \times 4.2(T-15)$$

$$690000 = (8450.6)(T-5) + 630(T-15)$$

$$690000 = 9080.6\,T - 42253 - 9450$$

$$T = \frac{741703}{9080.6}\ °C$$

$$T = 81.67°C$$

The final temperature is 81.67°C

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3.(a) $\psi = e^{\pm ikx}$

For particle in a box we have have function ψ is.

$$\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

$$\psi_n = \sqrt{\frac{2}{L}} e^{i\left(\frac{n\pi x}{L}\right)}$$

$$\therefore \psi_2 = \sqrt{\frac{2}{L}} e^{\frac{i2\pi x}{L}}$$

$$\psi_4 = \sqrt{\frac{2}{L}} e^{\frac{i4\pi x}{L}}$$

$$\psi_6 = \sqrt{\frac{2}{L}} e^{\frac{i6\pi x}{L}}$$

$$\text{Probability} = P_n = \int |\psi|^2 dx$$

$$P_n = \frac{2}{L} \int e^{(i2n\pi/L)x} dx$$

$$P = \frac{\frac{2}{L} e^{\left(\frac{2in\pi x}{L}\right)}}{2^{\frac{in\pi}{L}}}$$

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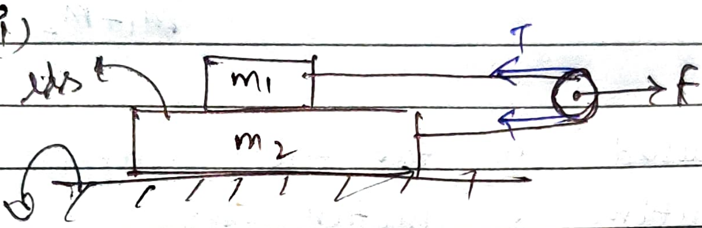
$$P_n = \frac{2}{L} \times \frac{L}{2in\pi} e^{\left(\frac{2in\pi \cdot x}{L}\right)}$$

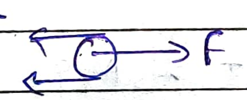
$$\boxed{\therefore \; P_n = \frac{1}{in\pi} e^{\left(\frac{2in\pi x}{L}\right)}}$$

3(b) (i)

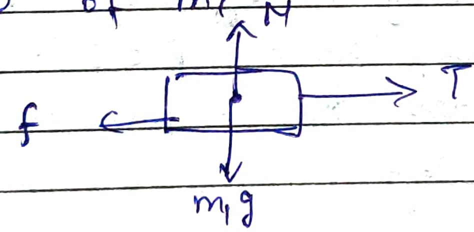


$\mu = 0$

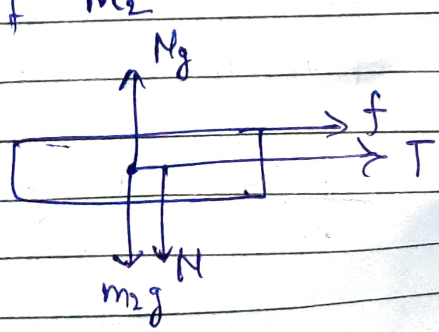
FBD of pulley



FBD of m_1



FBD of m_2



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(ii) If the blocks do not slip, then their acceleration is equal to the pulley a.

$$\therefore \quad a = \frac{F}{M_1 + M_2}$$

Hence, the acceleration is $\frac{F}{M_1 + M_2}$

(iii) Based on the force diagram, the equations of motion are

$$M_1 \ddot{x}_1 = T - \mu_s M_1 g$$

$$M_2 \ddot{x}_2 = T + \mu_s M_1 g$$

Because the pulley is effectively massless, $M_p a_p \approx 0$ and hence $F = 2T$. Thus alone provides the expression for the two pulleys

$$\therefore \quad a_1 = \frac{F}{2M_1} - \mu_s g$$

acceleration of m_1 is a_1

$$a_2 = \frac{F}{2M_2} + \mu_s \frac{M_1}{M_2} g$$

acceleration of m_2 is a_2.

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(d) ~~There are at~~

$$F > 2M_1 g(\mu_s + 1)$$

$$F > (M_1 + M_2)a$$

$$F > 2\mu_s M_1 g\left(\frac{M_1 + M_2}{M_2 - M_1}\right)$$