PHOTOELASTICITY

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"EXPERIMENTS IN SOLID STATE PHYSICS"

by

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3.6 PHOTO-ELASTICITY :

Introduction :

The photoelastic effect is the effect of stress on the optical behaviour of a crystal.

In cubic crystals, the refractive index is isotropic. In crystals with lower symmetry, it is not. In tetragonal, trigonal and hexagonal crystals, there are two principal refractive indices; these are called uniaxial crystals. In biaxial crystals, there are three refractive indices. Both, uniaxial and biaxial crystals are said be birefringent. If n_a, n_b and n_c are refractive indices in the three principal directions, the birefringence is $(n_a - n_c)$ in uniaxial crystals and $(n_a - n_b)$, $(n_b - n_c)$ and $(n_c - n_a)$ in biaxial crystals. Thus, sodium chloride (NaCl) is isotropic, quartz (SiO_2) is uniaxial and aragonite ($CaCO_3$) is biaxial.

When a crystal is stressed in one direction (uniaxial stress), the refractive index in that direction changes leaving the refractive indices in the perpendicular directions unchanged (or nearly so). Thus, an isotropic crystal becomes a uniaxial crystal and develops birefringence. A solid which has natural birefringence develops additional birefringence under the effect of stress. This is called piezobirefringence or photoelasticity.

Obviously, photelasticity relates the elastic properties of a crystal with its optical properties. The stress tensor (σ_{ij}) has six components. If the refractive index ellipsoid is inclined to the crystal axes, then the refractive indices also have six components. The most general relation between the refractive indices and the stress components is given by (see Nye, Krishnan).

$$\Delta B_{ij} = \pi_{ijkl}\,\sigma_{kl} \quad (i,j,k,l = 1,2,3) \qquad .. \qquad (3.13)$$

Here, ΔB_{ij} is the change in the coefficient in the indicatrix equation,

$$B_{ij}\,x_i x_j = 1 \quad \text{or}$$

$$\frac{1}{n^2_{ij}}\,x_i x_j = 1 \qquad .. \qquad (3.14)$$

In the matrix notation Equation (3.13) becomes

$$\Delta B_m = \pi_{mn}\,\sigma_n \qquad .. \qquad (3.15)$$

or

$$\Delta B_i = \pi_{ij}\,\sigma_i$$

The indices m, n or i and j take values 1 to 6. π_{ij} are called the photo-elastic constants. For triclinic crystals, the number of photoelastic constants is 36. This number reduces with higher symmetry. Cubic crystals with O, O_h and O_d symmetry (space groups 43m, 432 and m3m) have only three constants viz., π_{11}, π_{12} and π_{44}. The birefringence produced in these crystals on the application of stress in specific directions is given in Table 3.10.

Table 3.10
Birefringence produced in crystals of O, O_h and O_d classes

Orientation	Direction of stress	Direction of observation	Birefringence $(n_1 - n_2)$
I	[100]	[010]	$-1/2(n_o)^3 \sigma(\pi_{11} - \pi_{12})$
II	[111]	$\perp$ to [111]	$-1/2\,(n_o)^3\, \sigma\, \pi_{44}$

n_1 and n_2 refer to refractive indices parallel and perpendicular to the stress direction respectively. n_o is the refractive index of the crystal under zero stress.

It can be seen from Table 3.10 that by making measurements in two directions, we can get the constant π_{44} and the difference $(\pi_{11} - \pi_{12})$. To obtain π_{11} and π_{12} separately, additional experiments are necessary which are beyond our scope. Value of $(\pi_{11} - \pi_{12})$ and π_{44} for some crystals are given in Table 3.11.

Table 3.11
Photoelastic constant $(\pi_{11} - \pi_{12})$ and π_{44}
(in units of 10^{-12} mt^2/Nt) for some cubic crystals

Crystal	$(\pi_{11} - \pi_{12})$	π_{44}
Sodium chloride	- 1.21	- 0.85
Potassium chloride	1.70	- 4.31
Calcium fluoride	- 1.45	0.70

Objective :

To determine the photoelastic constants of a cubic crystal like sodium chloride.

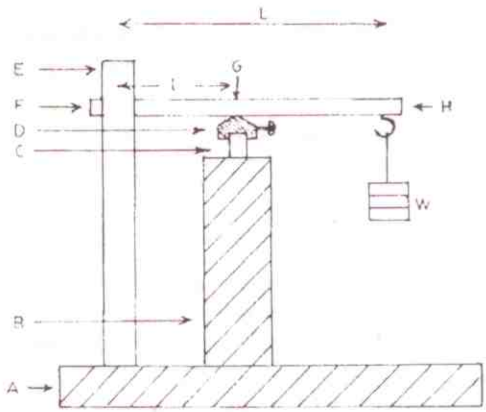

Figure 3.14 Crystal holder and stressing device for measurement of photoelasticity

Procedure :

The entire arrangement is shown in Figures 3.15a,b. In Figure 3.15a, S is the source of light, SL a slit and L_1 and L_2 are lenses. P is the polariser (polaroid or Nicol), C is the crystal holder-cum-stressing device and BC is the Babinet compensator.

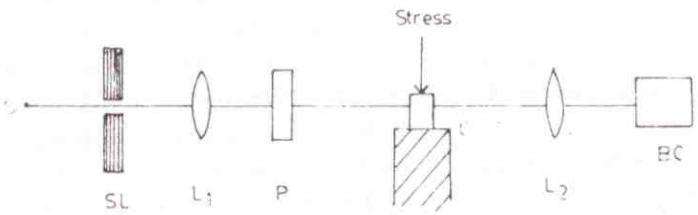

Figure 3.15 (a) Experimental set-up for photo elasticity (line diagram)

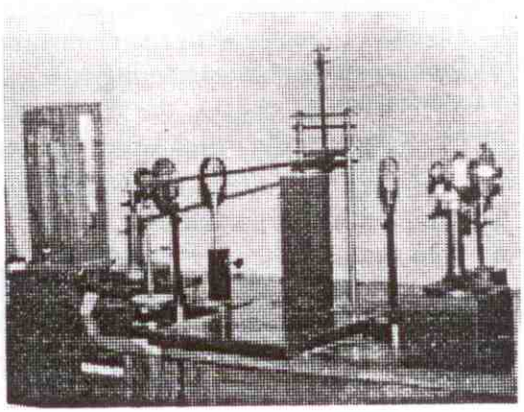

Figure 3.15 (b) Experimental set-up for photoelasticity (photograph)

Equipment :

The special accessories needed for this experiment are a Babinet compensator and a suitable platform to hold the crystal and to stress it. Besides, optical accessories like a sodium vapour lamp, polarisers, slits and lenses are required.

The Babinet compensator is an instrument useful in the measurement of birefringence. It consists of two wedgeshaped quartz prisms with a small angle. The two prisms are cut with their edges parallel and perpendicular to the optic axis (Fig. 3.13a). one wedge is moveable. When plane polarised light is incident normally on the wedge pair, with the vibration direction making an angle with the optic axis of the wedge, it splits into two waves, the O and E vibrations. These waves travel through the wedge in the same direction but with an optical path difference due to the difference in the refractive indices. This path difference varies from point to point along the length of the wedge. As a result, there is an interference fringe pattern in the field of view (Fig. 3.13b) when viewed through an eyepiece. Such a wedgepair constitutes the Babinet compensator. The path difference in the rays interfering at two dark fringes is λ. If there is relative displacement of the wedges, the fringe system moves. Thus, the compensator can be calibrated, the fringe width β being a measure of the path difference.

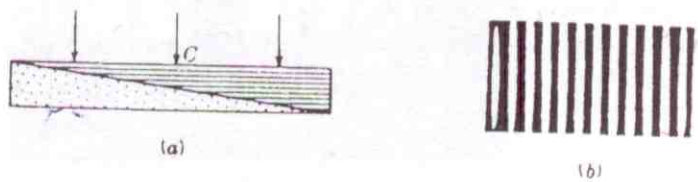

Figure 3.13 (a) The wedge in the Babinet compensator and (b) the fringe pattern

The crystal holder and stressing device is shown in Figure 3.14. It consists of a wooden base A which supports a wooden pedestal B. The crystal C is placed over this pedestal. The crystal is topped by a metal cap D which has a screw on a side. The base A also supports a steel frame E, which holds an iron arm FGH. The arm has its fulcrum at F. On the lower side of this arm, there is a notch at G. When the arm is in a horizontal position, the top of D just fits into the notch in the arm. At the H end there is a hook which carries the weights W (in steps of 1/2 kg). If the lengths FG and FH are l and L, it can be seen that the effective load on the crystal in WL/l.

The set-up is arranged without the crystal in position. A set of interference fringes are seen through the eyepiece of the Babinet compensator. The micrometer screw of the Babinet compensator is adjusted so that a dark fringe coincides with the vertical cross-wire. The screw is then manipulated such that two or three dark fringes cross the vertical cross wire. The micrometer readings are noted and fringewidth (β) is calculated.

The screw is again adjusted to bring the cross-wire in coincidence with a dark fringe. The crystal is now brought in position. The arm FGH is placed over D and the load W is attached. The screw on the side of D is gently released. The fringe pattern shows a shift. The screw of the Babinet compensator is adjusted so that the cross-wire coincides with the fringe in its new position. Thus, the shift (S) due to the load is measured. This shift is plotted against the applied load (Fig. 3.16) and the slope (S/W) is determined.

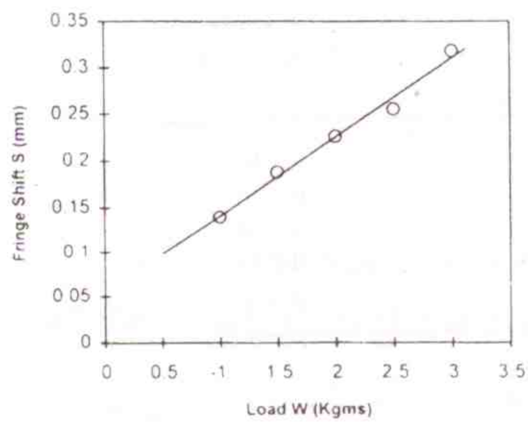

Figure 3.16 Plot of load and fringe-shift for NaCl crystal

If the crystal specimen is sodium chloride with orientation I (in Table 3.10), then the working equation is,

$$S\lambda/\beta = -(1/2)(n_o)^3 t(\pi_{11} - \pi_{12})\sigma \quad .. \quad (3.16)$$

where $$\sigma = Wg(L/l)(1/bt) \quad .. \quad (3.17)$$

Here, b and t are the width and thickness of the crystal. For the determination of π_{44}, a crystal cut to reveal the (111) faces has to be employed. Sample observations for an NaCl crystal in orientation I are given in Tables 3.12 and 3.13

Observations :

Table 3.12
Measurements to determine the width of fringes in Babinet compensator

Fringe No.	Reading of micrometer (mm)	Average fringe width (β)
1	9.680	
2	7.450	2.26 mm
3	5.210	
4	2.990	
5	0.375	

Table 3.13
Fringe shift for different applied loads

Load applied (W in kg)	Fringe shift (S) mm
1.0	0.140
1.5	0.188
2.0	0.226
2.5	0.255
3.0	0.318

Dimensions of the NaCl crystal = $8 \times 4.7 \times 5$ mm^3
L = 53 cm; l = 9 cm
Wavelength (λ) of sodium D line = 5893 A
Refractive index (n_o) of NaCl = 1.544
S/W (from Figure 3.16) = 1×10^{-5} cm/gm

$$\pi_{11} - \pi_{12} = - \frac{2\lambda l b}{(n_o)^3 L\beta g} (S/W)$$

$$= - 1.15 \times 10^{-13} \text{ cm}^2/\text{dyne}$$
$$(\text{or } - 1.15 \times 10^{-12} \text{ mt}^2/\text{Nt})$$