Related Topics

Law of inductance, Lenz's law, self-inductance, solenoids, transformer, coupled oscillatory circuit, resonance, damped oscillation, logarithmic decrement

Principle

A square wave voltage of low frequency is applied to an oscillating circuit comprising coil and capacitor of known capacitance. The sudden change of voltage at the both edges of the square wave signal induces a magnetic field in the primary coil which then couples into the solenoid and triggers a free damped oscillation in the secondary circuit. For different solenoids the natural frequencies of the circuits are measured and therewith the solenoids' inductances are calculated.

Material

1	Digital frequency generator	13654-99	1	Adapter, BNC- socket 4 mm	07542-26
1	Induction coils, set	11006-88	1	Measuring tape, $l = 2$ m	09963-00
1	Induction Coil, 75 turns, $d = 25$ mm	11006-07	1	Vernier caliper	03010-00
1	Coil, 1200 turns	06515-01	1	Connecting cord, red, $l = 250$ mm	07360-01
1	Capacitor, 0.47 µF, 100 V	39105-20	1	Connecting cord, blue, $l = 250$ mm	07360-04
1	Oscilloscope, 30 MHz, 2 channels	11459-95	2	Connecting cord, red, $l = 500$ mm	07361-01
1	Connection box	06030-23	2	Connecting cord, blue, $l = 500$ mm	07361-04

Figure 1: Experimental set-up

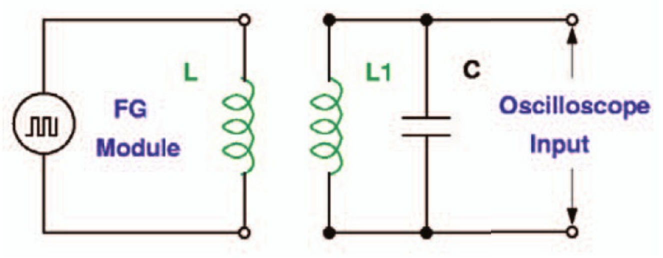

Figure 2: Schematic circuit of the set-up. **L** is the primary coil with 1200 turns, **L1** labels the solenoid in the secondary circuit.

Tasks

The natural frequency of the induced oscillation has to be measured each induction coil. From the natural frequency and the known capacitance calculate the inductances of the coils and determine the relationships between

1. inductance and number of turns,
2. inductance and solenoid length
3. as well as inductance and solenoid radius.

Set-up

Perform the experimental set-up according to Figs. 1 and 2. Set the digital function generator to an amplitude of 20 V, frequency of 500 Hz or lower and to the square wave signal with signal-type *out*. The solenoid has to be aligned carefully with the primary coil so that the magnetic field can couple efficiently from the primary coil into the solenoid. The distance between the two coils should be maximized so that the effect of the excitation coil on the resonant frequency can be disregarded. there should be no iron components in the vicinity of the coils.

The tolerance of the oscilloscope time-base is given as 4 %. If a higher degree of accuracy is required, the time-base can be calibrated for all measuring ranges with the digital function generator prior to these experiments.

Procedure

For each solenoid the settings of the oscilloscope have to be adjusted in such manner, that one damped oscillation can be fully seen on the screen and the peaks are clearly distinguishable as shown in fig. 3. The time between two peaks of the damped oscillation is the actual period length of the natural frequency and can be easily read from the screen by shifting the wave into appropriate positions. The lengths of the solenoids have to be measured with the measuring tape, the radius (i.e. the diameter) has

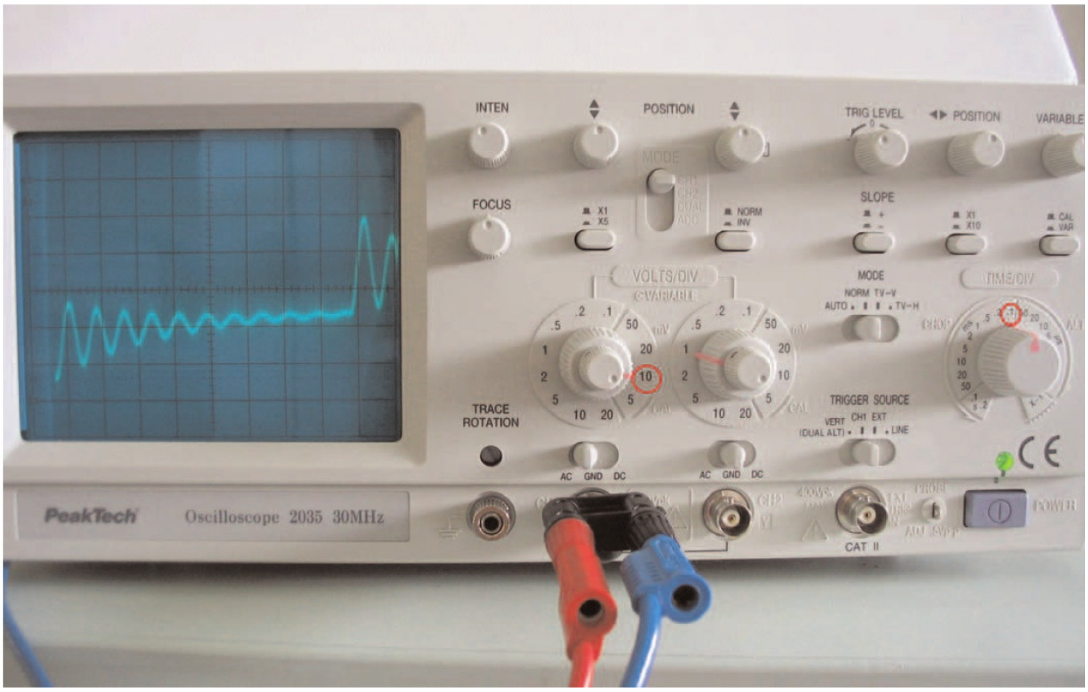

Figure 3: Damped oscillation on the oscilloscope. The actual settings can vary for differing coils.

to be determined with the vernier caliper and the numbers of turns are given.

Theory

If a current of strength I flows through a cylindrical coil (a.k.a. solenoid) of length l, cross sectional area $A = \pi \cdot r^2$, and number of turns N, a magnetic field is set up in the coil. When $l \gg r$ the magnetic field is uniform and the field strength H is given by

$$H = I \cdot \frac{N}{l} \quad (1)$$

The magnetic flux Φ is given by

$$\Phi = \mu_0 \, \mu \, \cdot H \cdot A \quad (2)$$

where μ_0 is the magnetic field constant and μ the absolute permeability oft he surrounding medium.
When this flux changes it induces a voltage between the ends of the coil,

$$\begin{aligned} U_{ind} &= -N \cdot \frac{d\Phi}{dt} \\ &= -N \cdot \mu_0 \, \mu \, \cdot A \cdot \frac{dH}{dt} \\ &= -L \cdot \frac{dI}{dt} \end{aligned} \quad (3)$$

where

$$L = \mu_0 \, \mu \, \cdot \pi \cdot \frac{N^2 \cdot r^2}{l} \quad (4)$$

ist he coefficient of self-induction (inductance) of the coil.
In practice the inductance of coils with $l > r$ can be calculated with greater accuracy by an approximation formula

$$L = 2.1 \cdot 10^{-6} \cdot N^2 \cdot r \cdot \left(\frac{r}{l}\right)^{3/4} \quad (5)$$

for $0 < \frac{r}{l} < 1$.

In the experiment the inductance of various coils is calculated from the natural frequency of an oscillating circuit:

$$\omega_0 = \frac{1}{\sqrt{LC_{tot}}}$$

$C_{tot} = C + C_i$ is the sum of the known capacitor and the input capacitance $C_i \approx 30 \text{ pF}$ of the oscilloscope, which exercises a damping effect on the oscillatory circuit and causes a negligible shift (approx. 1 %) in the resonance frequency.
The inductance is therefore represented by

$$L = (4\pi^2 \nu_0^2 C)^{-1} \quad (6)$$

Tab. 1: Coil data and calculated inductances (see eq. 5).

Coil N	N	$2r$ in mm	l in mm	L_{calc} in µH
1	300	41	160	830
2	300	33	160	568
3	300	26	160	374
4	200	41	105	506
5	100	41	50	221
6	150	26	160	93
7	75	26	160	23

Tab. 2: Measured natural frequencies and inductances from eqs. (5) and (6).

Coil No.	T in µs	ν in kHz	L_{exp} in µH	L_{calc} in µH
1	130	7.7	911	830
2	100	10	539	568
3	80	12.5	345	374
4	100	10	539	506
5	64	15.6	221	221
6	40	25	86	93
7	20	50	22	23

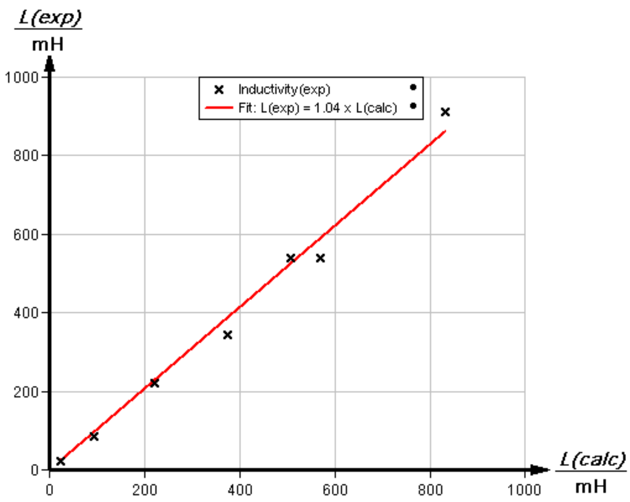

Fig. 4: Comparison of inductances L_{calc} with L_{exp} calculated from equations (5) and (6) respectively.

where $\nu_0 = \frac{\omega_0}{2\pi} = \frac{1}{T \cdot 2\pi}$ is the natural frequency with T being the period of the oscillation.

Evaluation and results

In the following the evaluation of the obtained values is described with the help of example values. Your results may vary from those presented here.

From equation (5) we obtain the theoretical inductance values of the used coils. These are listed in table 1.

The following coils provide the relationships between inductance and radius, length and number of turns that we are investigating:

1. no. 3, 6, 7
2. no. 1, 4, 5
3. no. 1, 2, 3

As a difference in length also means a difference in the number of turns, the relationship between inductance and number of turns found in task 1 must also be used to solve task 2.

In table 2 the measured natural frequencies are shown as well as the calculated values for L from relations (5) and (6) respectively. The experimental values L_{exp} are in good agreement with the theoretically expected values L_{calc} with a standard deviation of approximately 6%. The coils with higher inductances show rather considerable deviations. As the inductance is proportional to the square of the oscillation's period length, this scatter is hard to reduce because especially for long-period oscillations the oscilloscope's accuracy is limited.

1. Task: Determine the coils' relationships between inductance and number of turns.

To determine the relationship between inductance and number of turns consider coils with identical radius and length but different number of turns. The Coils no. 3, 6 and 7 meet these requirements.

In figure 5 the corresponding inductances are plotted in dependence of the number of turns.

Fitting the expression $L_{exp} = a \cdot n^b$ to the experimental values yields

$$b = 2.0003$$

which is in excellent agreement with the theoretical value $b_{theo} = 2$ (see eq. 5).

2. Task: Determine the coils' relationships between inductance and length of coil.

To determine the relationship between inductance and length of coil consider coils with identical radius but different lengths. The Coils no. 1, 4 and 5 meet these requirements. As the relation between inductance and number of turns is already known, the inductances can be normalized by the number of turns. Therefore consider the relationship between inductance normalized by turn number squared and the length of coil. In figure 6 the corresponding values are plotted in dependence of the coil length.

Fitting the expression ${L_{exp}}/{n^2} = a \cdot l^b$ to the experimental values yields

$$b = -0.67$$

which is in fair agreement with the theoretical value $b_{theo} = -0.75$ (see eq. 5).

3. Task: Determine the coils' relationships between inductance and radius of the coils.

To determine the relationship between inductance and radius of coil consider coils with identical lengths but different radii. The Coils no. 1, 2 and 3 meet these requirements. As the relation between inductance and number of turns is already known, the inductances can be normalized by the number of turns. Therefore consider the relationship between inductance normalized by turn number squared and the radius of the coils. In figure 7 the corresponding values are plotted in dependence of the coil radius.

Fitting the expression ${L_{exp}}/{n^2} = a \cdot l^b$ to the experimental values yields

$$b = 1.82$$

which is in good agreement with the theoretical value $b_{theo} = 1.75$ (see eq. 5).

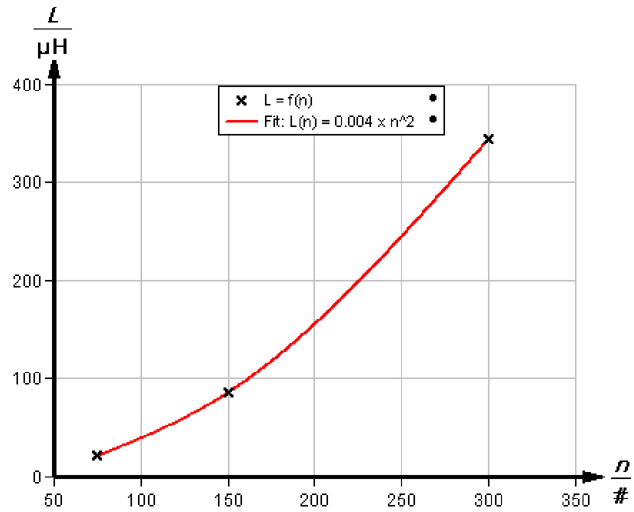

Fig. 5: Relation between inductance and number of turns.

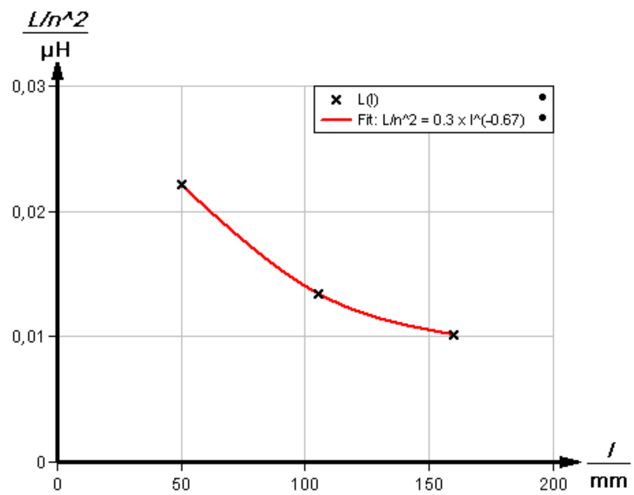

Fig.6: Relation between inductance and length of coil.

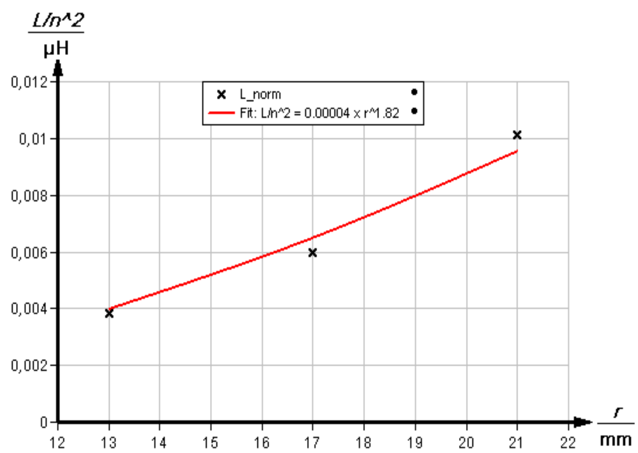

Fig. 7: Relation between inductance and radius of the coil.

