# Diffraction Grating

**Apparatus**

Spectrometer, transmission diffraction grating, sodium vapour lamp, mercury vapour lamp, power supply for spectral lamps, magnifying glass.

**Purpose of the Experiment**

(i) To calibrate the grating spectrometer using the known source (Hg source) of light and to calculate the grating constant.

(ii) Using the same grating, to calculate the wavelength of sodium doublet lines.

**Basic Methodology**

Light from a mercury source is incident normally on a diffraction grating mounted on a spectrometer. The diffraction angle of the diffracted light is measured for each spectral line of the Hg-source. Likewise for sodium source, the diffraction angle and angular separation $\Delta\theta$of the sodium doublet is measured.

## I Theory

A diffraction grating is a precise optical device for the study of spectra and is widely used in a large number of fields from Astronomy to Engineering, wherever there is need for detection of the presence of atomic elements.

A diffraction grating can be simply thought of as a set of identical and equally spaced slits separated by opaque strips. In practice, gratings are made by ruling fine grooves with a diamond point either on a plane glass surface to produce a transmission grating or on a metal mirror to produce a reflection grating. In a transmission grating, the grooves scatter light and so are opaque while the unruled surfaces transmit and act like slits. Typically a high quality grating (used for studying spectra in the visible range) has about 15,000 grooves per inch, which gives a slit spacing of the order of a micron.

The chief requirement of a good grating is that the lines be equally spaced over the width of the ruled surface, which can vary from 1 to 2.5 cm. After each groove has been ruled, the machine lifts the diamond point and moves the grating forward by a small rotation of a screw. For rulings of equal spacing, the screw must have a constant pitch. A typical groove profile is the triangular blaze profile shown in Fig.**??**. The angle $\phi$ is called the blaze angle. Large scale replication gratings can be made using a cast of the ruled surface with some transparent material. Replication gratings give satisfactory performance where very high resolving power is not required.

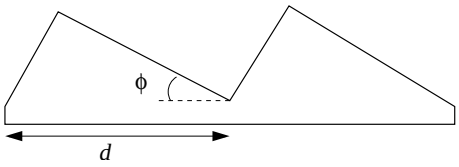

Figure 1: Groove profile of a standard transmission grating

**Theory of Diffraction Grating**

The phenomenon of diffraction is a typical consequence of the propagation of waves around obstructions. When a wave front is incident on a grating surface, light is transmitted through the slits and obstructed by the opaque portions. The secondary waves from the positions of the slit interfere with one another, similar to the interference of waves in Youngs experiment.

If the spacing between the lines is of the order of the wavelength of light then an appreciable deviation of the light is produced.

Consider the diffraction pattern produced by $N$ parallel slits, each of width $b$, separated by a distance $d$. (See Fig.2).

The electric field at any arbitrary point $P$ is a sum of $N$ terms (recall the derivation for the double slit),

$$\begin{aligned} E &= A\frac{\sin\beta}{\beta}\cos(\omega t-\beta)+A\frac{\sin\beta}{\beta}\cos(\omega t-\beta-\phi_1) \\ &\quad +...+A\frac{\sin\beta}{\beta}\cos(\omega t-\beta-(N-1)\phi_1) \qquad (1) \\ &= A\frac{\sin\beta}{\beta}\frac{\sin N\gamma}{\sin\gamma}\cos(\omega t-\beta-1/2(N-1)\phi_1) \quad (2) \end{aligned}$$

where $\beta=\frac{\pi b\sin\theta}{\lambda}$, $\gamma=\frac{\phi_1}{2}=\frac{\pi}{\lambda}d\sin\theta$ and $\phi_1$ is the phase difference between the light rays emanating from successive slits.

The corresponding intensity distribution will be

$$I=I_0\frac{\sin^2\beta}{\beta^2}\frac{\sin^2 N\gamma}{\sin^2\gamma}. \qquad (3)$$

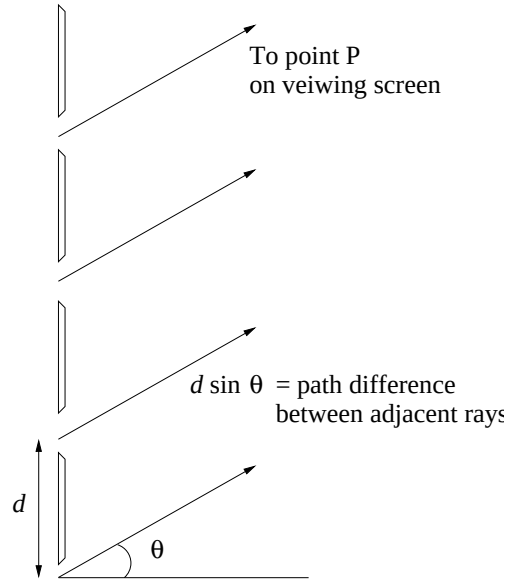


Figure 2:

The intensity distribution is a product of two terms, the first term $(\sin^2\beta)/\beta^2$ represents the diffraction pattern produced by a single slit and the second term $(\sin^2 N\gamma)/(\sin^2\gamma)$ represents the interference pattern produced by $N$ equally spaced slits. For $N=1$, eq. (3) reduces to the single slit diffraction pattern and for $N=2$, to the double slit diffraction pattern.

**Principal Maxima:** When the value of $N$ is very large, one obtains intense maxima at $\gamma=m\pi$ i.e., when

$$\boxed{d\sin\theta=m\lambda, \qquad m=0,1,2,... \text{ (maxima)}} \qquad (4)$$

This can be seen by noting that

$$\lim_{\gamma\to m\pi}\frac{\sin N\gamma}{\sin\gamma}=\lim_{\gamma\to m\pi}\frac{N\cos N\gamma}{\cos\gamma}=\pm N \qquad (5)$$

Thus, the resultant amplitude will be

$$E(\theta)=NA\frac{\sin\beta}{\sin\beta} \qquad (6)$$

and the corresponding intensity distributions are given by

$$I=N^2I_0\frac{\sin^2\beta}{\beta}, \qquad \text{where}\beta=\frac{\pi bm}{d}. \qquad (7)$$

Such maxima are known as principal maxima. Physically, at these maxima the fields produced by each of the slits are in phase and, therefore, they add up and the resultant field is $N$ times the field produced by each of the slits.

**Minima and Secondary Maxima:** To find the minima of the function $\frac{\sin^2 N\gamma}{\sin^2\gamma}$, we note that the numerator becomes zero at $N\gamma=0,\pi,2\pi$ or in general, $p\pi$ where $p$ is an integer. In

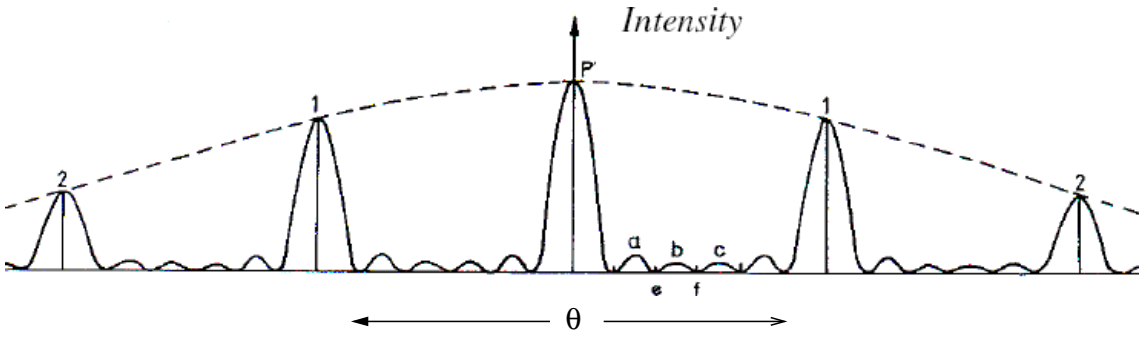


Figure 3:

the special case when $p = 0, N, 2N, \gamma$ will be $0, \pi, 2\pi$, For these values the denominator will also vanish, and we have the principal maxima described above. The other values of $p$ give zero intensity since for these the denominator does not vanish at the same time. Hence the condition for minima is $\gamma = p\pi/N$, excluding those values of $p$ for which $p = mN$, $m$ being the order. These values of $\gamma$ correspond to

$$d\sin\theta = \frac{\lambda}{N}, \frac{2\lambda}{N}, \frac{3\lambda}{N}, ... \frac{(N-1)\lambda}{N}, \frac{(N+1)\lambda}{N}, \tag{8}$$

omitting the values $0, N\lambda/N, 2N\lambda/N...$, for which $d\sin\theta = m\lambda$ corresponding to principal maxima. Thus, between two principal maxima we have $(N-1)$ minima. Between two such consecutive minima, the intensity has to have a maximum; these maxima are known as secondary maxima. These are of much lower intensity than principal maxima. The principal maxima are what correspond to the spectral lines visible through the grating.

The intensity distribution on the screen is shown in Fig.3. P corresponds to the position of the central maxima and 1, 2 etc. on the two sides of P represents the 1st, 2nd etc. principal maxima. a, b, c, etc. are secondary maxima and e, f etc. are the secondary minima. The intensity as well as the angular spacing of the secondary maxima and minima are so small in comparison to the principal maxima that they cannot be observed. This results in near uniform darkness between any two principal maxima.

**Sodium D Lines:**

The sodium doublet is responsible for the bright yellow light from a sodium lamp. The doublet arises from the 3p→3s transition in the sodium atom. The 3p level splits into two closely spaced levels with an energy spacing of 0.0021 eV. The splitting occurs due to the spin orbit effect. This can be crudely thought of as arising due to the internal magnetic field produced by the electrons circulation around the nucleus and the splitting takes place analogous to the Zeeman effect. Fig. 4 shows the 3p and 3s levels, their splitting and the radiative transition that produces the sodium doublets or D lines.

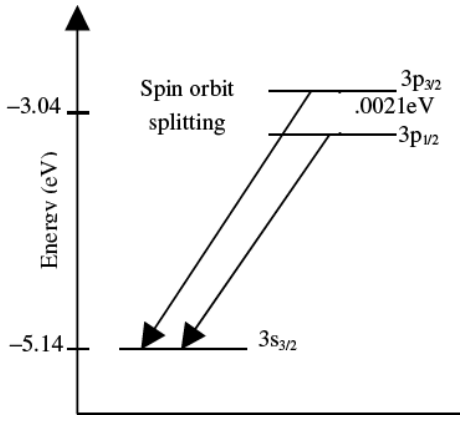


Figure 4:

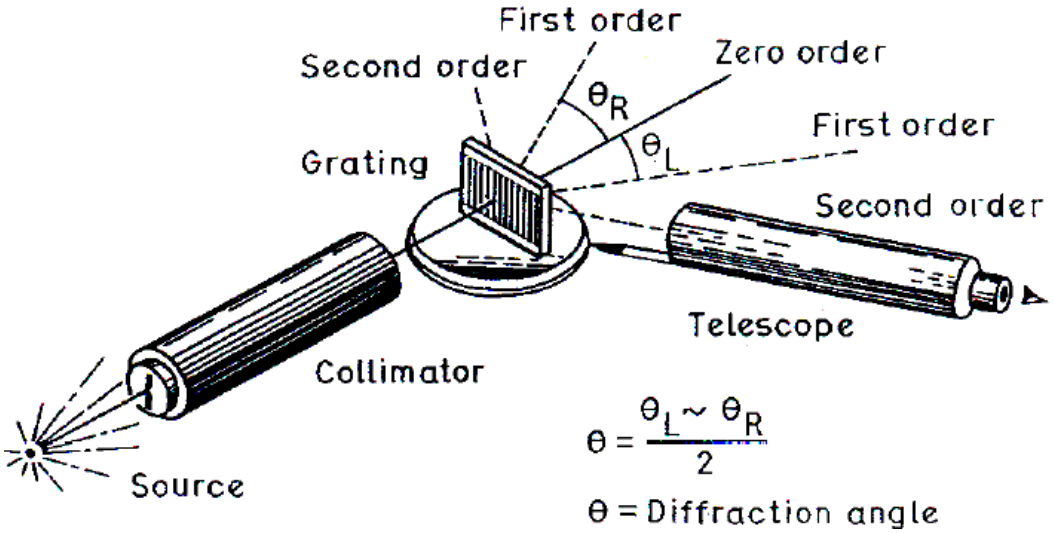


Figure 5:

## II Setup and Procedure

PART A: Calibration of the Diffraction Grating

1. Adjust telescope for parallel rays. This is done by focussing the telescope on an object at infinity, or some very distant object.

2. Level the spectrometer and prism table on which grating is mounted using a spirit level. Fig. 5 schematically shows the arrangement of the grating and the spectrometer.

3. Switch on the power supply for the spectral lamp. Wait for it to warm up before taking readings.

4. Mount the grating for normal incidence:

   (a) Mount the grating on the prism table, approximately normal to the collimator. Bring the slit into view through the telescope and note the reading of one of the vernier windows.

   (b) Add 90°to the reading obtained in step (a) and move the telescope to this position. The telescope is now perpendicular to the collimator.

   (c) Rotate the prism table till the slit is visible at the center of the telescope cross wires. At this position, light from the collimator is incident at 45°to the grating. Note down this reading.

   (d) Now subtract 45° from the reading in step (c) and rotate the prism table to this position. Now the light is incident normally to the grating surface.

   (e) Fix the prism table in this position with the tightening screw.

5. With the Mercury vapour source, observe the first order spectrum on one side.

6. Rotate the telescope so that cross-wire coincides with the extreme spectral line one one side and note down the angular position. Proceed across the spectrum all the way to the

| No. | Line | Wavelength (Å) |
|---|---|---|
| 1 | Violet 1 | 4047 |
| 2 | Violet 2 | 4078 |
| 3 | Blue | 4358 |
| 4 | Blue-green | 4916 |
| 5 | Green | 5461 |
| 6 | Yellow 1 | 5770 |
| 7 | Yellow 1 | 5791 |

Table 1: Wavelengths of main spectral lines of Mercury

other side of the zero order and note the angle of diffraction of each line. The diffraction angle is the difference between LHS and RHS observations divided by 2, for a particular spectral line. (See Fig. 5).

7. Plot a graph of $\lambda$ vs $\sin\theta$ and obtain the grating constant $1/d$ from the slope:

$$d\sin\theta = \lambda.$$

The wavelengths of the main spectral lines of Hg in the visible region are given in Table 1

PART B: Wavelengths of sodium D1 and D2 lines:

1. Repeat step 4 with the sodium source. You must observe the *second* order spectrum of sodium, in which the separation of the two yellow lines will be visible. Measure the angular position $\theta_L$ of the first yellow line (D1) on the left side. Use the micrometer screw to turn the telescope to align the crosswire at the second yellow line (D2) and read its angular position $\theta_L$.

2. Likewise measure $\theta_R$ on the RHS for second order D1 and D2 lines.

3. Calculate the wavelengths from $\sin\theta$ values using the grating constant obtained in Part A.

***Precautions:***

1. *Accustom yourself to the use of the circular vernier scale before taking readings. Note down the least count of the scale.*

2. *The experiment should be performed in a dark room.*

3. *Micrometer screw should be used for fine adjustment of the telescope. For fine adjustment the telescope should be first locked by means of the head screw.*

4. *The directions of rotation of the telescope micrometer screw should be maintained the same. Otherwise the play in the micrometer spindle will lead to backlash errors.*

5. *The spectral lamps should attain their full illuminating power after being warmed up for about 5 minutes, so the observations should be taken after 5 minutes.*

6. *One of the essential precautions for the success of this experiment is to set the grating normal to the incident rays. Small variation in the angle of incidence causes large error in the angle of diffraction. If the exact normality is not achieved, one finds that the angles of diffraction measured on the left and on the right are not exactly equal.*

7. *Read both the vernier scales to eliminate any errors due to non-coincidence of the centre of the circular scale with the axis of rotation of the telescope or table.*

## III Exercises and Viva Questions

1. What is a diffraction grating? How are gratings made? Name three different types of gratings.

2. Can a grating be used for studying spectra in the UV or infrared region? If so, what should be its characteristic? Can a prism be so used? What are the advantages of a grating over a prism?

3. The dispersion of a grating is defined as $D = \Delta\theta/\Delta\lambda$ where $\Delta\theta$ the angular separation of the principal maxima of two lines whose wavelengths differ by $\Delta\lambda$. Show that the dispersion of a grating is $D = m/(d\cos\theta)$ at the $m$th order. Calculate $D$ for the sodium doublet at the first order for your experiment.

4. The resolving power of a grating is defined as $R = \lambda_{avg}/\Delta\lambda$ where $\lambda_{avg}$ is the mean wavelength and $\Delta\lambda$, the difference in the wavelengths of two spectral lines which can just be resolved into two lines. It can be shown that $R = Nm$, where $N$ is the total number of rulings on the grating and $m$ is the order at which the two spectral lines can be resolved. Calculate the number of rulings required to resolve the sodium doublet at the first order.

5. Use the Bohr model for the frequency of light emitted in atomic transitions to calculate the wavelengths for the sodium doublet, using Fig. 4.

6. In the Hg spectrum, which lines are prominent and which are weak? What could be the reason for variation in intensities of spectral lines?

7. What would be the advantages and disadvantages of using the second order spectra in this experiment?

8. What is the mechanism by which the emission spectrum is produced in the spectral lamps (Na or Hg)? Look up Ref. 2 for details.

9. What will happen if the incident light does not fall normally on the grating? Show that if $\psi$ is the angle of incidence with respect to the normal to the grating, then the principal maxima occur at angles $\theta$ (w.r.t. the normal) such that $d(\sin\psi + \sin\theta) = m\lambda$. (This is problem 37.41 of Ref. 3).

10. Give examples of uses of gratings in Astronomy, Physics and Engineering.

## References

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3. *Fundamental of Physics*, D. Halliday, R. Resnick and J.A Walker, 6th ed., John Wiley & Sons, New York, 2001.

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