PHYS2090 OPTICAL PHYSICS
Laboratory – Fresnel Zone Plate

References

Hecht *Optics*, Addison-Wesley
Smith & King, *Optics and Photonics: An Introduction*, Wiley
Higbie, *Fresnel Zone Plate: Anomalous foci*, American Journal of Physics **44**(10), pp 929-930, 1976

Caution
This experiment uses a helium-neon laser to perform measurements. Although the laser is low power, care should be exercised in its use. Do not look directly into the beam. Keep the laser pointing along the rail and ensure that a beam block is used to stop the laser beam. Also beware of reflections from glass and other surfaces. Talk to a tutor about laser safety before using the laser.

You must sign the laser safety register before starting this experiment.

1. Introduction

The Huygens-Fresnel Principle states that
"every unobstructed point of a wavefront, at a given instant, serves as a source of spherical secondary wavelets (with the same frequency as that of the primary wave). The amplitude of the optical field at any point beyond is the superposition of all these wavelets (considering their amplitudes and relative phases."

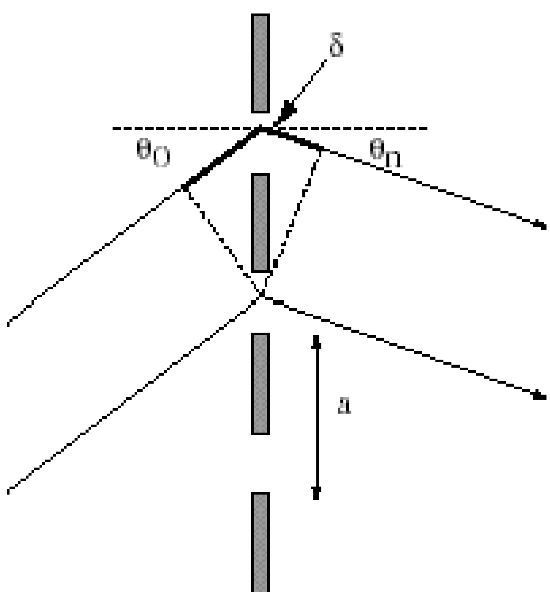

Figure 1. Interference between two rays.

Fresnel used this principle to describe near-field diffraction. He divided a spherical wavefront into many zones in which adjacent zones are out of phase and tend to cancel. By blocking alternate zones, it is possible to obtain a large increase in irradiance as a given point. Such a device is called a zone plate and will be studied in this experiment.

2. Theory

To understand a zone plate, we need to first consider a diffraction grating. Consider two parallel rays incident at an angle θ_0 and striking two adjacent slits, of separation, a, at the same relative location, as shown in figure 1. When the path length difference between the emerging rays is an integral number of wavelengths, constructive interference occurs and we get an interference maximum in the transmitted pattern. That is

$$a \sin\theta_n + a \sin\theta_0 = n\lambda \qquad (1)$$

where n is any integer known as the order and θ_n is the corresponding angle. Both n and θ_n can take on positive or negative values, depending on whether the rays bend down or up.

Although we can pair off all rays from adjacent slits in the above manner for an interference maximum, it may also happen that rays from within the same slit combine to give total destructive interference for the very same conditions. Thus the expected interference maximum would be obscured or 'suppressed' due to the single slit diffraction pattern. This is known as a missing order.

It is possible to derive a general equation for diffraction from many slits, called a diffraction grating (see Hecht). The intensity behind the grating for plane wave illumination is given by the equation

$$I(\theta) = \frac{I(0)}{N^2} \frac{\sin^2\beta}{\beta^2} \frac{\sin^2(N\alpha)}{\sin^2\alpha} \qquad (2)$$

where θ is the viewing angle, N is the number of slits, $\beta = \pi b \sin\theta / \lambda$, $\alpha = \pi a \sin\theta / \lambda$. Here b is the slit width. This equation says that, in general the interference pattern due to the regular spacing of the slits is modulated or multiplied by the pattern due to a single slit. This will be a recurring theme of diffraction phenomena: the pattern due to one feature of the diffracting aperture multiplying the pattern due to another feature. In the parlance of Fourier optics, this is a result of the convolution theorem. This can lead to the special case where the zeros of the single slit pattern fall exactly on the even orders of the pattern due to the regular spacing of the slits, generating missing maxima.

The above equation applies for plane wave illumination. However under illumination from a point source, the phase of the light across the grating will be varying as light further from the

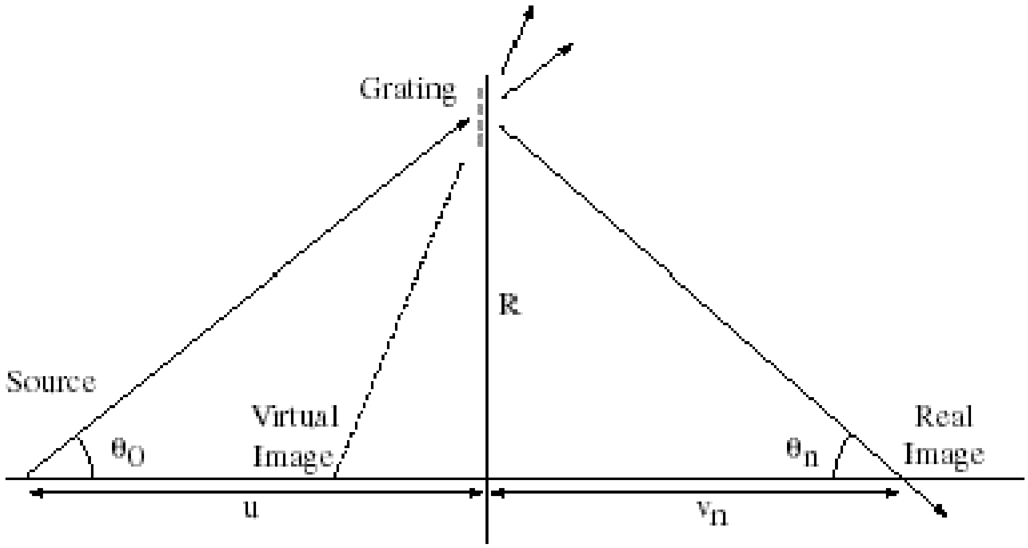

Figure 2. A grating illuminated by a spherical wave.

axis has a greater distance to travel. One can imagine generating a grating with a varying slit separation, d, to correct for this phase difference allowing an image of the source to be generated as shown in figure 2. Using geometry we can see that

$$\tan\theta_0 = \frac{R}{u}, \quad \tan\theta_n = \frac{R}{v}$$

We can then substitute this into equation (1) assuming paraxial rays so that tan θ ~ sin θ, to give

$$\frac{1}{u} + \frac{1}{v} = \frac{n\lambda}{dR} \qquad (3)$$

where the constant slit separation, a, of equation (1) has been replaced by the varying width, d, used here. Providing that the

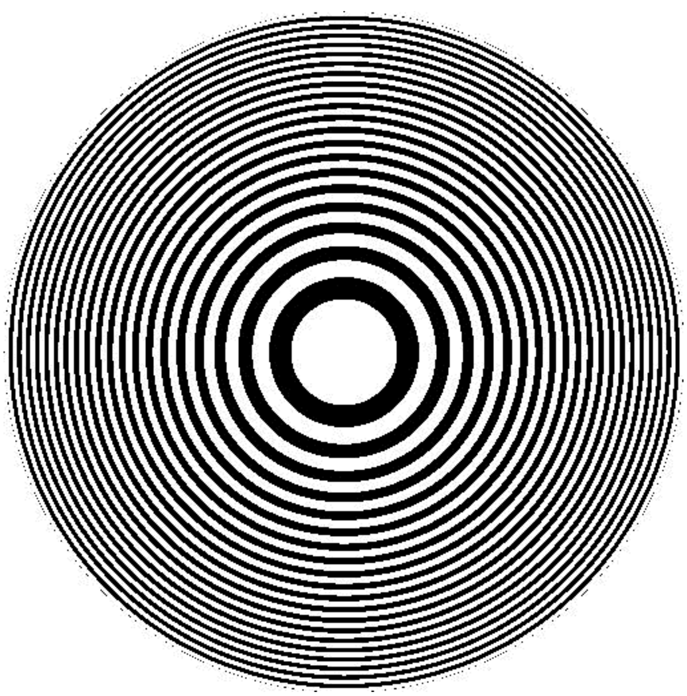

Figure 3. A Fresnel Zone Plate

spacing varies inversely with the distance from the axis, the right hand side of equation (3) is constant. Note that we are presuming a grating with a slit width that is half the slit separation. Diffraction effects then tell us that every second order will be missing. Hence n takes on only odd values in the above equation.

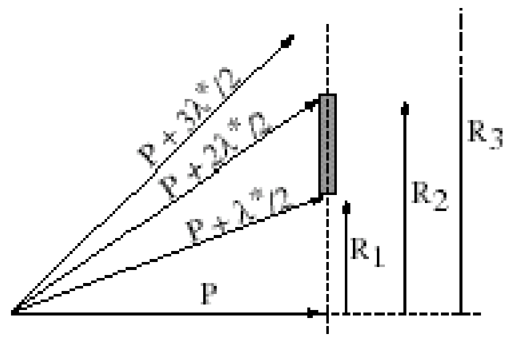

Figure 4. Construction of a zone plate.

Now consider a zone plate as seen in figure 3. It is a series of concentric circles on a flat plate with the alternate regions darkened. It is constructed by imagining that there is a light source of wavelength λ^* a distance P away from the plane of the plate. At a given instant in time, the light on the axis of the plate will have a certain phase. However, light on other portions of the plate will have a different phase since it had further to travel. The circles on the plate, which mark the zone boundaries, are those places where the light is exactly in phase or exactly out of phase with the light on the axis, as shown in figure 4. The Pythagorean Theorem gives:

$$R^2_m + P^2 = (P + m\lambda^*/2)^2 = P^2 + (m\lambda^*/2)^2 + mP\lambda^*$$

$$R^2_m = mP\lambda^* \quad (\text{for } m\lambda^* << P) \tag{4}$$

These are the Fresnel zone boundaries. In the way we have defined them, we see that what we have is essentially the interference pattern between a spherical wave and a plane wave a distance P from the source in the sense that if the two waves were in phase at the centre, at $R_{1,3...}$ they would be π out of phase – dark fringes – and at $R_{2,4,...}$ they would be in phase – bright fringes. However, the analogy is not exact. For the interference pattern, the dark and light regions will gradually merge into one another in a sinusoidal fashion whereas on the zone plate, the boundary transitions between light and dark are abrupt. This will give rise to an important difference between the point-source hologram, which is the above mentioned interference pattern, and the zone plate. The difference is due to the fact that a linear sinusoidal grating has only two diffracted rays whereas the normal linear grating has very many diffracted rays (interference maxima) as we have seen. This is a property that is very easy to demonstrate using Fourier transform techniques, as a binary (square wave) grating can be thought of as synthesised from sinusoidal gratings of multiple frequencies.

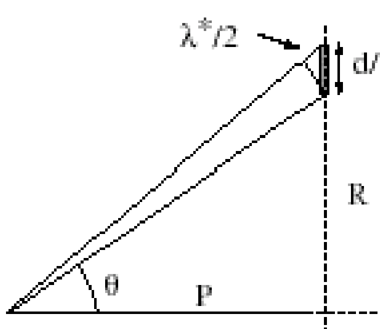

Figure 5. An element of the zone plate.

When we examine the zone plate out near its edge, we find that the light and dark regions are very close together and if we consider a very small surface element, we see that it looks like a tiny linear diffraction grating as discussed above. We now want to treat the zone plate as a collection of infinitesimal linear gratings with a spacing, d, which varies in a known way with the radius R. We can see that

$$d/2 = R_{m+1} - R_m$$

Restricting ourselves to paraxial rays, consideration of figure 5 gives:

$$R/P = \tan\theta \approx \sin\theta = \lambda^*/d$$

which shows that the slit separation varies inversely with radius. Substituting this expression into equation (3) yields the formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f_n} \qquad \text{where} \quad f_n = \frac{P}{n}\left(\frac{\lambda^*}{\lambda}\right) \tag{5}$$

This last formula is recognized as being the familiar Gaussian lens formula for a lens with a focal length f_n. That is, the zone plate will act just like a thin lens with a focal length f_n. Also, since n can take on negative values, the zone plate acts like a diverging lens as well as a converging lens. Furthermore, since n can take on several integral values, it will behave like very many diverging and converging lenses all rolled into one. Note that our theory said that n should take on only odd integral values. In practice even values are also observed because it is difficult to keep the slit width exactly half of the slit separation. Notice also that the focal length is wavelength dependent. This means that as a lens, the zone plate will exhibit chromatic aberration. See if you can observe this by viewing a distant object such as a fluorescent light through the zone plate.

3. Experiment

The zone plate is very delicate and easily damaged – please treat it with care. Illuminate the zone plate with a point source (use the microscope objective on the front of the helium-neon laser). Keep the source to zone plate distance constant and use a screen to find the foci. Plot $1/v_n$ against n to obtain a value for the primary focal length, f_1. Measure the zone plate constant as directed below (s = $\sqrt{\lambda^* P}$) and determine the wavelength of the helium neon laser.

To determine the illuminating wavelength, we need to measure the zone plate constant (s = $\sqrt{\lambda^{*}P}$). From equation (4) and the definition of the zone plate constant, we have $R_m = s\sqrt{m}$. Therefore, one can obtain s from a fit of R_m versus $\pm\sqrt{m}$. Measure the zone plate radii on the travelling microscope starting on the left-hand side of the tenth radius and proceeding to the right always moving the crank in the same direction in order to eliminate the mechanical backlash in the worm gear drive. Your readings will be: R(10), R(9),...,R(1), (centre), R(1),…, R(9), R(10), R(20), R(50), R(100),... etc. The further out you go, the better your result. Plot your direct data versus plus or minus the square root of m; ie, plot $R_m + K$ versus $\pm\sqrt{m}$ where K is an arbitrary constant. You need only do this measurement once and if your data is good, you won't be able to see that any of the measured points miss the fitted line.

Use the focal length and zone plate constant to determine the wavelength of the source.