Diffraction at a Single and Double Slit

Apparatus:
Optical bench, He-Ne Laser, single slit, double slit, photocell, microammeter.
Purpose of the experiment:
To measure the intensity distribution due to diffraction at single and double slits and use it to measure the slit width (d), and slit separation (a).
Basic methodology: Light from a He-Ne Laser source is diffracted by single and double slits. The resulting intensity variation is measured by a photocell whose output is read off as a current measurement.

I Theory:

Single Slit Diffraction:
When the diffracting object is illuminated by parallel rays (plane wavefronts) the situation is known as Fraunhofer diffraction. Consider a plane wave incident normally on a slit of width a. We wish to calculate the intensity distribution due to diffraction produced on the screen. We assume that the slit consists of a large number of equally spaced point sources and that each point on the slit is a source of Huygen's secondary wavelets which interfere with the wavelets emanating from other secondary points. Let the point sources be at $A_1, A_2, A_3, ...etc$ and let the distance between the consecutive points be Δ. (See Fig. 1).

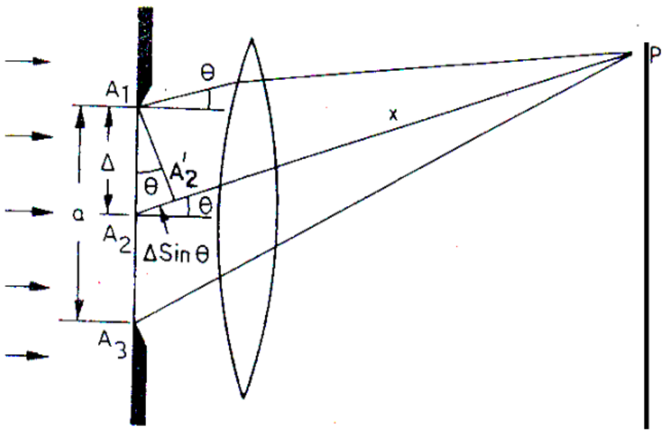

Figure 1: Fraunhofer diffraction at a slit

If the number of point sources is n, then

$$a = (n-1)\Delta \tag{1}$$

To find the intensity at a point P on the screen , we need to calculate the resultant field produced by these n sources. Since the slit actually consists of a continuous distribution of sources, we will in the final expression, take n to infinity and Δ to zero such that $n\Delta$tends to

a. The distance of the screen from the slits being very large in comparison to a, the amplitudes of the disturbances reaching point P from $A_1, A_2, ...etc$ will be very nearly the same. However, because of even slightly different path lengths to the point P, the field produced by A_1will differ in phase from the field produced by A_2. For an incident plane wave, the points A_1 and A_2 are in phase and, therefore,the phase difference between the wavelets emanating from these points is due to the additional path traversed by the disturbance emanating from the point A_2. If the diffracted rays make an angle θ with the normal to the slit, the path difference is

$$A_2 A_2{}' = \Delta \sin\theta \tag{2}$$

The corresponding phase difference ϕ is given by

$$\phi = \frac{2\pi}{\lambda}\Delta\sin\theta \tag{3}$$

Thus, if the electric field at the point P due to the disturbance emanating from the point A_1 is $E_0\cos(\omega t)$ then the field due to the disturbance emanating from A_2 is $E_0\cos(\omega t - \phi)$. Eq. 3 gives the difference in phases of the disturbance reaching P from every consecutive pair of points. Therefore, the resultant field at the point P is given by

$$E_P = E_0\left[\cos\omega t + \cos(\omega t - \phi) + \ldots + \cos\left(\omega t - (n-1)\phi\right)\right] \tag{4}$$

Using the trigonometric identity

$$\cos\omega t + \cos(\omega t - \phi) + \ldots + cos[\omega t + (n-1)\phi] = \frac{\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}}\cos\left[\omega t - \frac{1}{2}(n-1)\phi\right] \tag{5}$$

we get

$$E_P = E_\theta\cos\left[\omega t - \frac{1}{2}(n-1)\phi\right] \tag{6}$$

where the amplitude E_θ of the resultant field is given by

$$E_\theta = \frac{E_0\sin\frac{n\phi}{2}}{\sin\phi/2} \tag{7}$$

In the limit $n \to \infty$ and $\Delta \to 0$ in such a way that $n\Delta \to a$, we have $\frac{n\phi}{2} = \frac{n}{2}\frac{2\pi}{\lambda}\Delta\sin\theta$. Further, $\phi = \frac{2\pi}{\lambda}\Delta\sin\theta = \frac{2\pi}{\lambda}\frac{a}{n}\sin\theta$ tends to zero and we may therefore write

$$E_\theta = \frac{E_0\sin\frac{n\phi}{2}}{\phi/2} = nE_0\frac{\sin\left(\pi a\sin\theta/\lambda\right)}{\frac{\pi}{\lambda}a\sin\theta} = A\frac{\sin\beta}{\beta} \tag{8}$$

where $A = nE_0$ and $\beta = \pi a\sin\theta/\lambda$ Thus,

$$E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta) \tag{9}$$

The corresponding intensity is given by

$$I = I_0\frac{\sin^2\beta}{\beta^2} \tag{10}$$

where I_0 represent the intensity at $\theta = 0$.

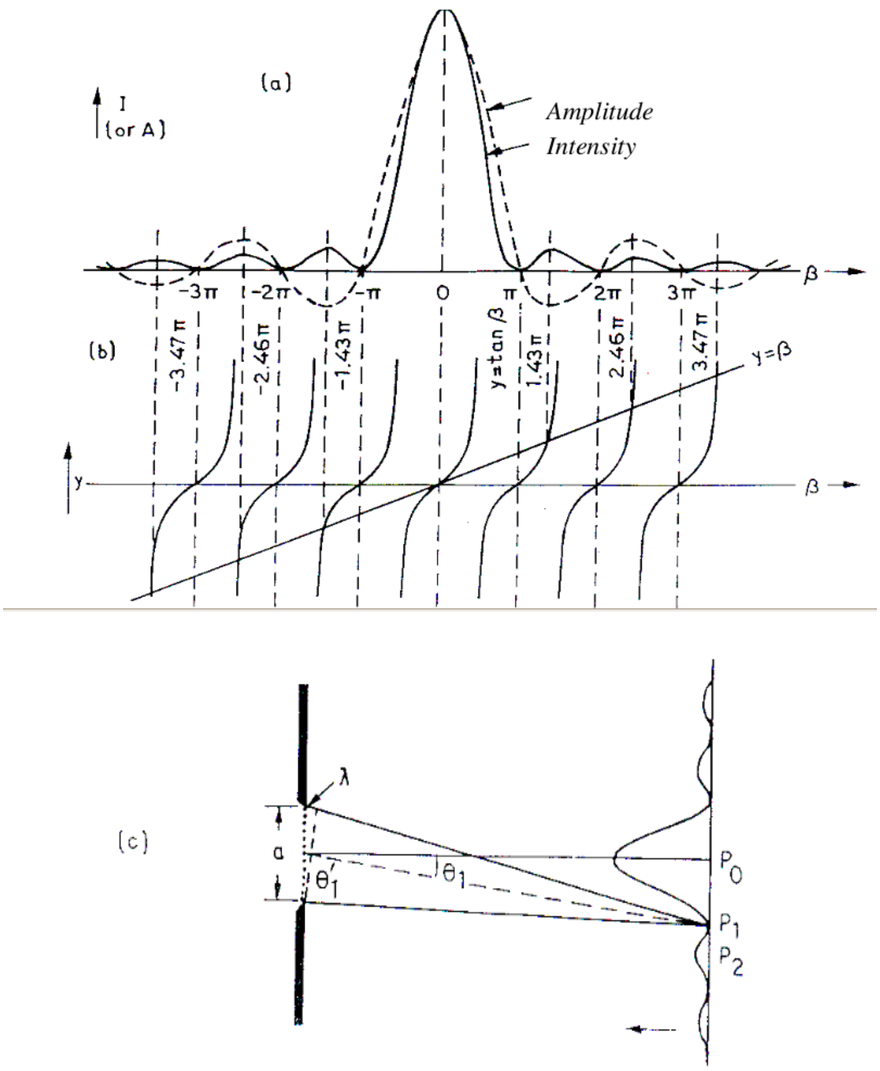

Figure 2: Intensity distribution of single slit diffraction

Positions of the Maxima and Minima:

The variation of the intensity with β is shown in Fig 2a. From eq. (10) it is obvious that the intensity is zero when $\beta = m\pi, \quad m \neq 0$ which gives the condition for minima

$$a \sin\theta = m\lambda \qquad m = \pm 1, \pm 2, \pm 3... \tag{11}$$

In order to determine the position of the maxima, we differentiate eq. (11) wrt β and set it equal to zero. This gives

$$\tan\beta = \beta \tag{12}$$

The root $\beta = 0$ corresponds to the central maximum. The other roots can be found by determining the points of intersections of the curves $y = \beta$ and $y = \tan\beta$ (Fig 2b, c). The intersections occur at $\beta = 1.43\pi, \beta = 2.46\pi$ etc., and are called the first, second maximum etc. Since $\left[\frac{\sin(1.43\pi)}{1.43\pi}\right]^2$ is about 0.0496, the intensity of the first maximum is about 4.96% of the central maximum. Similarly, the intensities of the second and third maxima are about 1.88% and 0.83% of the central maximum respectively.

From the width x of the central maximum of the single-slit diffraction pattern, the width a of the slit can be calculated:

$$\boxed{a = \lambda\sqrt{x^2 + D^2}/x} \tag{13}$$

where D is the distance of the screen from the slit plane.

Double Slit Diffraction Pattern:

In this section we will study the Fraunhofer diffraction pattern produced by two parallel slits (each of width a) separated by a distance d. We will find that the resultant intensity distribution is the product of single slit diffraction pattern and the interference pattern produced by two point sources separated by a distance d.

In order to calculate the diffraction pattern we use a method similar to that used for the case of a single slit and assume that the slits consist of a large number of equally spaced point sources and that each point on the slit is a source of Huygen's secondary wavelets. Let the point sources be at $A_1, A_2, A_3, ..$ (in the first slit) and at $B_1, B_2, B_3, ..$ (in the second slit) (see Fig 3). As before, we assume that the distance between two consecutive points in either of the slits is Δ. Then the path difference between the disturbances reaching the point P from two consecutive points in a slit is $\Delta \sin\theta$. The field produced by the first slit at the point P will,

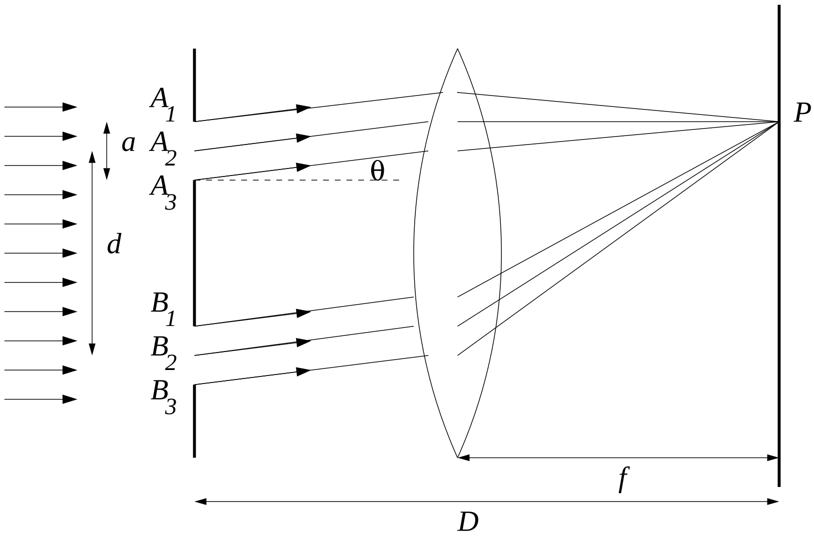

Figure 3: Fraunhofer Diffraction at two slits

therefore, be given by (see eq. 10)

$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta) \tag{14}$$

Similarly, the second slit will produce a field

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \phi_1) \tag{15}$$

at the point P, where $\phi_1 = \frac{2\pi}{\lambda} d\sin\theta$ is the phase difference between the disturbances from two corresponding points on the slits; by corresponding points we imply pair of points like $(A_1, B_1), (A2, B2),$ which are separated by a distance d. Hence the resultant field will be

$$E = E_1 + E_2 = A\frac{\sin\beta}{\beta}\left[\cos(\omega t - \beta) + \cos(\omega t - \beta - \phi_1)\right] \tag{16}$$

which represents the interference of two waves each of amplitude $A\frac{\sin\beta}{\beta}$ and differing in phase by ϕ_1. The above equation can be rewritten as

$$E = A\frac{\sin\beta}{\beta}\cos\gamma\cos(\omega t - \beta - \frac{\phi_1}{2}), \tag{17}$$

where $\gamma = \frac{\phi_1}{2} = \frac{\pi}{\lambda}d\sin\theta$.

The intensity distribution will be of the form

$$I = 4I_0\frac{\sin^2\beta}{\beta^2}\cos^2\gamma \tag{18}$$

where $I_0\frac{\sin^2\beta}{\beta^2}$ represents the intensity distribution produced by one of the slits. As can be seen, the intensity distribution is a produintensity of the diffraction pattern produced by a single slit of width a (first term) and the interference pattern produced by two point sources separated by a distance d (the second term $\cos^2\gamma$) (see Fig 4).

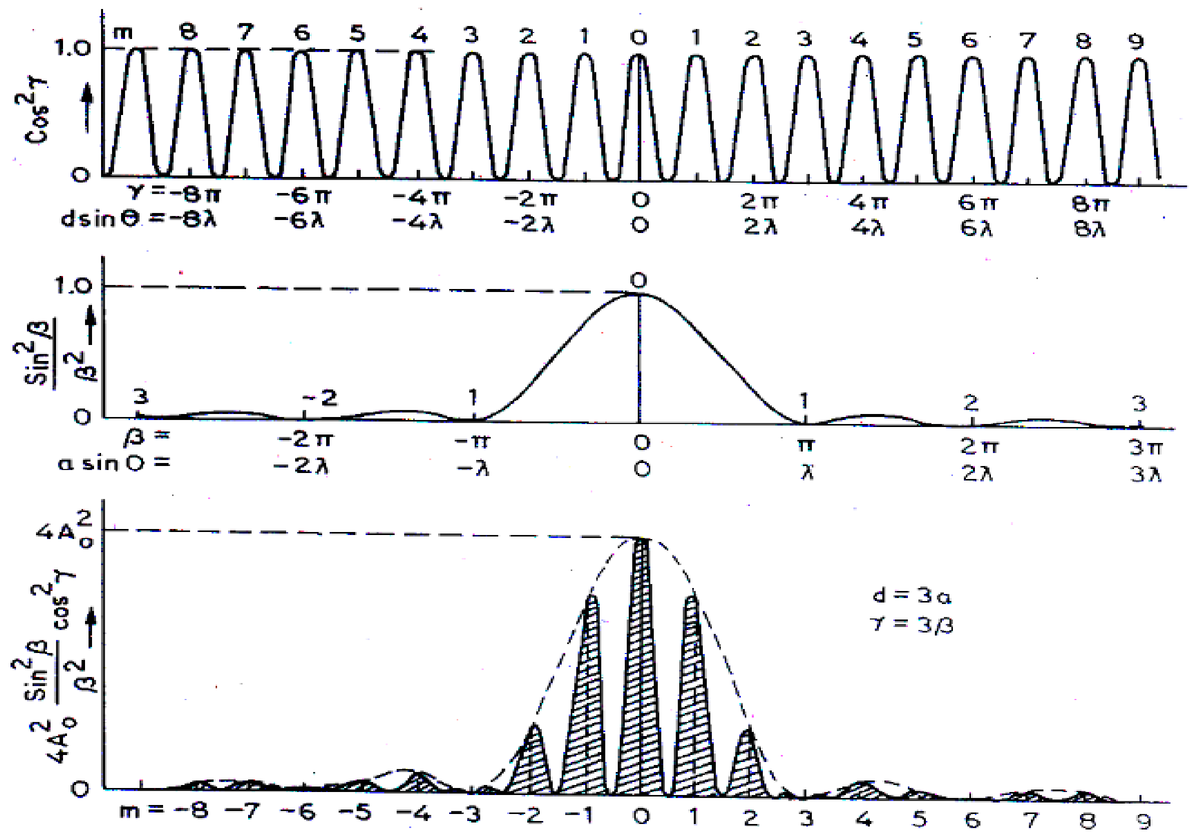

Figure 4: Intensity distribution of double-slit diffraction

Positions of Maxima and Minima:

Equation (17) tells us that the intensity is zero wherever $\beta = \pi, 2\pi, 3\pi$ or when $\gamma = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, ...$ The corresponding angles of diffraction will be given by

$$a\sin\theta = m\lambda\ ; \quad (m = 1, 2, 3, ... \tag{19}$$

$$d\sin\theta = \left[n + \frac{1}{2}\right]\lambda\ ; \quad (n = 0, 1, 2, ...) \tag{20}$$

Interference maxima occur when $\gamma = 0, \pi, 2\pi, ...$ or when

$$d \sin\theta = 0, \lambda, 2\lambda, ... \tag{21}$$

This pattern can be used to calculate the width a of the slits as well as the slit separation d. If x is the width of the central diffraction envelope, then as for the single slit, we have

$$\boxed{a = \lambda\sqrt{x^2 + D^2}/x.} \tag{22}$$

The fringe width y of the interference pattern inside the diffraction envelope is an indication of the slit separation:

$$\boxed{d = \lambda\sqrt{\frac{y^2}{4} + D^2}/y.} \tag{23}$$

II Set-up and Procedure:

1. Make sure the optical bench is leveled. Fix the laser source on the mount and connect to its stabilizer.
2. Switch on the laser source about 15 minutes before the experiment is due to start. This ensures the intensity of light from the laser source is constant.
3. Mount the slit on the saddle and align it so that it is illumined by the laser beam.
4. Mount the screen on its saddle at a large distance from the slit. Adjust the slit width to obtain a well-resolved diffraction pattern on this screen.
5. Replace the screen with the photocell.
6. The photocurrent is approximately proportional to intensity of the incident light. Record the photocurrent for different positions of the photocell across the diffraction pattern.
7. Repeat the same procedure for the double slit. Make sure you have a well-resolved diffraction pattern showing the interference minima within the central maximum.
8. Record the intensity pattern on both the sides of central maximum. Note that the interval between two consecutive positions of the photocell should be small enough to resolve the interference pattern.

Precautions:

1. *Never look at the laser beam directly as this may damage the eyes permanently.*
2. *The photocell should be as away from the slit as possible.*
3. *The measurements should be made moving the micrometer screw in the same direction.*
4. *The laser should be operated at a constant voltage 220V obtainable from a stabilizer. This prevents the flickering of the laser beam.*

III Exercises and Viva Questions

1. What are the characteristics of light produced by a laser? Can this experiment conducted by using any other source?
2. Verify eq. 4
3. For a traveling wave, derive the relation between path difference and phase differences.
4. What is the effect on the intensity distribution if the width a of the slit is changed? If the slit separation d is changed?
5. What would be the result if the experiment were to be carried out with white light?
6. What is the intensity distribution for a double slit ignoring diffraction effects?
7. Count the number of interference fringes observed within the envelope of central diffraction maximum. Give an explanation based on the experiment for the number of fringes seen.
8. What is the effect on the intensity pattern if the distance D between slit and photocell is changed?
9. How much should D change for a bright fringe at the photocell to be replaced by dark fringe?
10. What will be the intensity pattern for a 3-slit interference?

References:

1. *Fundamental of Optics*, F.A. Jenkins and H.E. White, McGraw-Hill International (4th edition), 1976.
2. *Optics*, A. Ghatak, Tata McGraw-Hill (2nd edition), 1992.
3. *Fundamentals of Physics*, D. Halliday, R. Resnick and J.A. Walker, John Wiley & Sons, 2001.