

**EEE F431: Simulation Assignment Report**  
**Joint Uplink and Downlink Resource Allocation for D2D**  
**Communications Underlying Cellular Networks**

November 28, 2020

**Ashwin Kumar K, Shashwat Avasti**  
ID - 2017B5A81034G, 2017B30459G

# 1 About the problem

The problem presented to us was an optimization problem that required the maximization of bit rate across all shareholders in the system, ie cellular users and DUEs. This problem by its very construction is nonlinear, to be specific, mixed integer nonlinear programming problem. The problem does not admit any closed form allotment solution, and the context in which the problem exists lends exhaustive search completely useless, since any allotment needs to be done within the coherence time of the channels which is of the order of microseconds. A detailed description of the problem is as follows.

## 1.1 System Model

We consider a single-cell network with one BS and a number of CUEs coexisting with multiple DUEs. The BS is responsible for allocating subcarrier resources to both CUEs and DUEs. We use  $j, i$  to represent the CUE  $j$ , DUE  $i$ , and  $j \in \mathcal{C} = \{1, 2, \dots, M\}$ ,  $i \in \mathcal{D} = \{1, 2, \dots, N\}$ , respectively. We assume that the cellular network adopts Frequency Division Duplexing (FDD) in which the uplink and the downlink subcarriers respectively occupy half of the whole spectrum. Moreover, we assume each CUE has been pre-allocated one orthogonal uplink subcarrier and one orthogonal downlink subcarrier. To manage interference, we assume each subcarrier (either uplink or downlink) could be reused by at most one DUE and each DUE could reuse at most one subcarrier. We assume that all links experience not only the slow shadowing and path loss but also the fast fading caused by multi-path propagation as in . That is, the instantaneous channel gain between CUE  $j$  and the BS is modeled as

$$g_{j,B} = G\beta_{j,B}\Gamma_{j,B}l_{j,B}^{-\alpha} \quad (1)$$

where  $G$ ,  $\beta_{j,B}$ ,  $\Gamma_{j,B}$ ,  $l_{j,B}$ , and  $\alpha$  are the path loss constant, fast fading gain with exponential distribution, slow fading gain with log-normal distribution, distance between CU and BS, and the path loss exponent, respectively. Similarly, we define  $g_{B,j}$ ,  $g_{i,B}$ ,  $g_{B,i}$ ,  $g_{j,i}$ ,  $g_{i,j}$ , and  $g_{i,i}$  as the channel gain between the BS and the CUE  $j$ , between the DUE  $i$  and the BS, between the BS and DUE  $i$ , between the CUE  $j$  and DUE  $i$ , between the DUE  $i$  and CUE  $j$ , between the D2D transmitter and the D2D receiver, respectively. We assume that the BS has the perfect channel state information (CSI) of all the links involved so that it can design a centralized resource allocation scheme. Define binary variables  $\rho_{i,j}^u$  and  $\rho_{i,j}^d$ , if DUE  $i$  reuses the uplink subcarrier of CUE  $j$  then  $\rho_{i,j}^u = 1$ , otherwise  $\rho_{i,j}^u = 0$ . Similarly, if DUE  $i$  reuses the downlink subcarrier of CUE  $j$  then  $\rho_{i,j}^d = 1$ , otherwise  $\rho_{i,j}^d = 0$ . The received signal to interference-plus-noise-ratio (SINR) for CUE  $j$  can be given by

$$\gamma_j^u = \frac{p_j g_{j,B}}{\sum_{i=1}^N \rho_{i,j}^u p_i g_{i,B} + N_0}, \gamma_j^d = \frac{p_{B,j} g_{B,j}}{\sum_{i=1}^N \rho_{i,j}^d p_i g_{i,j} + N_0} \quad (2)$$

where  $p_j$ ,  $p_i$ , and  $p_{B,j}$  denote the transmit power of CUE  $j$ , the transmit power of DUE  $i$ , and the transmit power from the BS to CUE  $j$ , respectively.  $N_0$  is the variance of zero mean Additive White Gaussian Noise (AWGN). Therefore, the achievable uplink data rate and the achievable downlink data rate for CUE  $j$  can be respectively expressed as

$$R_j^u = \log_2(1 + \gamma_j^u) \quad (3)$$

$$R_j^d = \log_2(1 + \gamma_j^d). \quad (4)$$

Similarly, the SINR of the DUE  $i$  can be expressed as

$$\gamma_i = \frac{p_i g_{i,i}}{\underbrace{\sum_{j=1}^N \rho_{i,j}^u p_j g_{j,i}}_{\text{if DUE } i \text{ reuses an uplink subcarrier}} + \underbrace{\sum_{j=1}^N \rho_{i,j}^d p_{B,j} g_{B,i}}_{\text{if DUE } i \text{ reuses a downlink subcarrier}} + N_0}, \quad (5)$$

and the achievable data rate of DUE  $i$  can be given by

$$R_i = \log_2(1 + \gamma_i), \quad (6)$$

and the sum data rate of the overall system is

$$R_{sum} = \sum_{j=1}^M R_j^u + \sum_{j=1}^M R_j^d + \sum_{i=1}^N R_i. \quad (7)$$

## 1.2 Mathematical modeling

We find that the optimization problem is essentially the following:

$$\begin{aligned} \mathcal{P1} : & \max_{\rho, \mathbf{p}} R_{sum} \\ s.t. \quad & C1 : \gamma_j^u \geq \gamma_j^{u,req}, \gamma_j^d \geq \gamma_j^{d,req} \quad \forall j \in \mathcal{C} \\ & C2 : \gamma_i \geq \gamma_i^{req}, \forall i \in \mathcal{D} \\ & C3 : 0 \leq p_i \leq p_i^{max}, \forall i \in \mathcal{D} \\ & C4 : 0 \leq p_j \leq p_j^{max}, \forall j \in \mathcal{C} \\ & C5 : 0 \leq p_{B,j} \leq p_{B,j}^{max}, \forall j \in \mathcal{C} \\ & C6 : 0 \leq \sum_{i=1}^N \rho_{i,j}^u \leq 1, 0 \leq \sum_{i=1}^N \rho_{i,j}^d \leq 1, \forall j \in \mathcal{C} \\ & C7 : \left( \sum_{j=1}^M \rho_{i,j}^u \right) \left( \sum_{j=1}^M \rho_{i,j}^d \right) = 0, \forall i \in \mathcal{D} \\ & C8 : \rho_{i,j}^u, \rho_{i,j}^d \in \{0, 1\}, \forall i \in \mathcal{D}, \forall j \in \mathcal{C}. \end{aligned} \quad (8)$$

In  $\mathcal{P1}$ ,  $\mathbf{p}$  is the set of transmit power including the transmit power of CUEs, DUEs, and the BS, and  $\rho$  is the set of binary variable indicating the subcarrier assignment of DUEs. Constraint  $C1$  guarantees the data rate requirements of CUEs, where  $\gamma_j^{u,req}$  and  $\gamma_j^{d,req}$  denote the uplink and downlink minimum SINR requirements of CUE  $j$ . Similarly,  $C2$  guarantees the data rate requirements of DUEs. The constraints  $C3$ ,  $C4$  and  $C5$  are the power constraints where  $p_i^{max}$ ,  $p_j^{max}$ , and  $p_{B,j}^{max}$  are maximal transmit power of DUE  $i$ , CUE  $j$ , and the BS, respectively. Constraint  $C6$  ensures that each uplink or downlink subcarrier of CUE  $j$  can be shared by at most one DUE, and  $C7$  represents that each DUE can only reuse one uplink or downlink subcarrier.

## 2 Our working process

Since any allotment needs to be done within the coherence time of the channels which is of the order of microseconds. It was therefore decided that we will use evolutionary algorithms for finding allotment vectors for resource allocation. These algorithms are much faster than exhaustive search but need not converge to the global optima. Having said that, evolutionary algorithms are able to reach close to optimal solutions. We decided to choose HMPSO, on the advice of our esteemed professor Nitin Sharma. HMPSO is an optimization algorithm that works to improve its score on an evaluation metric. This evaluation metric is called the penalty or fitness function. The fitness function rewards the solution if it meets its objective well, in our case this corresponds to the solution having high bit rate. The fitness function penalizes a solution if it violates some constraint of the optimization problem, in our case this corresponds to the solution violating power constraints and allotment constraints. The second aspect of HMPSO is finding a valid initial solution. This initial solution must satisfy all the constraints. We had a very hard time finding an initial solution.

### 3 Work done so far

The code for HMPSO and JUAD were available from GitHub and we needed to understand their implementation and incorporate the right fitness function in their HMPSO problem set. We were able to code the constraints and fitness function from eq. 8 above, however in order to integrate our needed a deeper understanding of the HMPSO framework. To this extent we consulted our esteemed professor Nitin Sharma on this and we were able to gain a insight into the working of HMPSO code.

A major challenge we faced during this implementation was taking into account the integer binary variables which were optimization variables but were not easy to incorporate in the HMPSO framework.

Post this we tried to incorporate a intermediate function that tries to interface between the HMPSO framework and our fitness function. The main objective of the said function would be to take a population that needs to be optimised and then assign each coordinate the equivalent physical property (like  $P_i, P_j, P_B, \rho^u, \rho^d$ ) and then map out the derived quantities and pass them on to our fitness function. We did attempt writing the interfacing function however the interfacing function wasn't working as expected at the time of writing this report.

With some more time at hand we would be able to complete this task.