Traveling Salesperson Problem

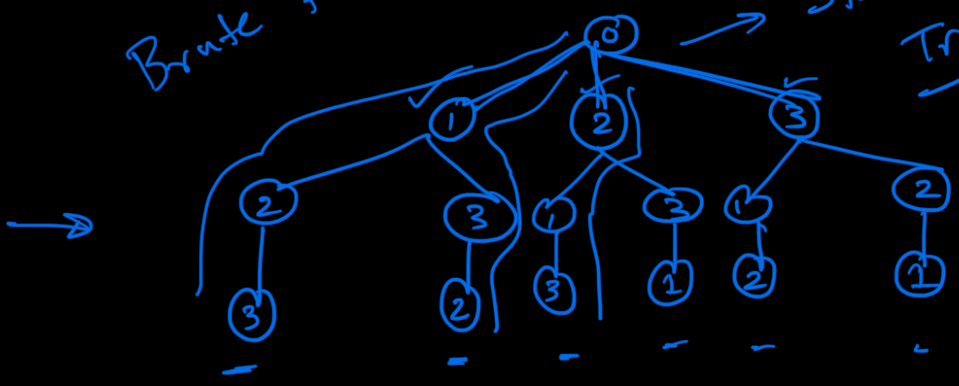
approach / BnB : Branch and Bound

1 1 1 1
1 2 3 4
1 2 4 3
1 3 2 4
1 3 4 2
1 4 3 2

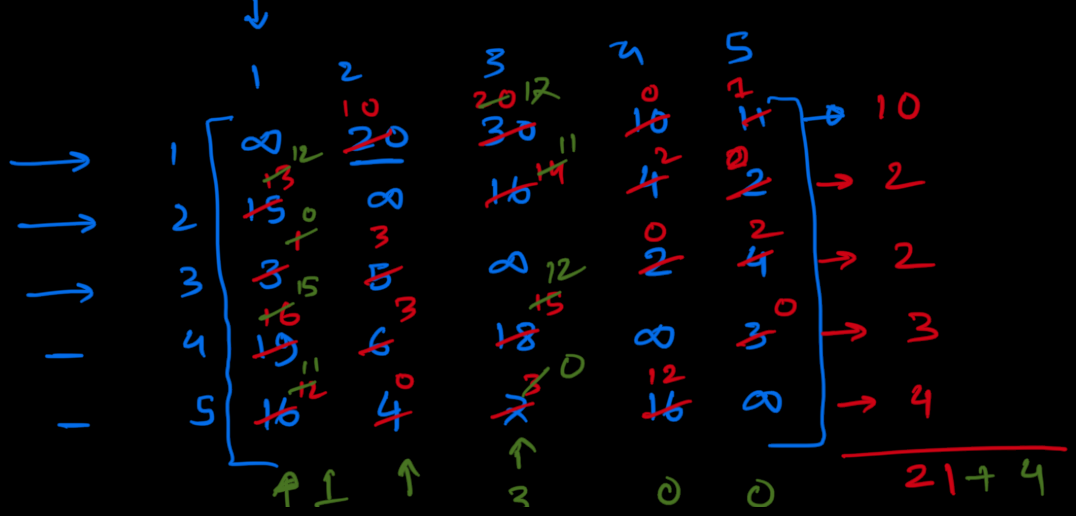
n! - 2

10
100
100 000

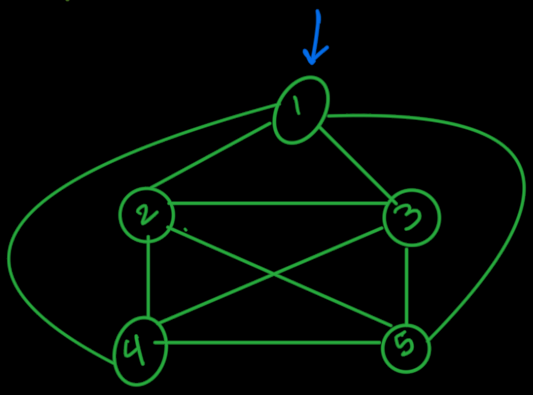
Brute Force → State Space Tree



100 + 101 + 102 = 107
102 + 100 + 101 = 107
100 + (0+1+2)/3
100 + 1 = 101



= 25



$$\begin{array}{c|ccccc|c}
 & 1 & 2 & 3 & 4 & 5 & \\
\hline
1 & \infty & \cancel{20}\ 10 & \cancel{30}\ 20 & \cancel{10}\ 0 & \cancel{11}\ 1 & 10 \\
2 & \cancel{15}\ \cancel{13}\ 12 & \infty & \cancel{16}\ 14 & \cancel{4}\ 2 & \cancel{2}\ 0 & 2 \\
3 & \cancel{3}\ 0 & \cancel{5}\ 15\ 3 & \infty & \cancel{2}\ 0 & \cancel{4}\ 2 & 2 \\
4 & \cancel{19}\ \cancel{16}\ 11 & \cancel{6}\ 3 & \cancel{18}\ 15 & \infty & \cancel{3}\ 0 & 3 \\
5 & \cancel{16}\ \cancel{12} & \cancel{4}\ 0 & \cancel{7}\ 3 & \cancel{16}\ 12 & \infty & 4 \\
\hline
 & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & 21 + 4 = 25 \\
 & \text{cost} & \text{matrix} & \uparrow & & & \\
 & 1 & 0 & 3 & 0 & 0 &
\end{array}$$

$$\begin{array}{c|ccccc|c}
 & 1 & 2 & 3 & 4 & 5 & \\
\hline
1 & \infty & \cancel{20}\ 10 & \cancel{30}\ 20 & \cancel{10}\ 0 & \cancel{11}\ 1 & 10 \\
2 & 13\ \cancel{15} & \infty & \cancel{16}\ 14 & \cancel{4}\ 2 & \cancel{2}\ 0 & 2 \\
3 & \cancel{3}\ 1 & \cancel{5}\ 3 & \infty & \cancel{2}\ 0 & \cancel{4}\ 2 & 2 \\
4 & 16\ \cancel{19} & \cancel{6}\ 3 & 18 & \infty & \cancel{3}\ 3 & 3 \\
5 & 12\ \cancel{16} & \cancel{4}\ 0 & \cancel{7}\ 3 & \cancel{16}\ 12 & \infty & 4 \\
\hline
 & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & 21 + 4 = 25 \\
 & \text{cost} & \text{matrix} & & & 0 & \\
 & 1 & 0 & 3 & 0 & 0 &
\end{array}$$

$$\begin{array}{c|ccccc|c}
 & & & & & & \\
\hline
 & \infty & \infty & \infty & \infty & \infty & 0 \\
 & 1\ \cancel{12} & \infty & \infty & 2 & 0 & 0 \\
 & \infty & 3 & \infty & 0 & 2 & 0 \\
 & 4\ \cancel{19} & 3 & \infty & \infty & 0 & 0 \\
 & 0\ \cancel{11} & 0 & \infty & 12 & \infty & 0 \\
\hline
 & \uparrow & \uparrow & \downarrow & \downarrow & \downarrow & = 11 \\
 & 11 & 0 & & 0 & 0 &
\end{array}$$

Reduced cost matrix

25

$$\begin{bmatrix}
\infty & 10 & 12 & 0 & 1 \\
12 & \infty & 11 & 2 & 0 \\
0 & 3 & \infty & 0 & 2 \\
15 & 3 & 12 & \infty & 0
\end{bmatrix}$$

Reduced Cost matrix

1-3

$[\infty\ \infty\ \infty\ \infty\ \infty]$

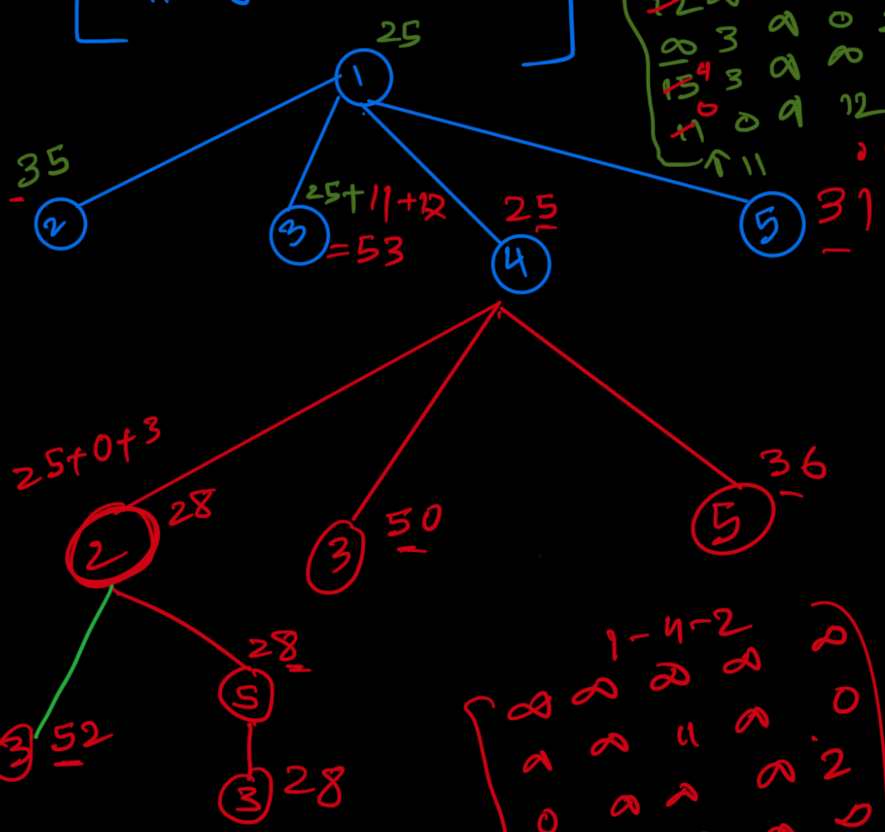
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} = 0$$

$25 + 0 + 10$

$$\begin{bmatrix} 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

$$\begin{bmatrix} \infty & \infty & \infty & 2 & 0 \\ \cancel{12}\,1 & \infty & \infty & 0 & 2 \\ \infty & 3 & \infty & \infty & 0 \\ \cancel{15}\,4 & 3 & \infty & \infty & 0 \\ \cancel{11}\,0 & 0 & \infty & 12 & \infty \end{bmatrix} \begin{matrix} -0 \\ -0 \\ -0 \\ -0 \\ -0 \end{matrix}$$

$\uparrow 11 \quad 0 \quad 0$



1-4

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & \boxed{3} & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

1-4-2

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \begin{matrix} -0 \\ -0 \\ -0 \\ -0 \\ -0 \end{matrix}$$

$\uparrow 1 \quad 1 \quad 1 \quad 1 \quad 1$

$0 \quad 0 \quad 0 \quad 0 \quad 0$

1-4-3

1-4-3

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \cancel{12}\,1 & \infty & \infty & \infty & 0 \\ \infty & \cancel{3}\,1 & \infty & \infty & \cancel{2}\,0 \\ \infty & \infty & \infty & \infty & \infty \\ \cancel{11}\,0 & 0 & \infty & \infty & \infty \end{bmatrix} \begin{matrix} \\ \\ -2 \\ \\ \\ \end{matrix}$$

$11 \quad 11+2=13$

$25+13+12=50$

Result

1-4-2-5-3-1

$\boxed{28}$

1-4-2-5-3-1 ✓

Result

User Defined Data Structure

For each node
Reduced Cost Matrix

Path
Cost

pag- 405
406
402 } Slahuri

Chapter 8.3

main
}

Reduced Cost Matrix
{
vector, cost
}
f (—)

}