Physics Cup 2019 Problem 3

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Abstract

The following is my solution to physics cup problem 3. We will first start by proving some auxilary theorems.

1 auxilary theorem 1

Let a projectile be fired from the ground at an angle θ we know that the equation of its trajectory can be written as follows:-

$$y = xtan\theta - \frac{gx^2 sec\theta}{2v_o^2}$$

Where the point of projection is taken as origin.This equation encodes all the info about the trajectory for eg the directrix of the projectile. To find the directrix of this projectile we just have to rearrange this equation in the form $(x-h)^2 = 4a(y-k)$ and use the properties of a parabola.

The directrix equation comes out to be:

$$y = \frac{V_o^2}{2g}$$

2 auxilary theorem 2

We can also prove that the equation of directrix is common for all parabolas formed by the bouncing ball on the incline.

It is not hard to prove that on the nth bounce the ball covers a distance of $ngsin\alpha * T^2$ where T is the time taken for each bounce.To find T we will work in the incline frame and the value of T comes out to be

$$T = \frac{2 * \sqrt{2gdcos\alpha}}{gcos\alpha}$$

Now the vertical height H of the of the ball just after the nth bounce can be written as

$$H = \frac{d}{cos\alpha} + \Sigma 8ndsin\alpha tan\alpha$$

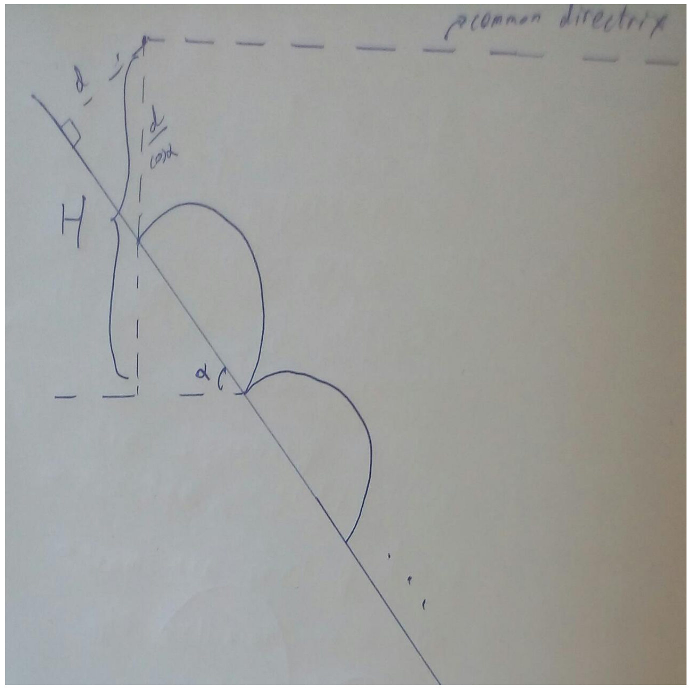

Figure 1: figure 1

which simplifies down to

$$H = \frac{d}{cos\alpha} + 4dtan\alpha sin\alpha(n^2 + n)$$

Now we will use auxillary theorem 1. After the nth bounce the square of net velocity of the ball is

$$v^2 = (\sqrt{2gdsin\alpha tan\alpha} + ngsin\alpha T)^2 + 2gdcos\alpha$$

on substituting the value of T and expanding the square we get

$$v^2 = \frac{2gd(1 + 4n^2sin^2\alpha + 4nsin^2\alpha)}{cos\alpha}$$

By auxilary theorem 1 the directrix of this projectile in at

$$\frac{v^2}{2g}$$

which we can see is equal to H hence proved.

3 auxilary theorem 3

We can also prove that there is a line passing through the initial point of dropping which touches all the parabolas. To prove this we will go into the incline frame and by the resolution of velocity vector we can see that the velocity component perpendicular to incline at the start of each bounce is $\sqrt{2gdcos\alpha}$ now the maximum height achieved by this in the inclined frame is $\frac{2gdcos\alpha}{2gcos\alpha}$ which is equal to the perpendicular distance of d from the incline hence proved.

4 locus of focii

We will solve this question by using geometry in the incline frame of reference. We know that the focus of each parabola must line on the line passing through its vertex and the vertex is equidistant from the directrix and the focus.This allows for the following geometric construction

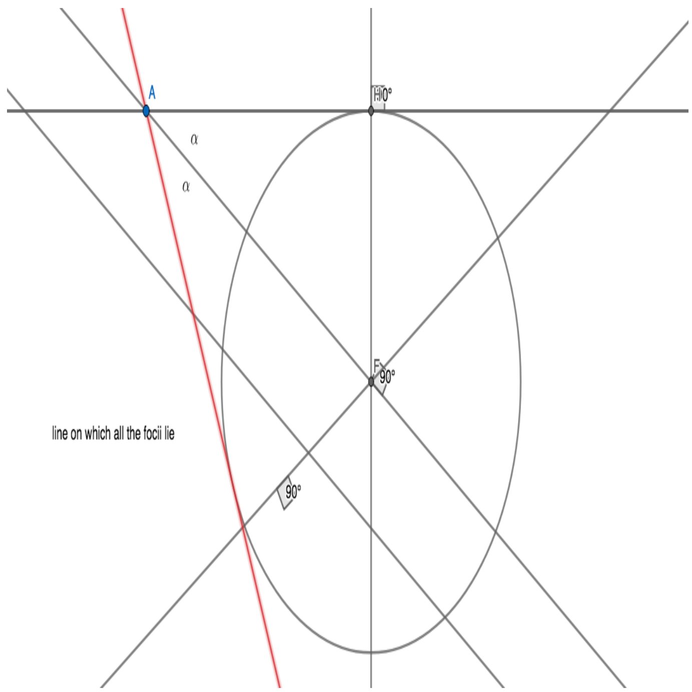

Figure 2: figure 2

Where the red line is the line on which all the focii lie where as the point F is some imaginary point which touches the line in aux theorem 2. To make this goemetric construction I have used the standard properties of parabola that if a tangent at P meets the directrix in K then KSP is a right angle where S is the focal point.