Introduction to Hash Table Data Structure

261217 Data Structures for Computer Engineers

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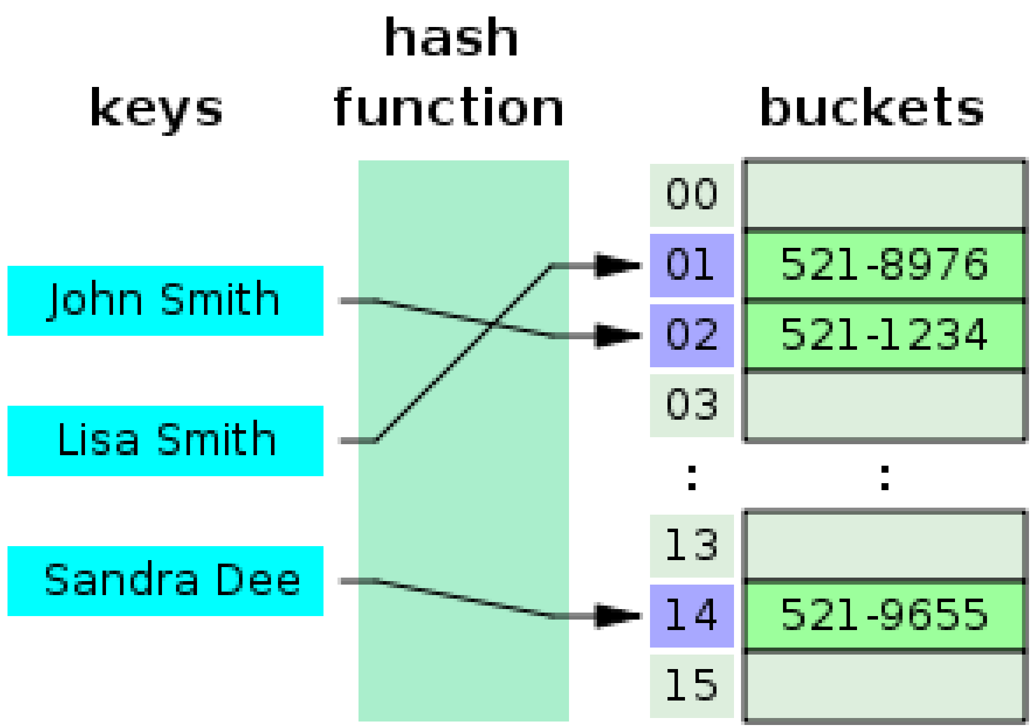
Hash Table Concept



Key Concept is the Hash Function
Hash function can map anything to an integer
The integer is an index of an array (table)

Hash function Application: Error checking

Bonus Time

$$x = \sum_{i=1}^{12} (14 - i) N_i \pmod{11}$$

$$x = (13N_1 + 12N_2 + 11N_3 + 10N_4 + 9N_5 + 8N_6 + 7N_7 + 6N_8 + 5N_9 + 4N_{10} + 3N_{11} + 2N_{12}) \pmod{11}$$

$$N_{13} = \begin{cases} 1 - x, & \text{if } x \leq 1 \\ 11 - x, & \text{if } x > 1 \end{cases}$$

Bonus Time

$$x = \sum_{i=1}^{12} (14 - i) N_i \pmod{11}$$

$$x = (13N_1 + 12N_2 + 11N_3 + 10N_4 + 9N_5 + 8N_6 + 7N_7 + 6N_8 + 5N_9 + 4N_{10} + 3N_{11} + 2N_{12}) \pmod{11}$$

$$N_{13} = \begin{cases} 1 - x, & \text{if } x \leq 1 \\ 11 - x, & \text{if } x > 1 \end{cases}$$

1 2345 67890 12 <u>3</u>

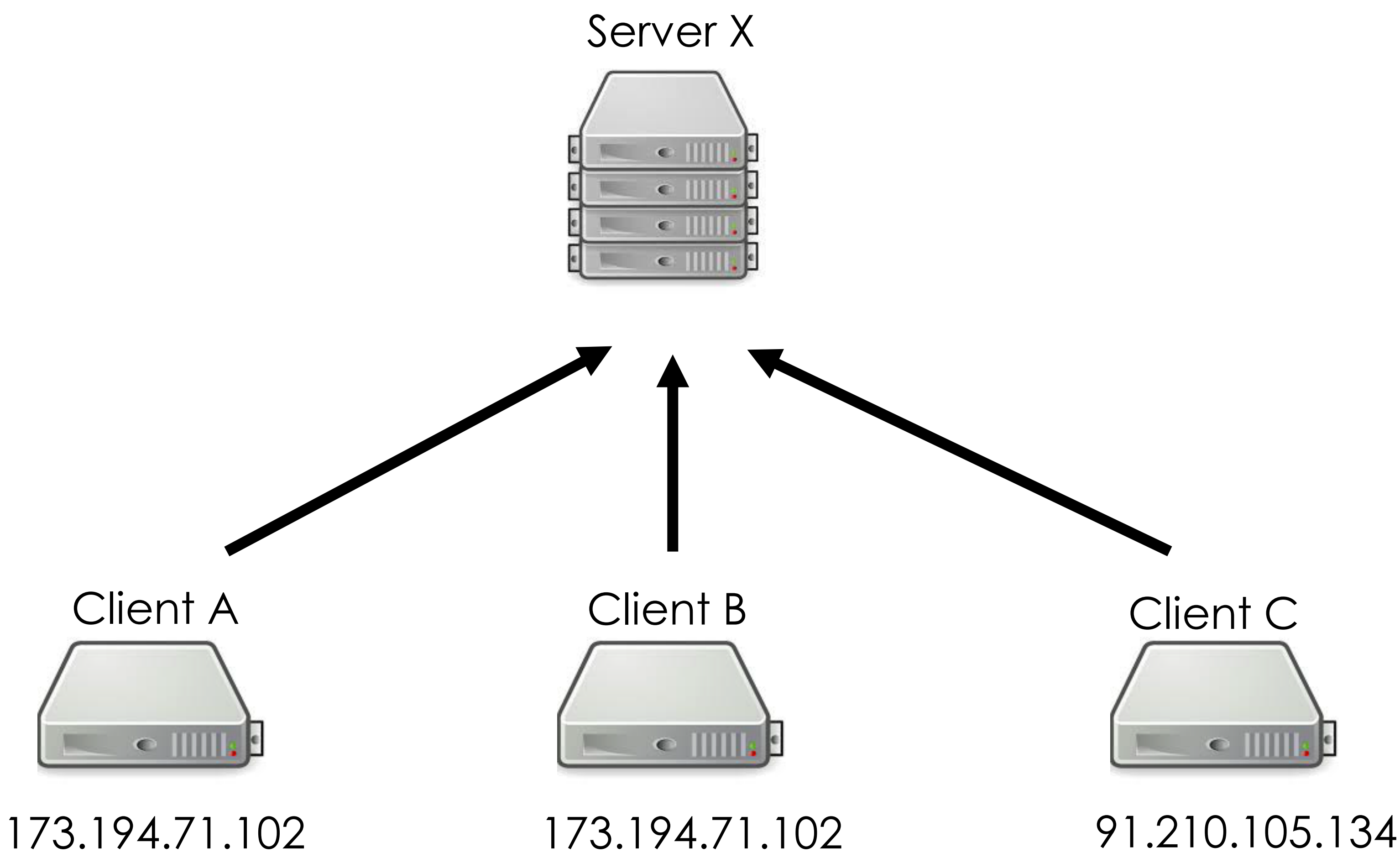
แสดงตัวเลขหลักที่ 13

Hash function Application: Password Storage

Hash function Application: Storage Optimization

Upload the file if the file is not uploaded yet
Guarantee No Duplication

Web Service


Server X
Client A
Client B
Client C
173.194.71.102
173.194.71.102
91.210.105.134

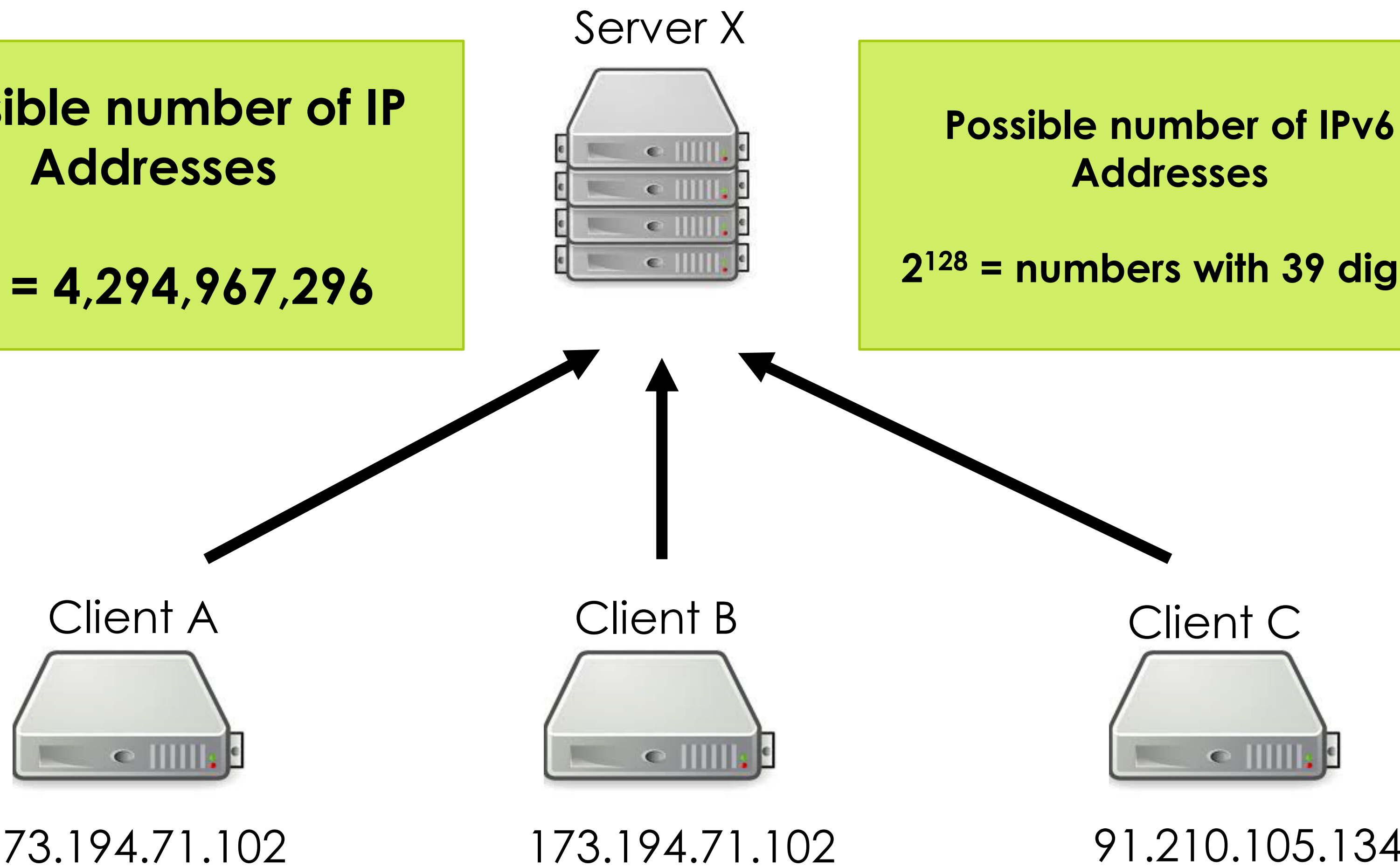
Web Service

Possible number of IP Addresses

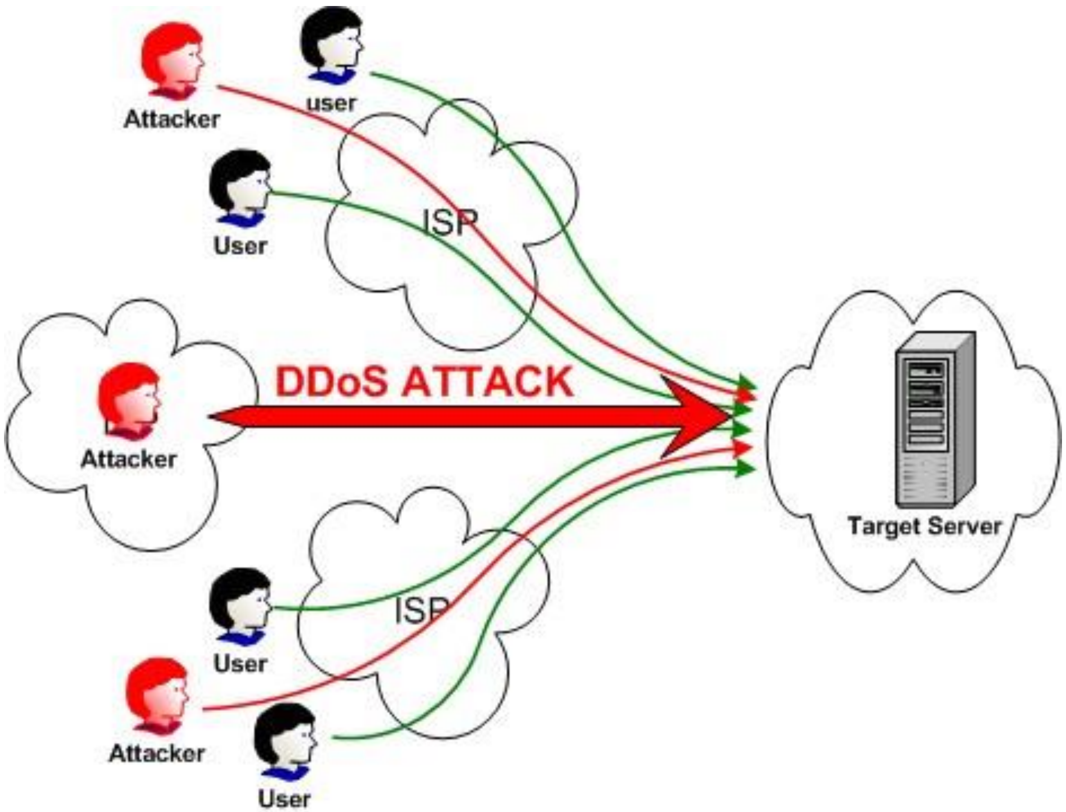
2^{32} = 4,294,967,296

Possible number of IPv6 Addresses

2^{128} = numbers with 39 digits



Denial of Service Attack



Access Log

Date	Time	IP address
09 Dec 2015	00:45:13	173.194.71.102
09 Dec 2015	00:45:15	69.171.230.68
...	...	...
...	...	...
09 Dec 2015	01:45:13	91.210.105.134

IP Access List

IP Access List

Analyse the access log and quickly answer queries: did anybody access the service from this *IP* during the last hour? How many times? How many *IP*s were used to access the service during the last hour?

Log Processing

- 1h of logs can contain millions of lines
- Too slow to process that for each query
- Keep count: how many times each IP appears in the last 1h of the access log
- **C** is some data structure to store the mapping from IP to counters
- We will learn later how to implement **C**

Log Processing

Decrease the counter

1 hour ago

Time	IP address
00:45:13	173.194.71.102
00:45:13	69.171.230.68
...	...
01:45:13	173.194.71.102
01:45:13	91.210.105.134

Now
(Need to update)

Increase the counter

Main Loop

log - array of log lines $(time, IP)$

C - mapping from IPs to counters

i - first unprocessed log line

j - first line in current 1h window

$i \leftarrow 0$

$j \leftarrow 0$

$C \leftarrow \emptyset$

Each second

  UpdateAccessList(log, i, j, C)

UpdateAccessList(log, i, j, C)

while $log[i].time \le Now()$:
  $C[log[i].IP] \leftarrow C[log[i].IP] + 1$
  $i \leftarrow i + 1$
while $log[j].time \le Now() - 3600$:
  $C[log[j].IP] \leftarrow C[log[j].IP] - 1$
  $j \leftarrow j + 1$

AccessedLastHour(IP, C)

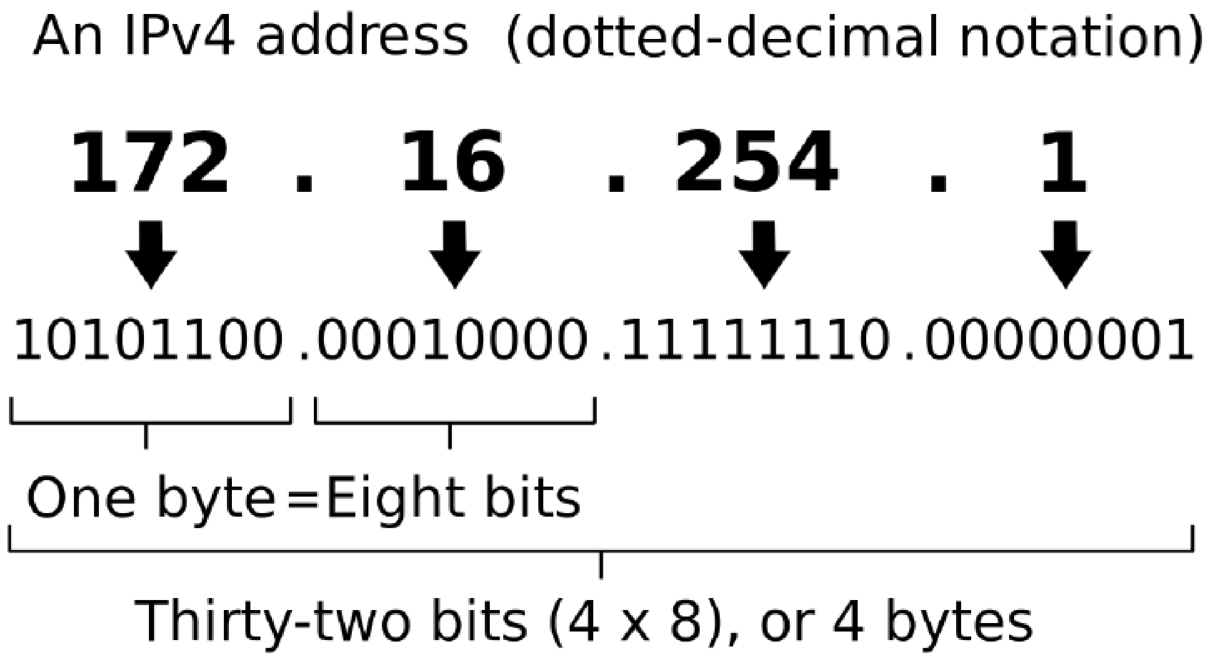
return $C[IP] > 0$

The Question is How to implement the mapping C?

Direct Addressing

- Need a data structure for **C**
- There are 2^{32} different IP(v4) addresses
  - $2^{32} = 4{,}294{,}967{,}296$
- Convert IP to 32-bit integer
- Create an integer array `A` of size 2^{32}
- Use `A[int(IP)]` as `C[IP]`

int(IP)



- int(0.0.0.1) = 1
- int(172.16.254.1) = 2,886,794,753
- int(69.171.230.68) = 1,168,893,508

$\mathtt{int}(IP)$

$\mathtt{return}\ \ IP[1]\cdot 2^{24} + IP[2]\cdot 2^{16} + IP[3]\cdot 2^{8} + IP[4]$

$\mathtt{UpdateAccessList}(log, i, j, A)$

$\mathtt{while}\ \ log[i].time \leq Now():$

$\quad A[\mathtt{int}(log[i].IP)] \leftarrow A[\mathtt{int}(log[i].IP)] + 1$

$\quad i \leftarrow i + 1$

$\mathtt{while}\ \ log[j].time \leq Now() - 3600:$

$\quad A[\mathtt{int}(log[j].IP)] \leftarrow A[\mathtt{int}(log[j].IP)] - 1$

$\quad j \leftarrow j + 1$

Is the server accessed by the IP in the last hour?

$$\texttt{AccessedLastHour}(IP)$$

$$\texttt{return}\ \ A[\texttt{int}(IP)] > 0$$

Big O Analysis of Direct Addressing Implementation

- UpdateAccessList is O(1) per log line
- AccessedLastHour is O(1)
- But need 2^{32} memory (4GB) even for few IPs
- IPv6: 2^{128} will not fit in memory
- In general: O(***U***) memory, ***U*** = number of possible IPs

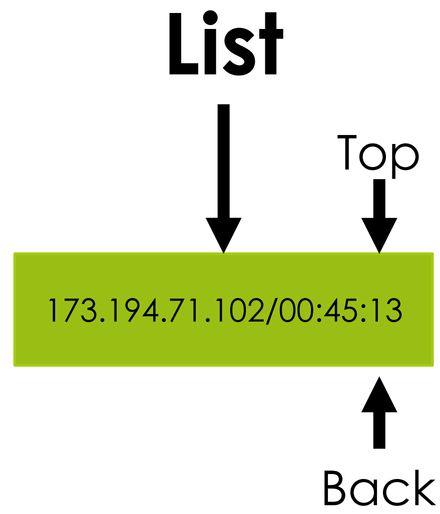
List-based Mapping

- Direct addressing requires too much memory
- Let’s store only active IPs
- Store them in a list
- Store only last occurrence of each IP
- Keep the order of occurrence

Access Log

Current Time: 00:45:15

Time	IP address
00:45:13	173.194.71.102
00:45:13	69.171.230.68
01:00:00	69.171.230.68
01:45:13	173.194.71.102
01:45:13	91.210.105.134



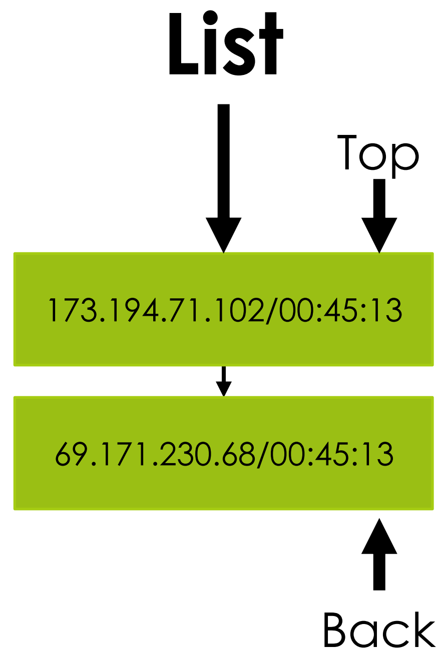
AccessedLastHour(69.171.230.68)?

AccessCountLastHour(69.171.230.68)?

Access Log

Current Time: 00:45:16

Time	IP address
00:45:13	173.194.71.102
00:45:13	69.171.230.68
01:00:00	69.171.230.68
01:45:13	173.194.71.102
01:45:13	91.210.105.134



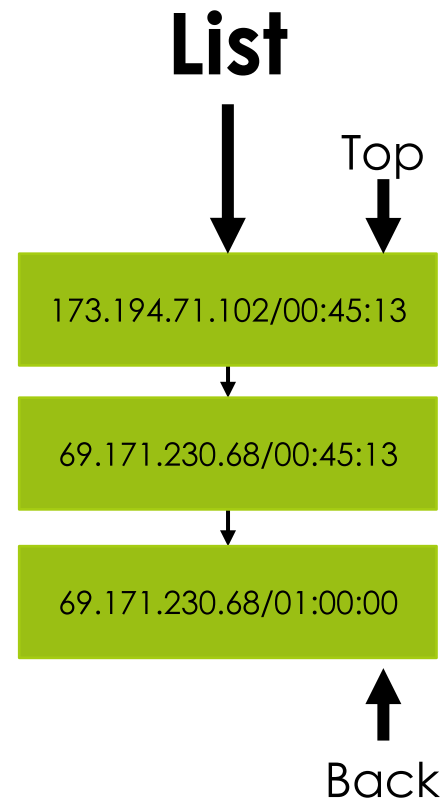
AccessedLastHour(69.171.230.68)?

AccessCountLastHour(69.171.230.68)?

Access Log

Current Time: 01:00:05

Time	IP address
00:45:13	173.194.71.102
00:45:13	69.171.230.68
01:00:00	69.171.230.68
01:45:13	173.194.71.102
01:45:13	91.210.105.134



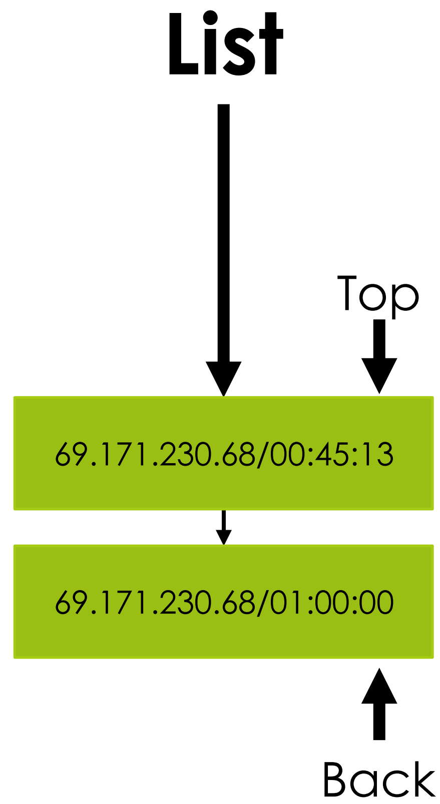
AccessedLastHour(69.171.230.68)?

AccessCountLastHour(69.171.230.68)?

Access Log

Current Time: 01:45:15

Time	IP address
00:45:13	173.194.71.102
00:45:13	69.171.230.68
01:00:00	69.171.230.68
01:45:13	173.194.71.102
01:45:13	91.210.105.134



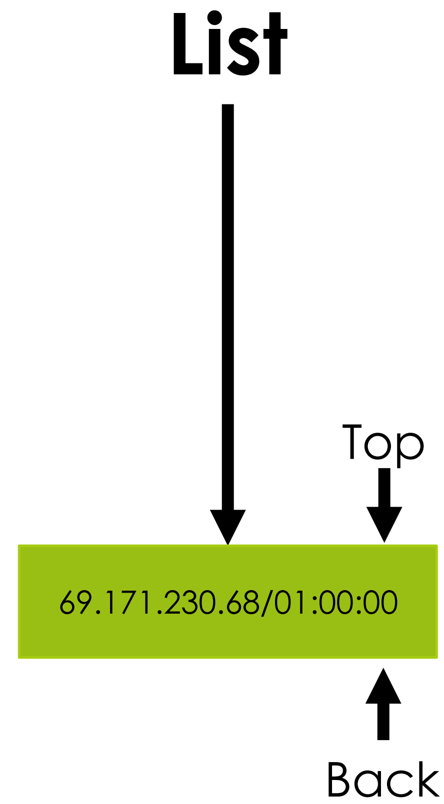
AccessedLastHour(69.171.230.68)?

AccessCountLastHour(69.171.230.68)?

Access Log

Current Time: 01:45:16

Time	IP address
00:45:13	173.194.71.102
00:45:13	69.171.230.68
01:00:00	69.171.230.68
01:45:13	173.194.71.102
01:45:13	91.210.105.134



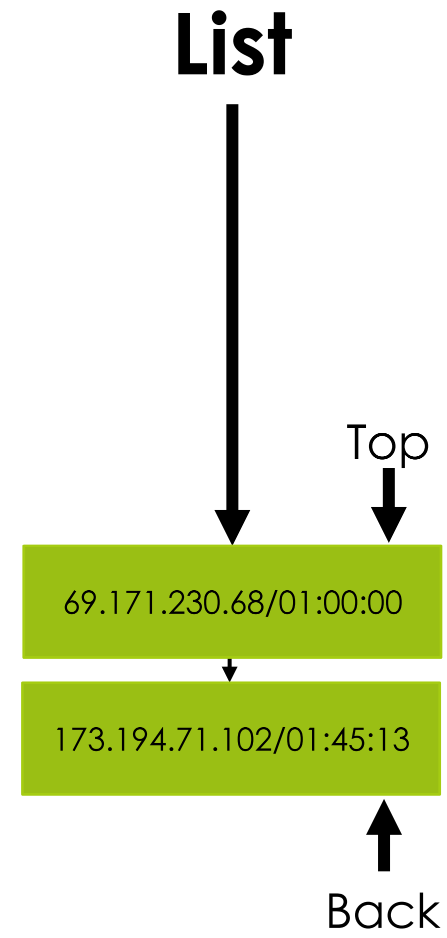
AccessedLastHour(69.171.230.68)?

AccessCountLastHour(69.171.230.68)?

Access Log

Current Time: 01:45:17

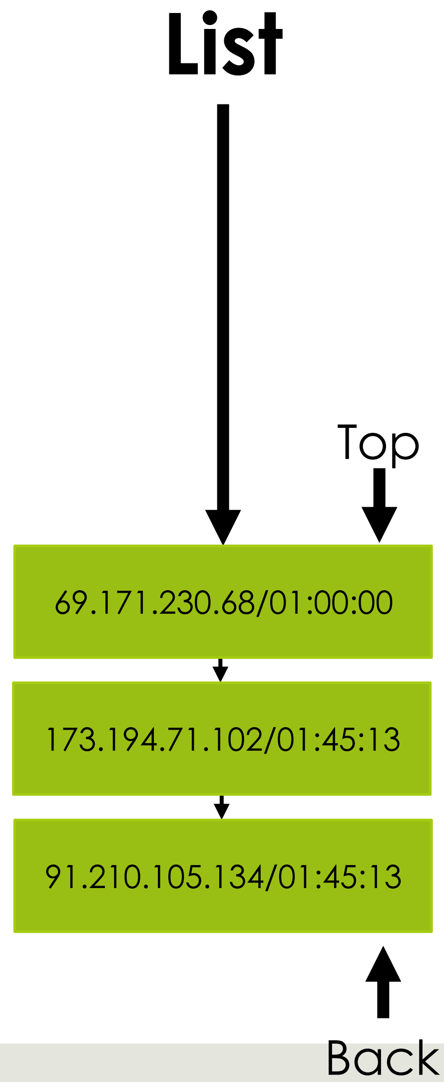
Time	IP address
00:45:13	173.194.71.102
00:45:13	69.171.230.68
01:00:00	69.171.230.68
01:45:13	173.194.71.102
01:45:13	91.210.105.134



Access Log

Current Time: 01:45:18

Time	IP address
00:45:13	173.194.71.102
00:45:13	69.171.230.68
01:00:00	69.171.230.68
01:45:13	173.194.71.102
01:45:13	91.210.105.134



UpdateAccessList(*log*, *i*, *L*)

UpdateAccessList(*log*, *i*, *L*)

```
while log[i].time ≤ Now()
     L.Append(log[i])
     i ← i + 1
while L.Top().time ≤ Now() - 3600
     L.Pop()
```

AccessedLastHour and AccessCountLastHour

AccessedLastHour(*IP*, *L*)

return *L*.FindByIP(*IP*) ≠ NULL

AccessCountLastHour(*IP*, *L*)

return *L*.CountIP(*IP*)

Big O Analysis of List-based Implementation

- n is number of active IPs
- Memory usage is O(n)
- `L.Append, K.Top, L.pop` are O(1)
- `UpdateAccessList` is O(1) per log line
- `L.FindByIP` and `L.CountIP` are O(n)
- `AccessedLastHour` and `AccessCountLastHour` are O(n)

Encoding (Hashing) IPs

- Encode/Hash IPs with small numbers
- For example, numbers from 0 to 999
- Different codes for currently active IPs

Hash Function

Definition

For any set of objects S and any integer $m > 0$, a function $h : S \to \{0, 1, \ldots, m-1\}$ is called a hash function.

Definition

m is called the cardinality of hash function h.

Bonus Time!!!

Desirable Properties

- **h** should be fast to compute
- Different values for different objects
- Direct addressing with $O(\mathbf{m})$ memory
- Want small cardinality **m**
- Impossible to have all different values if number of objects in *the universe* is more than **m**

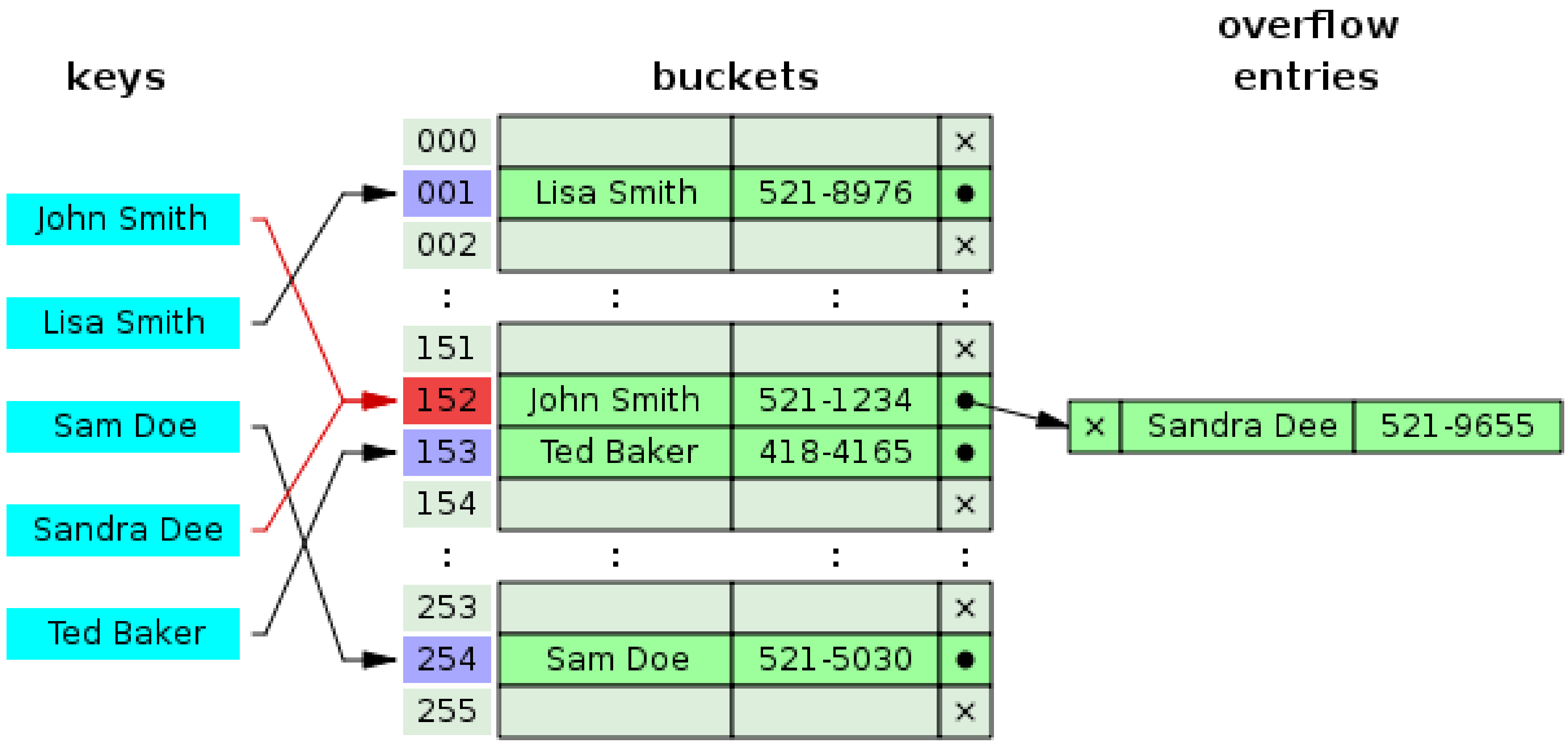
Collisions

Definition

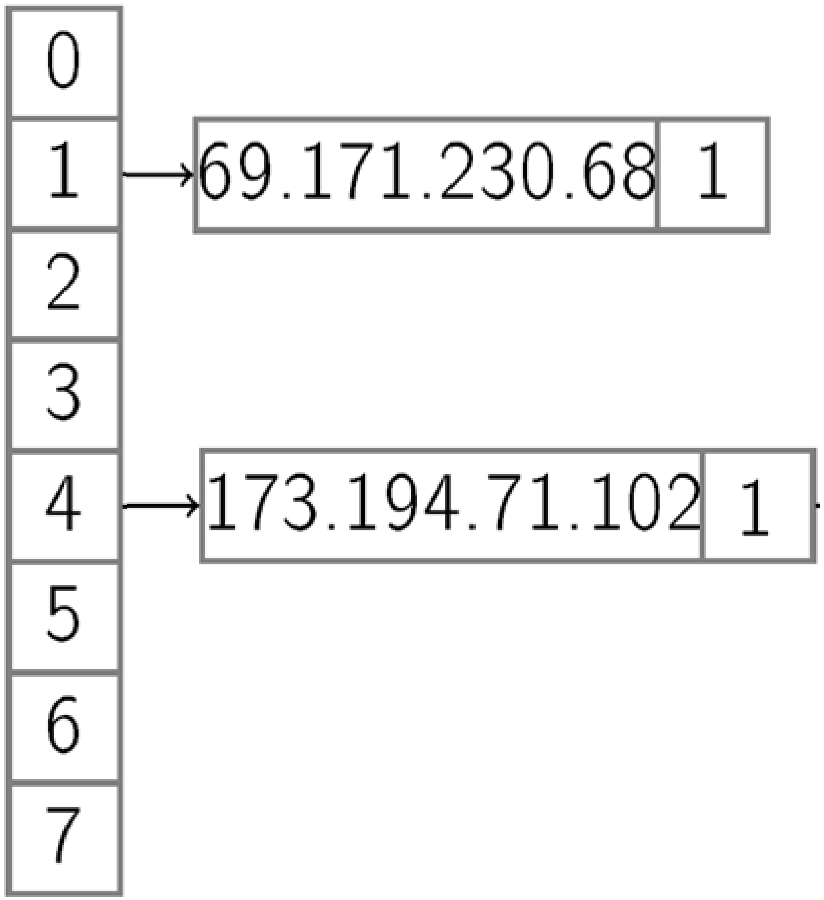
When $h(o_1) = h(o_2)$ and $o_1 \neq o_2$, this is a collision.

Collision Resolution of a Hash Table

Separate Chaining



Back to our example



$$h(173.194.71.102) = 4$$

$$h(69.171.230.68) = 1$$

$$h(173.194.71.102) = 4$$

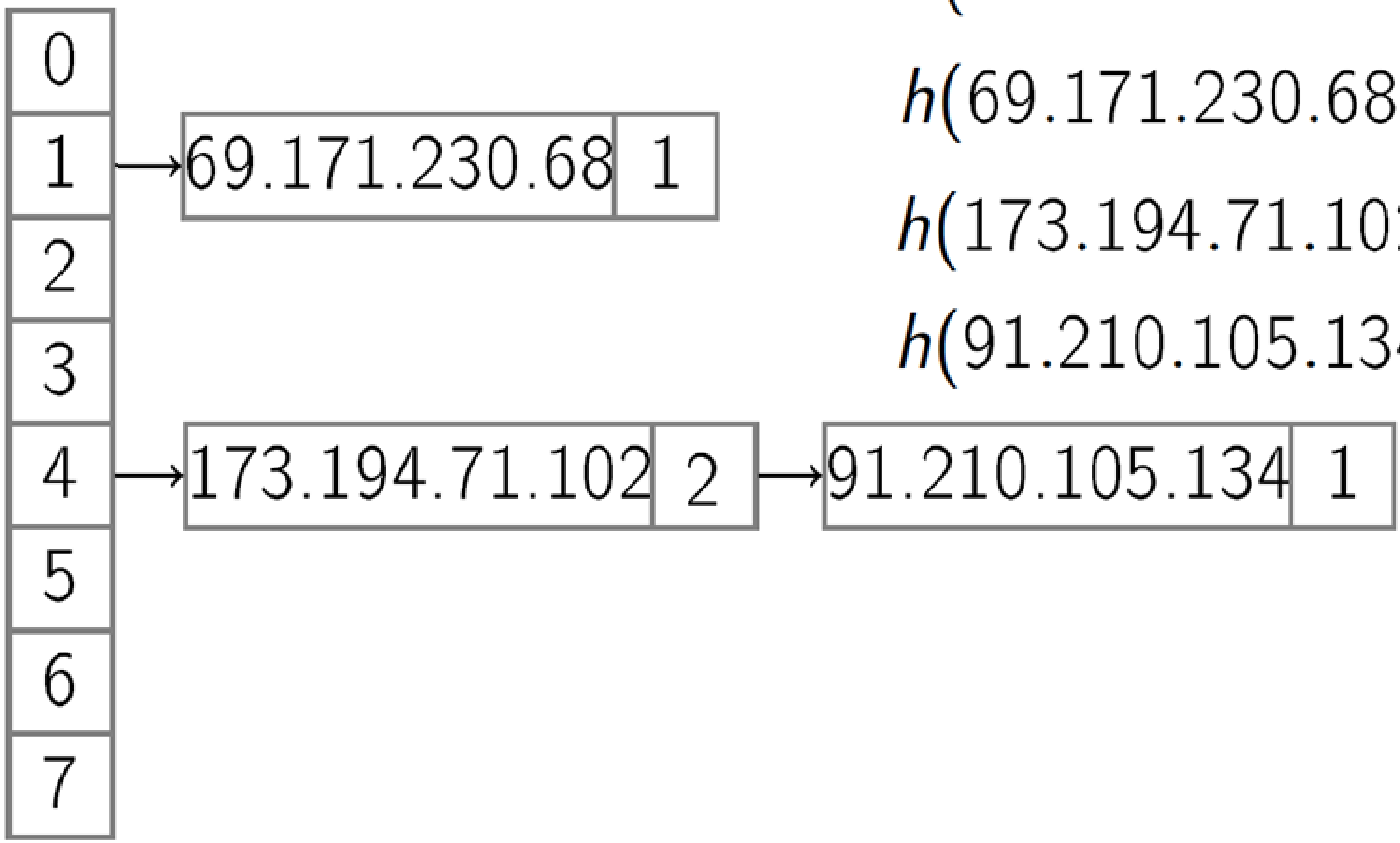
There is no collision

$$h(173.194.71.102) = 4$$

$$h(69.171.230.68) = 1$$

$$h(173.194.71.102) = 4$$

$$h(91.210.105.134) = 4$$



Map Data Structure

Dictionary Data Structure

Associative Array Data Structure

are Hash Table

Map (Dictionary, Associative Array)

- Arrays store items as an ordered collection and you can access thems with an index number.
- Maps store items in (Key, Value) pairs that you can access values by the corresponding keys (usually Strings)
  - Filename → location of the file on disk
  - Student ID → student name
  - Contact name → contact phone number
  - The first object is called "Key" or O
  - The second object is called "Value" or v
  - Map can have the following operations `HasKey(O)`, `Get(O)`, `Set(O,v)`

Map Data Structure

Map217

```
{
    {"580610615", "Kanokwan Pinthong"},
    {"580610616", "Kawewut Chujit"},
    {"580610618", "Kittitorn Rakpanyakeaw"}
}
```

- HasKey("580610615")
- HasKey("580610617")
- Get("580610618")
- Set("580610618", "Kittitat Boonkarn")
- Get("580610618")

Demo: Java HashMap

https://www.w3schools.com/java/java_hashmap.asp

Map Data Structure

Dictionary Data Structure

Associative Array Data Structure

can be implemented using Hash Table

$h : S \to \{0, 1, \ldots, m-1\}$

$O, O' \in S$

$v, v' \in V$

$A \leftarrow$ array of m lists (chains) of pairs (O, v)

`HasKey`(O)

$L \leftarrow A[h(O)]$

```
for (O′, v′) in L:
  if O′ == O:
    return true
return false
```

Get(O)

```
L ← A[h(O)]
for (O', v') in L:
    if O' == O:
        return v'
return n/a
```

$\texttt{Set}(O, v)$

$L \leftarrow A[h(O)]$

$\texttt{for } p \texttt{ in } L\texttt{:}$

$\quad \texttt{if } p.O == O\texttt{:}$

$\quad\quad p.v \leftarrow v$

$\quad\quad \texttt{return}$

$L.\texttt{Append}(O, v)$

Runtime Analysis

Lemma

Let c be the length of the longest chain in A. Then the running time of `HasKey`, `Get`, `Set` is $O(c+1)$.

Runtime Analysis

Lemma

Let n be the number of different keys O currently in the map and m be the cardinality of the hash function. Then the memory consumption for chaining is $\Theta(n + m)$.

Set

Definition

Set is a data structure with methods `Add`(O), `Remove`(O), `Find`(O).

Examples

- IPs accessed during last hour
- Students on campus
- Keywords in a programming language

Implementing Set

$h : S \to \{0, 1, \ldots, m-1\}$

$O, O' \in S$

$A \leftarrow$ array of m lists (chains) of objects O

Find(O)

$L \leftarrow A[h(O)]$

for O' in L:

  if $O' == O$:

    return true

return false

Implementing Set

$\mathtt{Add}(O)$

$L \leftarrow A[h(O)]$

$\mathtt{for}\ O'\ \mathtt{in}\ L:$

$\quad \mathtt{if}\ O' == O:$

$\qquad \mathtt{return}$

$L.\mathtt{Append}(O)$

Implementing Set

$\texttt{Remove}(O)$

$\texttt{if not Find}(O)\texttt{:}$

  $\texttt{return}$

$L \leftarrow A[h(O)]$

$L.\texttt{Erase}(O)$

Hash Table

Definition

An implementation of a set or a map using hashing is called a hash table.

Programming Languages

Set:

- `unordered_set` in C++
- `HashSet` in Java
- `set` in Python

Map:

- `unordered_map` in C++
- `HashMap` in Java
- `dict` in Python

Conclusion

- Chaining is a technique to implement a hash table
- Memory consumption is $O(n + m)$
- Operations works in time $O(c + 1)$
- How to make both m and c small?