Today's content

- GCD intro
- properties of G.C.D
- GCD function
- GCD problems
  - GCD of entire array.
  - Delete one.
  - Pubg

GCD → Greatest Common Divisor / Highest Common Factor

gcd(a,b) → greatest factor which divides both a and b.

gcd(a,b) = x ⇒ a%x = 0 b%x = 0 x → highest no. following this property.

gcd(20,65)

1	1
2	5
4	13
5	65
10	
20	

gcd(-10,20)

~~-10~~	1
~~-5~~	2
~~-2~~	4
~~-1~~	5
1	10
2	20
5	
10	

gcd(-10,-5)

~~-10~~	~~-5~~
~~-5~~	~~-1~~
~~-2~~	1
-1	5
1	
2	
5	
10	

gcd(0,8)

1	1
2	2
3	4
4	8
5	
6	
7	
8	
9	
⋮	
∞	

gcd(0,-10)

1	~~-10~~
2	~~-5~~
3	~~-2~~
4	~~-1~~
5	1
6	2
7	5
8	10
9	
10	
⋮	
∞	

Properties of GCD

① $gcd(a,b) = gcd(b,a)$

② $gcd(a,b) = gcd(|a|,|b|)$

③ $gcd(0,a) = |a|$

④ $gcd(1,a) = 1$

⑤ $gcd(a,b,c) = gcd(gcd(a,b),c) = gcd(gcd(a,c),b)$

$= gcd(gcd(b,c),a)$

⑥ Special Property

$A, B > 0$ and $A > B$

Given $gcd(A,B) = x \Rightarrow A\%x = 0$ and $B\%x = 0$

$x \rightarrow$ highest factor

$gcd(A-B,B) = x$

$(A-B)\%x = 0$ $\quad$ $B\%x = 0$

$x \rightarrow$ highest factor to hold this property

$\Rightarrow (A\%x - B\%x + x)\%x$ ($A\%x \rightarrow 0$, $B\%x \rightarrow 0$)

$\Rightarrow 0$

$gcd(23,5) \rightarrow gcd(18,5) \rightarrow gcd(13,5) \rightarrow gcd(8,5) \rightarrow gcd(3,5)$

$\therefore gcd(23,5) = gcd(3,5)$

$A, B \geq 0$. $A \geq B$.

$$gcd(A, B) = gcd(B, A-B)$$
$$= gcd(B, A-2B)$$
$$= gcd(B, A-3B)$$
$$= gcd(B, A-4B)$$
$$\vdots$$
$$gcd(B, A - xB)$$

x → quo div

$A - xB$ → dvd − quo div

$$\boxed{gcd(A, B) = gcd(B, A \% B)}$$

$gcd(15, 6) \rightarrow 3$ (A = 15, B = 6)

$gcd(6, 3) \rightarrow 3$ (A = 6, B = 3)

$gcd(3, 0)$ (A = 3, B = 0)

```
int gcd(int a, int b){
    if (b==0) { return a }
    return gcd(b, a%b)
}
```

① What if B > A ?

gcd(A=6, B=21) → 3 ans.

gcd(A=21, B=6) → 3

gcd(A=6, B=3) → 3

gcd(A=3, B=0)

② Both base conditions required ?

gcd(A=7, B=0) → return 7

gcd(A=0, B=8) → return 8.

gcd(A=8, B=0)

③

```
int gcd(int a, int b){
    if(a==0){return b}
    if(b==0){return a}
    return gcd(a%b, b)
}
```

X

gcd(A=15, B=6)

gcd(A=3, B=6)

gcd(A=3, B=6)

gcd(A=3, B=6)

gcd(3, 6)

1

1

infinite Recursion

Stack overflow

N → N/2 → N/4 → N/8 → N/16 → - -

iterations = $\log_2 N$

$A > B$, $A, B > 0$

$$gcd(a, b) = gcd(\underbrace{a \% b}_{< b}, b)$$

$b < a/2$	$b = a/2$	$b > a/2$
$a \% b < b < a/2$ ⇓ $\boxed{a \% b < a/2}$	$a \% b < b$ $\boxed{a \% b < a/2}$	$b > a/2$ $2b - a > 0$ multiply both sides -1 $a - 2b < 0$ add a on both sides $a + a - 2b < a$ $2a - 2b < a$ $2(a - b) < a$ $a - b < a/2$ $\boxed{a \% b < a/2}$

$$a \% b = \begin{cases} a - b \\ a - 2b < 0 \\ a - 3b < 0 \\ \vdots \\ a - xb < 0 \end{cases}$$

$\boxed{\text{T.C} \rightarrow O(\log_2 \max(A, B)) \quad \text{S.C} \rightarrow O(\log_2 \max(A, B))}$

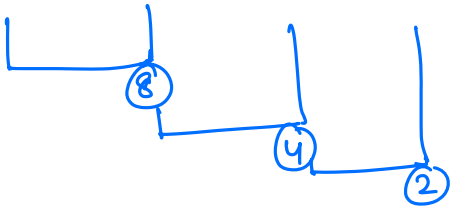
[Break → 10:44 → 10:49]

Q: Given arr[N]. Calculate GCD of entire array.

arr[3]: [6 12 15]

ans = 3.

arr[4]: [8 16 12 10]



ans = 2.

code.

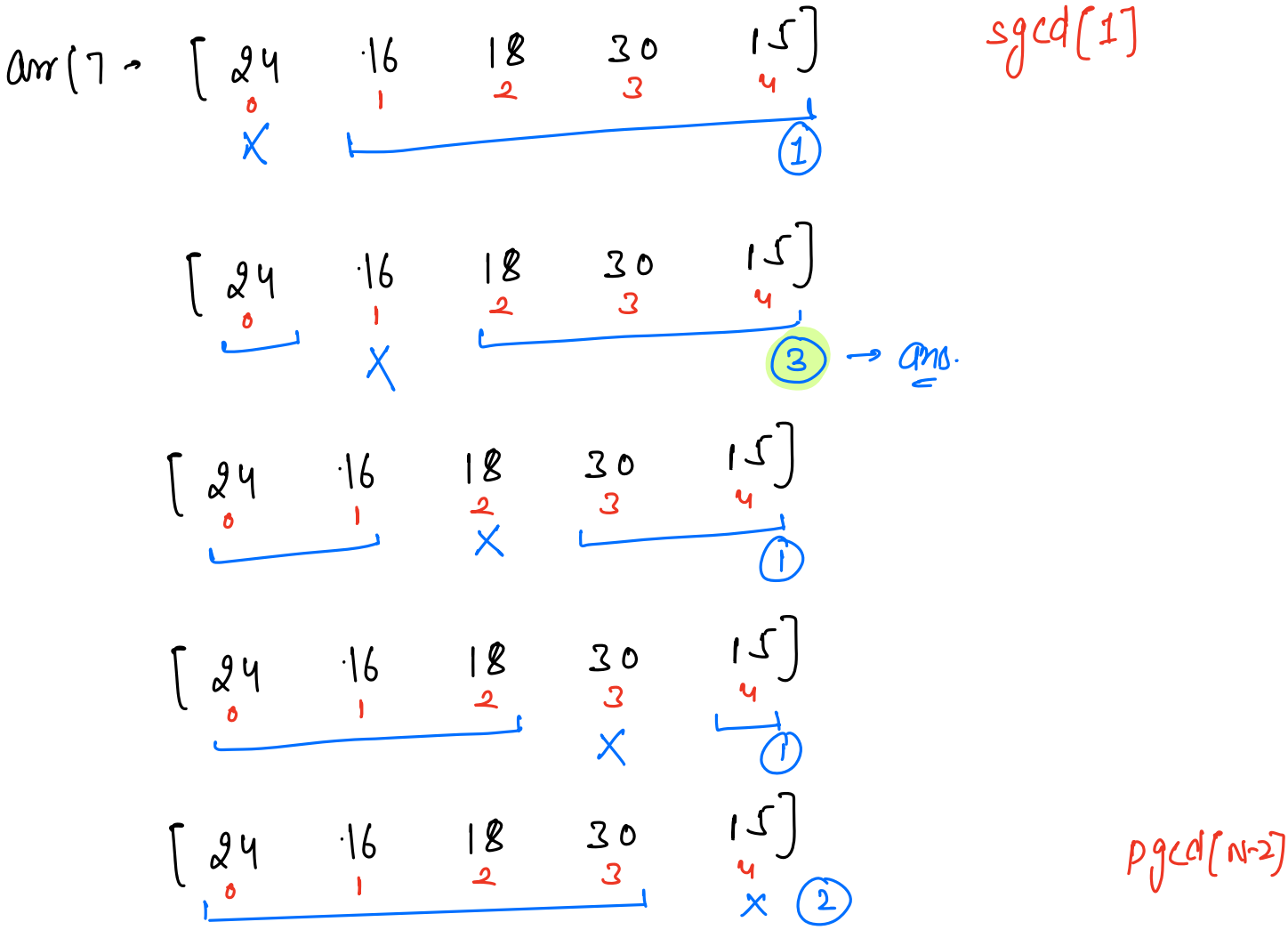
```
ans = arr[0];
for( i = 1; i < N; i++){
    ans = gcd( ans, arr[i]);
}
return ans;
```

T.C → $O(N \cdot \log_2 max)$

S.C → $O(\log_2 max)$

Delete One.

Given N elements, we have to delete 1 element, such that gcd of remaining elements is maximum.



idea → use prefix gcd and suffix gcd.

Pseudo-code:

```
pgcd[N], pgcd[0] = arr[0];

for( i = 1; i < N; i++){
    pgcd[i] = gcd( pgcd[i-1], arr[i]);
}

sgcd[N] , sgcd[N-1] = arr[N-1]

for( i = N-2; i >= 0; i--){
    sgcd[i] = gcd( sgcd[i+1] , arr[i]);
}

//try removing all the elements one by one
ans = max( sgcd[1], pgcd[N-1]); //edge case

for( i = 1; i < N-1; i++){
    left = pgcd[i-1]
    right = sgcd[i+1]
    ans = Max( ans, gcd(left, right));
}

return ans;
```

Need to remove 1 value that's why sgcd[1]

$$T.C \rightarrow O(N \log_2 max)$$
$$S.C \rightarrow O(N + \log_2 max)$$

Q. N players playing a game & each player has a health of A[i] (for i^{th} player).

If player i attacks player j, then →

a) if (A[i] >= A[j]) → player j will die.

b) if (A[i] < A[j]) → A[j] = A[j] - A[i]

Find the minimum health of last surviving player.

① ②
arr = [10, 6]

1 attacks → 2 player 2 will die

health of last player → 10

2 attacks → 1

① ②
[4, 6]

② attacks ① player 1 will die

health of last player → 6

observation:

Player with less health should attack the player with greater health.

① attacks ②

① ②
[4, 2]

① attacks ② player 2 will die

health of last player → 4

② attacks ①

① ②
[2, 2]

↳ ① attacks ② player 2 will die
health of last player = 2

② attacks ① player 1 will die
health of last player = 2

arr[] → [9, 6, 15]

↓

[3, 6, 9] → [3, 3, 6] → [3 3] → [3]

a, b. → a-b, b. } ⇒ multiple times → gcd(a, b)

ans → G.C.D of entire array

```
ans = arr[0];
for( i = 1; i < N; i++){
    ans = gcd( ans, arr[i]);
}
return ans;
```

T.C → $O(N \cdot \log_2 max)$
S.C → $O(\log_2 max)$

Problem Solving Session [Doubts Arrays & B.M]

→ questions which are least solved

→ Very large power

5th Aug. 2023 → 11AM

$$\left\{ \max(\log A, \log B) \Leftrightarrow \left[\log_2 \max(A, B) \right] \right\}$$

$N \leq 10^5$

$A[i] \leq 10^9$

$1 \leq B \leq 10^6$

[15-20) [30-40)

Inter → Adv.

E-M M-H.

→ [Take hints
→ video explanation]

4×7 → 28 problem.

3 sections → 7 avg. ⇒ 21-23 problem

int a = 10^5.

long A = ((long)a * a) + 2 ✓

$10^5 * 10^5$ = 10^{10} ⇒ overflow