Time Complexity-1

$\boxed{9{:}05}$

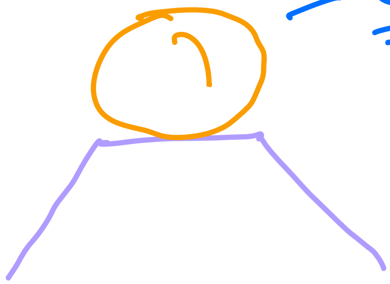
Vinay Neekhra

Senior Instructor & Mentor

Reachable in Scaler Lounge

"Big O notation is the ruler that
measures the growth of algorithms, reminding us that
efficiency is not just a number, but a mathematical marvel"

(*) magic ball, which can answer any question you have.


Oracle

toss a coin, will I get a tail or head,

→ Head, tail

→ will this magic ball be always powerful?

Hint: toss a coin, magic ball answers as after the coin already landed

⇒ time

Any question?

1. → question tab.
2. answers → private chat
→

Q. $[a, b]$ ⇒ both a and b are inclusive

(a, b) ⇒ a, b are excluded

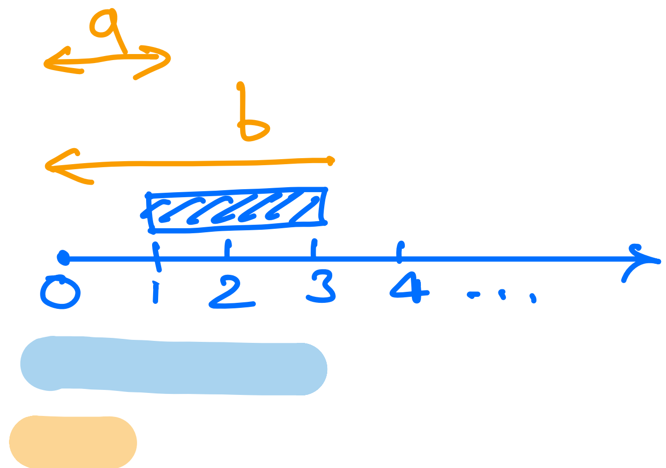
$[a,b) \Rightarrow$ a is included, b is excluded.

$\Rightarrow \boxed{[a, b] \Rightarrow b-a+1}$

$\Rightarrow [1, 3] \Rightarrow 3-1+1 = 3$

$\downarrow$

1, 2, 3



$[3, 10] \Rightarrow 10-3$

$7+1 = 8$

Q. $\log_2 N \Rightarrow$ the number, when it's placed as power of 2, gives N.

$\Rightarrow \log_2 4 = 2$

$2^x = 4 \Rightarrow x = ?$

$\log_2 N \Rightarrow$ the no. of times we can divide N by 2 till it becomes 1.

$\log_2 1024 = 10$ $\rightarrow$ ~~1000~~, $\underline{1024}$

$\log_2 51 = 5$ $[\; 2^5 \Rightarrow 32$

$2^6 \Rightarrow 64$

$2^{5.\sim} = 51$

$[0, 100] \Rightarrow 101$ $[1, 100)$

$[0)$

⊛ Arithmatic Progression

$$\underset{\longleftrightarrow 3}{4,} \quad \underset{\longleftrightarrow 3}{7,} \quad \underset{\longleftrightarrow 3}{10,} \quad \underset{\longleftrightarrow 3}{13,} \quad \underset{\longleftrightarrow 3}{16,} \quad 19$$

$[a + 0d \rightarrow 1$

$a + 1d \rightarrow 2$

$a + (n-1)d \rightarrow \underline{n^{th}}$

$a, \quad a+d, \quad a+2d, \; \ldots\ldots \quad a+(n-1)*d$

$1^{st}, \quad 2^{nd}, \quad 3^{rd} \quad \ldots\ldots \quad N^{th}$ term

$a \rightarrow$ first term

$d \rightarrow$ difference

$N \rightarrow$ number of items

$$\boxed{\text{Sum of AP for N terms} = \frac{N}{2} * [2a + (n-1)*d]}$$

Sum of the first natural numbers. $(1 \rightarrow \infty)$

$[1, 2, 3 \ldots N]$

$\Rightarrow a = 1$

$\Rightarrow d = 1$

N

$$\Rightarrow \frac{N}{2} * \left(2*1 + (n-1)*1\right)$$

$$= \frac{N}{2} * (N+1)$$

$$= \frac{n(n+1)}{2}$$

(*) Geometric Progression:

$$5 \xrightarrow{\times 2} 10 \xrightarrow{\times 2} 20 \xrightarrow{\times 2} 40 \xrightarrow{\times 2} 80 \xrightarrow{\times 2} 160 \ldots$$

$a, \; ar, \; ar^2 \ldots \quad a \cdot r^{n-1}$

$a \rightarrow$ first term
$r \rightarrow$ ratio
$N \rightarrow$ number of terms

Sum of GP series with N terms

$$= \frac{a\,(r^n - 1)}{(r-1)}$$

area of circle
$= \pi r^2$

$$\Rightarrow \int_0^{2\pi} \frac{1}{2} r^2 \, d\theta$$

$$\Rightarrow \frac{1}{2} r^2 [\theta]_0^{2\pi}$$

$$\Rightarrow \frac{1}{2} r^2 [2\pi - 0]$$

$$\Rightarrow \pi r^2$$

Quiz

```
int fn (N) {
    S=0
    for (i=1; i<=N; i++) {
        S += i
    }
    return S
```

instructions
$a = b$
$a = 5+3$
$a = 3+5$

$i \rightarrow \overbrace{(1, 2, 3, \ldots N)}^{N}$

}

no. of iterations = N times

O(N)

Q.
```
int fn (N) {
    s = 0
    for (i = 1; i <= N; i = i+2) {
        s = s + i;
    }
    return s;
}
```

$i \rightarrow 1, 3, 5, 7, 9, \ldots$ (+2, +2)

iterations = all the odd no.(s) b/n [1, N]

$\approx N/2$

$\Rightarrow \frac{1}{2} \times \boxed{N} = O(N)$

$N = 7 \Rightarrow [1, 2, 3, 4, 5, 6, 7] \Rightarrow 4 \quad \left(\frac{N+1}{2}\right)$

$\left(\frac{7+1}{2}\right) = 4$

$N = 6 \Rightarrow [1, 2, 3, 4, 5, 6) \Rightarrow 3 \rightarrow \left(\frac{6+1}{2}\right) = 3$

Q. fn(N){

```
fn(N){
    for (i=0; i<=100; i++) {
        S=S+i+i^2
    }
    return S
}
```

iterations = 101

$= 101 * N^0$

$= O(1)$

Constant TC.

Quiz

```
for (i=1; i*i <=N; i++) {
    ___
}
```

$\sqrt{N}$ times

$\Rightarrow O(\sqrt{N})$

$i*i <= N$

$i^2 <= N$

$\Rightarrow \boxed{i <= \sqrt{N}}$

$i_{max} = \sqrt{N}$

$i \rightarrow [1 \rightarrow \sqrt{N}]$

iterations $= \sqrt{N}$

Q: fn (N) {

```
i = N
while (i > 1) {
    i = i / 2
}
}
```

i_{before}	iterations	i_{after}
N	1^{st}	$N/2 \rightarrow N/2^1$
$N/2$	2^{nd}	$N/4$ $\quad N/2^2$
$N/4$	3^{rd}	$N/8$
$N/8$	4^{th}	$N/16$
	⋮	⋮
	K	$1 \rightarrow \frac{N}{2^K}$

$$N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \frac{N}{8} \rightarrow \dots \quad 1$$

$\log N$

$$1 = \frac{N}{2^K}$$

$$\Rightarrow 2^K = N$$

$$\Rightarrow \log_2 2^K = \log_2 N \quad \rightarrow 2^? = 2^K$$

$$\Rightarrow K = \log N$$

$\Rightarrow O(N)$

$\Rightarrow O(\sqrt{N})$

02

iterations = $\log_2 N \Rightarrow O(\log N)$

$O(N) \Rightarrow$ N = 100, → x time

→ N = 200 → 2x

$O(\sqrt{N}) \Rightarrow$ N = 100 → x time

→ N = 10000 → x^2 time

Count of factors:

Algo 1 → 10^{18} → 317 years

Algo 2 → 10^{18} → 10 secs.

10^9 → 1 sec

10^9 → 1 msec

N

$\log_2(1024) \Rightarrow 10$ + 1 unit

$\log_2(2048) \Rightarrow 11$

10:34

$O(N) \rightarrow 2 * N$

$O(\frac{1}{2}N) \rightarrow O(N)$

N > 0

Q. for (i=0; $\boxed{i<=N}$; i = i*2){
}

i_b	iteration	i_{after}
0	1st	0
0	2nd	0
0	3rd	0
0	4th	0
	⋮	
	∞	

TLE

Q. for (i=1; $\boxed{i<=N}$; i = i*2){
}

i_b	iteration	i_{after}
1	1st	2^1
2	2nd	$4 \to 2^2$
4	3rd	$8 \to 2^3$
	⋮	
	K	$\to 2^K$

$$2^K = N$$

$$\Rightarrow \log_2 2^K = N$$

$$\Rightarrow K = \log_2 N$$

iterations = $\log_2 N$

$N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \frac{N}{8} \rightarrow \dots \quad 1$

$\log N$

Q. for (i=1; i<=N; i++) {
  for(J=1; J<=N; J=J*2) {
    ...
  }
}

i	J	# iteration
1	(1, log N)	log N +
2	(1, log N)	log N +
⋮		+
N	(1, log N)	log N

iterations = ⇒ $N \log N$ = $O(N \log N)$

Q for (i=1; i<=10; i++){

```
for (J=1; J<=N; j++) {
    —
}
}
```

i	J	# iteration
1	[1, N]	N +
2	[1, N]	N +
⋮		
10	[1_N]	N +
		10N

iterations = 10N ⇒ O(N)

Q
```
for (i=1; i<=N; i++){
    for (J=1; J<=N; j++) {
        —
    }
}
```

i	J	# iteration
1	[1, N]	N +
2	[1, N]	N +
⋮	⋮	⋮
N	[1,N]	N +
N+1	0	0 iteration

iterations = $N * N = N^2 \Rightarrow O(N^2)$

Q

```
for (i=1; i<=N; i++ ){
    for (J=1; J<=2^i; J++ ) {
        ...
    }
}
```

i	J	# iterations
1	[1, 2]	2
2	[1, 4]	4
3	[1, 8]	8
⋮	⋮	
N	$[1, 2^N]$	2^N

$$\#\text{iterations} = 2 + 4 + 8 + \dots + 2^N$$

$$= 2^1 + 2^2 + 2^3 + \dots + 2^N$$

$$\text{Sum} = \frac{a(r^n - 1)}{r-1} = \frac{2(2^N - 1)}{(2-1)} = 2^*(2^N - 1)$$

$$2 \cdot 2^N - 2 \Rightarrow O(2^N)$$

Q.

```
for (i=N; i>0; i=i/2) {
    for (J=1; J<=i; J++) {
        ...
    }
}
```

i	J	# iterations
N	[1-N]	N +
N/2	[1-N/2]	N/2 +

$N/4$ | $[1, N/4]$ | $N/4 +$
$\vdots$
1 | $[1,1]$ | 1 +

$$\#\text{iterations} = \overbrace{N + \frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \dots + 1}^{\text{no. of terms}} \rightarrow \text{How?}$$

$O(N)$

Sum?

$$\Rightarrow N\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\right] \approx 2N$$

$$\boxed{N + \frac{N}{2} + \frac{N}{3} + \frac{N}{4} + \dots}$$

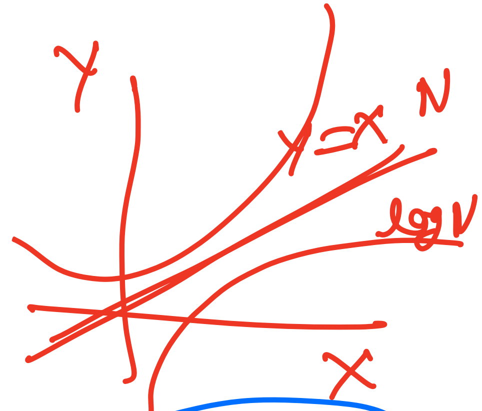
$$\underbrace{O(\quad)}_{\text{Big-O}}$$

$\Rightarrow$ How to calculate big-O from from the no. of iterations

1. neglect all lower order terms (+ addition)
   (i.e keep only highest power of N)
2. Neglect all constant coefficients → (× multiplication)

iterations $= 4N^2 + 2N + 100$

$\rightarrow O(N^2)$



Q. $4N^2 + 3N + 6\sqrt{N} + 9\log N + 10$

$\Downarrow$

$O(N^2)$

$N > \log N$

$N = 10 \qquad \log N$

$10^{N} \qquad \log_{10} 10$

$10 > 1$

$10^6 > 6 \log_{10} 10^6$

100

$\rightarrow \quad \sqrt{N} \quad N$

$\qquad 10 \quad 100$

$$\boxed{\log N < \sqrt{N} < N < N\log N < N\sqrt{N} < N^2 < N^3 < 2^N < N! < N^N}$$

$N \rightarrow N+1$

Growth of function

$10^{10} < 11^{11}$

these

Doubt session:

① log issues → NCERT log chapter

$N^2 < 2^N$

$\Rightarrow N \to 2N$

$\Rightarrow 4N^2 \quad 2^{2N}$

$N = 100$

$N^N \quad 2^N$

$100^{100} > 2^{100}$

$N^2 \log N$

$\Rightarrow O(N^2 \log N)$

$N^2 < N^3 < 2^N$

N big Number

$\Rightarrow$ 100 $\quad 100^2 \quad 100^3 \quad 2^{100}$

$N^2 < N^3 < 2^N$

$N^{1\,million} < 2^N$

very large N

$N^N ? 2^N$

$N > 2 \qquad N^N > 2^N$

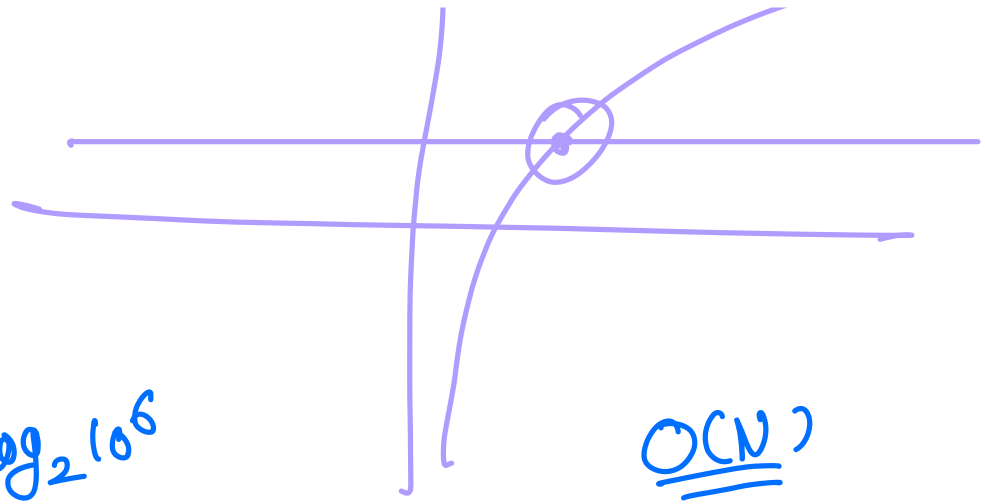
$\cancel{N} \log N > \cancel{N} \log 2$

$\Rightarrow \log N > \log 2$

N

$\Rightarrow$ A2 $\Rightarrow$ α $N \log N$

A1 $\Rightarrow$ α N



$N \Rightarrow 10^6$ $\Rightarrow$ A_2 : $10^6 \log_2 10^6$

A_1 = 10^6

$O(N)$

$\Rightarrow 10N$

A_2 : $10^7 \cdot \log_2 10^7$

$= 10^7$

(growing more than 10 times

$O(\sqrt{N})$

$10^{18} \rightarrow 10^9$ - 1olee

$O(N^2)$

$\Rightarrow 10^6 * N^2$

N

$N/2$

A_1 $\qquad$ A_2

$10^6 N^2$ $\qquad$ N^3

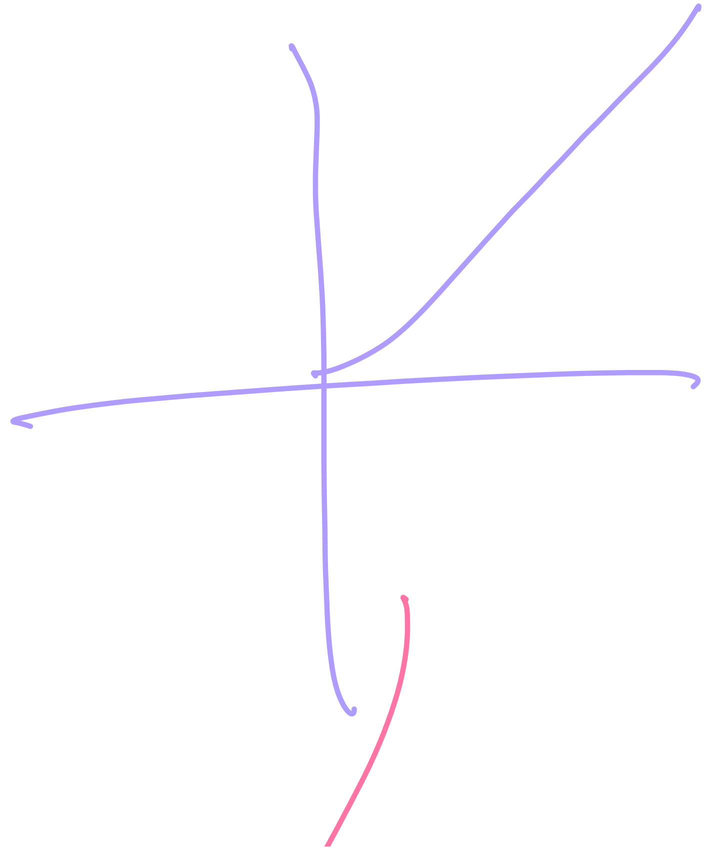
$N \Rightarrow 1$ $\qquad$ $\boxed{10^6}$ $\qquad$ 1

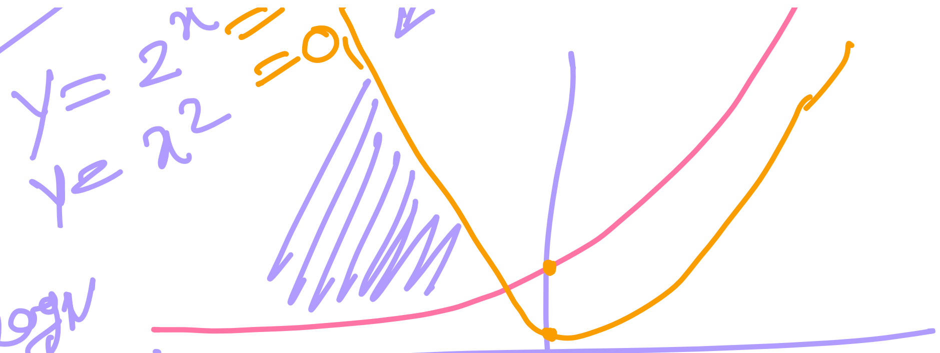
$\boxed{N \Rightarrow 10^9}$ $\qquad$ $10^6 \times 10^{18} \Rightarrow \boxed{10^{24}} < 10^{27}$ $\qquad$ 1000 times more time

$N! \qquad N^{N-1} < N^N$

$N! \qquad N^{N-1}$

$\Rightarrow \log(N!) \qquad \underline{N-1 \log N}$





$\Rightarrow \log N + \log N-1 + \log N-2 + \log N$

N

$\log N$

$\log(a \cdot b) \Rightarrow \log a + \log b$

$\Rightarrow (N-1) \log N$

$< \log N$

$< \log N$

$\Rightarrow$

V

$< N \log N$

$(N-1) \log N$

a

$$\boxed{N^a < 2^N}$$

$\Rightarrow$ chen $\rightarrow$ 8×8 $\boxed{64}$

1 percent

$(1.01)^{365} = 31.$

$(0.99)^{365} = 0.0001$

$\Rightarrow$ $\boxed{1, 2, 4, 8}$, 16 . . .

$\Rightarrow$ $2^{64} >$

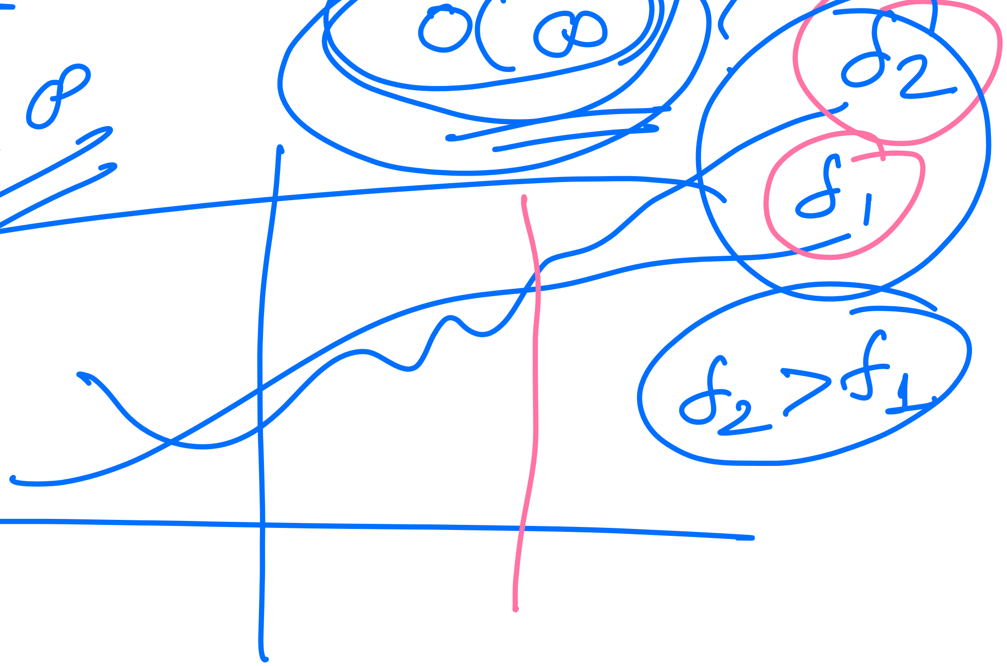
$\approx \infty$

$f_1 = O(f_2)$

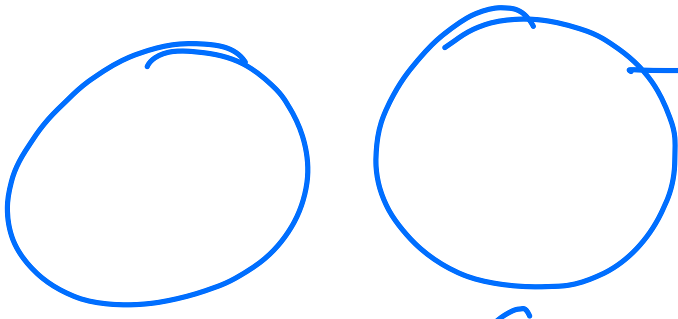
$= O(\infty)$

$2\infty = \infty$

$O(\infty)$



$f_2 > f_1$



S_1 S_2

∞

$S_1 \Rightarrow$ positive numbers

$S_2 \Rightarrow$ all the Integers. $\rightarrow \boxed{\infty}$

$S_2 > S_1$