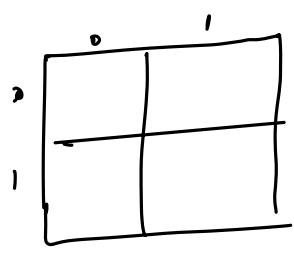
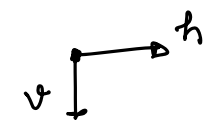
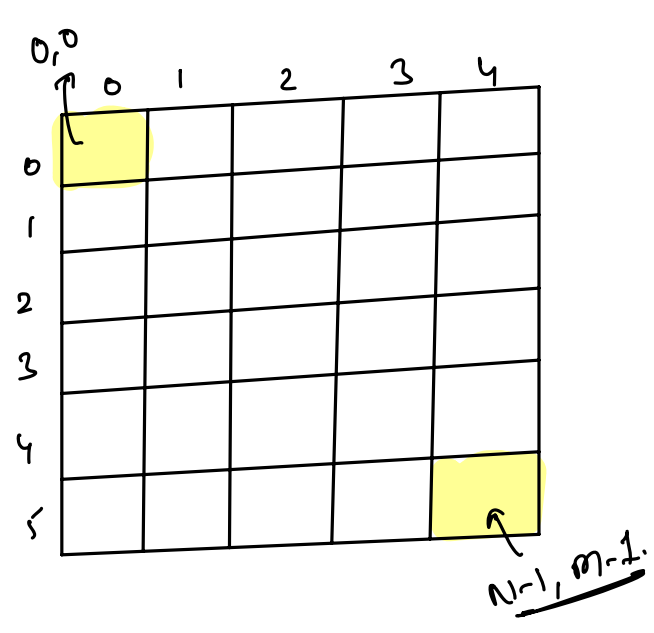
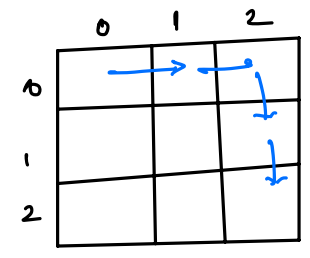


Q1 Given mat[N][M], Total no. of ways from (0,0) to (N-1, M-1).

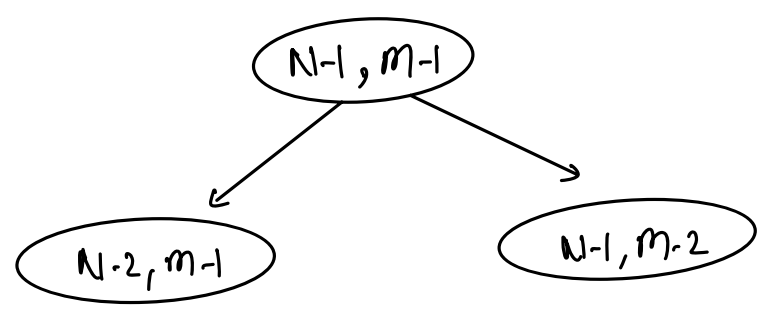


hv, vh.  
Ans = 2

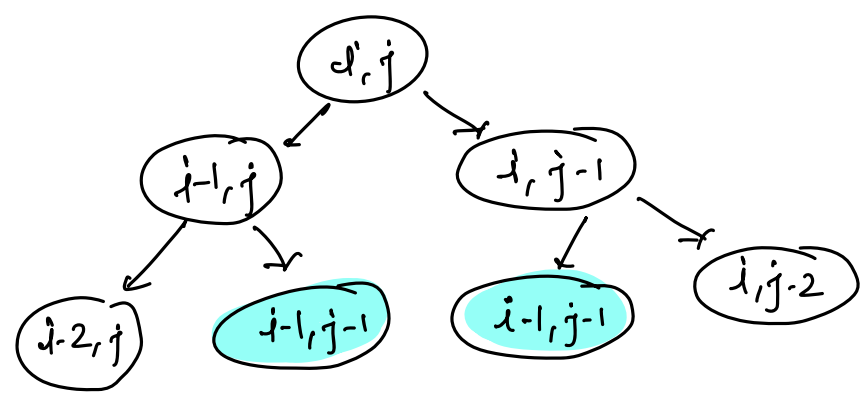


hhvv  
hvhv  
huvh  
vhhv  
vhvh  
vrhh  
ans = 6.

Bf idea. → Try all possible ways to reach N-1, M-1 starting from (0,0).



$$\text{ways}(i, j) = \text{ways}(i-1, j) + \text{ways}(i, j-1)$$



optimal substructure ✓  
overlapping subproblems ✓

∴  
D.P.

# code →

```
int ways ( N-1 i, m-1 j ) {  
    if ( i == 0 || j == 0 ) { return 1 }  
    return ways( i-1, j ) + ways( i, j-1 );  
}
```

$T.C \rightarrow O(2^{N+m})$   
 $S.C \rightarrow O(N+m)$

dp[N][m] //  $\forall dp[i][j] = -1$

```
int ways ( N-1 i, m-1 j, dp[N][m] ) {  
    if ( i == 0 || j == 0 ) { return 1 }  
    if ( dp[i][j] != -1 ) { return dp[i][j] }  
    dp[i][j] = ways( i-1, j, dp ) + ways( i, j-1, dp )  
    return dp[i][j];  
}
```

$T.C \rightarrow O(N \cdot m)$   
 $S.C \rightarrow O(N \cdot m)$

	0	1	2	3	4
0	1	1	1	1	1
1	1	2	3	4	5
2	1	3	6	10	15
3	1	4	10	20	35
4	1	5	15	35	70
5	1	6	21	56	126

ans.

total no. of ways to reach (0,0)  
starting from (0,0)?

↓  
~~0~~ → You can't reach (0,0)  
starting from (0,0)

✓ 1 → Do nothing

$dp[i][j]$  = total no. of ways to reach (i,j) starting from (0,0)

#code:-

initialise row → 0 with 1  
 initialise col → 0 with 1

```

for (i = 1; i < N; i++) {
    for (j = 1; j < m; j++) {
        dp[i][j] = dp[i-1][j] + dp[i][j-1];
    }
}
return dp[N-1][m-1];

```

$T.C \rightarrow O(N \cdot m)$   
 $S.C \rightarrow O(N \cdot m)$

further SC optimisation?

	1	1	1	1	1
1	1	2	3	4	5
2	1	3	6	10	15
3	1	4	10	20	30
4	1	5	15	35	70
5	1	6	21	56	126

Apply psum  $n-1$  times.

# final code

```
int dp[m];     $\forall i, dp[i] = 1;$ 
```

```
for( i = 1; i < n; i++) {
```

```
    for( j = 1; j < m; j++) {  
        dp[j] = dp[j-1] + dp[j];  
    }
```

```
    return dp[m-1];
```

$T.C \rightarrow O(N \times m)$   
 $S.C \rightarrow O(m)$

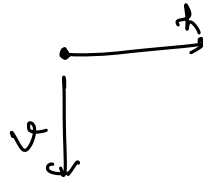
Q2

	0	1	2	3
0	1	1	1	1
1	1	0	1	0
2	0	1	1	1
3	1	0	1	1
4	1	1	1	1

arr[N][M]

arr[i][j] = 0  $\Rightarrow$  cell is blocked

arr[i][j] = 1  $\Rightarrow$  cell is empty.



initialise row  $\rightarrow 0$  with 1  
initialise col  $\rightarrow 0$  with 1

for (i = 1; i < N; i++) {

for (j = 1; j < M; j++) {

if (arr[i][j] == 0) { dp[i][j] = 0 }

else { dp[i][j] = dp[i-1][j] + dp[i][j-1]; }

}

return dp[N-1][M-1];


	0	1	2	3
0	1	1	1	1
1	1	0	1	0
2	0	1	1	1
3	1	0	1	1
4	1	1	1	1




	0	1	2	3
0	1	1	1	1
1	1	0	1	0
2	0	0	1	1
3	0	0	1	2
4	0	0	1	3

$\rightarrow$  # todo  
s.c optimisation

# Dungeons & Princess



	0	1	2	3
0	-3	+2	+4	-5
1	-6	+5	-4	+6
2	-15	-7	+5	-2
3	+2	+10	-3	-4



→  
↓



+ve




-ve

Health level drops to less than or equal to 0, then the person dies.

→ What should the minimum health level with which you can enter top-left cell so that you save the princess.

①



-3	-2
-4	1

X


$$h.l = 1$$

$$1 - 2 = -1 \quad X$$


$$1 - 4 = -3 \quad X$$

② Path with minimum sum

X



	0	1	2	3
0	-3	+2	+4	-5
1	-6	+5	-4	+6
2	-15	-7	+5	-2
3	+2	+10	-3	-4



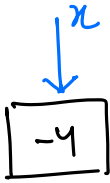
### ③ Path with maximum sum

-2	-8	100
-1	-3	1

91

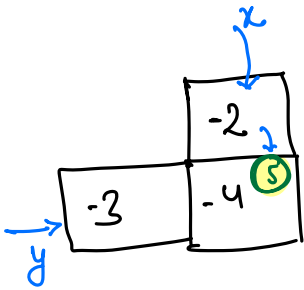


Let's start with smallest problem.



$$x + (-4) = 1$$

$$x = 1 + 4 = 5$$

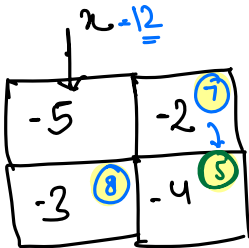


$$x + (-2) = 5$$

$$x = 5 + 2 = 7$$

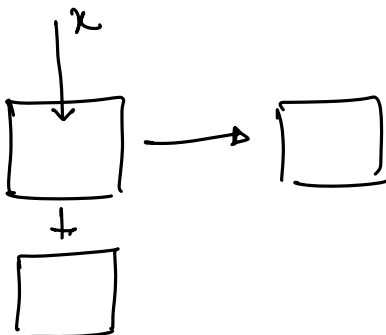
$$y + (-3) = 5$$

$$y = 5 + 3 = 8$$



$$x + (-5) = \min(7, 8)$$

$$x = 7 + 5 = 12$$



$$x + arr[i][j] = \text{Min} \left[ \begin{array}{l} \text{min Energy required to enter } [i+1][j], \\ \text{min Energy required to enter } [i][j+1] \end{array} \right]$$

$$x = \text{Max} \left( 1, \text{Min} \left[ \begin{array}{l} \text{min Energy required to enter } [i+1][j], \\ \text{min Energy required to enter } [i][j+1] \end{array} \right] - arr[i][j] \right)$$

	<u>arr</u>			
	0	1	2	3
0	-3 (4)	+2 (1)	+4 (1)	-5 (6)
1	-6 (7)	+5 (1)	-4 (5)	+6 (1)
2	-15 (16)	-7 (8)	+5 (2)	-2 (7)
3	+2 (1)	+10 (1)	-3 (8)	-4 (5)

$$x + 5 = \text{MM}(7, 8)$$

$$x = 7 - 5 = (2)$$

# code →

dp[N][m];

if (arr[N-1][m-1] > 0) { dp[N-1][m-1] = 1 }

else { dp[N-1][m-1] = Abs(arr[N-1][m-1]) + 1 }

// last row

for (j = m-2; j ≥ 0; j--) {

{ dp[i][j] = Max(1, dp[N-1][j+1] - arr[N-1][j])

}



// last col

```
for( i = N-2; i ≥ 0; i--){  
    dp[i][m-1] = Max(1, dp[i+1][m-1] - arr[i][m-1]);  
}
```

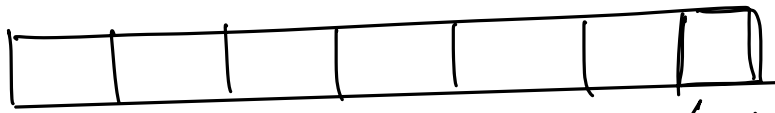
```
for( i = N-2; i ≥ 0; i--){  
    for( j = m-2; j ≥ 0; j--){  
        dp[i][j] = Max(1, Min(dp[i+1][j], dp[i][j+1]) - arr[i][j]);  
    }  
}
```

return dp[0][0];

$T.C \rightarrow O(N \cdot m)$   
 $S.C \rightarrow O(N \cdot m)$

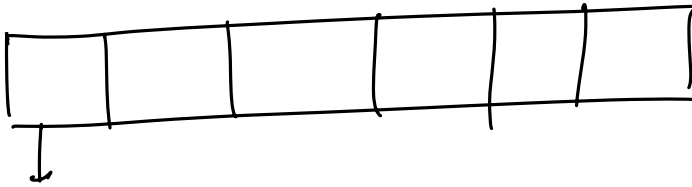
$S.C \rightarrow O(m)$

\_\_\_\_\_x\_\_\_\_\_x\_\_\_\_\_



$$\text{arr}[N-1] + \text{maxSum}(\text{arr}, N-2)$$

$$\text{maxSum}(\text{arr}, N-1)$$



$$(0, \text{arr}[0])$$



$$\forall i \rightarrow 0, N-1 \quad dp[i] = \max \begin{pmatrix} 0 + dp[i-1] \\ \text{arr}[i] + dp[i-2] \end{pmatrix}$$