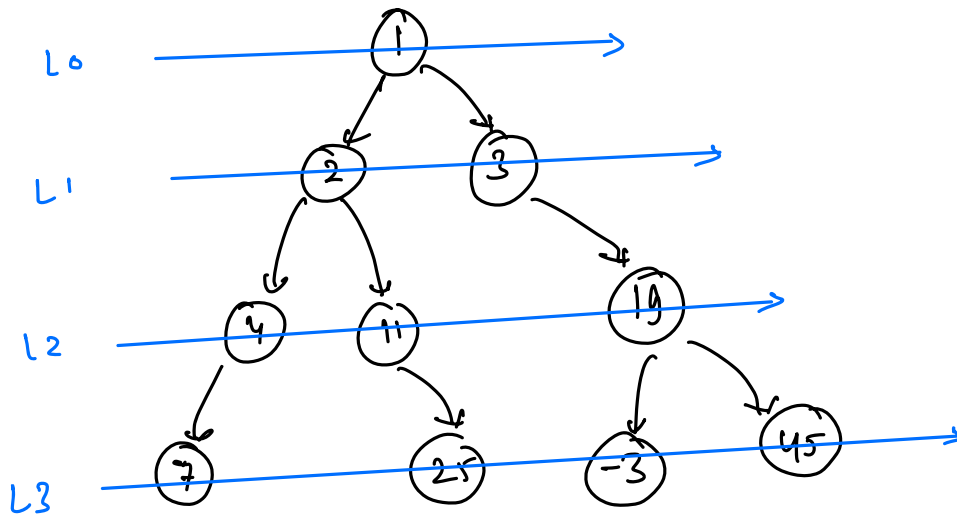


Level Order Traversal

o/p. →

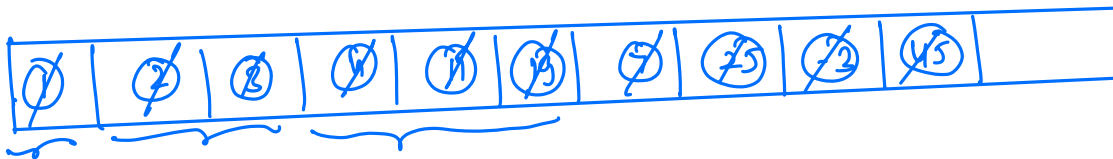


1

2 3

4 11 19

7 25 -3 45



{
1
→ 2 3
→ 4 11 19
→ 7 25 -3 45
} → output.

#code.

```
Queue<Node> q;
```

```
q.enqueue(root);
```

```
while (q.isEmpty() == false) {
```

```
    int sz = q.size();
```

```
    for (i = 1; i ≤ sz; i++) {
```

```
        Node x = q.dequeue();
```

```
        print(x.val);
```

```
        if (x.left != null) { q.enqueue(x.left); }
```

```
        if (x.right != null) { q.enqueue(x.right); }
```

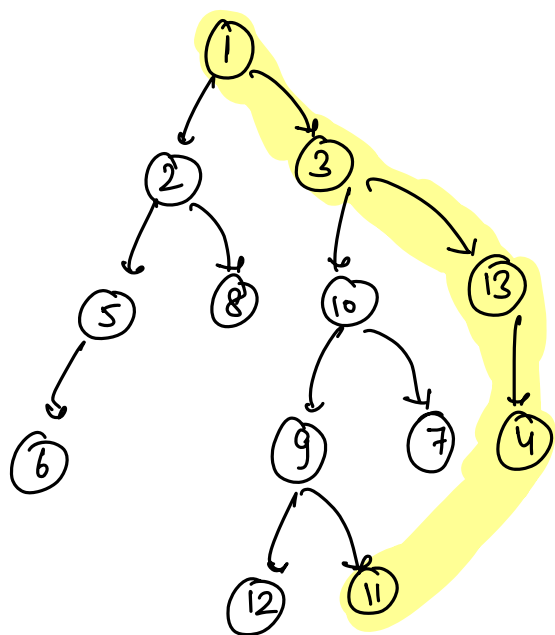
```
    }
```

```
    print("\n");
```

```
}
```

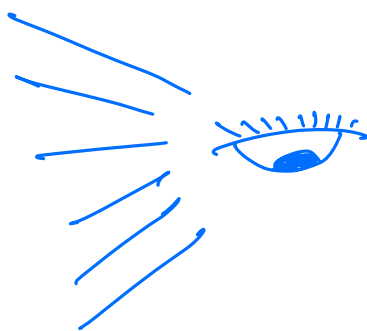
$T.C \rightarrow O(N)$
 $S.C \rightarrow O(N)$

Q: Find right view of binary tree.



o/p. →

1, 3, 13, 4, 11



Solution - Print the last element of every level.

```
Queue < Node > q;
```

```
q.enqueue(root);
```

```
while (q.isEmpty() == false) {
```

```
    int sz = q.size();
```

```
    for (i = 1; i ≤ sz; i++) {
```

```
        Node x = q.dequeue();
```

```
        if (i == sz) { print(x.val); }
```

```
        if (x.left != null) { q.enqueue(x.left); }
```

```
        if (x.right != null) { q.enqueue(x.right); }
```

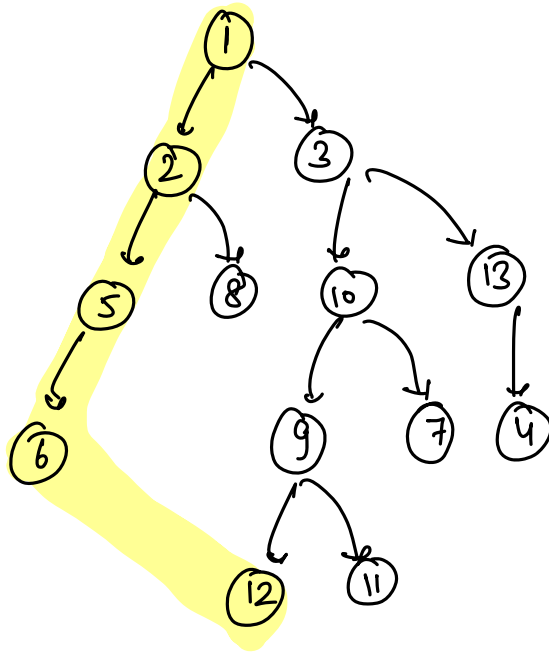
```
    }
```

```
    print("\n");
```

```
}
```

$T.C \rightarrow O(N)$
 $S.C \rightarrow O(N)$

Q1 Left View of Binary tree



o/p. →

1, 2, 5, 6, 12.

Solution → print first element of every level.

```
Queue <Node> q;
```

```
q.enqueue(root);
```

```
while (q.isEmpty() == false) {
```

```
    int sz = q.size();
```

```
    for (i = 1; i ≤ sz; i++) {
```

```
        Node x = q.dequeue();
```

```
        if (i == 1) { print(x.val); }
```

```
        if (x.left != null) { q.enqueue(x.left); }
```

```
        if (x.right != null) { q.enqueue(x.right); }
```

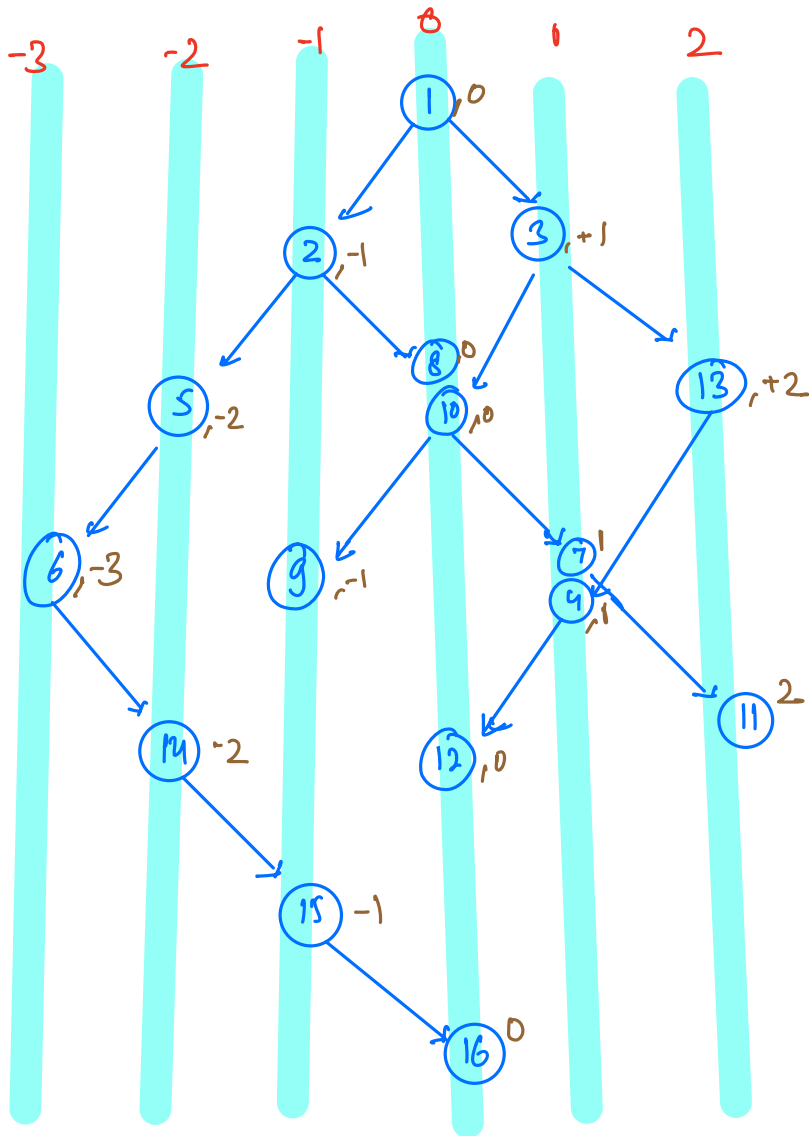
```
    }
```

```
    print("\n");
```

```
}
```

[T.C → $O(N)$
S.C → $O(N)$]

Vertical Order Traversal of Binary Tree



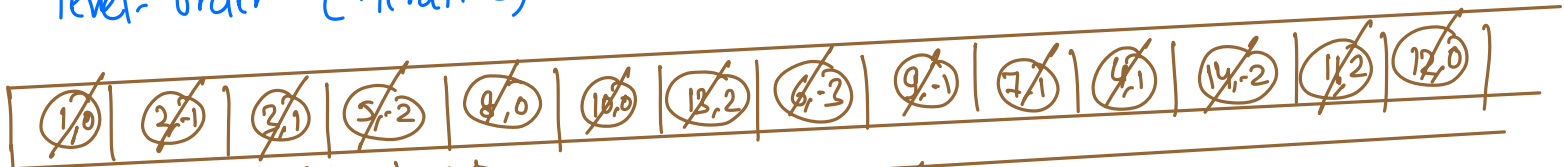
o/p. →

6				
5	14			
2	9	15		
1	8	10	12	16
3	7	4		
13	11			

v.l	All nodes
0	→ 1, 16, 8, 10, 12
-1	→ 2, 9, 15
-2	→ 5, 14
-3	→ 6
1	→ 3, 7, 4
2	→ 11, 13

pre-order (recursion) X

level-order (iterative) ✓



v.l → All elements

0	→ 1, 8, 10, 12, 16
-1	→ 2, 9, 15
1	→ 3, 7, 4
-2	→ 5, 14
2	→ 13, 11
-3	→ 6

Pair < Node, v.l >

```

class Pair {
    Node n;
    int v.l;
}
    
```

#code.→

```
Pair rootpair = new Pair( root, 0);
```

```
Queue < Pair > q ;    q.enqueue( rootpair );
```

```
HashMap < int, List < int > > map ;    minvl = 0 , maxvl = 0
```

```
while ( q.isEmpty() == false ) {
```

```
    Pair rp = q.dequeue();
```

```
    minvl = Min( minvl, rp.vl ) , maxvl = Max( maxvl, rp.vl );
```

```
    Insert rp.node.val in the hashmap against rp.vl
```

```
    if ( rp.node.left != null ) {
```

```
        [ q.enqueue( new Pair( rp.node.left, rp.vl - 1 ) );
```

```
    if ( rp.node.right != null ) {
```

```
        [ q.enqueue( new Pair( rp.node.right, rp.vl + 1 ) );
```

```
    }
```

```
for( i = minvl ; i ≤ maxvl ; i++ ) {
```

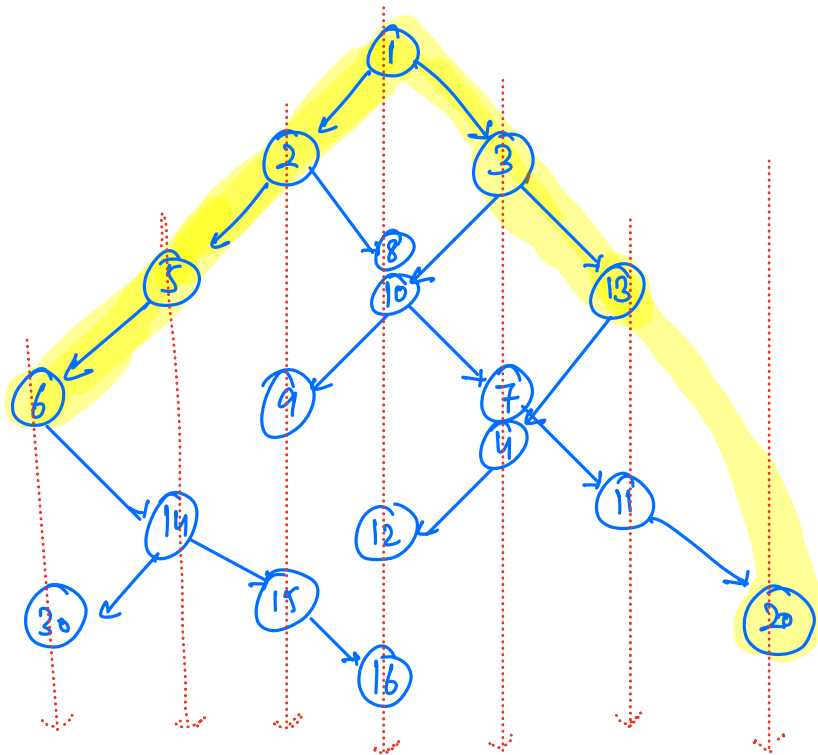
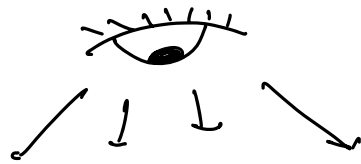
```
    [ print( map[i] );
```

```
    [ print( "\n" );
```

```
    }
```

$T.C \rightarrow O(N)$
 $S.C \rightarrow O(N)$

Q1 Top View of Binary Tree



o/p. →

[6, 5, 2, 1, 3, 13, 20]

v.l	→	All elements
0	→	1, 8, 10, 12, 16
-1	→	2, 9, 15
1	→	3, 7, 4
-2	→	5, 14
2	→	13, 11
-3	→	6
3	→	20

solution → print first node of every vertical level.

Bottom View of a Binary tree

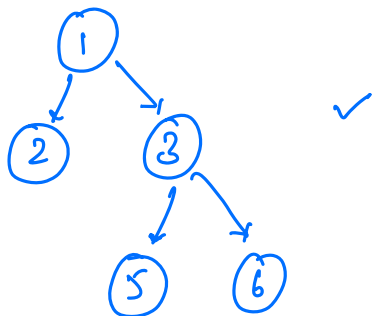
solution → print last node of every vertical level.

Types of Binary Tree (structure)

① Proper / Full Binary Tree

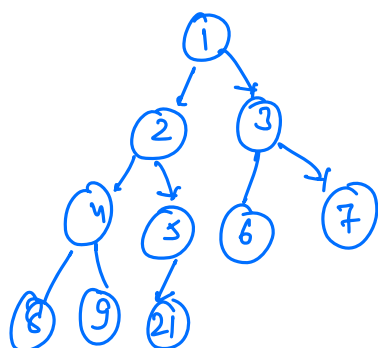
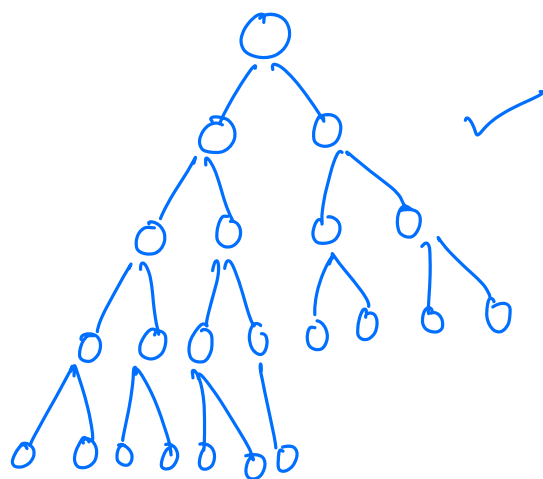
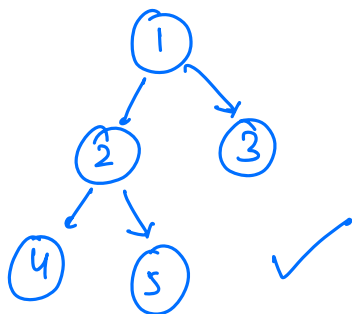
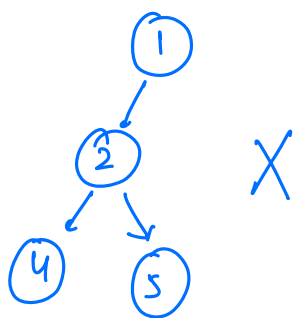
For every node, either 0 or 2 children.

Ex -



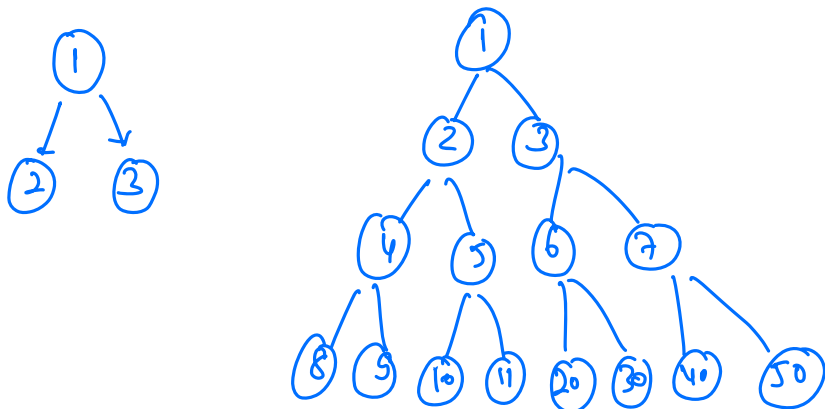
② Complete Binary Tree (C.B.T)

Every level must be completely filled except maybe the last level. In the last level, all nodes are filled from left to right.



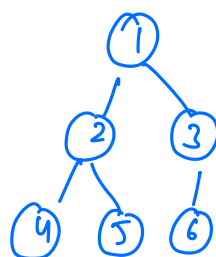
3. Perfect Binary Tree

All levels must be completely filled.



13

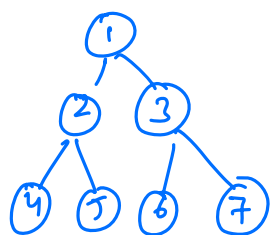
① Are all complete binary trees, also proper tree? No



C.B.T ✓

Proper ✗

② Are all perfect binary trees, also complete tree? Yes.



Perfect Binary Tree ✓

C.B.T ✓

③ Are all perfect binary trees, also proper tree?

Yes.

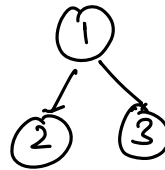
Q. Given a perfect binary tree with N nodes.
Find height of the tree.

$N=1$

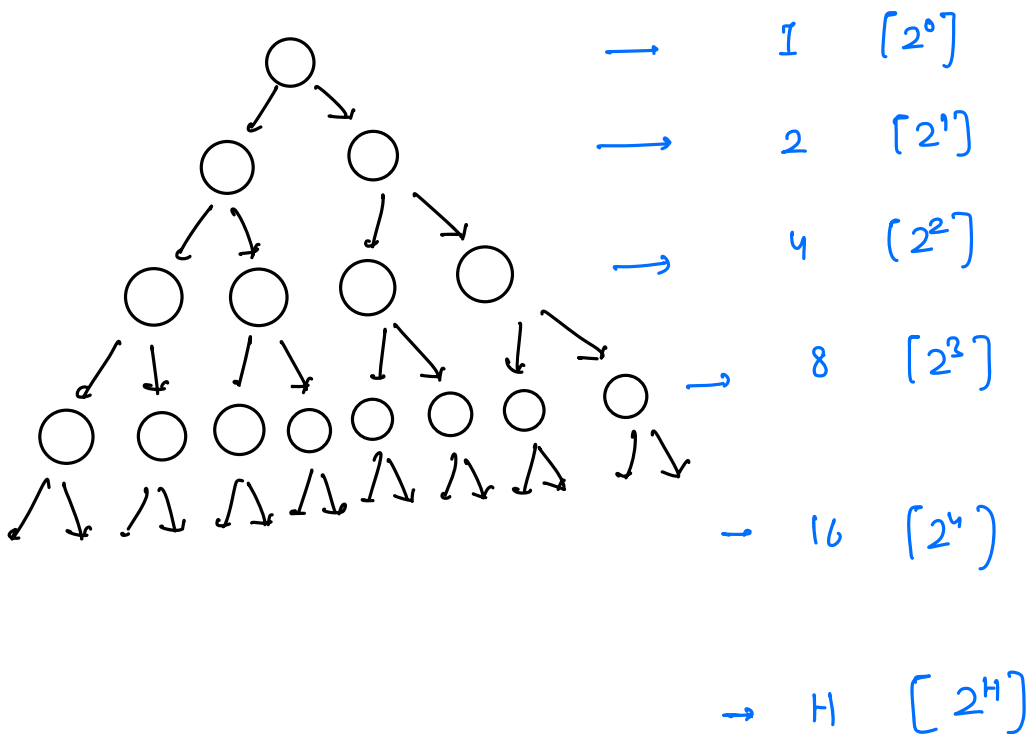


ht = 0

$N=3$



ht = 1



$$2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^H = N$$

$$\frac{2^{H+1} - 1}{(2-1)} = N$$

$a=1, r=2$

no. of terms = $H+1$
terms

$$2^{H+1} = N+1$$

$$H+1 = \log_2(N+1) \Rightarrow$$

$$H = \log_2(N+1) - 1$$

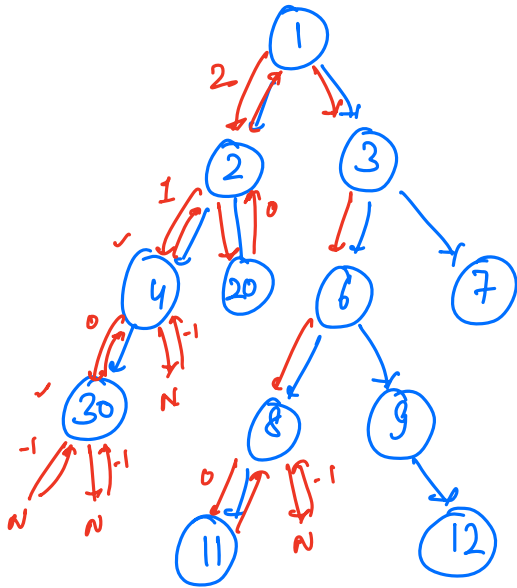
$$[\log N \leq \text{Ht. of tree} < N]$$

Balanced Binary Tree

≠ nodes

$$\left| \begin{array}{cc} \text{nt. of} & \text{nt. of} \\ \text{left} & \text{right} \\ \text{child} & \text{child} \end{array} \right| \leq 1$$

Q1 Given a binary tree, check if it's balanced or not.



Not balanced

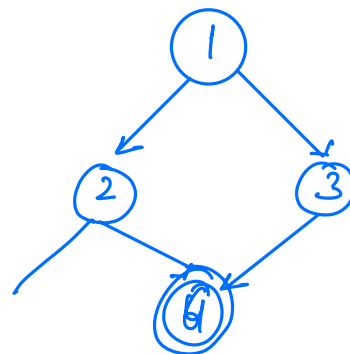
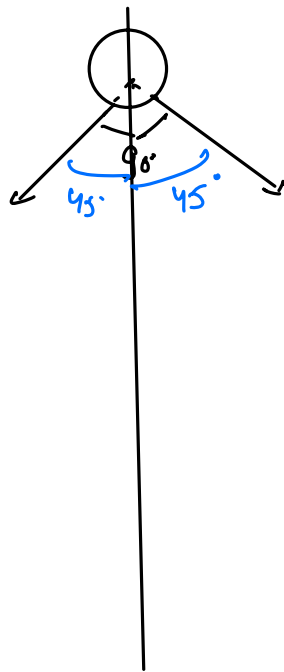
boolean isbalanced \rightarrow true

```
int height(Node root) {  
    if (root == NULL) { return -1; }  
    lht = height(root.left);  
    rht = height(root.right);  
    if (Abs(lht - rht) > 1) { isbalanced = false; }  
    return Max(lht, rht) + 1;  
}
```

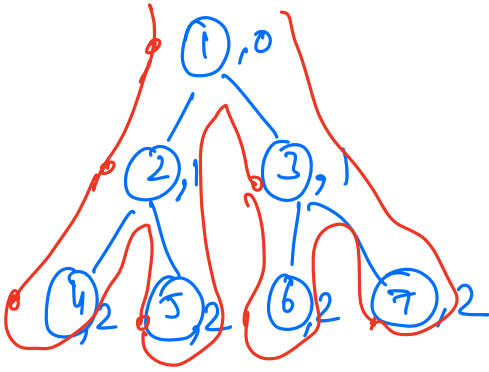
— utilise this function
to check whether
the tree is
balanced or not.

(T.C $\rightarrow O(N)$
S.C $\rightarrow O(h \text{ of tree})$)

#dry-run



level order with recursion?



0 \rightarrow 1

1 \rightarrow 2, 3

2 \rightarrow 4, 5, 6, 7

\rightarrow In-order / pre-order traversal T.C $\rightarrow O(N)$
S.C $\rightarrow O(1)$

\Downarrow
Morris traversal.