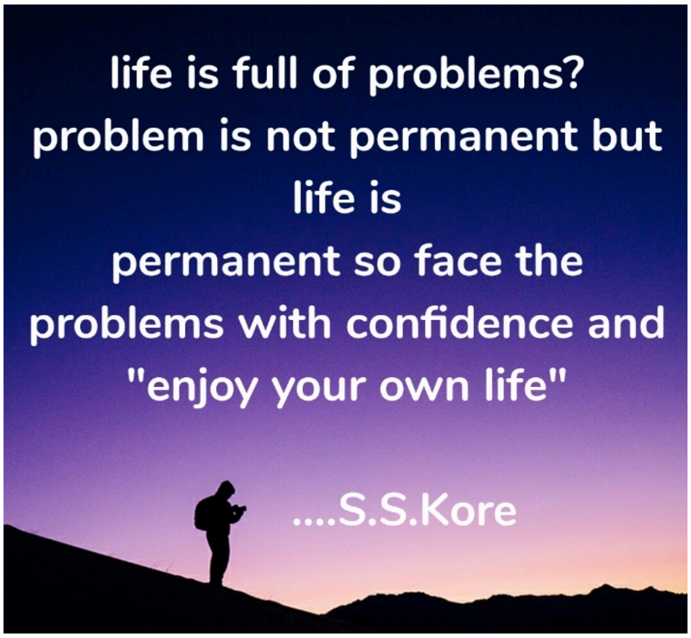


Today's content

- Addition & multiplication rule
- Permutation basics
- Combination basics & properties
  - → $^{N}C_r \% P$, $^{N}C_r \% m$
  - → Excel Column Title.

Given 3 T/F questions, every question have to be answered. In how many ways can we answer all the questions?

F F F
F F T
F T F
F T T
T F F
T F T
T T F
T T T

8 ways.

$\underline{2}\quad \underline{2}\quad \underline{2} = \underline{8}.$

T F   T F   T F

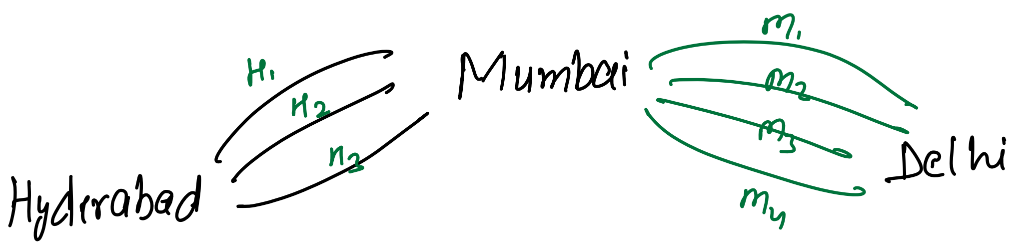
Q → Given 10 Girls & 7 Boys. How many different pairs?

pair → [1G, 1B]

G_1	B_1
G_2	B_2
G_3	B_3
G_4	B_4
G_5	B_5
G_6	B_6
G_7	B_7
G_8	
G_9	
G_{10}	

70 ways.

Eg



No. of ways to reach Delhi from Hyderabad via Mumbai?

$\underbrace{\text{Hyd} \to \text{Mumbai}}_{3}$ and $\underbrace{\text{Mumbai} \to \text{Delhi}}_{4}$

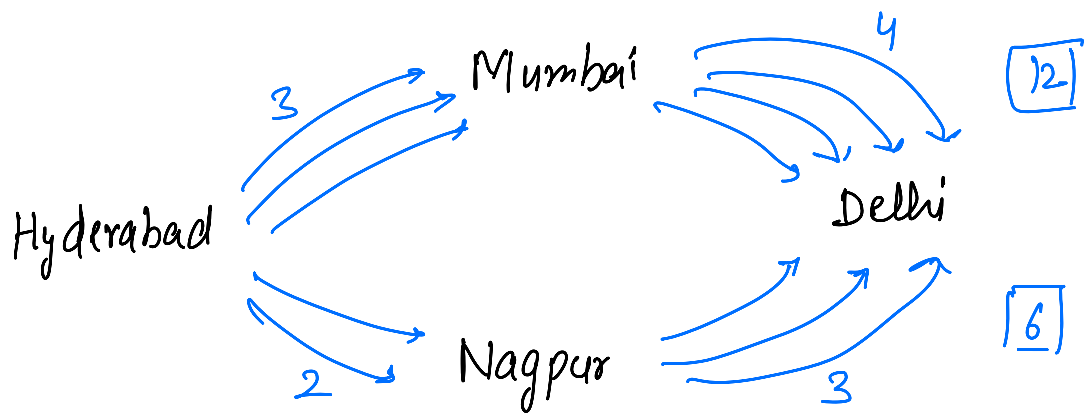
3 * 4 = 12 ways

Q:

Total no. of ways from Hyd. to Delhi?

2 * 3 = 6 ways.

Q:



No. of ways to reach Delhi from Hyderabad.

18 ways.

// Valentine's Gift.

3 Pen | 7 Flowers | 3 Rings

5 Books | 3 Chocolates

① 1 Pen and 1 book ② 1 flower & 1 chocolate ③ 1 ring.

Either one of these 3 gifts.

$(3 * 5) + (7 * 3) + 3 = 39$ ways.

[OR → addition.
AND – multiplication]

Permutations → arrangement of objects.

In general, $(i, j) \neq (j, i)$: order matters.

Q1 Given 3 distinct characters. In how many ways, we can arrange them?

S = "acd"

$\underline{3} * \underline{2} * \underline{1} = 3! = \underline{6}.$

acd
adc
cad
cda
dac
dca

Q2 In how many ways, you can arrange 4 distinct characters?

S = "abcd"

$\underline{4} * \underline{3} * \underline{2} * \underline{1} = 4! = \underline{24}.$

Q) In how many ways n distinct characters can be arranged?

$$N * (N-1) * (N-2) * \_\_\_\_\_\_ 3 * 2 * 1 = \underline{N!}$$

Q) Given 5 distinct characters, in how many ways can we arrange 2 characters?

[a b c d e]

$\_ \; \_$

$5 * 4 = 20$ ways

a {b c d e}
b {a c d e}
c {a b d e}
d {a b c e}
e {a b c d}

Q) 3 distinct characters $N * (N-1) * (N-2)$

Q) 4 characters out of N distinct characters?

$$N * (N-1) * (N-2) * (N-3)$$

Q1 Given N distinct characters. In how many ways can we arrange r characters?

$$\frac{N * (N-1) * (N-2) * \text{---} \text{---} (N-r+1) * (N-r) * (N-r-1) * (N-r-2) * \text{---} 2*1}{(N-r) * (N-r-1) * (N-r-2) * \text{---} 2*1}$$

$\Rightarrow$ $\boxed{\frac{N!}{(N-r)!} = {}^{N}P_{r}}$ $\Rightarrow$ no. of ways of arranging r distinct characters out of N distinct characters.

Combinations → [selection] → no. of ways to select

$(i, j) = (j, i) \Rightarrow$ order doesn't matter.

Q) In how many ways can we select 3 players from a pool of 4 players. $[P_1 \; P_2 \; P_3 \; P_4]$

$P_1 \; P_2 \; P_3$

$P_1 \; P_2 \; P_4$

$P_1 \; P_3 \; P_4$

$P_2 \; P_3 \; P_4$

ans = 4.

Q) No. of ways to arrange the players in 3 slots.

Given 4 players → $[P_1 \; P_2 \; P_3 \; P_4]$

$P_1 \; P_2 \; P_3$	$P_1 \; P_2 \; P_4$	$P_1 \; P_3 \; P_4$	$P_2 \; P_3 \; P_4$
$P_1 \; P_3 \; P_2$	$P_1 \; P_4 \; P_2$	$P_1 \; P_4 \; P_3$	$P_2 \; P_4 \; P_3$
$P_2 \; P_1 \; P_3$	$P_2 \; P_1 \; P_4$	$P_3 \; P_1 \; P_4$	$P_3 \; P_2 \; P_4$
$P_2 \; P_3 \; P_1$	$P_2 \; P_4 \; P_1$	$P_3 \; P_4 \; P_1$	$P_3 \; P_4 \; P_2$
$P_3 \; P_1 \; P_2$	$P_4 \; P_1 \; P_2$	$P_4 \; P_1 \; P_3$	$P_4 \; P_2 \; P_3$
$P_3 \; P_2 \; P_1$	$P_4 \; P_2 \; P_1$	$P_4 \; P_3 \; P_1$	$P_4 \; P_3 \; P_2$

$${}^{N}P_r \Rightarrow {}^{4}P_3 \Rightarrow \frac{4!}{(4-3)!} = \frac{4!}{1!} = 24. \quad = {}^{N}C_r * r!$$

$$\therefore \quad {}^{N}P_r = {}^{N}C_r * r!$$

$$\frac{{}^{N}P_r}{r!} = {}^{N}C_r$$

$$\boxed{\frac{N!}{(N-r)! * r!} = {}^{N}C_r}$$

Properties.

① $${}^{N}C_r = \frac{N!}{(N-r)! * r!}$$

$${}^{N}C_{N-r} = \frac{N!}{(N-(N-r))! * (N-r)!}$$

$$= \frac{N!}{r! * (N-r)!}$$

$$\therefore \boxed{{}^{N}C_r = {}^{N}C_{N-r}}$$

② $$\boxed{{}^{N}C_r = {}^{N-1}C_r + {}^{N-1}C_{r-1}}$$

Q: Given N, R, P. Calculate ${}^{N}C_R \% P$

Note → p is prime no

Constraints.

$$\left\{\begin{array}{c} 1 \le (N, R) \le 10^5 \\ R < N < P \\ p \text{ is prime} \end{array}\right\}$$

$p \to 10^9+7$

$${}^{N}C_r \% p = \left(\frac{N!}{(N-r)! * r!}\right) \% p$$

$$= \left(N! * (N-r)!^{-1} * r!^{-1}\right) \% p$$

$$= \left[(N! \% p) * (N-r)!^{-1} \% p * r!^{-1} \% p\right] \% p.$$

$$= \left[\left(N! \% p * \underbrace{(N-r)!^{-1} \% p}_{a^{-1} \% p}\right) \% p * \underbrace{r!^{-1} \% p}_{b^{-1} \% p}\right] \% p.$$

$$\left[a^{-1} \% p \xrightarrow[p \to \text{prime no.}]{\gcd(a,p)=1} a^{p-2} \% p\right]$$ Fermat's theorem

```
long nf = 1
for( i = 1; i <= N; i++){
        nf = (nf * i) % p
}
```

$\underline{(N-R)!^{-1} \% p.}$

$gcd(p, (N-r)!) = 1$

$p \rightarrow$ prime no.

$N < p$

$N - R < p \Rightarrow$ [p is a prime no. greater than $N-R$]

$(N-R)! = 1 * 2 * 3 * 4 * 5 * \text{——} \text{——} \text{——} * (N-R)$

$$(N-R)!^{-1} \% p = \left((N-R)!\right)^{p-2} \% p$$

$$= \big(\underbrace{(N-R)! \% p}_{x}\big)^{p-2} \% p.$$

$\underline{x^{p-2} \% p}$ [using fast-power]

$$(R!)^{-1} \% p = (R!)^{p-2} \% p$$

$$= (\underbrace{R! \% p}_{y})^{p-2} \% p$$

$y^{p-2} \% p$ [using fast power]

code :

```
long nf = 1
for( i = 1; i <= N; i++) {
        nf = (nf * i) % p ;
}
```

N! % p

```
long nmrf = 1
for( i = 1; i <= N-R; i++) {
        nmrf = (nmrf * i) % p ;
}
```

(N-R)! % p

```
long rf = 1
for( i = 1; i <= r; i++) {
        rf = (rf * i) % p
}
```

(R! % p)

```
long b = fastpower( nmrf , p-2 , p);
long c = fastpower( rf, p-2, p);
long ans = ((nf * b) % p * c) % p.
return ans;
```

T.C → O(N + log p)
S.C → O(log p)

```python
class Solution:
    # @param A : integer
    # @param B : integer
    # @param C : integer
    # @return an integer
    def solve(self, A, B, C):
        if A==1 and B==1 and C==1:
            return 0
        import sys
        sys.setrecursionlimit(10**6)
        def fast_power(a,n,c):
            if n==0:
                return 1
            p=fast_power(a,n//2,c)
            if n%2==0:
                return (p*p)%c
            else:
                return (((p*p)%c)*a)%c
        nf=1
        rf=1
        min_d=min(B,(A-B))
        for i in range(1,min_d+1):
            nf=(nf*(A-i+1))%C
            rf=(rf*i)%C
        y=(fast_power(rf,C-2,C))%C
        ans=(nf*y)%C
        return ans
```

Q: Given N, R, M $\left({}^{N}C_{R} \% M\right)$ M → not 0 prime no.

$$\boxed{{}^{N}C_{r} = {}^{N-1}C_{r} + {}^{N-1}C_{r-1}}$$

$$\left[{}^{N}C_{r} \% m = \left({}^{N-1}C_{r} \% m + {}^{N-1}C_{r-1} \% m\right) \% m\right]$$

1.0NC(if r is 1 to n) if we have n==0 then we will get 0.some value some it is 0.
2. (1 to n)NC0 if we have r==0 then we will get ncr==1.

N=4, R=3.

N \ R	0	1	2	3
0	1	0	0	0
1	1	1	0	0
2	1	2	1	0
3	1	3	3	1
4	1	4	6	4 ans

$(i, j) \Rightarrow {}^{i}C_{j}$

$${}^{N}C_{R} = {}^{N-1}C_{R} + {}^{N-1}C_{R-1}$$

$${}^{1}C_{1} = {}^{0}C_{1} + {}^{0}C_{0}$$

$${}^{1}C_{2} = {}^{0}C_{2} + {}^{0}C_{1}$$

$${}^{2}C_{1} = {}^{1}C_{1} + {}^{1}C_{0}$$

$${}^{4}C_{3} \rightarrow {}^{3}C_{2} \rightarrow {}^{2}C_{1} \rightarrow {}^{1}C_{0}$$

{ mat[i][j] = mat[i-1][j] + mat[i-1][j-1]. }

code.

```
long mat[N+1][R+1];

for( j = 0; j <= R ; j++) {
        mat[0][j] = 0;
}
for( i = 0; i <= N; i++){
        mat[i][0] = 1;
}

for( i = 1; i <= N; i++){
        for( j = 1; j <= R; j++){
                mat[i][j] = (mat[i-1][j] + mat[i-1][j-1]) % m
        }
}

return mat[N][R];
```

T.C → O(N*R)
S.C → O(N*R)

Excel →

Find N^{th} column title.

N =	1	2	3	26	27	28	29	--	52	53	54	--
	A	B	C	Z	AA	AB	AC	--	AZ	BA	BB	BZ

N=30 ⇒ AD

N=100 "CV"

N=50 ⇒ AX

1 → A	AA	BA	CA	---	ZA
2 → B	AB				
3 → C	AC				
4 → D	AD				
5 → E					
24 → X					
25 → Y	AY				
26 → Z	AZ	BZ	CZ		ZZ

0	10	20	30	40	90
1	11	21	31		
2	12	22			
3	13	23			
4	14	24			
5	15	25		—	
6	16	26			
7	17	27			
8	18	28			
9	19	29	39	49	99

$$
\begin{array}{r|l}
2 & 34 \\ \hline
2 & 17 \rightarrow 0 \\ \hline
2 & 8 \rightarrow 1 \\ \hline
2 & 4 \rightarrow 0 \\ \hline
2 & 2 \rightarrow 0 \\ \hline
2 & 1 \rightarrow 0 \\ \hline
 & 0 \rightarrow 1
\end{array}
$$

$$(34)_{10} = (100010)_2$$

$$
\begin{array}{r|l}
26 & 30 \\ \hline
26 & 1 \rightarrow 4\,[D] \\ \hline
 & 0 \rightarrow 1\,[A]
\end{array}
$$

$$(30)_{10} = (AD)_{26}$$

$$
\begin{array}{r|l}
26 & 50 \\ \hline
26 & 1 \rightarrow 24\ [X] \\ \hline
 & 0 \rightarrow 1\ [A]
\end{array}
$$

$$(50)_{10} = (AX)_{26}$$

$$
\begin{array}{r|l}
26 & 100 \\ \hline
26 & 3 \rightarrow 22\ [V] \\ \hline
 & 0 \rightarrow 3\ [C]
\end{array}
$$

$$(100)_{10} = (CV)_{26}$$

$$
\begin{array}{r|l}
26 & 456 \\ \hline
26 & 17 \rightarrow 14 \; [N] \\ \hline
 & 0 \rightarrow 17 \; [Q]
\end{array}
$$

Q.N

676

678 ✓

$$
\begin{array}{r|l}
26 & 52 \\ \hline
26 & \cancel{2}\ 1 \quad 0 \times 26 ✓ \; [Z] \\ \hline
 & 0 \quad 1 ✓ \; [A]
\end{array}
$$

AZ

A → 1
B → 2
C → 3
X → 24
Y → 25
Z → 26

Remainder → [0 → 25]

$$
\begin{array}{r|l}
26 & 78 \\ \hline
26 & \cancel{3}2 \rightarrow [0] \times \rightarrow \underline{26} \quad [Z] \\ \hline
 & 0 \rightarrow 2 \checkmark \quad [B]
\end{array}
$$

$$\boxed{(78)_{10} = (BZ)_{26}}$$

code →

1 → A
2 → B
3 → C
4 → D
|
26 → Z

map.

```
String ans = "";
while( N != 0){
    rem = N % 26
    N = N/26 ;
    if (rem == 0){ rem = 26 , N = N-1 }
    ans += map.get(rem);
}
return ans.reverse();
```

$$\boxed{\begin{array}{l} T.C \rightarrow O(\log_{26} N) \\ S.C \rightarrow O(1) \end{array}}$$

Maths NCERT → 11th Ch-7

mississippi

$$\frac{11!}{4!\,4!\,2!}$$ ⇒ arrangements when there are duplicates.