Bit Manipulation - 1

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“Do your bit, get your bite!”

$\{1, 2, 3, 4 \ldots\ 9, 10 \ldots\ 11 \ldots\ \} \Rightarrow$ Decimal system

↳ ten

$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

digit

$342 = 300 + 40 + 2$

$= 3 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$

$a_n x^n + a_{n-1} x^{n-1}$

3   10   2

$2563 = 2000 + 500 + 60 + 3$

I
II
III
IV
V
VI
L
M

§. Binary system: $\{0, 1\}$

$(110)_2 = 1 \times 2^2 + 1 * 2^1 + 0 \times 2^0$

$= 4 + 2 + 0 = 6$

$(\overset{3}{1}\overset{2}{0}\overset{1}{1}\overset{0}{1})_2 = 1 \times 2^3 + 0 \times 2^2 + 1 * 2^1 + 1 * 2^0$

$= 8 + 0 + 2 + 1 = 11$

0	0	0	$\Rightarrow$	0	0
0	0	1	$\Rightarrow$	1	1
0	1	0	$\Rightarrow$	2	2
0	1	1	$\Rightarrow$	3	3
1	0	0	$\Rightarrow$	4	— 0
1	0	1	$\Rightarrow$	5	— 1 $\Rightarrow 1*2^2 + 0*2^1 + 1*2^0 = 5$
1	1	0	$\Rightarrow$	6	— 2
1	1	1	$\Rightarrow$	7	— 3

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$\boxed{2^4 = 16}$
$\boxed{2^5 = 32}$
$\boxed{2^6 = 64}$
$2^7 = 128$
$2^8 = 256$
$2^9 = 512$
$\boxed{2^{10} = 1024}$

- Binary to decimal:

0	1	0	1	1	0	
5	4	3	2	1	0	$= 22$
32	16	8	4	2	1	

Quiz: $(1011010)_2$

1	$\cancel{0}$	1	1	$\cancel{0}$	1	$\cancel{0}$
64	32	16	8	4	2	1

$\Rightarrow \underline{64+16} + \underline{8+2} = \underline{\underline{90}}$

§. Decimal to Binary:

long-division method.

2	20	0
2	10	0
2	5	1
2	2	0
2	1	1
	0	

$\Rightarrow$ 10100 = 20.

4 3 2 1 0

16 8 4 2 1

Quiz 45

2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
	0	

101101

32 16 8 4 2 1

13 → 8 → 5

32

$\boxed{101101}$

Q. H.W. Why modern computers are binary system based and not (say 3 bit (0,1,2) or 4 bit (0,1,2,3) based?

§. Addition of decimal numbers:

$$
\begin{array}{cccc}
 & 1 & 1 & \\
 & 3 & 6 & 8 \\
+ & 4 & 5 & 3 \\
\hline
 & 8 & 2 & 1 \\
\hline
\end{array}
$$

Algo.

$(d_1 + d_2) \% 10 \Rightarrow$ digit

$(d_1 + d_2) / 10 \Rightarrow$ carry

§. Addition of Binary numbers:

$$
\begin{array}{cccccccc}
 & & 1 & 1 & 1 & & & \\
 & & 1 & 0 & 0 & 1 & 1 & 0 \\
+ & & 1 & 0 & 1 & 0 & 1 & 1 \\
\hline
 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
\end{array}
$$

$1 + 1 \Rightarrow 2 \Rightarrow 10$

$$
\begin{array}{c}
1 \\
+1 \\
+1 \\
\hline
\end{array}
$$

$$
\begin{array}{ccc}
1 & & \\
0 & 1 & 2 \\
0 & 1 & 2 \\
\hline
\end{array}
\Rightarrow
\begin{array}{c}
1 \\
1 \\
\hline
10
\end{array}
$$

$$
\begin{array}{cc}
1 & 0 \\
0 & 1 \\
\hline
1 & 1
\end{array}
$$

Quiz:

$$
\begin{array}{ccccc}
 & & 1 & 1 & \\
1 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 \\
\hline
\end{array}
$$

11101

§ Bitwise operators:

① ⇒ are operated on individual bits

② computer programming

{ AND, OR, XOR, NOT, left shift, right shift }

AND	OR	XOR	NOT	left shift	right shift
&	|	^	! / ~	<<	>>

NOT:

$0 \to 1$

$1 \to 0$

$0 \to$ false (unset)

$1 \to$ true (set)

A	B	A & B	A | B	A ^ B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

$0 + 0 = \boxed{0}$ $\quad 0 + 1 = \boxed{1}$ $\quad 1 + 0 = \boxed{1}$

$1 + 1 = 1\boxed{0}$

bitwise addition

Obs^n: A ^ B ⇒ 0 if bits are same

1 if bits are different

§. Bitwise operations on number

$2+3=5$

I. $5 \& 6 = 4$

$$\begin{array}{r} 101 \\ 110 \\ \hline 100 \end{array} = 4$$

II. $20 \& 45 = 4$

$$\begin{array}{r} 010100 \\ 101101 \\ \hline 000100 \end{array} = 4$$

III. $20 \& 45$

$$\begin{array}{r} 010100 \\ 101101 \\ \hline \boxed{111001} \end{array} = \underline{57}$$

⊛ Properties:

1) $A \,\&\, 1 = ?$

$A = 10 \rightarrow 1010$

$1 \rightarrow 0001$

$\overline{\quad 0000 \quad}$

$A = 9 \quad 1001$

$1 \quad 0001$

$\overline{\quad \boxed{0001} \quad}$

$A \,\&\, 1 \rightarrow 0$, if last bit is 0, $\Rightarrow$ A is even

$ \rightarrow 1$, if last bit is 1, $\Rightarrow$ A is odd

0th bit

$\cdots \; 16 \;\; 8 \;\; 4 \;\; 2 \;\; 1$

$\cdots \; 1 \;\; 0 \;\; 1 \;\; 1 \;\; 1$

even + odd = odd

$\cdots \; 16 \;\; 8 \;\; 4 \;\; 2 \;\; 1$

$\cdots \; 1 \;\; 0 \;\; 1 \;\; 1 \;\; 0$

even + even = even

$(A \% 2 == 0)$

$A \,\&\, 1 == 0$

Faster

2) $A \,\&\, 0 = 0$

$$\begin{array}{r} A \rightarrow 101 \\ 0 \rightarrow 000 \\ \hline 000 \end{array} \quad , \quad \begin{array}{r} A \rightarrow 101 \\ A \rightarrow 101 \\ \hline A \Rightarrow 101 \end{array}$$

3) $A \,\&\, A = A$

4) $A \mid 0 = A$

$$\begin{array}{r} A \rightarrow 101 \\ 0 \rightarrow 000 \\ \hline 101 \end{array} \qquad \begin{array}{r} A \rightarrow 101 \\ A \rightarrow 101 \\ \hline 101 \end{array}$$

5) $A \mid A = A$

6) $A \wedge 0 = A$

$$\begin{array}{r} A \rightarrow 101 \\ 0 \rightarrow 000 \\ \hline 101 \end{array} \qquad \begin{array}{r} A \Rightarrow 101 \\ A \Rightarrow 101 \\ \hline 000 \end{array}$$

7) $A \wedge A = 0$

---

8) Commutative property:

$$a \,\&\, b = b \,\&\, a$$
$$a \mid b = b \mid a$$
$$a \wedge b = b \wedge a$$

$$\left.\begin{array}{c} a \,\&\, b \,\&\, c \\ \\ b \,\&\, a \,\&\, c \\ \\ c \,\&\, b \,\&\, a \end{array}\right\} \text{equal}$$

9) Associative property:

$$(a \& b) \& c = a \& (b \& c)$$

$$(a | b) | c = a | (b | c)$$

$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$

Quiz1 $a \wedge b \wedge a \wedge d \wedge b \rightarrow (a \wedge a) \wedge (b \wedge b) \wedge d$

$\rightarrow 0 \wedge d \Rightarrow d$

Quiz: $1 \wedge 3 \wedge 5 \wedge 3 \wedge 2 \wedge 1 \wedge 5 \rightarrow (1 \wedge 1) \wedge (3 \wedge 3) \wedge (5 \wedge 5) \wedge 2$

$2 \rightarrow 7 \rightarrow 4 \rightarrow 6 \rightarrow 7 \rightarrow 2$

$0 \wedge 0 \wedge 0 \wedge 2$

$= 2$

$1 \rightarrow 001$
$2 \rightarrow 010$
$3 \rightarrow 011$
$4 \rightarrow 100$
$5 \rightarrow 101$
$6 \rightarrow 110$
$7 \rightarrow 111$

Q. Given an array of integers where every element appears twice except for one element which appears only once. Find the unique element.

ex. A = [6, 9, 6, 10, 9] , ans = 10

$\boxed{10:33}$

A = [2, 3, 5, 6, 3, 6, 2] , ans = 5

Obs^n: take XOR of all the numbers

....001010
....001001

```
int num = 0
for(int i=0; i<N; i++){
        num = num ^ A[i]   // num ^= A[i]
}
return num;
```

Q. left shift: int : 4 Bytes : 4*8 bits = 32 bits

8 bit number

*2 with every shift

	128	64	32	16	8	4	2	1
discarded	7	6	5	4	3	2	1	0

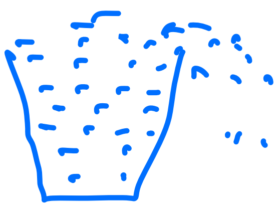
$a=45$		0	0	1	0	1	1	0	1	$\rightarrow$ 45
$a<<1$		0	1	0	1	1	0	1	0	$\rightarrow$ 90
$a<<2$		1	0	1	1	0	1	0	0	$\rightarrow$ 180
$a<<3$	1	0	1	1	0	1	0	0	0	$\rightarrow$ 104

$45 \xrightarrow{*2} 90 \xrightarrow{*2} 180 \xrightarrow{*2} \boxed{360}$

360

$64+32+8 = 104$

360 is too large to be stored via 8 bits.



Overflow

$$\boxed{\begin{aligned} a<<n &= a*2^n \\ 1<<n &= 1*2^n = 2^n \end{aligned}}$$

$16 \Rightarrow 2^4$ $\quad \boxed{1<<4}$

8 bit number

1	1	1	1	1	1	1	1
128	64	32	16	8	4	2	1
6	7	5	4	3	2	1	0

$= 2^7+2^6+2^5+2^4+2^3+2^2+2^1+2^0$

$= 2^8-1 = 255$

$(0-255) \Rightarrow 256$ numbers

§. Right shift:

	7	6	5	4	3	2	1	0		
	128	64	32	16	8	4	2	1		
a = 20,	0	0	0	1	0	1	0	0		⇒ 20
(a>>1)	0	0	0	0	1	0	1	0	discarded	⇒ 10
(a>>2)	0	0	0	0	0	1	0	1	discarded	⇒ 5
(a>>3)	0	0	0	0	0	0	1	0	discarded	⇒ 2
(a>>4)	0	0	0	0	0	0	0	1	discarded	⇒ 1
(a>>5)	0	0	0	0	0	0	0	0	discarded	⇒ 0

/2 with each shift

$5/2 = 2$

$1/2 = 0$

$$\boxed{a >> n = a/2^n}$$

(-1) >> 1

$-1 << 1 \Rightarrow -2$

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1. Calendly link: 1:1 × problems 10:15 min.
2. office hours: study-room.

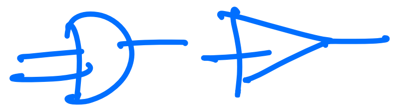
Q

$\boxed{a << 32}$ → int ✓ long long int ✗ overflow

% operator.

$\Rightarrow 2 \oplus 3$

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



$$\begin{array}{r} {}^{1}\ \ \\ 0\,1\,0 \\ 0\,1\,1 \\ \hline \end{array}$$

$= \boxed{1\,0\,1} = 5$

XOR

AND

1

$A * B \Rightarrow$

A / B

$\boxed{A \% B} \neq A/B \Rightarrow a - a*(a/b)$