

Multiple Features:

$$h_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \dots + \theta_n x_n$$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix} \quad x_0 = 1 \quad \rightarrow n+1 \text{ dimension}$$

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_n \end{bmatrix}$$

$$h_{\theta}(x) = \theta^T x = \begin{bmatrix} \theta_0 & \theta_1 & \dots & \theta_n \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$$= \theta_0 + \theta_1 x_1 + \dots + \theta_n x_n$$

Now, $\theta \rightarrow n+1$ dimensional vector

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

New Algo for gradient descent:
repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x^{(i)}$$

} simultaneously update θ_j for $j=0 \dots n$

Feature Scaling: It is needed to make the gradient descent work well.

We will try to make every feature into approximately $-1 \leq x_i \leq 1$ range.

$$0 \leq x \leq 3 \quad \checkmark$$

$$-2 \leq x \leq 0.5 \quad \checkmark$$

$$-100 \leq u \leq 200 \quad \times$$

$$0.0001 \leq x \leq 0.00001 \quad \times$$

Mean normalization:

$$x_i = \frac{x_i - \mu_i}{s_i} \rightarrow \begin{array}{l} \text{mean/average} \\ \text{range (max-min)} \\ \text{standard deviation} \end{array}$$

Debugging: Making sure gradient descent is working correctly.

Declare convergence if $J(\theta)$ decreases by less than 10^{-3} in one iteration

If α is too

Small: slow convergence

large: may not decrease on every iteration
and thus may not converge

Polynomial Regression:

We can change the behaviour of curve of our hypothesis by making it quadric, cube or square or any form.

$$h_0(x) = \theta_0 + \theta_1 x$$

$$\left[\begin{array}{l} h_0(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 \\ h_0(x) = \theta_0 + \theta_1 x + \theta_2 \sqrt{x} \end{array} \right] \rightarrow \begin{array}{l} x_1 = 1 \rightarrow 10 \\ x_2 = 1 \rightarrow 100 \\ x_3 = 1 \rightarrow 1000 \\ x_1 = 1 \rightarrow 100 \\ x_2 = \sqrt{x} = 1 \rightarrow 10 \end{array}$$

Normal Equation:

Without taking derivative for many times,
we can use the formula,

$$\theta = (X^T X)^{-1} X^T y \quad X = \text{design matrix}$$

→ Here, feature scaling is not important.

→ Need to compute $(X^T X)^{-1} \rightarrow O(n^3)$
slow if n is very large

If n is large, gradient descent is better
 $O(kn^2) \approx$

If $(X^T X)$ is singular/degenerate NE $\rightarrow X$
 $(X^T X)$ is non invertible

- Some features are redundant \rightarrow linearly dependent
- Too much features

Solⁿ: delete some features

In octave 'pinve' function will give inverse, though $(X^T X)$ is non-invertible.
So we will use pinv to find NE.

Octave Tutorial :

$= =$ equal

$\sim =$ not equal

$\text{xor}(1,0) \rightarrow 1$

semicolon suppresses output

`disp(- - -);`

↳ output

`disp(sprintf ('2 decimals: %0.2f', pi));`

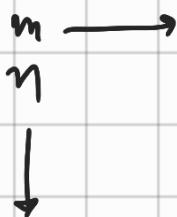
↳ 2 decimals: 3.14

format long

`a = pi;`

`a → 3.141592653589793`

ones ($n \times m$)



zeros

rand

randn

sqrt

eye

help

size

length

load

who

whos

clear

save hello.txt ✓ -ascii % to save as txt file

A(3,2) → an element

A(2,:) → second row

A(:,2) → second column

A([1 3], :) → everything from 1st & 3rd row

A(:, 2) = [10; 11; 12] → assignment, them to 2nd column

$A = [A, [100; 111; 122]]$; % to append new column vector to right

$V = A(:)$ % put all element of A into a single vector

$C = [A \ B]$ % concatenate A & B matrix

$C = [A; B]$ % concatenate vertically

$A * C$

$A.*B$ → elementwise multiplication

$A.^2$ → " square

$A./V$ → " division

$\log(V)$ → " logarithm

$\exp(V)$ → " e^v

$\text{abs}(V)$ → " absolute value

$-V$ → negative v

$V + \text{ones}(\text{length}(V), 1) == V + 1$

A' → Transpose

$\max(A)$

$[val, ind] = \max(A)$

value ↳ index

$a < 3 \rightarrow$ elementwise comparison

$\text{find}(a < 3) \rightarrow$ indexs of true values

$\text{magic}(3)$

$[r, c] = \text{find}(A >= 7)$

$\text{sum}(a)$

$\text{prod}(a)$

$\text{floor}(a)$

$\text{ceil}(a)$

$\max(\text{rand}(3), \text{rand}(3))$

$\max(A, [], 1) \rightarrow$ columnwise max

$\text{sum}(a, 1) \rightarrow$ columnwise summation

$\text{sum}(a, 2) \rightarrow$ rowise summation

$\text{sqrt}(\text{sum}(A * \text{eye}(9)))$

$\text{flipud}(\text{eye}(9)) \rightarrow$ Anti diagonal Identity matrix of 9×9

pinv

```
sin  
plot (x, y);  
hold on;  
plot (x, y, 'r');  
%red color  
xlabel ('time')  
ylabel ('value')  
legend ('sin', 'cos')  
title ('my plot')  
print -dpng 'myplot.png'  
close  
  
figure (1); plot (x, y1);  
figure (2); plot (x, y2);  
subplot (1, 2, 1) %divide plot a 1x2 grid.  
access first element  
plot ---  
subplot (1, 2, 2) %access second element  
plot ---  
axis ([0.5 1 -1 1])
```

clf;

imagesc(A)

imagesc(A), colorbar, colormap gray;

for i=1:4,

$$v(i) = 2^i$$

end;

$$\begin{array}{rcl} v & \rightarrow & 2 \\ & & 4 \\ & & 8 \\ & & 16 \end{array}$$

indices = 1:10;

for i = indices,

disp(i);

end;

i = 1

while true,

v(i) = 999;

if i == 6,

break;

end;

end;

```
if v(1) == 1,  
    disp(---);  
elseif v(1) == 2,  
    disp(---);  
else  
    disp(---);  
end;
```

Function:

```
=> return  
function y = squareThisNumber(x)  
    y = x^2; < body  
    addpath('c:\ . - . ')
```

name parameter
↓
x

```
function [y1, y2] = _____
```

↳ return two numbers

vectorization → use LA.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

3×2

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

2×3

$$A' = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

2×3