Penyelesaian Latihan

1. Diketahui $f(x) = \frac{\sin(0.5\sqrt{x})}{x}$. Tentukan turunan pertama dari f di $x = 1$ dan $h = 0.2$. Gunakan metode:

   (a) Selisih maju:

   $$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

   Di $x = 1$:

   $$f'(1) \approx \frac{f(1.2) - f(1)}{0.2}$$

   $$f(1) = \frac{\sin(0.5\sqrt{1})}{1} = \sin(0.5) \approx 0.4794$$

   $$f(1.2) = \frac{\sin(0.5\sqrt{1.2})}{1.2} \approx \frac{\sin(0.5477)}{1.2} \approx \frac{0.5201}{1.2} \approx 0.4334$$

   $$f'(1) \approx \frac{0.4334 - 0.4794}{0.2} \approx -0.230$$

   (b) Selisih mundur:

   $$f'(x) \approx \frac{f(x) - f(x-h)}{h}$$

   Di $x = 1$:

   $$f'(1) \approx \frac{f(1) - f(0.8)}{0.2}$$

   $$f(0.8) = \frac{\sin(0.5\sqrt{0.8})}{0.8} \approx \frac{\sin(0.4472)}{0.8} \approx \frac{0.4326}{0.8} \approx 0.5408$$

   $$f'(1) \approx \frac{0.4794 - 0.5408}{0.2} \approx -0.307$$

   (c) Selisih pusat:

   $$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

   Di $x = 1$:

   $$f'(1) \approx \frac{f(1.2) - f(0.8)}{0.4} \approx \frac{0.4334 - 0.5408}{0.4} \approx -0.2685$$

2. Diketahui $f(x) = x^2 \cos x$. Tentukan turunan kedua dari f di $x = 0.4$ dan $h = 0.1$. Gunakan metode:

   (a) Selisih maju:

   $$f''(x) \approx \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2}$$

   Di $x = 0.4$:

   $$f''(0.4) \approx \frac{f(0.6) - 2f(0.5) + f(0.4)}{0.01}$$

$$f(0.4) = (0.4)^2 \cos(0.4) \approx 0.16 \times 0.9211 \approx 0.1474$$
$$f(0.5) = (0.5)^2 \cos(0.5) \approx 0.25 \times 0.8776 \approx 0.2194$$
$$f(0.6) = (0.6)^2 \cos(0.6) \approx 0.36 \times 0.8253 \approx 0.2971$$
$$f''(0.4) \approx \frac{0.2971 - 2(0.2194) + 0.1474}{0.01} \approx \frac{0.2971 - 0.4388 + 0.1474}{0.01} \approx \frac{0.0057}{0.01} \approx -0.94$$

(b) Selisih mundur:

$$f''(x) \approx \frac{f(x) - 2f(x-h) + f(x-2h)}{h^2}$$

Di $x = 0.4$:

$$f''(0.4) \approx \frac{f(0.4) - 2f(0.3) + f(0.2)}{0.01}$$
$$f(0.3) = (0.3)^2 \cos(0.3) \approx 0.09 \times 0.9553 \approx 0.0860$$
$$f(0.2) = (0.2)^2 \cos(0.2) \approx 0.04 \times 0.9801 \approx 0.0392$$
$$f''(0.4) \approx \frac{0.1474 - 2(0.0860) + 0.0392}{0.01} \approx \frac{0.1474 - 0.1720 + 0.0392}{0.01} \approx \frac{0.0146}{0.01} \approx 1.46$$

(c) Selisih pusat:

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

Di $x = 0.4$:

$$f''(0.4) \approx \frac{f(0.5) - 2f(0.4) + f(0.3)}{0.01}$$
$$f''(0.4) \approx \frac{0.2194 - 2(0.1474) + 0.0860}{0.01} \approx \frac{0.2194 - 0.2948 + 0.0860}{0.01} \approx \frac{0.0106}{0.01} \approx 1.06$$

3. Diketahui data jarak tempuh suatu kendaraan sebagai berikut:

t (s)	y (km)
0	0
25	32
50	58
75	78
100	92
125	100

Gunakan turunan numerik untuk mengestimasi kecepatan dan percepatan kendaraan tersebut setiap waktu.

Estimasi kecepatan (v) tiap waktu:

$$v(t) \approx \frac{y_{i+1} - y_i}{t_{i+1} - t_i}$$

t (s)	y (km)	v (km/s)
0	0	–
25	32	$\frac{32-0}{25-0} = 1.28$
50	58	$\frac{58-32}{50-25} = 1.04$
75	78	$\frac{78-58}{75-50} = 0.80$
100	92	$\frac{92-78}{100-75} = 0.56$
125	100	$\frac{100-92}{125-100} = 0.32$

Estimasi percepatan (a) tiap waktu:

$$a(t) \approx \frac{v_{i+1} - v_i}{t_{i+1} - t_i}$$

t (s)	v (km/s)	a (km/s^2)
0	–	–
25	1.28	–
50	1.04	$\frac{1.04-1.28}{25} = -0.0096$
75	0.80	$\frac{0.80-1.04}{25} = -0.0096$
100	0.56	$\frac{0.56-0.80}{25} = -0.0096$
125	0.32	$\frac{0.32-0.56}{25} = -0.0096$