Date-

Assignment No. :

Problem Statement:

Program in C to find the minimum spanning tree from a given graph G by Prim's Algorithm.

Theory:

Like Kruskal's algorithm, Prim's algorithm is also a Greedy algorithm. It starts with an empty spanning tree. The idea is to maintain two sets of vertices. The first set contains the vertices already included in the MST, the other set contains the vertices not yet included. At every step, it considers all the edges that connect the two sets, and picks the minimum weight edge from these edges. After picking the edge, it moves the other endpoint of the edge to the set containing MST. A group of edges that connects two set of vertices in a graph is called cut in graph theory. **So, at every step of Prim's algorithm, we find a cut (of two sets, one contains the vertices already included in MST and other contains rest of the verices), pick the minimum weight edge from the cut and include this vertex to MST Set (the set that contains already included vertices).**

The idea behind Prim's algorithm is simple, a spanning tree means all vertices must be connected. So the two disjoint subsets of vertices must be connected to make a Spanning Tree. And they must be connected with the minimum weight edge to make it a Minimum Spanning Tree.

The algorithm may informally be described as performing the following steps:

1. Initialize a tree with a single vertex, chosen arbitrarily from the graph.

2. Grow the tree by one edge: of the edges that connect the tree to vertices not yet in the tree, find the minimum-weight edge, and transfer it to the tree.
3. Repeat step 2 (until all vertices are in the tree).

As described above, the starting vertex for the algorithm will be chosen arbitrarily, because the first iteration of the main loop of the algorithm will have a set of vertices in Q that all have equal weights, and the algorithm will automatically start a new tree in F when it completes a spanning tree of each connected component of the input graph. The algorithm may be modified to start with any particular vertex s by setting C[s] to be a number smaller than the other values of C (for instance, zero), and it may be modified to only find a single spanning tree rather than an entire spanning forest (matching more closely the informal description) by stopping whenever it encounters another vertex flagged as having no associated edge.

Algorithm:

Input specification:

1. I : The incidence matrix of dimension (n x n) of the given graph
2. v_S : The source vertex to start the search from

Output specification:

1. A two dimensional array I[1..n][1..n] whose starting index is 1 and ending index is n, size of the array being (n x n).
2. A stack to store the intermediate vertices, say S.

Steps:

Algorithm for method main():

Step 1 : Print "Enter number of vertices : "
Step 2 : Input n
Step 3 : Set gptr = make_2d(n) //make_2d() is a function to make a 2d array
//of order n and returns the address of the first element and here gptr is a
//pointer to pointer of integer type
Step 4 : Set tree = NULL //tree is a pointer to pointer of integer type

```
Step 5 :    Print "(If any two vertices is not connected by an edge, enter 0)"
Step 6 :    Repeat through Step 7 to Step13 For i = 0 to n-1
Step 7 :    Repeat through Step 8 to Step 12 For j = 0 to n-1
Step 8 :    Print "Enter the weight between vertices"(i + 1)"and" j + 1": "
Step 9 :    Input gptr[i][j]
Step 10 :   If(gptr[i][j] = 0) Then
Step 11 :   Set gptr[i][j] = INT_MAX
            [End of If structure]
Step 12 :   Set j=j+1
            [End of inner For loop]
Step 13 :   Set i=i+1
            [End of outer For loop]
Step 14 :   Print "Enter the starting vertex : "
Step 15 :   Input v0
Step 16 :   If(v0 < 1 OR v0 > n) Then
Step 17 :   Print "Invalid staring vertex"v0
Step 18 :   goto Step 22
            [End of If structure]
Step 19 :   Print "The weighted adjacency matrix of the given graph is : "
Step 20 :   print_2d(gptr, n)//print2d is a function to print the elements of a 2d
    //array of order n
Step 21 :   tree = prims(gptr, n, v0)
Step 22 :   Print "The adjacency matrix of the minimal spanning tree of the given
    graph is : "
Step 23 :   print_2d(tree, n)
Step 24 :   Repeat through Step 7 to Step 28 For i = 0 to n-1
Step 25 :   free(gptr[i])//free() is a function to frees up allocated memory of the
    //given argument
Step 26 :   If(tree != NULL) Then
Step 27 :   free(tree[i])
            [End of If structure]
Step 28 :   i=i+1
            [End of For loop]
Step 29 :   free(gptr)
Step 30 :   If(tree != NULL) Then
Step 31 :   free(tree)
            [End of If structure]
```

Algorithm for method Prims():

Step 1 : Repeat through Step 2 to Step For i=1 to N
Step 2 : Selected[i]=FALSE
Step 3 : Set i=i+1
[End of For loop]
Step 4 : Repeat through Step 2 to Step For i=1 to N
Step 5 : Repeat through Step 2 to Step For j=1 to N
Step 6 : Tree[i][j]=0
Step 7 : Set j=j+1
[End of inner For loop]
Step 8 : Set i=i+1
[End of outer For loop]
Step 9 : Selected[1]=TRUE, ne=1
Step 10 : Repeat through Step 11 to Step While(ne<N)
Step 11 : min=Infinite
Step 12 : Repeat through Step 2 to Step 20 For i=1 to N
Step 13 : If(Selected[i]=TRUE) Then
Step 14 : Repeat through Step 2 to Step 19 For j=1 to N
Step 15 : If(Selected[j]=FALSE) Then
Step 16 : If(min>Gptr[i][j]) Then
Step 17 : Set min=Gptr[i][j]
Step 18 : Set x=i ,y=j
[End of inner If structure]
[End of outer If structure]
Step 19 : Set j=j+1
[End of inner For loop]
[End of If structure]
Step 20 : Set i=i+1
[End of outer For loop]
Step 21 : Tree[x][y]=1

Step 22 : Selected[y]=TRUE
Step 23 : Ne=ne+1
[End of While loop]
Step 24 : Return (TREE)

Source Code:

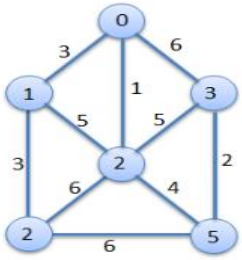
```
#include <stdio.h>
#include <stdbool.h>
#include <stdlib.h>
#include <limits.h>

#define inf INT_MAX

int ** make_2d(int n){//Making a 2d array by dynamic memory allocation
   int ** matrix = (int **)malloc(sizeof(int *) * n);
   for(int i = 0;i < n;i++)
      matrix[i] = (int *)malloc(sizeof(int) * n);
   return matrix;
}

int ** prims(int ** gptr, int n, int v0){
   bool selected[n];
   int ** tree = make_2d(n), i = 0, j = 0, ne = 0, min, x, y;
   // Initializations
   while(i < n)
      selected[i++] = false;
   for(i = 0;i < n;i++)
      for(j = 0;j < n;j++)
         tree[i][j] = 0;
   selected[v0] = true, ne = 1;
```

```
   // Finding the nearest neighbour of the selected vertex
   while(ne < n){
      min = inf;
      for(i = 0;i < n;i++){
         if(selected[i] == true){
            for(j = 0;j < n;j++){
               if(selected[j] == false){
                  if(min > gptr[i][j]){
                     min = gptr[i][j];
                     x = i, y = j;
                  }
               }
            }
         }
      }
      tree[x][y] = 1;
      selected[y] = true;
      ne++;
   }
   return tree;
}

void print_2d(int ** matrix, int n){//Printing the adjacency matrix
   for(int i = 0;i < n;i++){
      for(int j = 0;j < n;j++){
         if(matrix[i][j] == INT_MAX)
            printf("-- ");
         else
            printf("%2d ", matrix[i][j]);
      }
      printf("\n");
   }
}

int main(){
   int n, v0;
```

```
    printf("\nEnter number of vertices : ");
    scanf("%d", &n);
    int ** gptr = make_2d(n), ** tree = NULL;
    printf("\n(If any two vertices is not connected by an edge, enter 0)");
    printf("\n\n");
    for(int i = 0;i < n;i++){
        for(int j = 0;j < n;j++){
            printf("\b\rEnter the weight between vertices %d and %d : ", (i + 1), (j + 1));
            scanf("%d", &gptr[i][j]);
            if(gptr[i][j] == 0)
                gptr[i][j] = INT_MAX;
        }
    }
    printf("\nEnter the starting vertex : ");
    scanf("%d", &v0);
    if(v0 < 1 || v0 > n){
        printf("\nInvalid staring vertex %d!", v0);
        goto freeall;
    }
    printf("\nThe weighted adjacency matrix of the given graph is : \n");
    print_2d(gptr, n);
    tree = prims(gptr, n, v0);
    printf("\nThe adjacency matrix of the minimal spanning tree of the given graph
is : \n");
    print_2d(tree, n);
freeall:
    for(int i = 0;i < n;i++){
        free(gptr[i]);
        if(tree != NULL)
            free(tree[i]);
    }
    free(gptr);
    if(tree != NULL)
        free(tree);
    printf("\n");
    return 0;
}
```

Input & Output:

Input graph:



A Simple Weighted Graph

Output of program:

```
Enter number of vertices : 6

(If any two vertices is not connected by an edge, enter 0)

Enter the weight between vertices 1 and 1 : 0
Enter the weight between vertices 1 and 2 : 3
Enter the weight between vertices 1 and 3 : 1
Enter the weight between vertices 1 and 4 : 6
Enter the weight between vertices 1 and 5 : 0
Enter the weight between vertices 1 and 6 : 0
Enter the weight between vertices 2 and 1 : 3
Enter the weight between vertices 2 and 2 : 0
Enter the weight between vertices 2 and 3 : 5
Enter the weight between vertices 2 and 4 : 0
Enter the weight between vertices 2 and 5 : 3
Enter the weight between vertices 2 and 6 : 0
Enter the weight between vertices 3 and 1 : 1
Enter the weight between vertices 3 and 2 : 5
Enter the weight between vertices 3 and 3 : 0
Enter the weight between vertices 3 and 4 : 5
Enter the weight between vertices 3 and 5 : 6
Enter the weight between vertices 3 and 6 : 4
Enter the weight between vertices 4 and 1 : 6
Enter the weight between vertices 4 and 2 : 0
Enter the weight between vertices 4 and 3 : 5
Enter the weight between vertices 4 and 4 : 0
Enter the weight between vertices 4 and 5 : 0
Enter the weight between vertices 4 and 6 : 2
Enter the weight between vertices 5 and 1 : 0
Enter the weight between vertices 5 and 2 : 3
Enter the weight between vertices 5 and 3 : 6
Enter the weight between vertices 5 and 4 : 0
Enter the weight between vertices 5 and 5 : 0
Enter the weight between vertices 5 and 6 : 6
Enter the weight between vertices 6 and 1 : 0
Enter the weight between vertices 6 and 2 : 0
Enter the weight between vertices 6 and 3 : 4
Enter the weight between vertices 6 and 4 : 2
Enter the weight between vertices 6 and 5 : 6
Enter the weight between vertices 6 and 6 : 0

Enter the starting vertex : 3
```

```
The weighted adjacency matrix of the given graph is :
--  3  1  6 --  --
 3 --  5 --  3 --
 1  5 --  5  6  4
 6 --  5 --  --  2
--  3  6 --  --  6
--  --  4  2  6 --

The adjacency matrix of the minimal spanning tree of the given graph is :
 0  1  0  0  0  0
 0  0  0  0  1  0
 1  0  0  0  0  0
 0  0  0  0  0  1
 0  0  0  0  0  0
 0  0  1  0  0  0
```

Discussion:

1. In this algorithm an adjacency matrix or an adjacency list graph representation and linearly searching an array of weights to find the minimum weight edge to add, requires $O(|V|^2)$ running time. However, this running time can be greatly improved further by using heaps to implement finding minimum weight edges in the algorithm's inner loop.
2. At every iteration of Prim's algorithm, an edge must be found that connects a vertex in a sub graph to a vertex outside the sub graph. Since P is connected, there will always be a path to every vertex. The output Y of Prim's algorithm is a tree, because the edge and vertex added to tree Y are connected.
3. At the iteration when edge e was added to tree Y, edge f could also have been added and it would be added instead of edge e if its weight was less than e, and since edge f was not added, we conclude that,

   $$w(f) \geq w(e)$$
4. Using a simple binary heap data structure, Prim's algorithm can now be shown to run in time $O(|E| \log |V|)$ where $|E|$ is the number of edges and $|V|$ is the number of vertices. Using a more sophisticated Fibonacci heap, this can be brought down to $O(|E| + |V| \log |V|)$, which is asymptotically faster when the graph is dense enough that $|E|$ is $\omega(|V|)$, and linear time when $|E|$ is at least $|V| \log |V|$. For graphs of even greater density (having at least $|V|^c$ edges for some $c > 1$), Prim's algorithm can be made to run in linear time even more simply, by using a *d*-ary heap in place of a Fibonacci heap.