Problem 8

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1 Solution

We can find the total amount of ways 5 Xs can be in a 3 by 3 grid by breaking it up into smaller problems, that of the rows, columns, diagonals, and then adding the union and subtracting the intersections which double count.

For the rows:

$$\begin{matrix} X & X & X \\ * & * & * \\ * & * & * \end{matrix}$$

There are 3 rows and no two rows can be occupied by the 5 Xs at once, so there are $3\binom{6}{2}$ ways of filling rows.

For the columns:

$$\begin{matrix} X & * & * \\ X & * & * \\ X & * & * \end{matrix}$$

There are 3 columns and no two can be occupied by the 5 Xs at once, but for each column, we have already counted the occupation of that column plus the 3 rows, hence there are $3(\binom{6}{2} - 3) = 3\binom{6}{2} - 9$ ways to fill the columns that have not already been counted.

For the diagonals:

$$\begin{matrix} X & * & * \\ * & X & * \\ * & * & X \end{matrix}$$

There are again $\binom{6}{2}$ total ways, but 6 have already been counted, so there are $\binom{6}{2} - 6$ ways for one diagonal. For the other, it is the same, except the case where both diagonals are occupied is subtracted (since it has already been counted), resulting in $\binom{6}{2} - 7$ ways.

This totals to: $3\binom{6}{2} + 3\binom{6}{2} - 9 + \binom{6}{2} - 6 + \binom{6}{2} - 7 = 8\binom{6}{2} - 22 = 98$ ways.